MAT324: Real Analysis – Fall 2016
Assignment 3

Due Thursday, **September 29**, in class.

Problem 1: Suppose that $E_1, E_2, \dots$ are measurable subsets of $\mathbb{R}$. Show that if $E_k \supset E_{k+1}$ for all $k \geq 1$ and $m(E_{2016}) < \infty$, then

$$m\left(\bigcap_{k=1}^{\infty} E_k\right) = \lim_{k\to\infty} m(E_k).$$

Problem 2: Let $\mathcal{N} \subset [0,1]$ be a non-measurable set. Determine whether the function $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -x & \text{if} \quad x \in \mathcal{N} \\ x & \text{if} \quad x \notin \mathcal{N} \end{cases}$$

is measurable. Explain.

Problem 3: Suppose that, for each rational number q, the set $\{x \mid f(x) > q\}$ is measurable. Can we conclude that f is measurable?

Problem 4: Suppose $f, g : E \to \mathbb{R}$ are measurable functions on $E \in \mathcal{M}$. Show that $h : E \to \mathbb{R}$ defined by

$$h(x) = \begin{cases} \dfrac{f(x)}{g(x)} & \text{if} \quad g(x) \neq 0 \\ 0 & \text{if} \quad g(x) = 0 \end{cases}$$

is measurable.

Problem 5: Let $f : (a,b) \to \mathbb{R}$. If f has a finite derivative at all points then show that f' is measurable.

Problem 6: (Extra Credit - 5p) Prove that any measurable set E with $m(E) > 0$ has a nonmeasurable subset $\mathcal{N} \subset E$.