MAT 341 – Applied Real Analysis
Spring 2015

Midterm 1 – March 10, 2015

NAME: ______________________________

Please turn off your cell phone and put it away. You are **NOT** allowed to use a calculator.

Please show your work! To receive full credit, you must explain your reasoning and neatly write the steps which led you to your final answer. If you need extra space, you can use the other side of each page.

Academic integrity is expected of all students of Stony Brook University at all times, whether in the presence or absence of members of the faculty.

PROBLEM	**SCORE**
1	
2	
3	
4	
TOTAL	

Problem 1: (22 points) Suppose that the Fourier cosine series of a given function $f(x)$ is

$$f(x) = \frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{1 + 2(-1)^n}{2015n^3} \cos\left(\frac{n\pi x}{4}\right).$$

a) Show that $f(x) = f(x+8)$.

b) Does this Fourier cosine series converge uniformly? Explain.

c) Find the Fourier cosine series of $1 - 5f(x)$.

d) Find the Fourier cosine series of $f'(x)$ if it exists. If it does not exist, explain why it does not exist.

Problem 2: (30 points) Consider the function

$$f(x) = \begin{cases} 1 & \text{if} \quad -\pi \le x < 0, \\ x & \text{if} \quad 0 \le x < \pi; \end{cases} \qquad\qquad f(x + 2\pi) = f(x).$$

a) Sketch the graph of f on the interval $[0, 4\pi]$.

b) Find the Fourier series for f.

c) To what value does the Fourier series converge at:

i) $x = 0$; ii) $x = \frac{\pi}{2}$; iii) $x = 3\pi$? Explain.

d) Does the Fourier series of f converges uniformly on the interval $[0, \pi]$? Does it converge uniformly on the interval $[0, 4\pi]$? Explain.

Problem 3: (24 points) Consider the heat conduction problem in a bar that is in thermal contact with an external heat source. Then the modified heat conduction equation is

$$\frac{\partial^2 u}{\partial x^2} + s(x) = \frac{1}{k}\frac{\partial u}{\partial t}$$

where the term $s(x)$ describes the effect of the external agency; $s(x)$ is positive for a source. Suppose that the boundary conditions are

$$u(0,t) = T_0, \quad u(a,t) = T_1$$

and the initial condition is $u(x,0) = f(x)$.

a) Write $u(x,t) = w(x,t) + v(x)$, where $w(x,t)$ and $v(x)$ are the transient and steady state parts of the solution, respectively. State the boundary value problems that $v(x)$ and $w(x,t)$, respectively, satisfy.

b) Suppose $k = 1$ and $s(x) = 6x$. Find $v(x)$.

Problem 4: (24 points) Find the solution of the heat problem

$$\frac{\partial^2 u}{\partial x^2} = 4\frac{\partial u}{\partial t}, \qquad 0 < x < 2, \qquad t > 0;$$

$$u(0,t) = 0, \quad u(2,t) = \pi, \qquad t > 0;$$

$$u(x,0) = \frac{\pi x}{2} - 3\sin(\pi x) + 5\sin(2\pi x), \quad 0 \le x \le 2.$$

Some useful formulas

$$\int x\cos(ax)\,dx = \frac{\cos(ax)}{a^2} + \frac{x\sin(ax)}{a} + C$$

$$\int x\sin(ax)\,dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a} + C$$