ECE1371 Advanced Analog Circuits
Lecture 2

Higher-Order Modulators: MOD2 and MOD*N*

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Course Goals

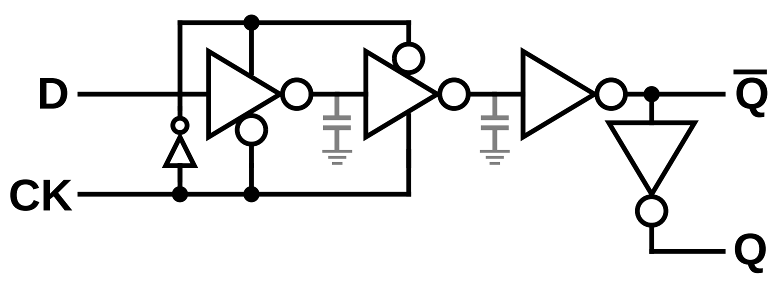
- **Deepen understanding of CMOS analog circuit design through a top-down study of a modern analog system— a delta-sigma ADC**
- **Develop circuit insight through brief peeks at some nifty little circuits**

  The circuit world is filled with many little gems that every competent designer ought to know.

Date	Lecture			Ref	Homework
2012-01-12	RS	1	Introduction: MOD1	ST 2, A	1: MOD1 in Matlab
2012-01-19	RS	2	MOD2 & MOD*N*	ST 3, 4, B	2: MOD2 in Matlab
2012-01-26	RS	3	Example Design: Part 1	ST 9.1, CCJM 14	3: Sw.-level MOD2
2012-02-02	TC	4	SC Circuits	R 12, CCJM 14	4: SC Integrator
2012-02-09	TC	5	Amplifier Design		
2012-02-16	TC	6	Amplifier Design		5: SC Int w/ Amp
2012-02-23	Reading Week + ISSCC– No Lecture				
2012-03-01	RS	7	Example Design: Part 2	CCJM 18	Start Project
2012-03-08	RS	8	Comparator & Flash ADC	CCJM 10	
2012-03-15	TC	9	Noise in SC Circuits	ST C	
2012-03-22	TC	10	Matching & MM-Shaping	ST 6.3-6.5, +	
2012-03-29	RS	11	Advanced ΔΣ	ST 6.6, 9.4	
2012-04-05	TC	12	Pipeline and SAR ADCs	CCJM 15, 17	
2012-04-12	No Lecture				
2012-04-19	Project Presentation				

NLCOTD: Dynamic Flip-Flop

- **Standard CMOS version**



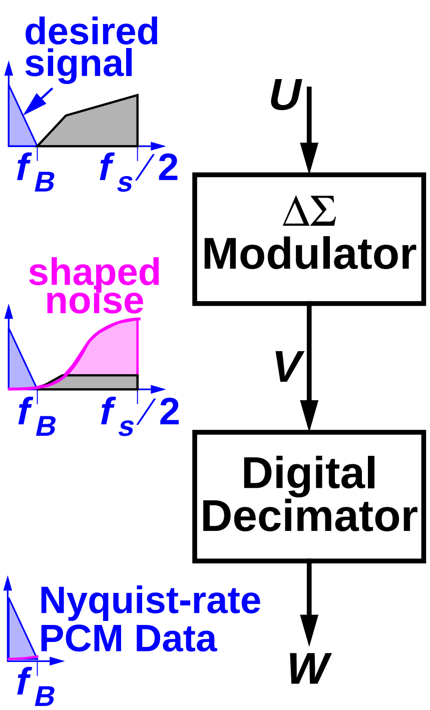
- **Can the circuit be simplified?**
  Is a complementarty clock necessary?

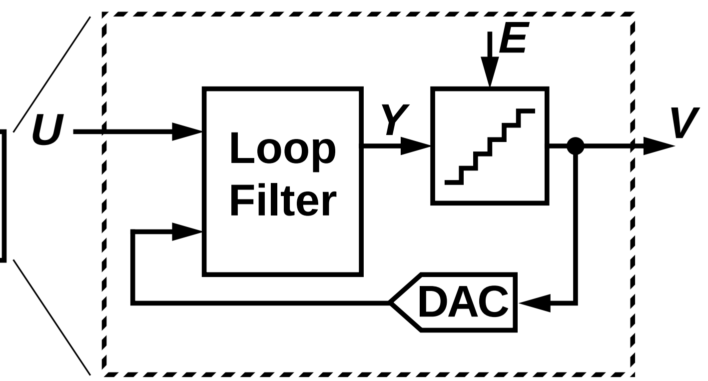
Highlights

(i.e. What you will learn today)

1. **Second-order modulator (MOD2)**
2. **Arbitrary-order modulator (MOD*N*) design with the ΔΣ Toolbox**

Review: A ΔΣ ADC System

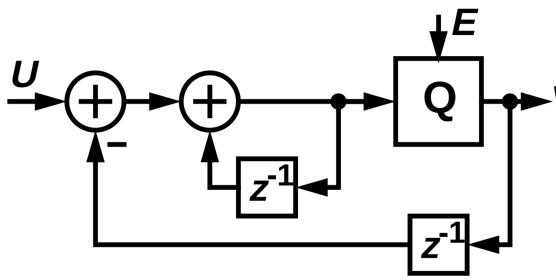




$$V(z) = STF(z)U(z) + NTF(z)E(z)$$

$STF(z)$: signal transfer function
$NTF(z)$: noise transfer function
$E(z)$: quantization error

Review: MOD1

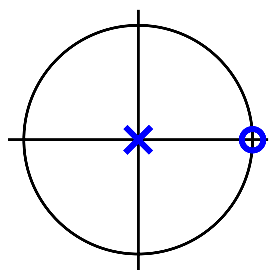


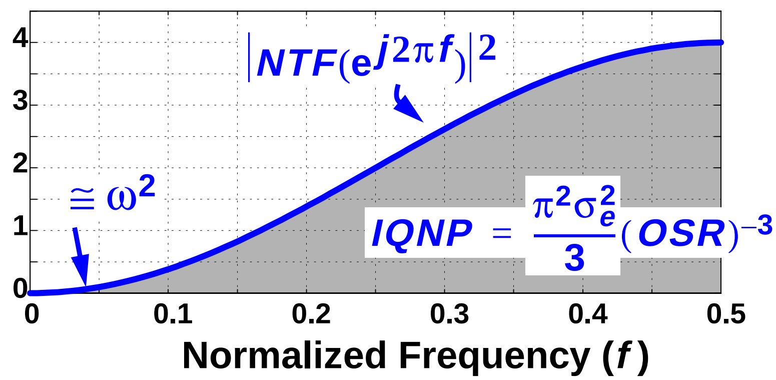
$$V(z) = NTF(z)E(z) + STF(z)U(z)$$

$$NTF(z) = (1 - z^{-1})$$

$$STF(z) = 1$$

NTF poles & zeros:





Review Summary

- $\Delta\Sigma$ works by spectrally separating the quantization noise from the signal

  Requires oversampling. $OSR \equiv f_s / (2f_B)$.

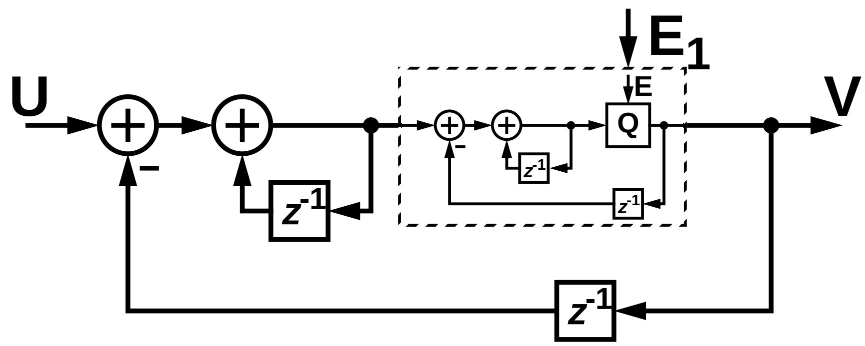
- Noise-shaping is achieved by the use of *filtering* and *feedback*
- A binary DAC is *inherently linear,* and thus a binary $\Delta\Sigma$ modulator is too
- MOD1 has $NTF(z) = 1 - z^{-1}$

  $\Rightarrow$ Arbitrary accuracy for DC inputs.
  9 dB/octave SQNR-OSR trade-off.

- MOD1-CT has *inherent anti-aliasing*

MOD2: 2nd-Order ΔΣ Modulator
[Ch. 3 of Schreier & Temes]

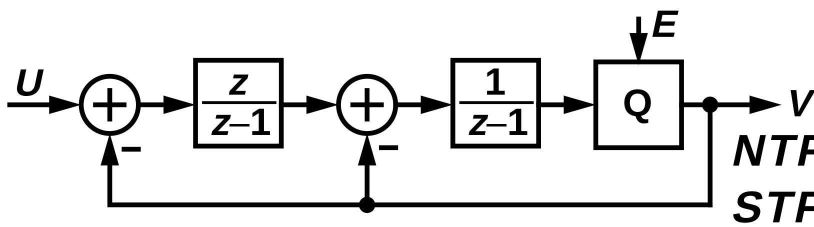
- **Replace the quantizer in MOD1 with another copy of MOD1 in a recursive fashion:**



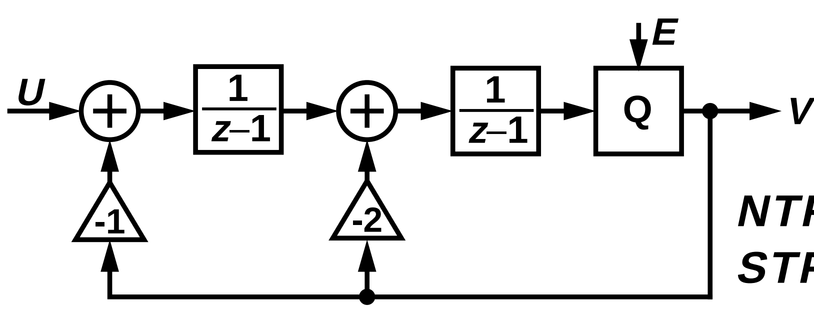
$$V(z) = U(z) + (1 - z^{-1})E_1(z),\ E_1(z) = (1 - z^{-1})E(z)$$

$$\Rightarrow V(z) = U(z) + (1 - z^{-1})^2 E(z)$$

Simplified Block Diagrams

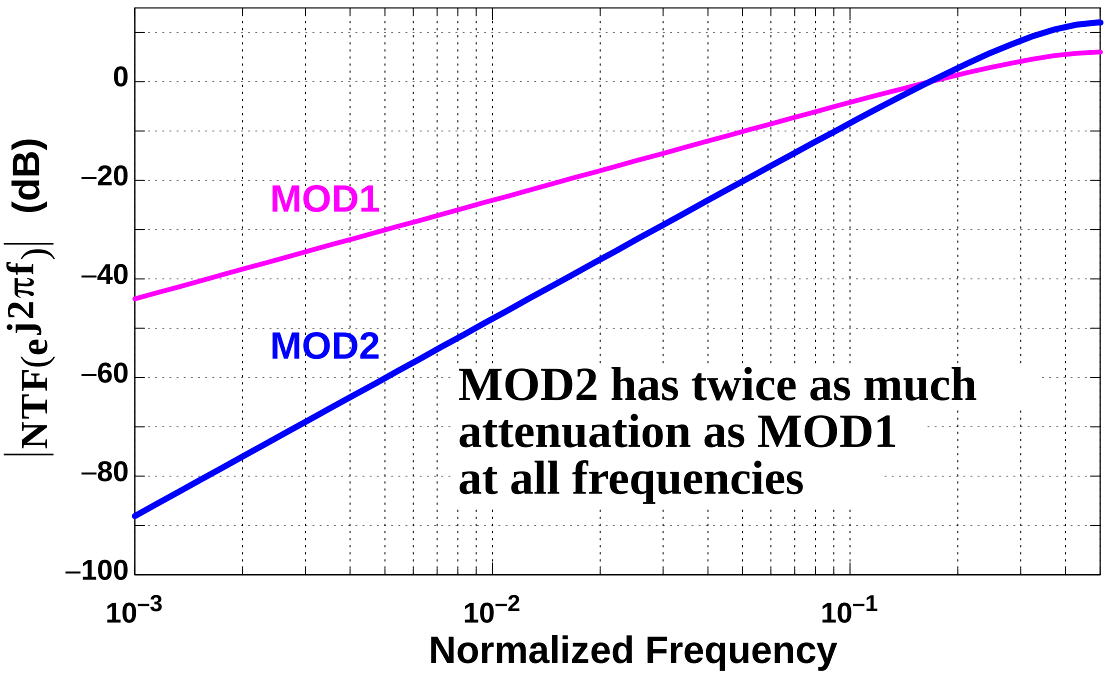


$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-1}$$



$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-2}$$

NTF Comparison



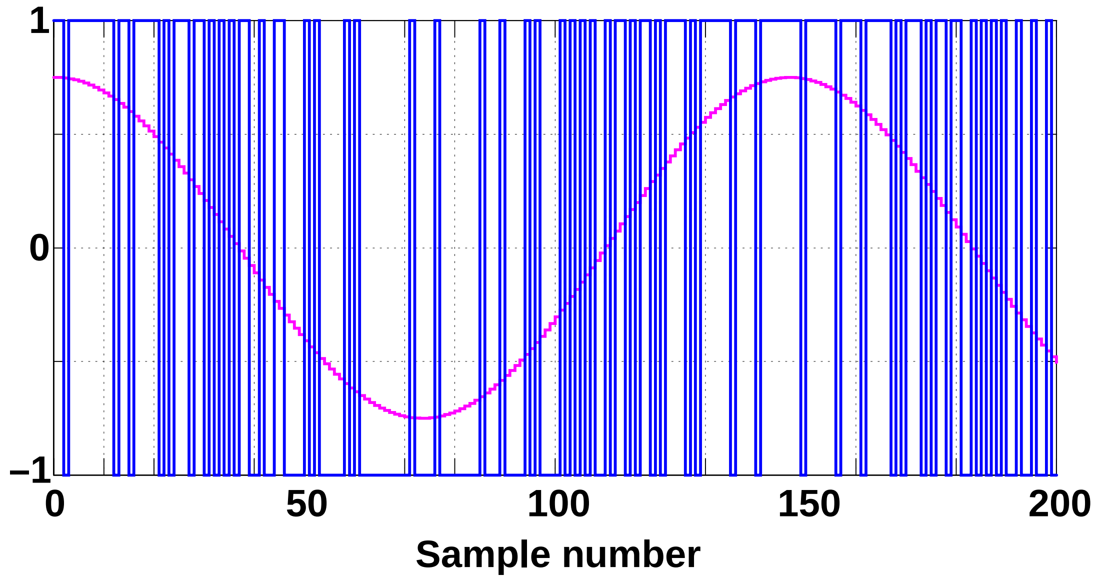
In-band Quant. Noise Power

- **For MOD2,** $\left|H(e^{j\omega})\right|^2 \approx \omega^4$
- **As before,** $IQNP = \int_0^{\omega_B} \left|H(e^{j\omega})\right|^2 S_{ee}(\omega)\,d\omega$ **and** $S_{ee}(\omega) = \sigma_e^2/\pi$
- **So now** $IQNP = \frac{\pi^4\sigma_e^2}{5}(OSR)^{-5}$ $*$

  With binary quantization to ± 1**,**
  $\Delta = 2$ **and thus** $\sigma_e^2 = \Delta^2/12 = 1/3$.
- **"An octave increase in *OSR* increases MOD2's SQNR by 15 dB (2.5 bits)"**

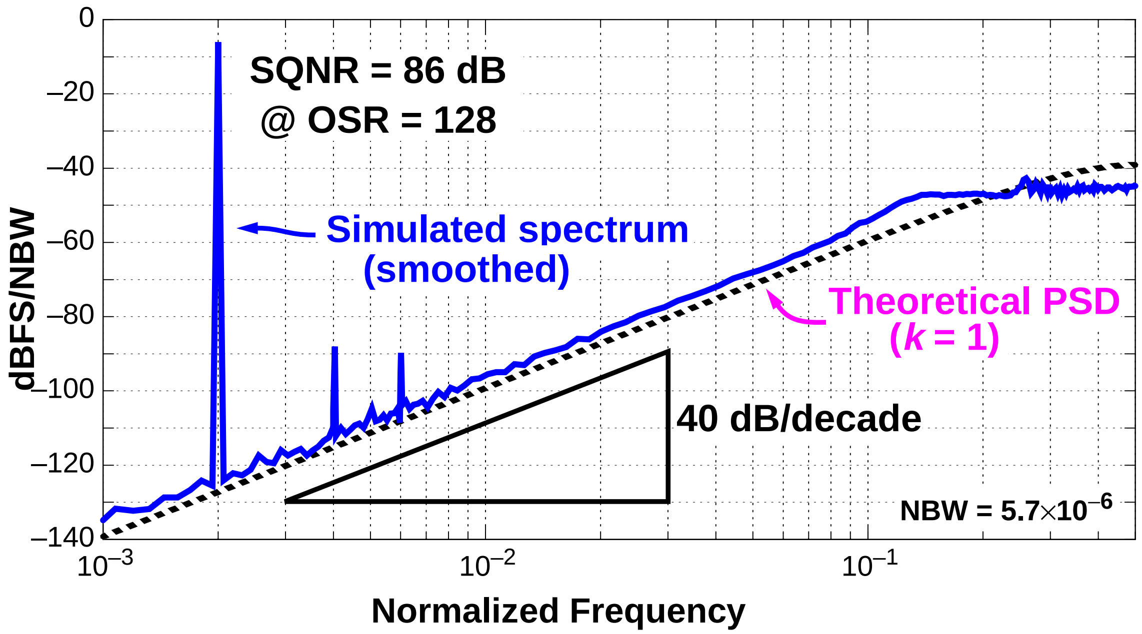
Simulation Example

Input at 75% of FullScale



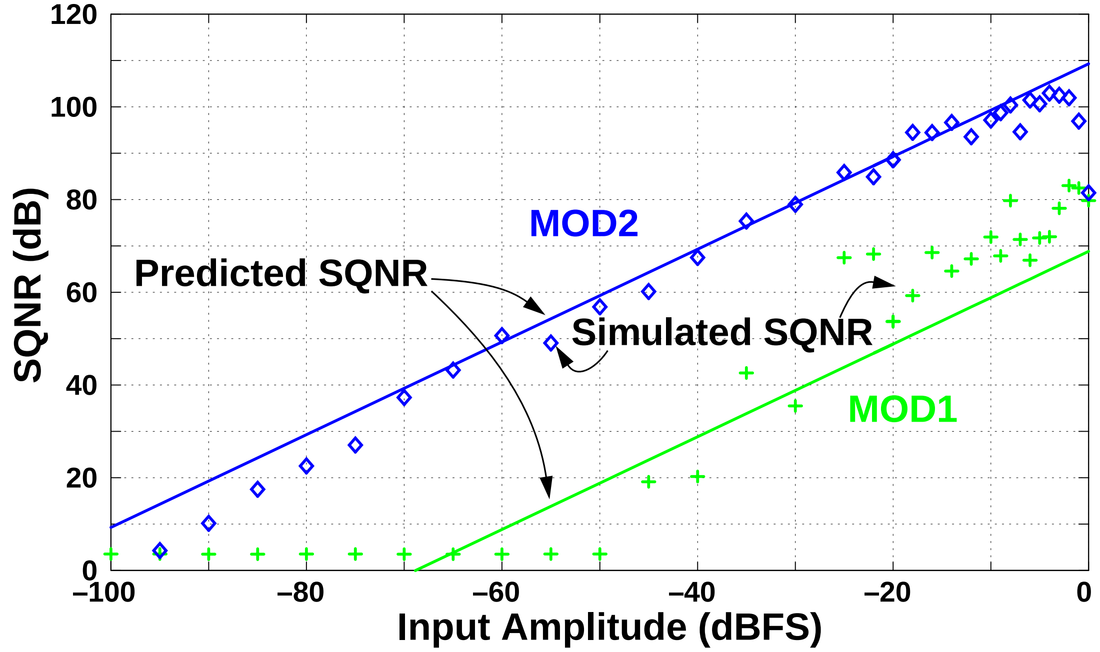
Simulated MOD2 PSD

Input at 50% of FullScale

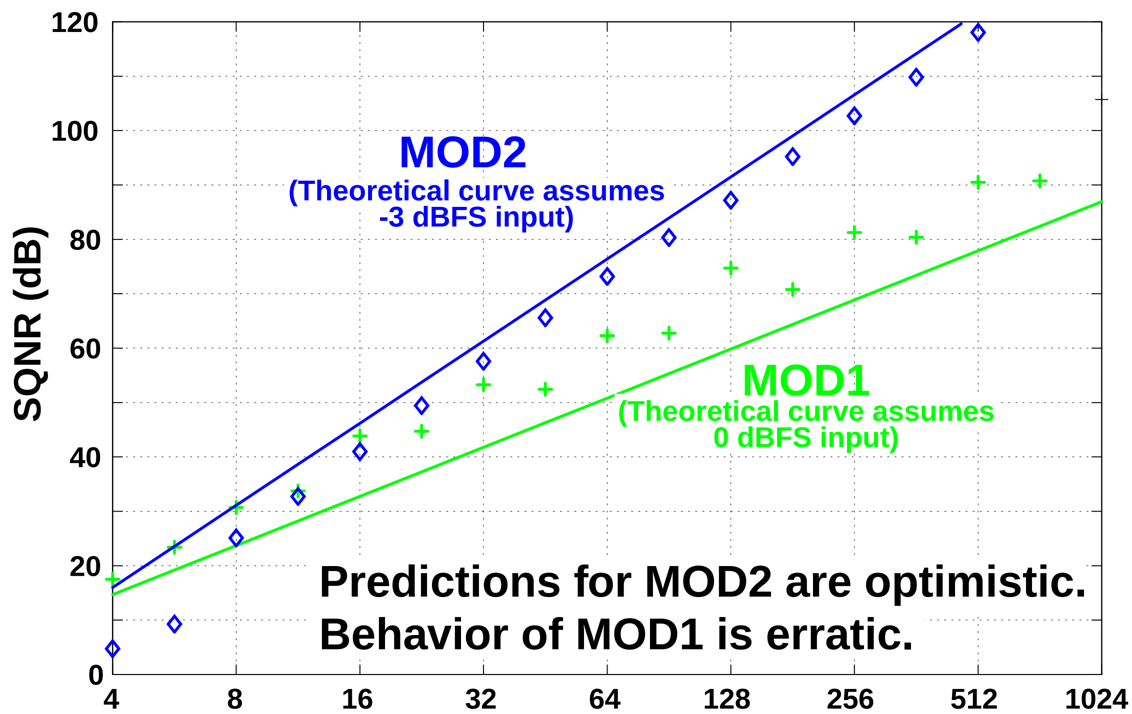


SQNR vs. Input Amplitude

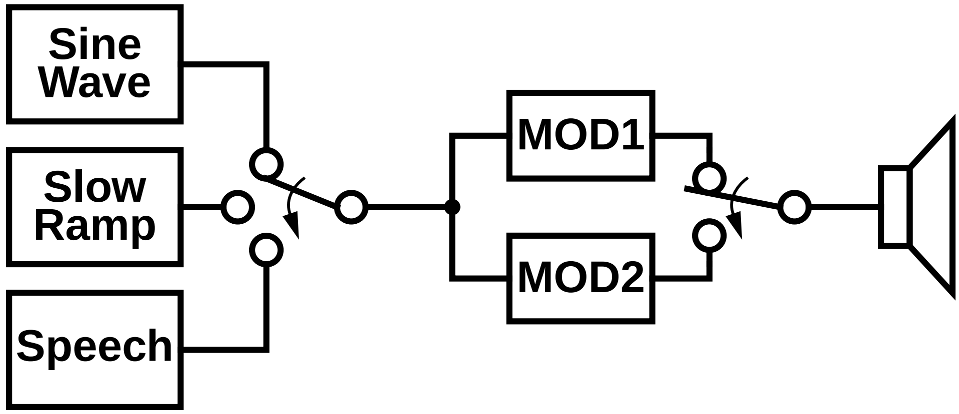
MOD1 & MOD2 @ OSR = 256



SQNR vs. OSR



Audio Demo: MOD1 vs. MOD2
[dsdemo4]

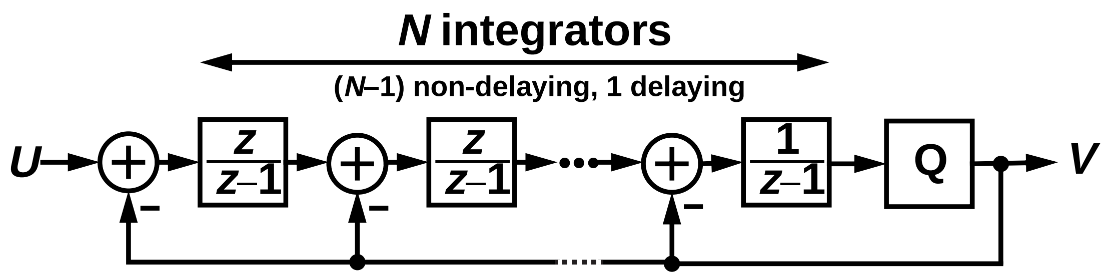


MOD1 + MOD2 Summary

- **ΔΣ ADCs rely on filtering and feedback to achieve high SNR despite coarse quantization**
  They also rely on digital signal processing.
  ΔΣ ADCs need to be followed by a digital decimation filter and ΔΣ DACs need to be preceded by a digital interpolation filter.
- **Oversampling eases analog filtering requirements**
  Anti-alias filter in an ADC; image filter in a DAC.
- **Binary quantization yields inherent linearity**
- **MOD2 is better than MOD1**
  15 dB/octave vs. 9 dB/octave SQNR-OSR trade-off.
  Quantization noise more white.
  Higher-order modulators are even better.

MODN

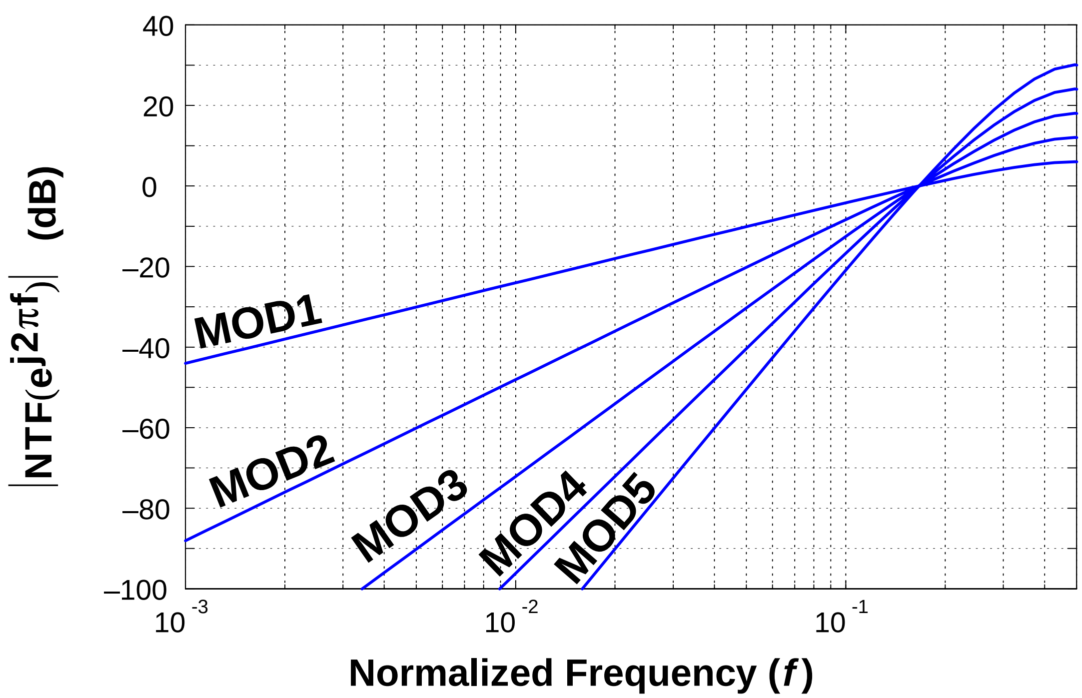
[Ch. 4 of Schreier & Temes]



$$STF(z) = z^{-1}$$
$$NTF(z) = (1 - z^{-1})^N$$

- MODN's NTF is the N^{th} power of MOD1's NTF

NTF Comparison



Predicted Performance

- **In-band quantization noise power**

$$IQNP = \int_0^{0.5/OSR} \left|NTF(e^{j2\pi f})\right|^2 \cdot S_{ee}(f)\ df$$

$$\approx \int_0^{0.5/OSR} (2\pi f)^{2N} \cdot 2\sigma_e^2 df$$

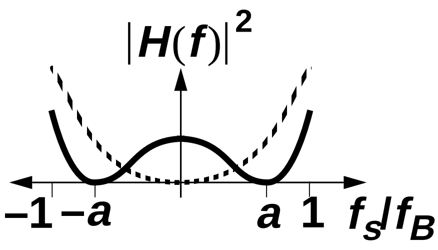
$$= \frac{\pi^{2N}}{(2N+1)(OSR)^{2N+1}}\sigma_e^2$$

- **Quantization noise drops as the $(2N+1)^{th}$ power of OSR— ($6N$+3) dB/octave SQNR-OSR trade-off**

Improving NTF Performance– NTF Zero Optimization

- **Minimize the integral of $|NTF|^2$ over the passband**

  Normalize passband edge to 1 for ease of calculation:



Need to find the a_i which minimize the integral

$$\int_{-1}^{1}(x^2 - a_1^2)^2 dx\ , \qquad n = 2$$

$$\int_{-1}^{1} x^2(x^2 - a_1^2)^2 dx\ , \qquad n = 3$$

$$\int_{-1}^{1}(x^2 - a_1^2)^2(x^2 - a_2^2)^2 dx\ , \qquad n = 4$$

$$\vdots$$

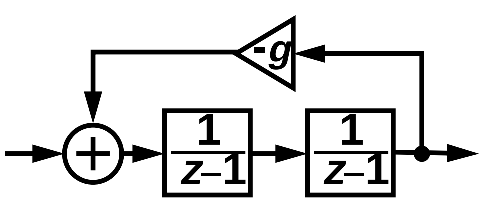
Solutions Up to Order = 8

Order	Optimal Zero Placement Relative to f_B	SQNR Improvement
1	0	0 dB
2	$\pm 1/\sqrt{3}$	3.5 dB
3	$0, \pm\sqrt{3/5}$	8 dB
4	$\pm\sqrt{3/7 \pm \sqrt{(3/7)^2 - 3/35}}$	13 dB
5	$0, \pm\sqrt{5/9 \pm \sqrt{(5/9)^2 - 5/21}}$ [Y. Yang]	18 dB
6	$\pm$0.23862, $\pm$0.66121, $\pm$0.93247	23 dB
7	0, $\pm$0.40585, $\pm$0.74153, $\pm$0.94911	28 dB
8	$\pm$0.18343, $\pm$0.52553, $\pm$0.79667, $\pm$0.96029	34 dB

Topological Implication

- **Feedback around pairs integrators:**

2 Delaying Integrators



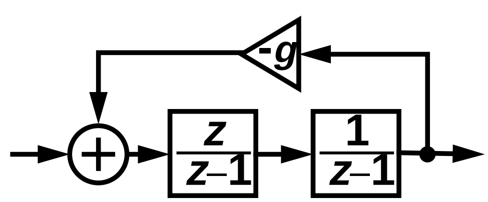
Poles are the roots of

$$1 + g\left(\frac{1}{z-1}\right)^2 = 0$$

i.e. $z = 1 \pm j\sqrt{g}$

Not quite on the unit circle, but fairly close if g<<1.

Non-delaying + Delaying Integrators (LDI Loop)



Poles are the roots of

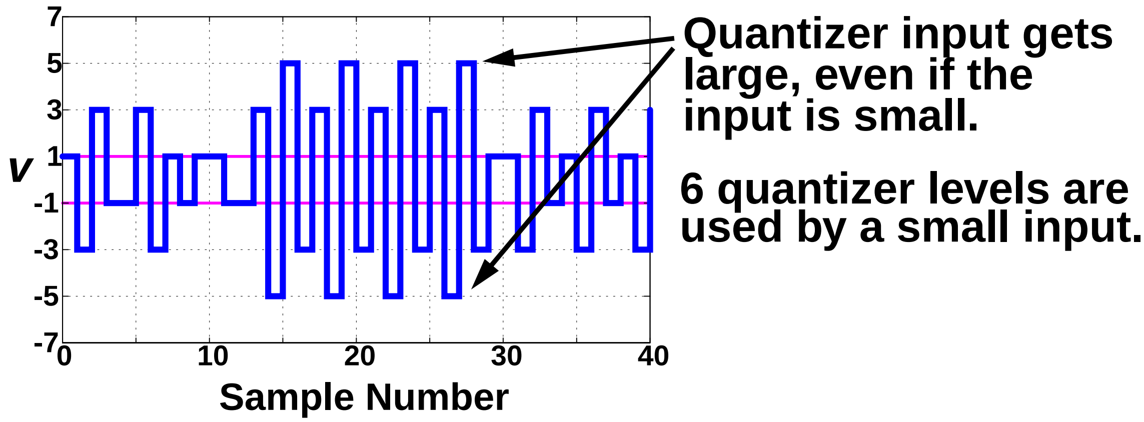
$$1 + \frac{gz}{(z-1)^2} = 0$$

i.e. $z = e^{\pm j\theta}$, $\cos\theta = 1 - g/2$

Precisely on the unit circle, regardless of the value of g.

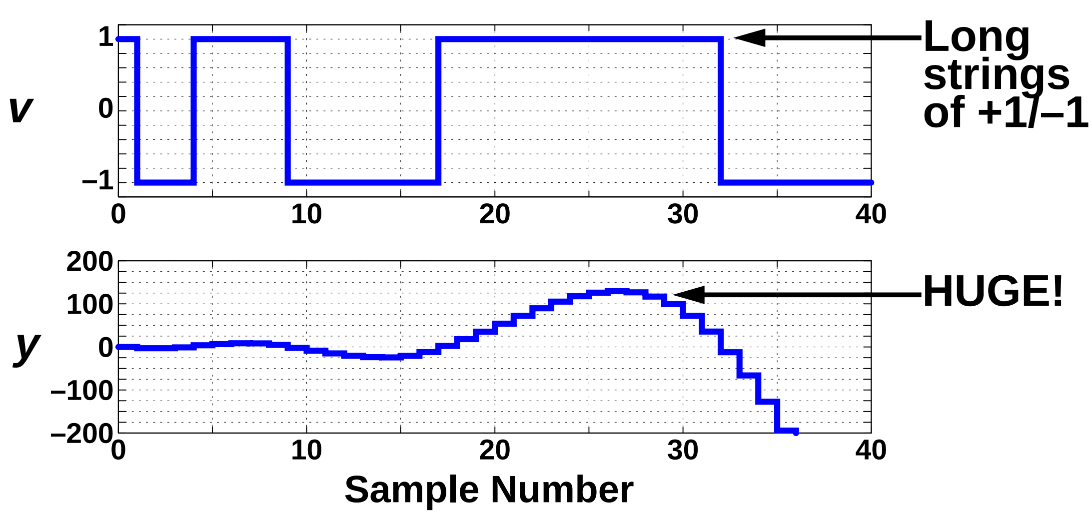
Problem: A High-Order Modulator Wants a Multi-bit Quantizer

E.g. MOD3 with an Infinite Quantizer and Zero Input



Simulation of MOD3-1b

(MOD3 with a Binary Quantizer)



- **MOD3-1b is unstable, even with zero input!**

Solutions to the Stability Problem
Historical Order

1 **Multi-bit quantization**

Initially considered undesirable because we lose the inherent linearity of a 1-bit DAC.

2 **More general NTF (not pure differentiation)**

Lower the NTF gain so that quantization error is amplified less.

Unfortunately, reducing the NTF gain reduces the amount by which quantization noise is attenuated.

3 **Multi-stage (MASH) architecture**

- **Combinations of the above are possible**

Multi-bit Quantization

A modulator with *NTF* = *H* and *STF* = 1 is guaranteed to be stable if $|u| < u_{max}$ at all times, where $u_{max} = nlev + 1 - \|h\|_1$ and $\|h\|_1 = \sum_{i=0}^{\infty} |h(i)|$

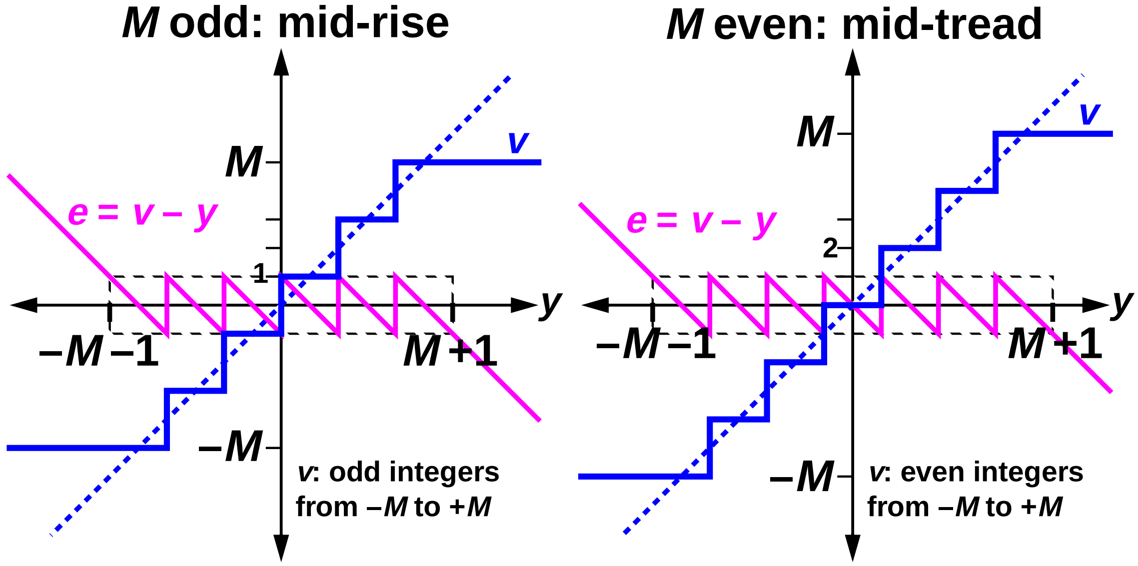
- **In MOD*N* $H(z) = (1 - z^{-1})^N$, so $h(n) = \{1, -a_1, a_2, -a_3, \ldots (-1)^N a_N, 0 \ldots\}$, $a_i > 0$ and thus $\|h\|_1 = H(-1) = 2^N$**
- **$nlev = 2^N$ implies $u_{max} = nlev + 1 - \|h\|_1 = 1$**

  MOD*N* is guaranteed to be stable with an *N*-bit quantizer if the input magnitude is less than Δ/2 = 1.

  This result is quite conservative.
- **Similarly, $nlev = 2^{N+1}$ guarantees that MOD*N* is stable for inputs up to 50% of full-scale**

M-Step Symmetric Quantizer

$\Delta = 2$, ($nlev = M + 1$)



- **No-overload range:** $|y| \le nlev \Rightarrow |e| \le \Delta/2 = 1$

Inductive Proof of $\|h\|_1$ Criterion

- **Assume STF = 1 and** $(\forall n)(|u(n)| \le u_{max})$
- **Assume** $|e(i)| \le 1$ **for** $i < n$.**[Induction Hypothesis]**

$$|y(n)| = \left|u(n) + \sum_{i=1}^{\infty} h(i)e(n-i)\right|$$
$$\le u_{max} + \sum_{i=1}^{\infty} |h(i)||e(n-i)|$$
$$\le u_{max} + \sum_{i=1}^{\infty} |h(i)| = u_{max} + \|h\|_1 - 1$$

Then $u_{max} = nlev + 1 - \|h\|_1$

$\Rightarrow \quad |y(n)| \le nlev$

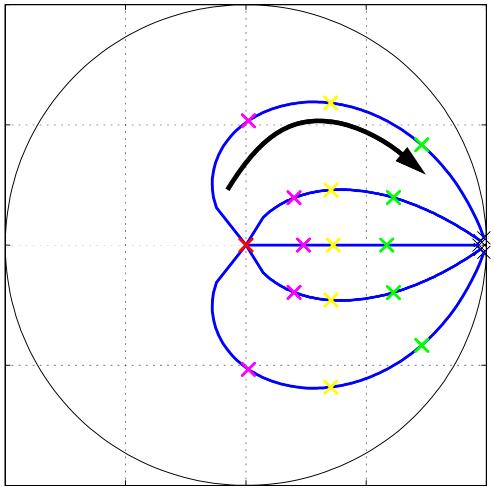
$\Rightarrow \quad |e(n)| \le 1$

- **So by induction** $|e(i)| \le 1$ **for all** $i > 0$

More General NTF

- Instead of $NTF(z) = A(z)/B(z)$ with $B(z) = z^n$, use a more general $B(z)$

  Roots of B are the poles of the NTF and must be inside the unit circle.



Moving the poles away from $z = 1$ toward $z = 0$ makes the gain of the NTF approach unity.

The Lee Criterion for Stability in a 1-bit Modulator: $\|H\|_\infty \le 2$

[Wai Lee, 1987]

- The measure of the "gain" of H is the maximum magnitude of H over frequency, aka the *infinity-norm* of H: $\|H\|_\infty \equiv \max_{\omega \in [0, 2\pi]} (H(e^{j\omega}))$

Q: Is the Lee criterion <u>necessary</u> for stability?

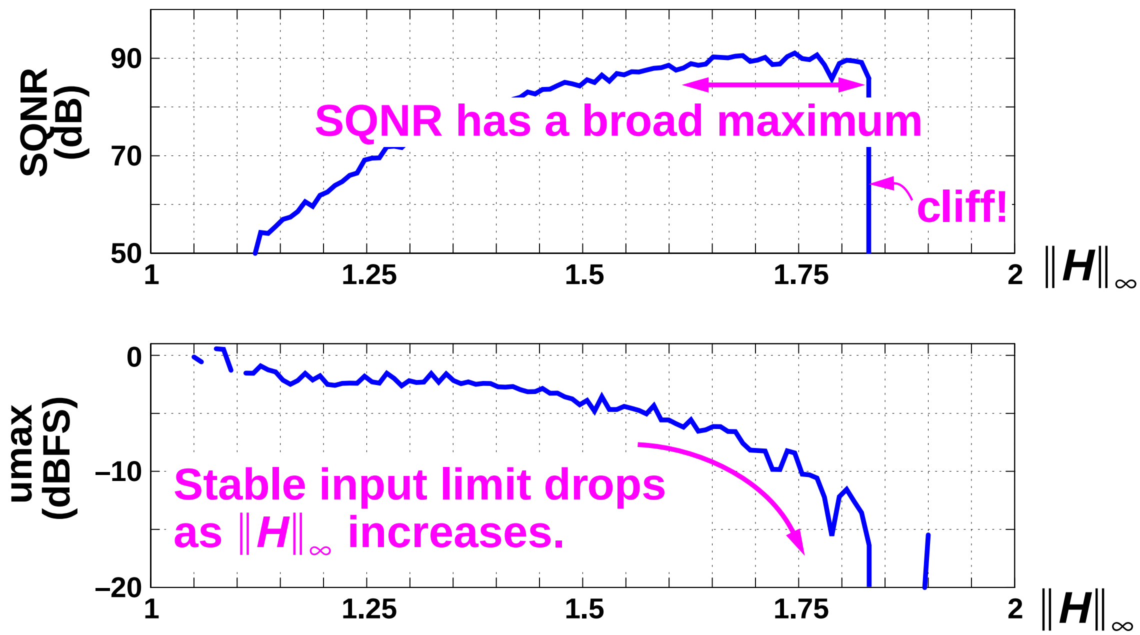
No. MOD2 is stable (for DC inputs less than FS) but $\|H\|_\infty = 4$.

Q: Is the Lee criterion <u>sufficient</u> to ensure stability?

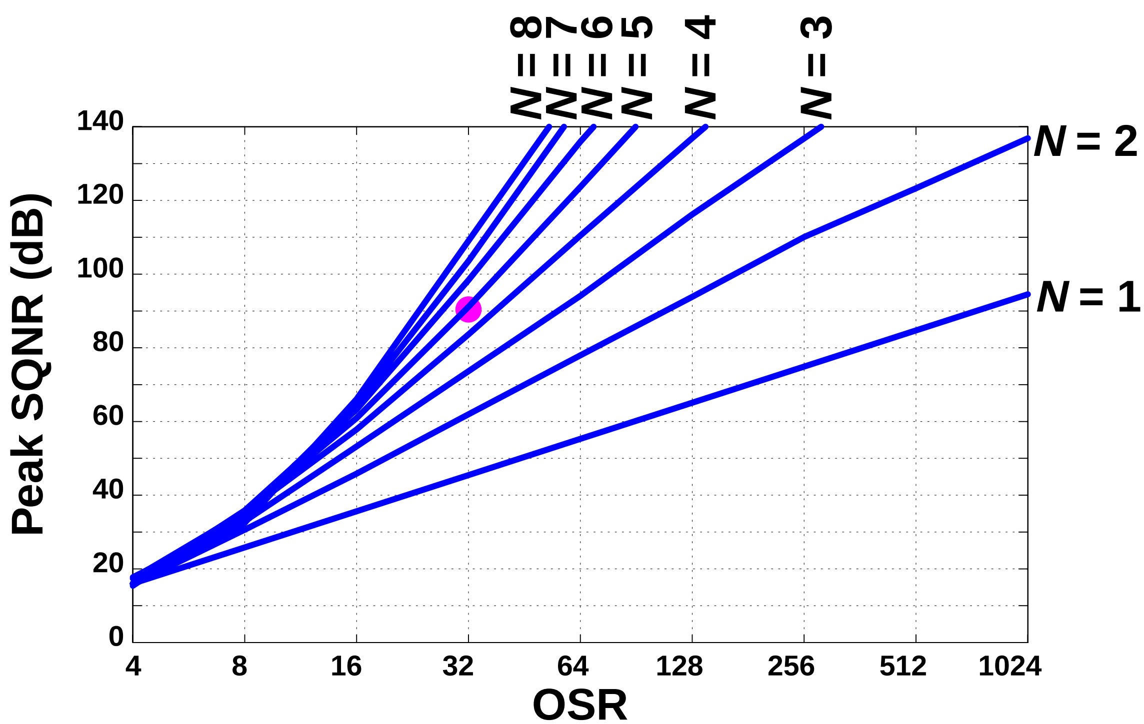
No. There are lots of counter-examples, but $\|H\|_\infty \le 1.5$ often works.

Simulated SQNR vs. $\|H\|_\infty$

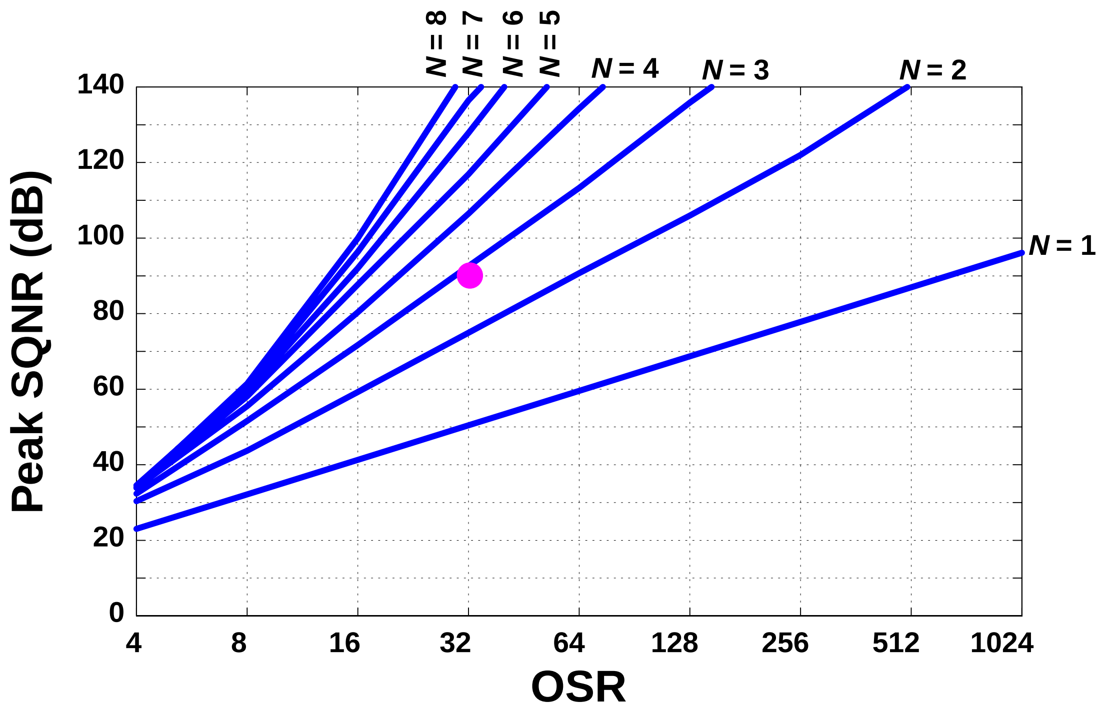
5^{th}-order NTFs; 1-b Quant.; OSR = 32



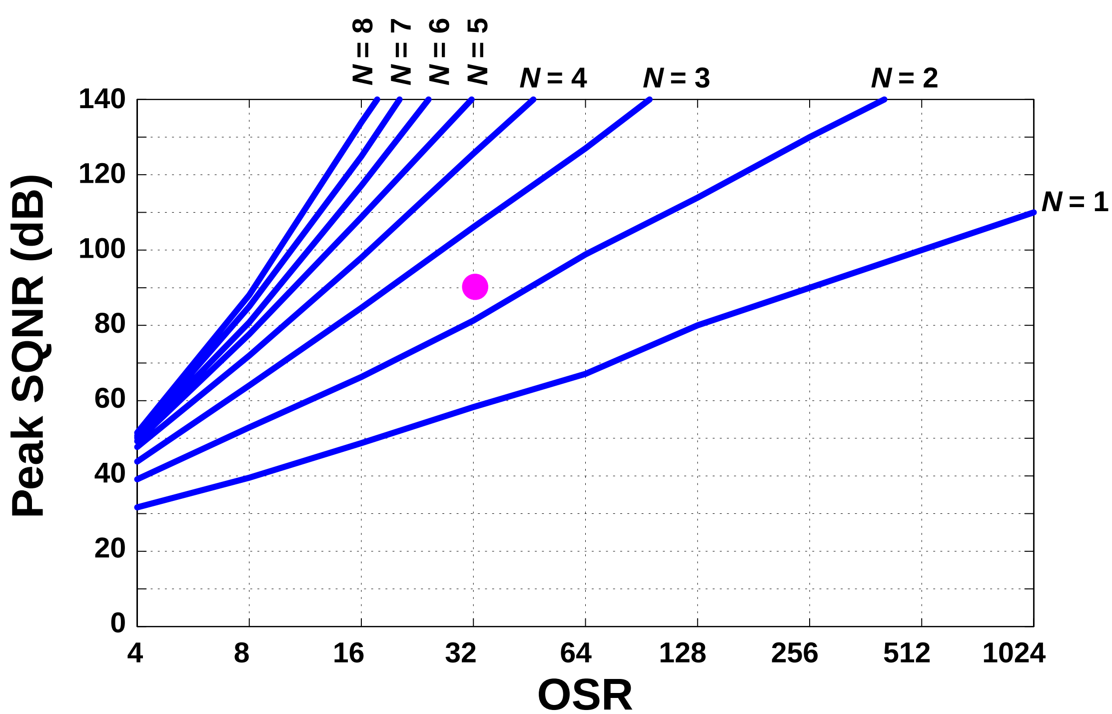
SQNR Limits— 1-bit Modulation



SQNR Limits for 2-bit Modulators

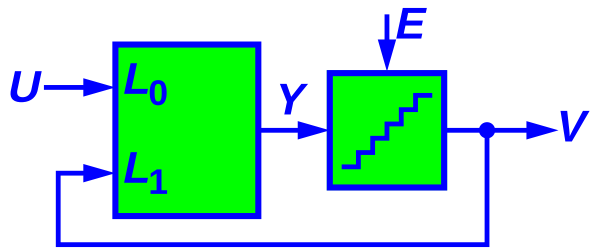


SQNR Limits for 3-bit Modulators



Generic Single-Loop ΔΣ ADC

- **Linear Loop Filter + Nonlinear Quantizer:**



$$Y = L_0 U + L_1 V$$
$$V = Y + E$$
$$\Longrightarrow \boxed{V = STF \cdot U + NTF \cdot E}, \text{ where}$$

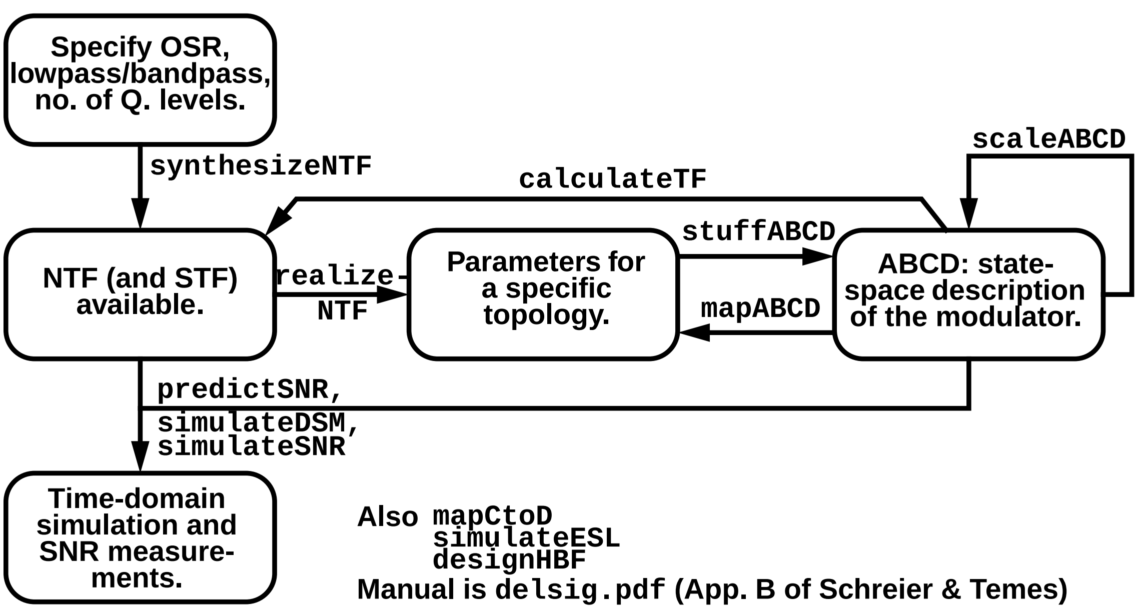
$$NTF = \frac{1}{1 - L_1} \quad \& \quad STF = L_0 \cdot NTF$$

Inverse Relations:
$L_1 = 1 - 1/NTF$, $L_0 = STF / NTF$

ΔΣ Toolbox

`http://www.mathworks.com/matlabcentral/fileexchange`

Search for “Delta Sigma Toolbox”

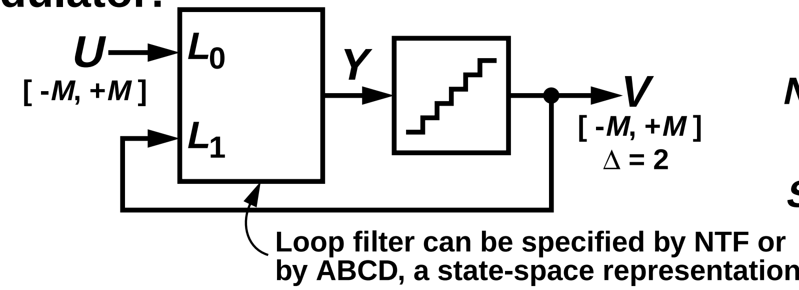


Also `mapCtoD`
`simulateESL`
`designHBF`

Manual is `delsig.pdf` (App. B of Schreier & Temes)

ΔΣ Toolbox Modulator Model

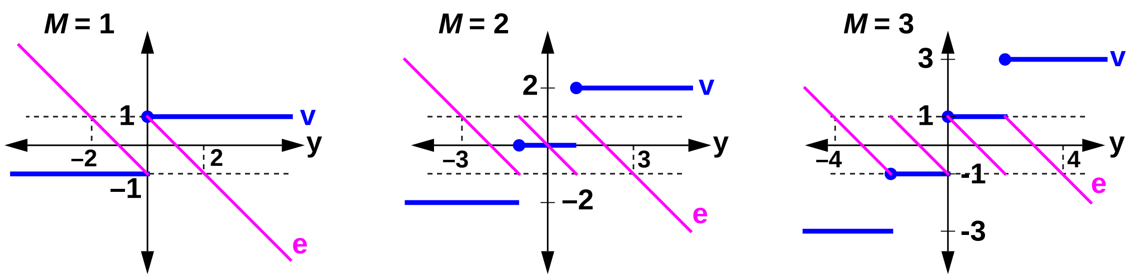
Modulator:



$$NTF = \frac{1}{1 - L_1}$$

$$STF = \frac{L_0}{1 - L_1}$$

Quantizer:



Mid-tread quantizer;
v: even integers [–*M*,+*M*]

Mid-rise quantizer;
v: odd integers [–*M*,+*M*]

NTF Synthesis

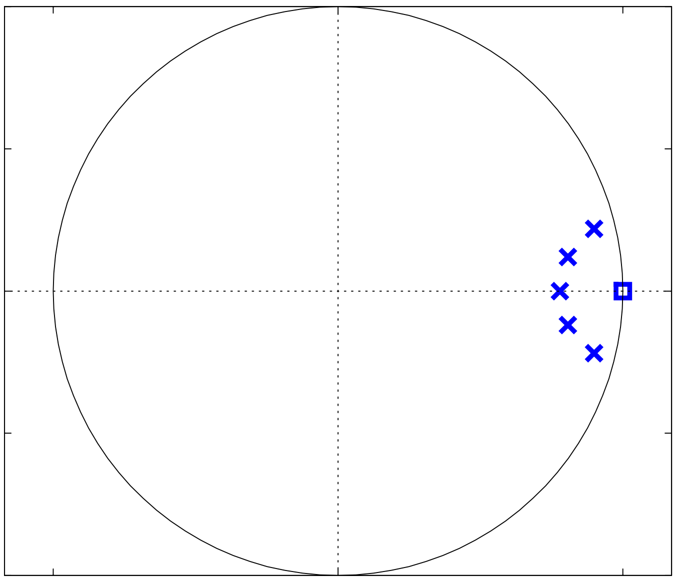
`synthesizeNTF`

- **Not all NTFs are realizable**
  Causality requires $h(0) = 1$, or, in the frequency domain, $H(\infty) = 1$. Recall $H(z) = h(0)z^0 + h(1)z^{-1} + \ldots$
- **Not all NTFs yield stable modulators**
  Rule of thumb for single-bit modulators:
  $\|H\|_\infty < 1.5$ [Lee].
- **Can optimize NTF zeros to minimize the mean-square value of *H* in the passband**
- **The NTF and STF share poles, and in some modulator topologies the STF zeros are not arbitrary**
  Restrict the NTF such that an all-pole STF is maximally flat. (Almost the same as Butterworth poles.)

Lowpass Example [dsdemo1]

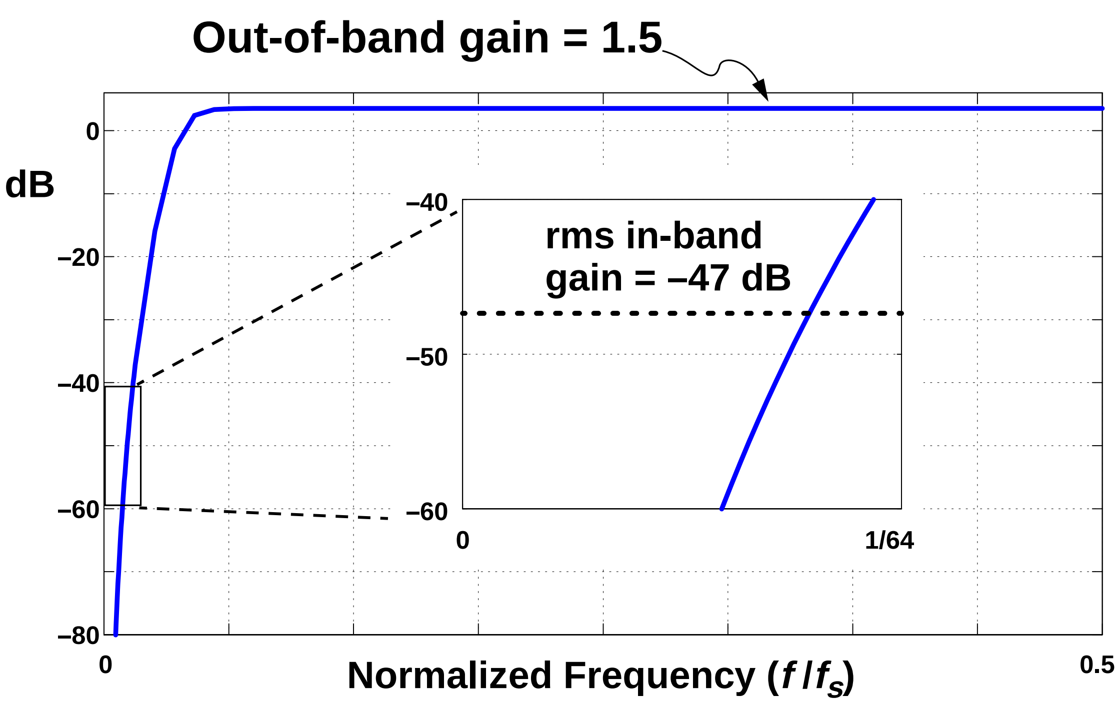
5th-order NTF, all zeros at DC

- **Pole/Zero diagram:**

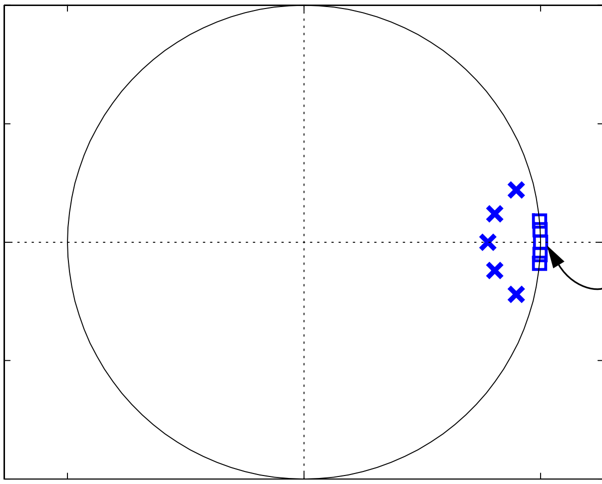


```
OSR = 32;
H = synthesizeNTF(5);
plotPZ(H);

f = linspace(0,0.5);
z = exp(2i*pi*f);
H_z = evalTF(H,z);
plot(f,dbv(H_z));
g = rmsGain(H,0,0.5/OSR)
```


Lowpass NTF



Improved 5th-Order Lowpass NTF
Zeros optimized for OSR=32

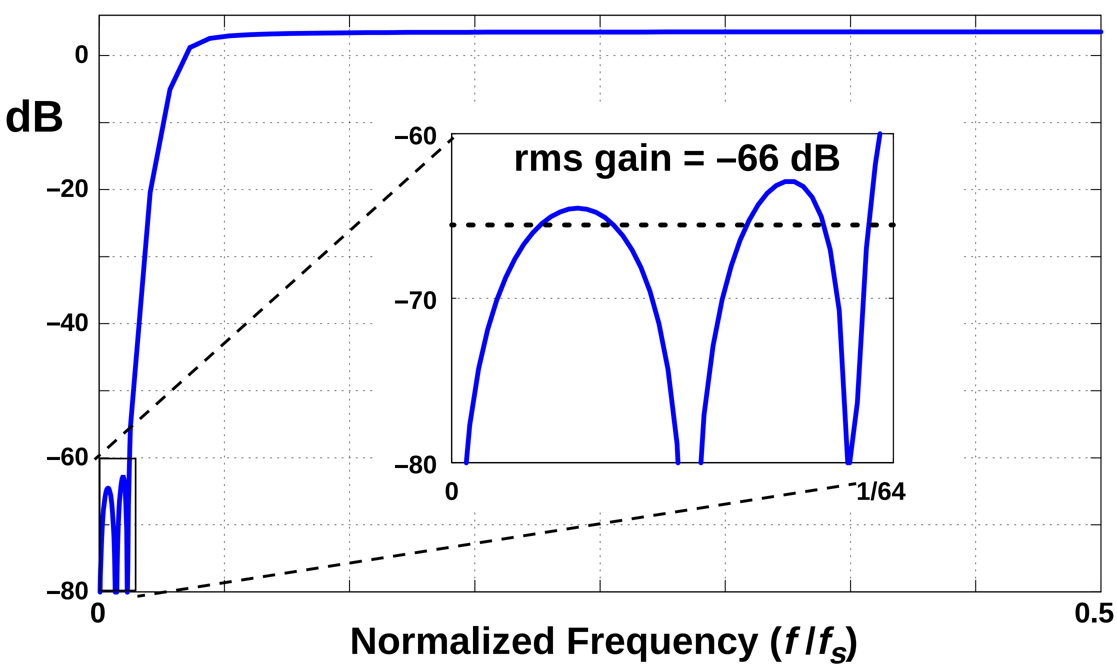


optimization flag

```
OSR = 32;
H = synthesizeNTF(5,OSR,1);
...
```

Zeros spread across the band-of-interest to minimize the rms value of the NTF.

Improved NTF

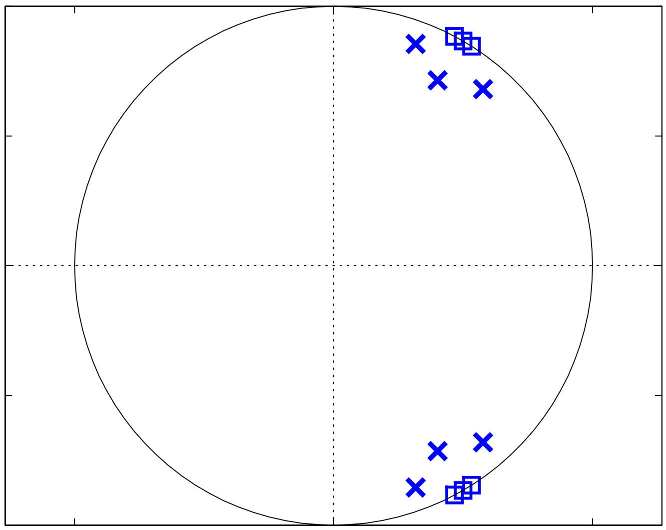


Bandpass Example

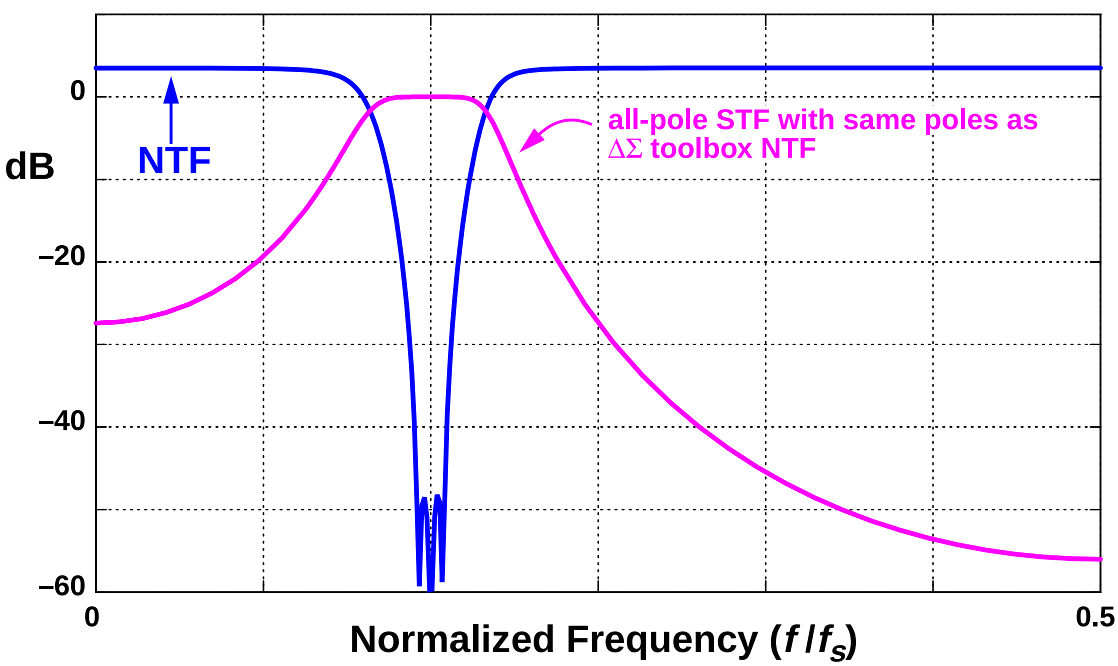
```
OSR = 64;
f0 = 1/6;
H=synthesizeNTF(6,OSR,1,[],f0);...
```

center frequency

[] or NaN means use default value, i.e. Hinf = 1.5



Bandpass NTF and STF



Summary: NTF Selection

- **If OSR is high, a single-bit modulator may work**

- **To improve SQNR,**

  Optimize zeros,

  Increase $\|H\|_\infty$, or

  Increase order.

- **If SQNR is insufficient, must use a multi-bit design**

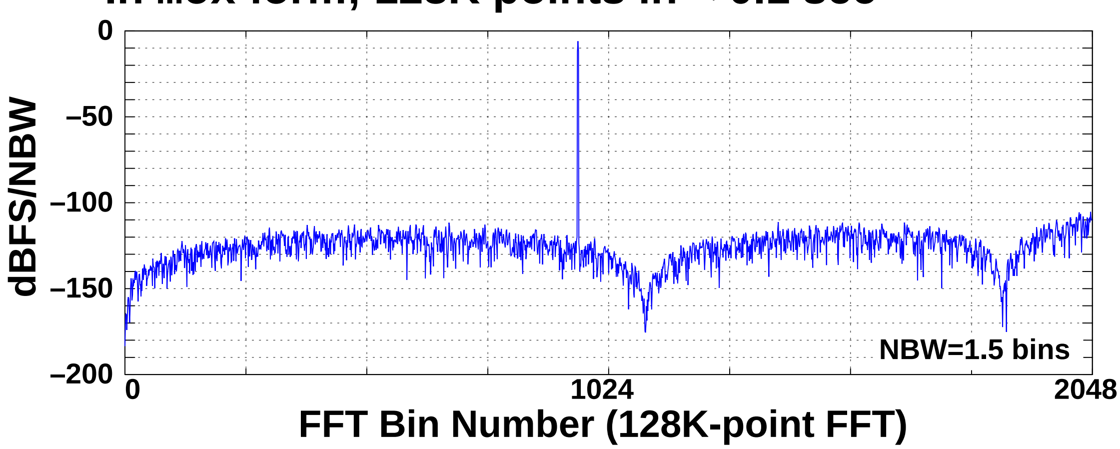
  Can turn all the above knobs to enhance performance.

- **Feedback DAC assumed to be ideal**

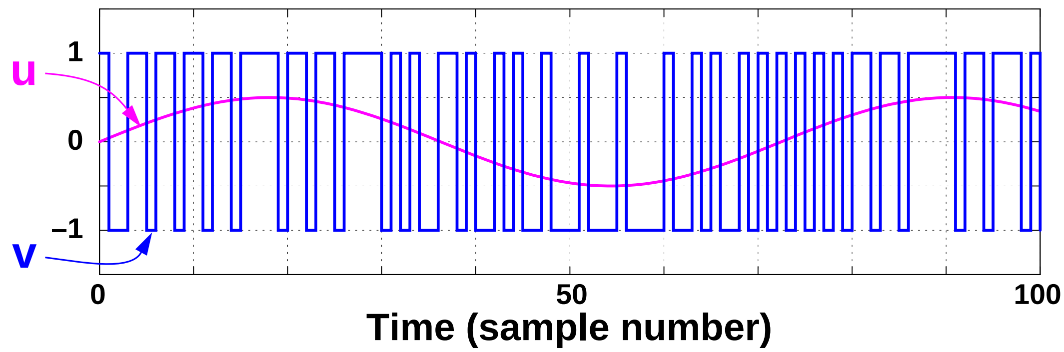
NTF-Based Simulation [dsdemo2]

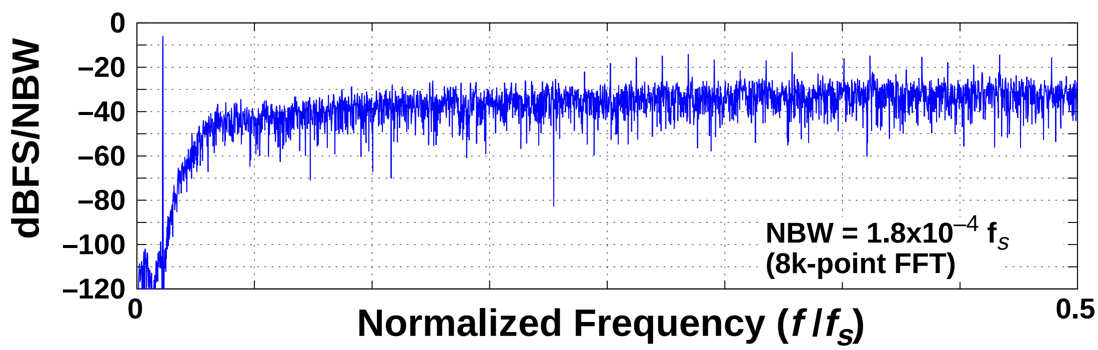
```
order=5; OSR=32;
ntf = synthesizeNTF(order,OSR,1);
N=2^17; fbin=959; A=0.5; % 128K points
input = A*sin(2*pi*fbin/N*[0:N-1]);
output = simulateDSM(input,ntf);
spec = fft(output.*ds_hann(N)/(N/4));
plot(dbv(spec(1:N/(2*OSR))));
```

- **In mex form; 128K points in < 0.1 sec**



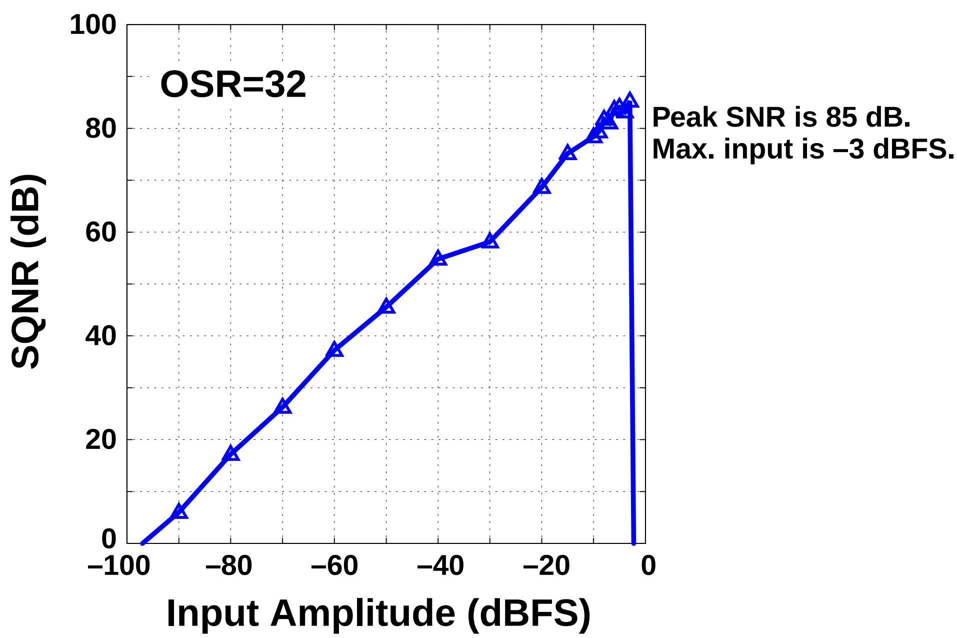
Simulation Example Cont'd





SNR vs. Amplitude: simulateSNR

```
[snr amp] = simulateSNR(ntf,OSR);
plot(amp,snr,'b-^');
```



Homework #2– Part A

Create a Matlab function that computes MOD2's output sequence given a vector of input samples and exercise your function in the following ways:

1. **Plot the error $\overline{v} - u$ as a function of u using 100-point rectangular and triangular* averages for $\overline{v}$.**
2. **Produce a spectral plot like that on Slide 14.**
3. **Compare the in-band quantization noise of your system with a half-scale sine-wave input against the relation given on Slide 12 for OSR in $[2^3, 2^{10}]$.**

*. $\overline{v_{tri}} = \sum_{n=0}^{N-1} tri(n) v(n)$, $tri = rect_{(N/2)} * rect_{(N/2)}$

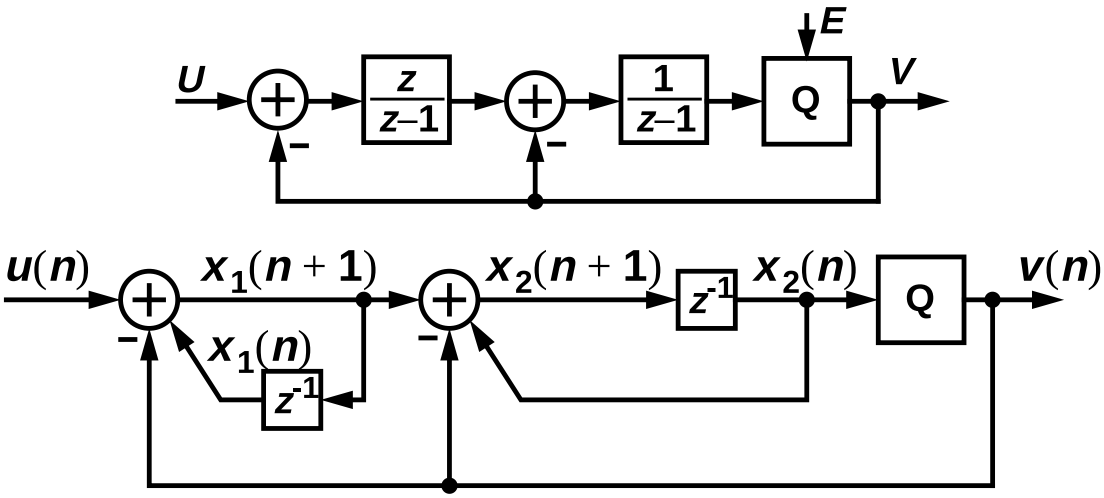
Homework #2– Part B

Extract code from `dsdemo1` & `dsdemo2` and modify it to:

1. **Create a 3^{rd}-order NTF with zeros optimized for OSR = 32. Plot the poles/zeros and frequency response of your NTF.**
2. **Simulate a 3-level ΔΣ modulator with this NTF. Plot example input and output waveforms. Plot a spectrum and the predicted noise curve.† Plot the SQNR vs. input amplitude curve and note the maximum stable input.**

†. **Beware that with an *M*-step modulator, the full-scale is *M* and this must be accounted for in normalizing the FFT and in computing the expected noise curve. You ought to check your code using a large value for *M*, say *M* = 20.**

MOD2 Expanded



Difference Equations:

$$v(n) = Q(x_2(n))$$

$$x_1(n+1) = x_1(n) - v(n) + u(n)$$

$$x_2(n+1) = x_2(n) - v(n) + x_1(n+1)$$

Example Matlab Code

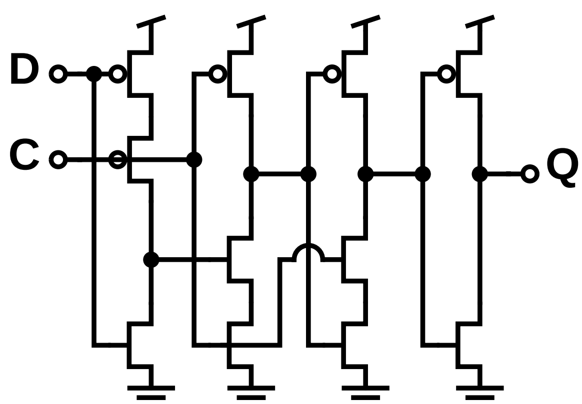
```
function [v] = simulateMOD2(u)
    x1 = 0;
    x2 = 0;
    for i = 1:length(u)
        v(i) = quantize( x2 );
        x1 = x1 + u(i) - v(i);
        x2 = x2 + x1 - v(i);
    end
return

function v = quantize( y )
    if y>=0
        v = 1;
    else
        v = -1;
    end
return
```

What You Learned Today

And what the homework should solidify

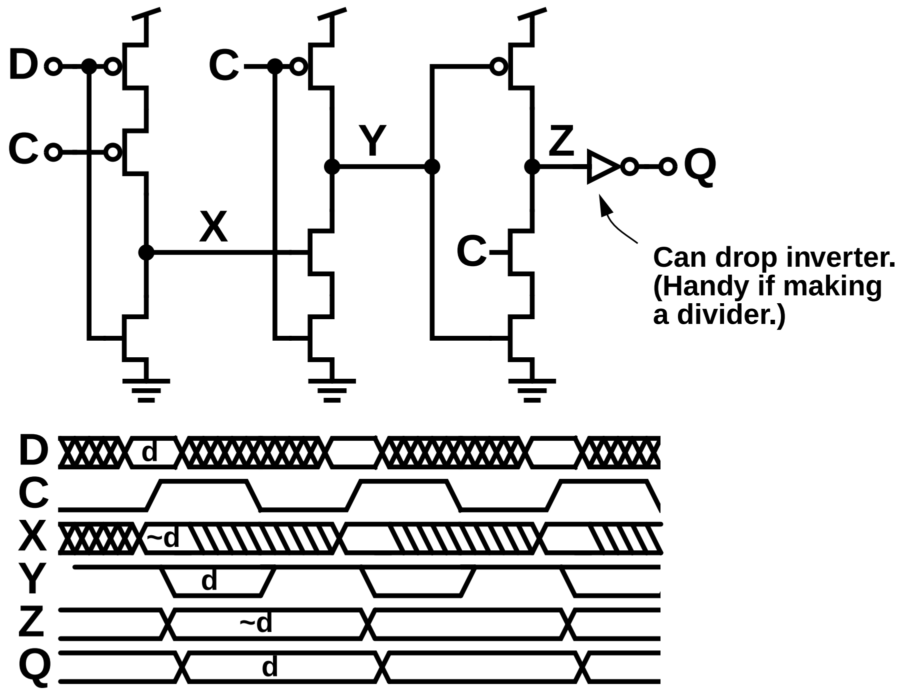
1. Second-order modulator (MOD2)
2. Arbitrary-order modulator (MOD*N*) design with the ΔΣ Toolbox

NLCOTD: True Single-Phase Dynamic FF



+ Clock not inverted anywhere
+ Small
+ Fast

TSPFF Operation



TSPFF Gotchas

- **Leakage:**
  Won't work if clock is too slow.
  Possible high current if clock is stopped.

  Need to add devices that hold the dynamic nodes at a safe value.

- **No positive feedback**

  Vulnerable to metastability.