MAT 351 Differential Equations: Dynamics & Chaos
Spring 2016

Assignment 1

Due Thursday, **February 11**, in class.

Problem 1: Determine the equilibrium points of the following differential equations and discuss their stability.

a) $\dot{x} = 3x(1 - x)$

b) $\dot{x} = \cos^2(x)$

c) $\dot{x} = r + x - x^3$, for various values of r. It may be useful to look at the Lyapunov function.

Problem 2: The growth of cancerous tumors can be modeled by the Gompertz law $\dot{N} = -aN \log(bN)$, where $N(t)$ is proportional to the number of cells in the tumor and $a, b > 0$ are parameters. Find the fixed points of this model and discuss their stability. Sketch the graph of the solution $N(t)$ based at $1/(2b)$.

Problem 3: Consider the equation $\dot{x} = rx + x^3$, where $r > 0$ is fixed. Show that $|x(t)| \to \infty$ in finite time, starting from any initial condition $x_0 \neq 0$.

Problem 4: Let p and q be positive integers with no common factors. Consider the initial value problem $\dot{x} = |x|^{p/q}$, $x(0) = 0$.

a) Show that there are an infinite number of solutions if $p < q$.

b) Show that there is a unique solution if $p > q$.

Problem 5: A solution $x(t)$ is a *periodic solution* of the differential equation $\dot{x} = f(x)$ if there exists $T > 0$ such that $x(t) = x(t + T)$ for all time t, but $x(t) \neq x(t + s)$ for all $0 < s < T$. Show that there are no periodic solutions to $\dot{x} = f(x)$ on the real line.