MAT 341: Applied Real Analysis – Spring 2017

HW9 – Comments

Sec. 4.1 – Problem 2: The sketch of the surfaces should look like the graphs below.

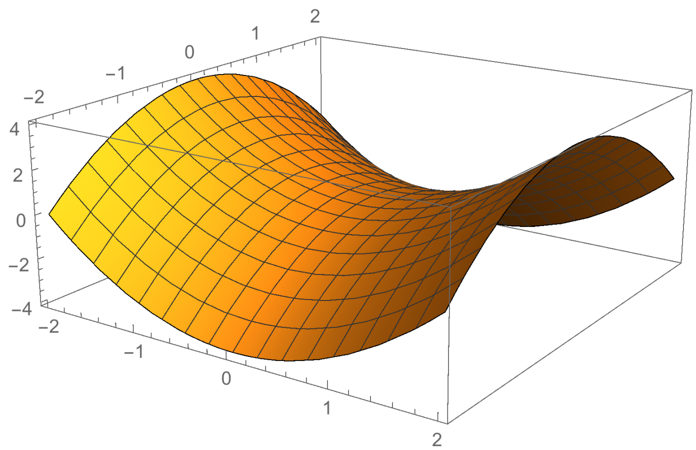

Figure 1: A sketch of the surface $z = x^2 - y^2$.

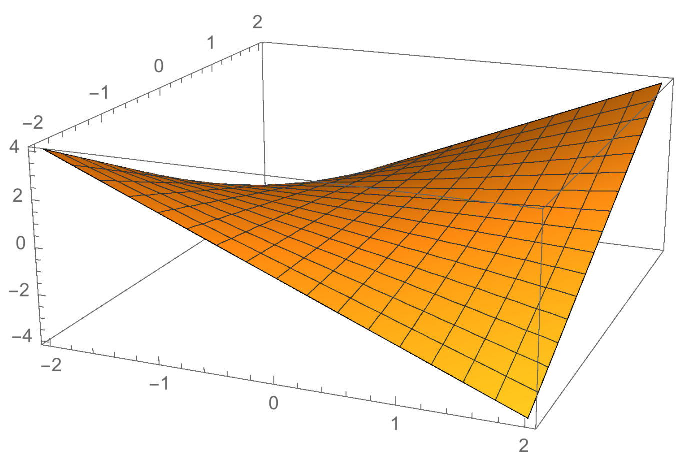

Figure 2: A sketch of the surface $z = xy$.

Regarding the boundary conditions: you have to evaluate $u(x, y)$, $\frac{\partial u}{\partial x}(x, y)$ and $\frac{\partial u}{\partial y}(x, y)$ at the given values. For example, if $u(x, y) = xy$ then $u(0, b) = 0$ and $u_x(0, b) = b$, $u_y(0, b) = 0$.

Sec. 4.2 – Problem 5: You are asked to solve the following PDE:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 1, \quad 0 < y < b$$

$$u(0, y) = 0, \quad u(1, y) = 0, \qquad 0 < y < b$$

$$u(x, 0) = 0, \quad u(x, b) = \sin(3\pi x), \qquad 0 < x < 1$$

You may assume that b is any constant. However, once you reach a formula for $u(x, y)$ as in Equation 9 (page 266) there is no need to compute the coefficients, simply use the fact that you already have $\sin(3\pi x)$ as a Fourier series and look for the coefficient of $n = 3$ (the rest are all zeros). To sketch the level curves, one has to do as in Figure 2, page 268 (see next page).

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(*Problem 4.2:5 Sketch of level curve. *)
a = 1;
b = 0.5;
const = 0.1;
ContourPlot[Sinh[3 * π * y] / Sinh[3 * π * b] Sin[3 π * x] == const, {x, 0, a}, {y, 0, b}]
```

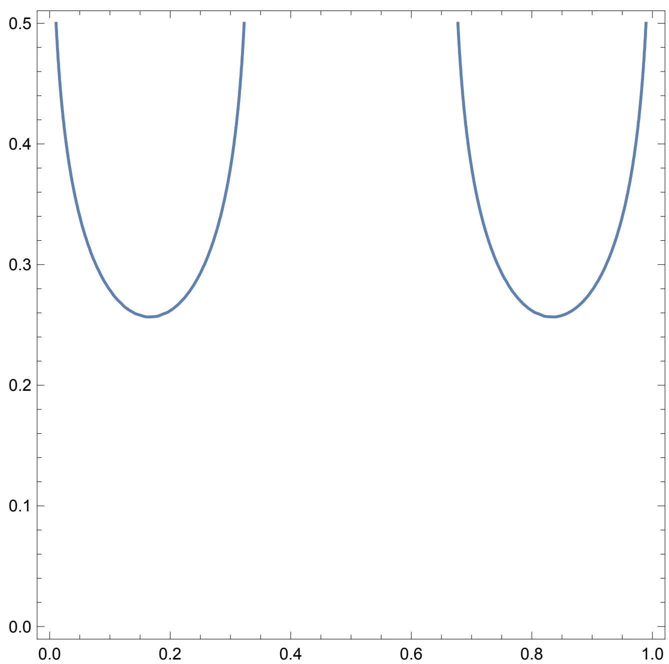

Figure 3: Level curves $u(x, y) =$ const. drawn in Mathematica.

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Plot3D[Sinh[3*π*y]/Sinh[3*π*b] Sin[3π*x], {x, 0, a}, {y, 0, b}]
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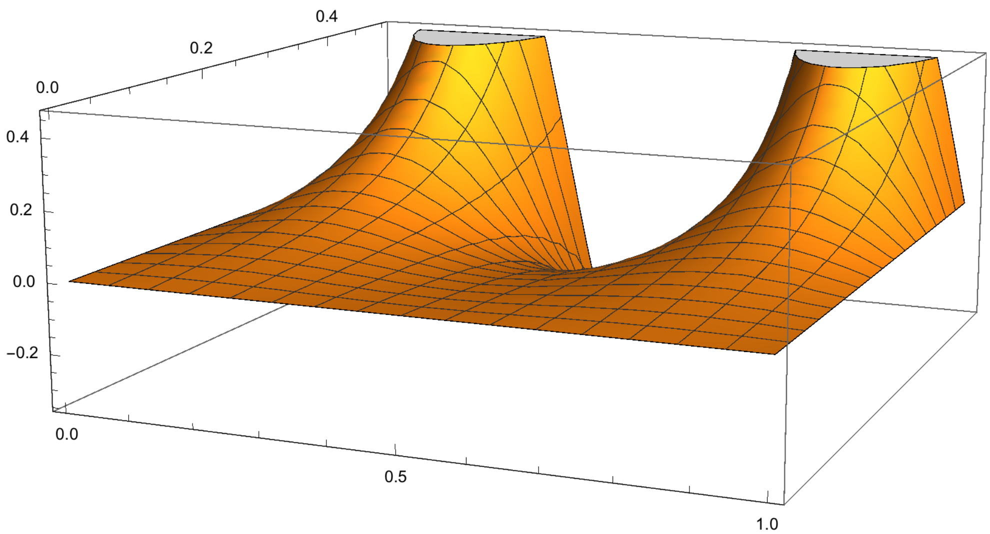

Figure 4: The surface $z = u(x, y)$. The level curves are obtained by cutting the level surface by a plane transversely.

Sec. 4.2 – Problem 6: You are asked to solve the following PDE:

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < a, \quad 0 < y < b \\
&u(0, y) = 0, \quad u(a, y) = 1, \qquad 0 < y < b \\
&u(x, 0) = 0, \quad u(x, b) = 0, \qquad 0 < x < a
\end{aligned}
$$