MAT 351 Differential Equations: Dynamics & Chaos

Spring 2016

Practice problems

Problem 1: Determine the equilibrium points of the following one-dimensional differential equations and discuss their stability:

1. $\dot{x} = e^{-x}\sin(x)$
2. $\dot{x} = x^2(6-x)$
3. $\dot{x} = -\sinh(x)$

Problem 2: Find the critical value of λ in which bifurcations occur in the following systems. Sketch the phase portrait for various values of λ and the bifurcation diagram. Classify the bifurcation.

1. $\dot{x} = x^3 - 5x^2 - (\lambda-8)x + \lambda - 4.$
2. $\dot{x} = 1 + \lambda x + x^2$
3. $\dot{x} = 5 - \lambda e^{-x^2}$

Problem 3: Describe the local stability behavior near equilibrium points of the following nonlinear systems. Also, draw the phase portrait near the equilibrium point.

1. $\dot{x} = y^2 - x + 2$, $\dot{y} = x^2 - y^2$
2. $\dot{x} = y + x^3$, $\dot{y} = x - y^3$ (also find a Lyapunov function)
3. $\ddot{x} + x + 4x^3 = 0$

Problem 4: Consider the system

$$\begin{aligned}\dot{x} &= -y + x(1-2x^2-3y^2)\\ \dot{y} &= x + y(1-2x^2-3y^2)\end{aligned}$$

1. Find all fixed points of the system and define their stability.
2. Transfer the system into polar coordinates (r,θ).
3. Find a trapping region of the form $R = \{(r,\theta) : a \le r \le b\}$ and then use Poincaré-Bendixson theorem to prove that the system has a limit cycle in the region R.

Problem 5: Discuss the local and global behavior of solutions of

$$\begin{aligned} \dot{r} &= r(a - r^2) \\ \dot{\theta} &= -1 \end{aligned}$$

as the parameter a passes through 0. Does the system have a limit cycle for some value of a? What type of bifurcation does this system undergo?