MAT257 RSG 9 Sketch Solutions

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1. No idea

2. Using Fubini's

$$\begin{aligned}\int_{x=0}^{x=1}\left(\int_{y=0}^{y=1} x\cos(xy)dy\right)dx &= \int_{x=0}^{x=1} \sin(xy)|_{y=0}^{y=1} dx \\ &= \int_{x=0}^{x=1} \sin(x)dx \\ &= (-\cos(x))|_{x=0}^{x=1} \\ &= 1-\cos(1)\end{aligned}$$

3. Let Q be some rectangle containing S. We define $f_S : Q \to \mathbb{R}$ by

$$f_S(x) = \begin{cases} f(x), x \in S \\ 0, \text{ otherwise}\end{cases}$$

   Similarly define $f_{S'}$.

   Suppose f_S is continuous at x. If $x \in \text{int}(S)$ then $f_S = f_{S'}$ on a neighbourhood of x hence $f_{S'}$ is also continuous at x. The same holds true if $x \in (S)$ (in particular both functions are 0 in some neighbourhood). Now suppose $x \in \text{Bd}(S)$. As f_S is continuous at x and every neighbourhood x contains points not in S, it must be true that $f_S(x) = 0$. For any y in some neighbourhood of x, we have that either $f_{S'}(y) = f_S(y)$ or $f_{S'}(y) = 0$. Therefore the $f_{S'}(y) \to 0$ as $y \to x$ and since $x \notin S'$ we know $f_{S'} = 0$ so $f_{S'}$ is continuous at x. Then we consider $f_S - f_{S'}$ which is 0 nearly everywhere hence we are done.