MAT 303: Calculus IV with Applications

Homework 11 - Instructions

HW 11 contains only two exercises from Section 5.7 and several exercises from Section 6.2 of the textbook. Please read the lecture notes and browse the Mathematica tutorial before getting started. Please solve Exercises 5.7.24 and 5.7.31 completely, as follows:

1) Please compute the exponential of matrix A, given in the problem, both analytically and by using Mathematica. Analytically, this can be done as in the examples from the lecture notes:

   a) find the eigenvalues of the matrix, and their geometric and algebraic multiplicities. Write the Jordan canonical form J of A.

   b) Use the formula for the exponential of a Jordan block to compute e^{Jt}.

   c) Find a complete set of eigenvectors and generalized eigenvectors for A, and write them columnwise (in the proper order) in a matrix S, as in the Lecture Notes. At this moment, you know that $S^{-1}AS = J$.

   d) Compute $\Phi(t) = Se^{Jt}$ and $e^{At} = Se^{Jt}S^{-1}$. Both $\Phi(t)$ and e^{At} are fundamental matrices of the homogeneous system $x' = Ax$.

2) Find the solution of the nonhomogeneous system by using the method of Variation of Parameters. Two equivalent (but not identical ways) of writing the solution are:

$$\begin{aligned} x(t) &= e^{At} \int e^{-At} f(t) dt \\ x(t) &= \Phi(t) \int \Phi(t)^{-1} f(t) dt. \end{aligned}$$

   Here we take the most general antiderivative of the integrand, $\int g(t)dt = G(t) + c$. If we keep the constant of integration c, then $x(t)$ is the general solution of the nonhomogeneous system. If we ignore the constant of integration c, then we get a particular solution of the nonhomogeneous system. If we have an initial condition in the problem, like $x(t_0) = x_0$, then we can obtain the unique solution as

$$x(t) = e^{At} \left(x_0 + \int_{t_0}^{t} e^{-As} f(s) ds \right) = \Phi(t) \left(\Phi^{-1}(t_0) x_0 + \int_{t_0}^{t} \Phi(s)^{-1} f(s) ds \right).$$

   Use Mathematica to check the computations. You will need the functions MatrixExp[..] and Integrate[..], as in the tutorial. Matrix Multiplication in Mathematica is a dot "."

For the problems from Section 6.2, use Mathematica as instructed in the textbook.