MAT257 RSG 9

Rishibh Prakash

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1. Find a smooth step function θ different from the one defined in lecture. Recall that a step function is defined as follows:

$$\theta(x) = \begin{cases} 0, x \leq 0 \\ 1, x \geq 1 \end{cases}$$

2. Let $f(x, y) = x\cos(xy)$. Find

$$\int_{[0,1]\times[0,1]} f$$

3. Let $f \in C^2(\mathbb{R}^2)$. Let $Q = [a, b] \times [c, d]$. Find

$$\int_Q D_1 D_2 f$$

4. Let $A \subset \mathbb{R}^n$ be a rectangle and suppose $f, g : A \to \mathbb{R}$ are integrable functions that differ on a measure 0 set. Show that

$$\int_A f = \int_A g$$

   Does the above hold if we drop the assumption that A is a rectangle?

5. Suppose $f : S \to \mathbb{R}$ is integrable, where S is bounded but not necessarily a rectangle. Let $S' = \operatorname{int}(S)$. Show that $f|_{S'}$ is also integrable and that

$$\int_S f = \int_{S'} f|_{S'}$$

   (*Hint*: Extend both functions to be defined on a rectangle and show that these extended functions are continuous at the same points.)

6. Recall that we say a set S is "Jordan measurable" if its characteristic function χ_S is integrable. If S is Jordan measurable, we define its volume $v(S)$ as the integral of χ_S. Show that if S is a non-empty open set that is Jordan measurable then $v(S) > 0$.

7. Find examples of open, closed and compact sets that are *not* Jordan measurable.

8. Suppose $S \subset \mathbb{R}^n$ is Jordan measureable. Show that $\overline{S}$ is also Jordan measureable and that $v(S) = v(\overline{S})$. Give an example where $\overline{S}$ and $\mathrm{int}(S)$ are Jordan measurable but S is not.