

**MAT324: Real Analysis – Fall 2016**  
ASSIGNMENT 9

Due Tuesday, **November 29**, in class.

**Problem 1:** Let  $\mathcal{N} \subset [0, 1]$  be a non-measurable set and let  $\mathcal{C} \subset [0, 1]$  be the Cantor middle-thirds set. Decide whether the following are true or false and explain your answer.

- a)  $\mathcal{N} \times \mathcal{C}$  is a Borel set;
- b)  $\mathcal{N} \times \mathcal{C}$  is a Lebesgue measurable set;
- c)  $\mathcal{N} \times \mathcal{C}$  is not measurable with respect to  $m_2$ , the Lebesgue measure on  $\mathbb{R}^2$ .

**Problem 2:** Consider the function

$$g(x, y) = \begin{cases} \frac{1}{x^2} & \text{if } 0 < y < x < 1 \\ -\frac{1}{y^2} & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Show that  $\int_0^1 \int_0^1 g(x, y) \, dx dy = -1$  and  $\int_0^1 \int_0^1 g(x, y) \, dy dx = 1$ . Is  $g$  an integrable function?

**Problem 3:** Compute

$$\int_{(0, \infty) \times (0, 1)} y \sin(x) e^{-xy} \, dx dy,$$

and explain why Fubini's theorem is applicable.