MAT324: Real Analysis – Fall 2016 Assignment 1

Due Thursday, September 8, in class.

Problem 1: Let C be the Cantor middle-thirds set constructed in the textbook. Show that C is compact, uncountable, and a null set.

Problem 2: Let A be the subset of [0, 1] which consists of all numbers which do not have the digit 4 appearing in their decimal expansion. Find m(A).

Problem 3: Let A be a null set. Show that $m^*(A \cup B) = m^*(B)$ for any set B.

Problem 4: Let E_1, E_2, \ldots, E_n be disjoint measurable sets. Show that for all $A \subseteq \mathbb{R}$, we have

$$m^*\left(A\cap\left(\bigcup_{j=1}^n E_j\right)\right) = \sum_{j=1}^n m^*\left(A\cap E_j\right).$$