ECE1371 Advanced Analog Circuits
Lecture 1

INTRODUCTION TO DELTA-SIGMA ADCS

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Course Goals

- **Deepen understanding of CMOS analog circuit design through a top-down study of a modern analog system— a delta-sigma ADC**
- **Develop circuit insight through brief peeks at some nifty little circuits**

  The circuit world is filled with many little gems that every competent designer ought to know.

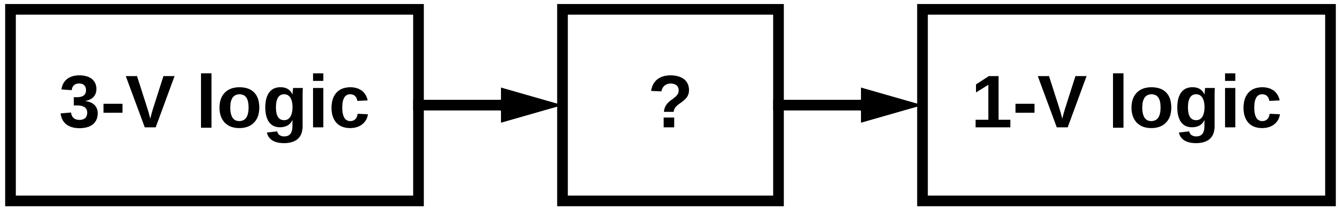
Logistics

- **Format:**
  Meet Mondays 1:00-3:00 PM (not Feb 16, Apr 6)
  12 2-hr lectures plus proj. presentation
- **Grading:**
  30% homework
  40% project
  30% exam
- **References:**
  Schreier & Temes, "Understanding $\Delta\Sigma$ …"
  Chan-Carusone, Johns & Martin, "Analog IC …"
  Razavi, "Design of Analog CMOS ICs"
- **Lecture Plan:**

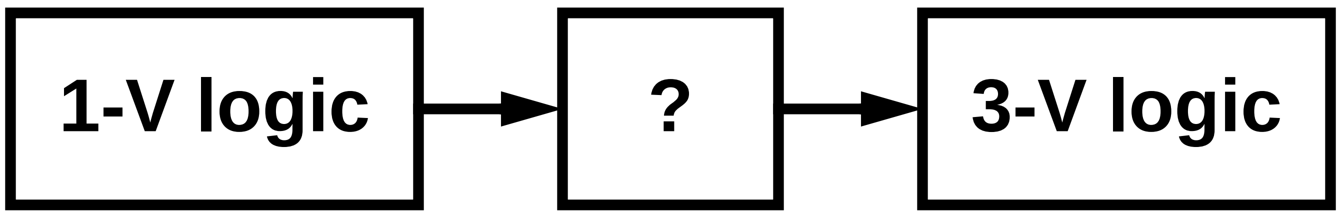
Date	Lecture (M 13:00-15:00)			Ref	Homework
2015-01-05	RS	1	MOD1 & MOD2	ST 2, 3, A	1: Matlab MOD1&2
2015-01-12	RS	2	MOD*N* + ΔΣ Toolbox	ST 4, B	2: ΔΣ Toolbox
2015-01-19	RS	3	Example Design: Part 1	ST 9.1, CCJM 14	3: Sw.-level MOD2
2015-01-26	RS	4	Example Design: Part 2	CCJM 18	
2015-02-02	TC	5	SC Circuits	R 12, CCJM 14	4: SC Integrator
2015-02-09	TC	6	Amplifier Design		
2015-02-16	Reading Week– No Lecture				
2015-02-23	TC	7	Amplifier Design		5: SC Int w/ Amp
2015-03-02	RS	8	Comparator & Flash ADC	CCJM 10	Project
2015-03-09	TC	9	Noise in SC Circuits	ST C	
2015-03-16	RS	10	Advanced ΔΣ	ST 6.6, 9.4	
2015-03-23	TC	11	Matching & MM-Shaping	ST 6.3-6.5, +	
2015-03-30	TC	12	Pipeline and SAR ADCs	CCJM 15, 17	
2015-04-06	Exam			Proj. Report Due Friday April 10	
2015-04-13	Project Presentation				

NLCOTD: Level Translator

VDD1 > VDD2, e.g.



- **VDD1 < VDD2, e.g.**



- **Constraints:** **CMOS**
  1-V and 3-V devices
  no static current

Highlights
(i.e. What you will learn today)

1 MOD1: 1st-order $\Delta\Sigma$ modulator
   Structure and theory of operation
2 Inherent linearity of binary modulators
3 Inherent anti-aliasing of continuous-time modulators
4 MOD2: 2nd-order $\Delta\Sigma$ modulator
5 Good FFT practice

0. Background
(Stuff you already know)

- **The SQNR* of an ideal *n*-bit ADC with a full-scale sine-wave input is (6.02*n* + 1.76) dB**

  "6 dB = 1 bit."

- **The PSD at the output of a linear system is the product of the input's PSD and the squared magnitude of the system's frequency response**

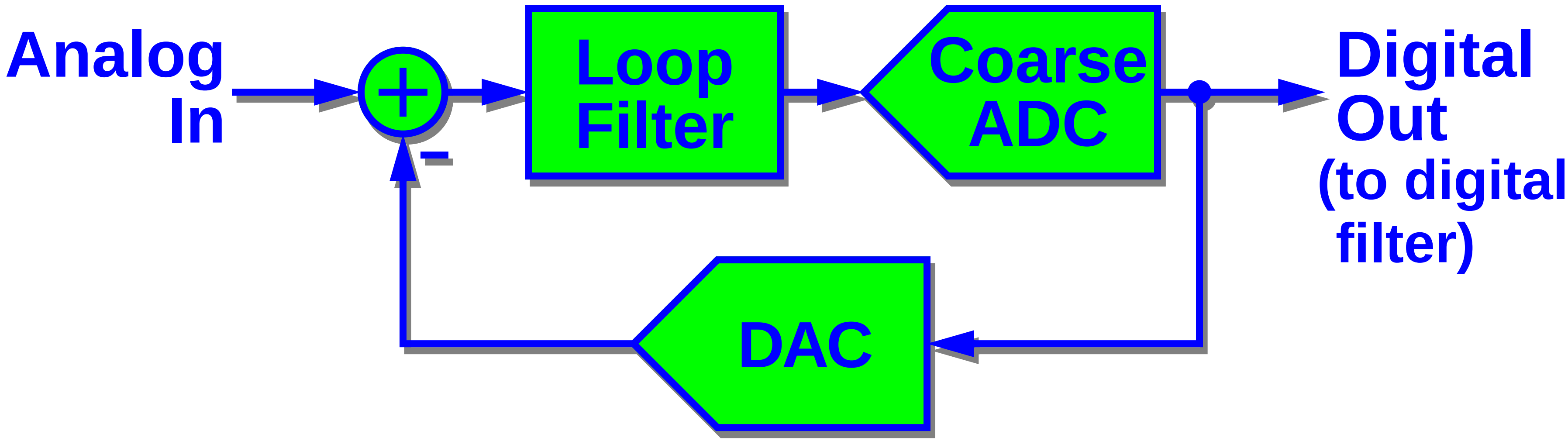
  i.e. $X \rightarrow \boxed{H(z)} \rightarrow Y \qquad S_{yy}(f) = |H(e^{j2\pi f})|^2 \cdot S_{xx}(f)$

- **The power in any frequency band is the integral of the PSD over that band**

***. SQNR = Signal-to-Quantization-Noise Ratio**

1. What is ΔΣ?

- **ΔΣ is NOT a fraternity**
- **Simplified ΔΣ ADC structure:**



- **Key features: coarse quantization, filtering, feedback and oversampling**

  Quantization is often *quite* coarse (1 bit!), but the effective resolution can still be as high as 22 bits.

What is Oversampling?

- **Oversampling is sampling faster than required by the Nyquist criterion**

  For a lowpass signal containing energy in the frequency range $(0, f_B)$, the minimum sample rate required for perfect reconstruction is $f_s = 2f_B$

- **The *oversampling ratio* is $OSR \equiv f_s / (2f_B)$**

- **For a regular ADC, $OSR \sim 2 - 3$**

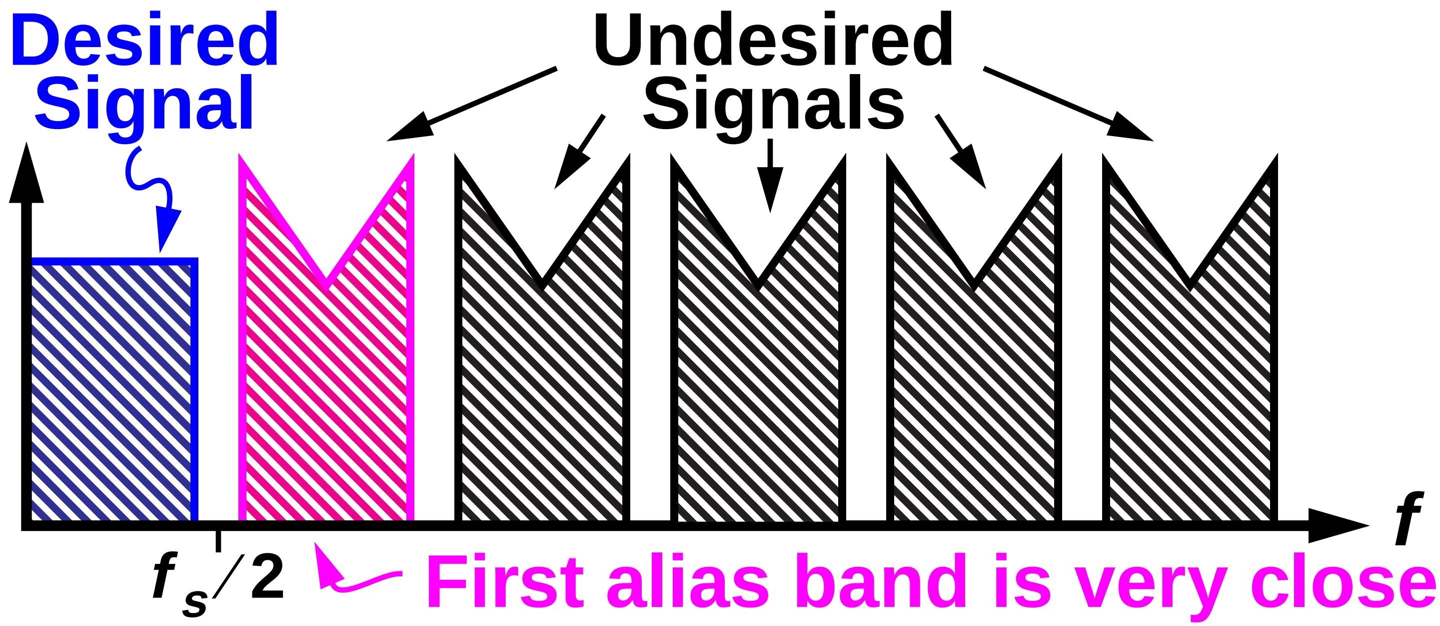
  To make the anti-alias filter (AAF) feasible

- **For a $\Delta\Sigma$ ADC, $OSR \sim 30$**

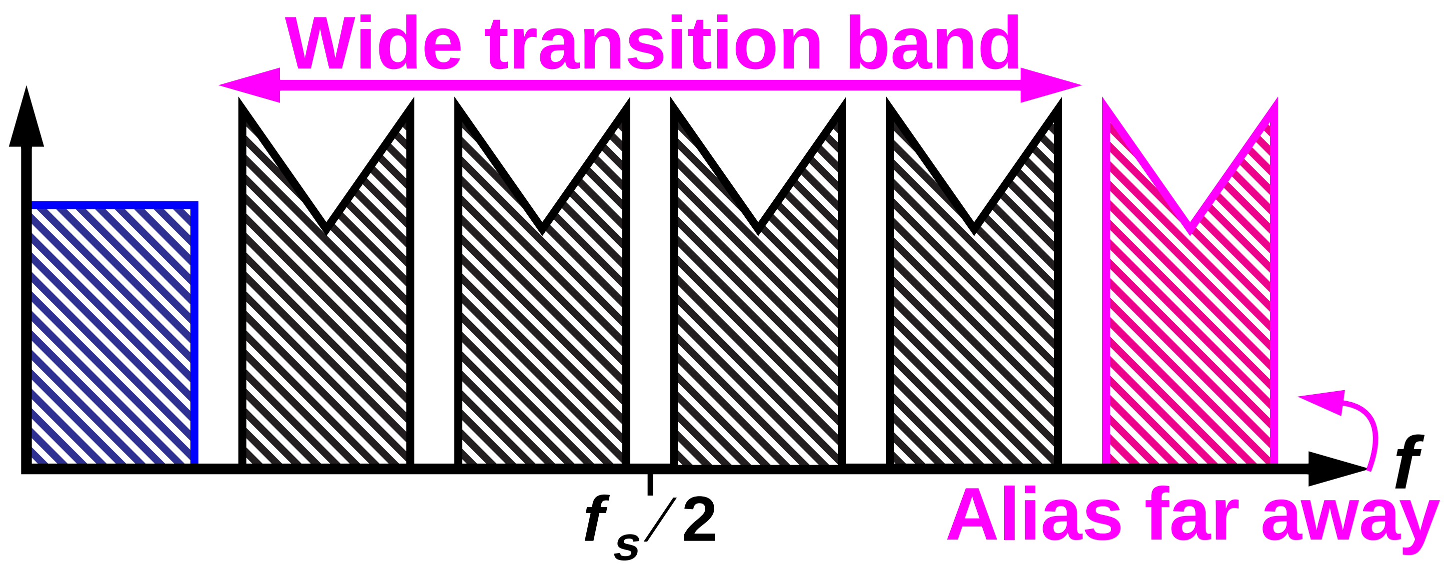
  To get adequate quantization noise suppression. Signals between f_B and $\sim f_s$ are removed digitally.

Oversampling Simplifies AAF

OSR ~ 1:

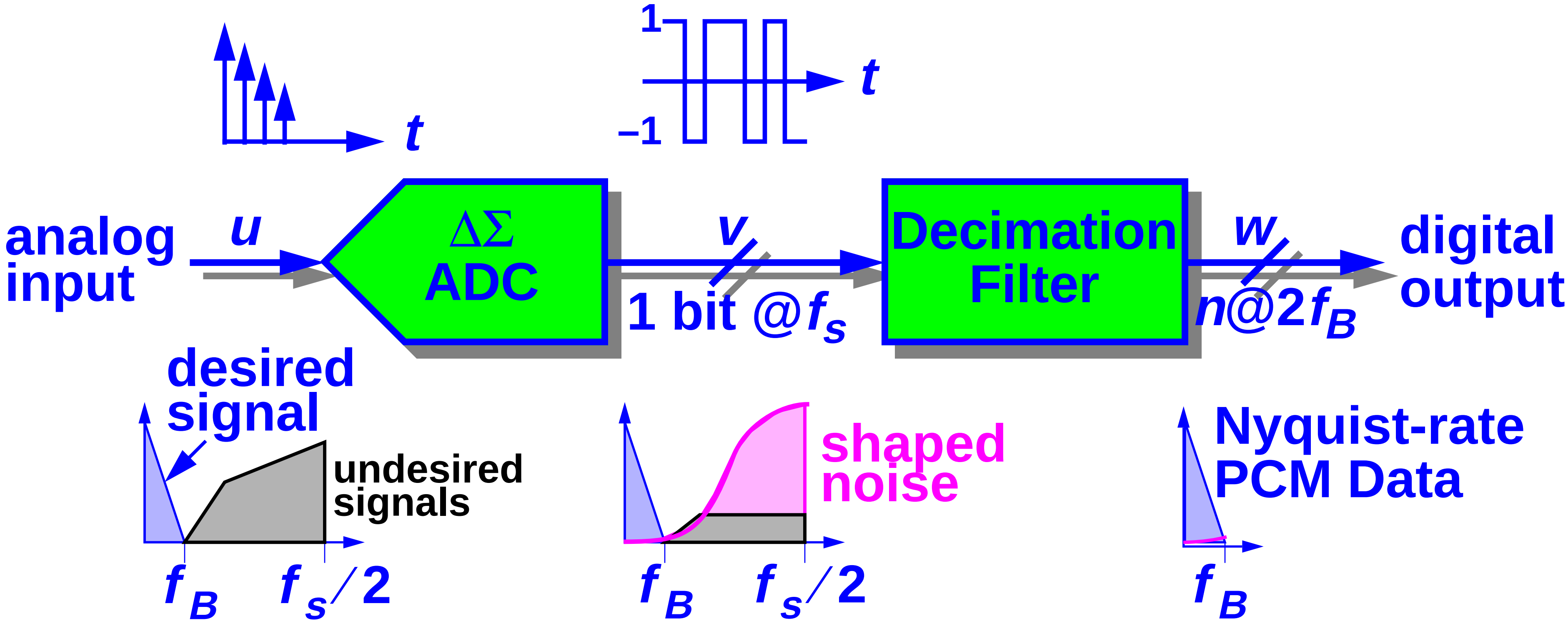


OSR = 3:

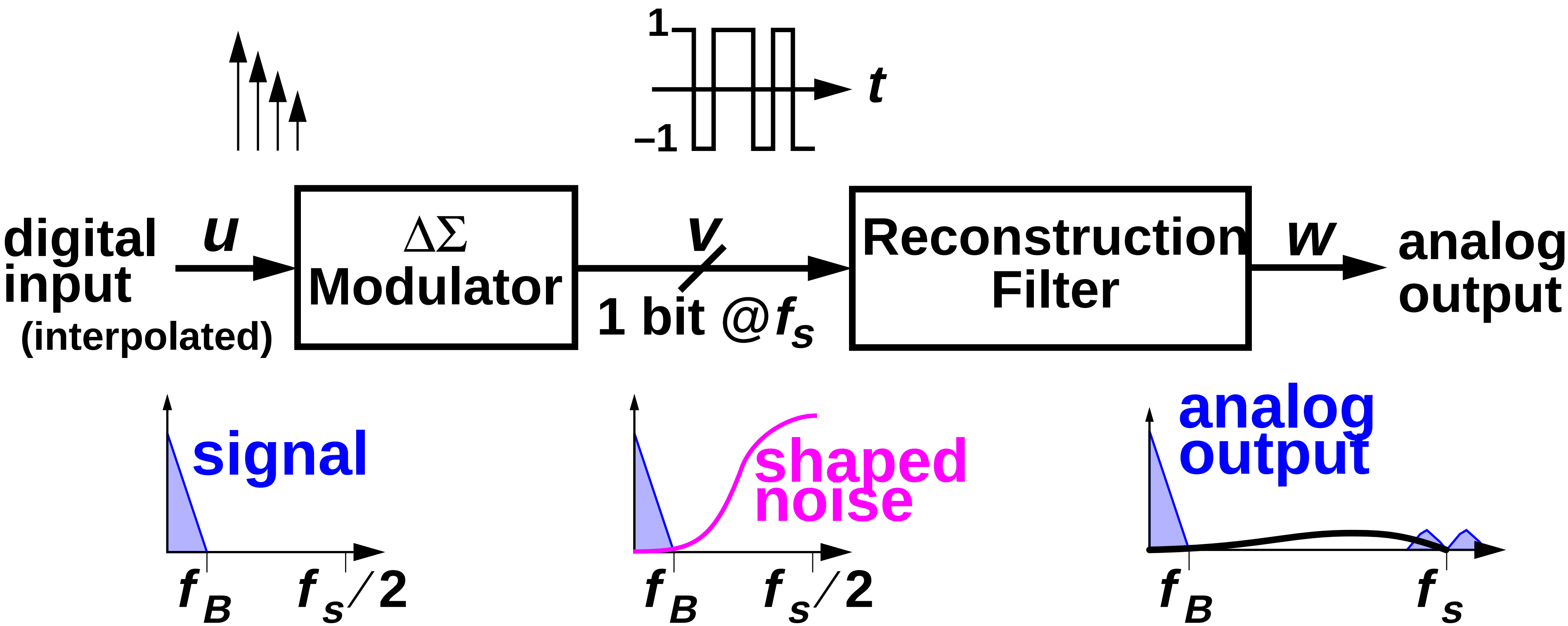


How Does A ΔΣ ADC Work?

- **Coarse quantization ⇒ lots of quantization error. So how can a ΔΣ ADC achieve 22-bit resolution?**
- **A ΔΣ ADC spectrally separates the quantization error from the signal through *noise-shaping***



A ΔΣ DAC System



- **Mathematically similar to an ADC system**

  Except that now the modulator is digital and drives a low-resolution DAC, and that the out-of-band noise is handled by an analog reconstruction filter.

Why Do It The ΔΣ Way?

- **ADC: Simplified Anti-Alias Filter**

  Since the input is oversampled, only very high frequencies alias to the passband.
  A simple RC section often suffices.

  If a continuous-time loop filter is used, the anti-alias filter can often be eliminated altogether.

- **DAC: Simplified Reconstruction Filter**

  The nearby images present in Nyquist-rate reconstruction can be removed digitally.

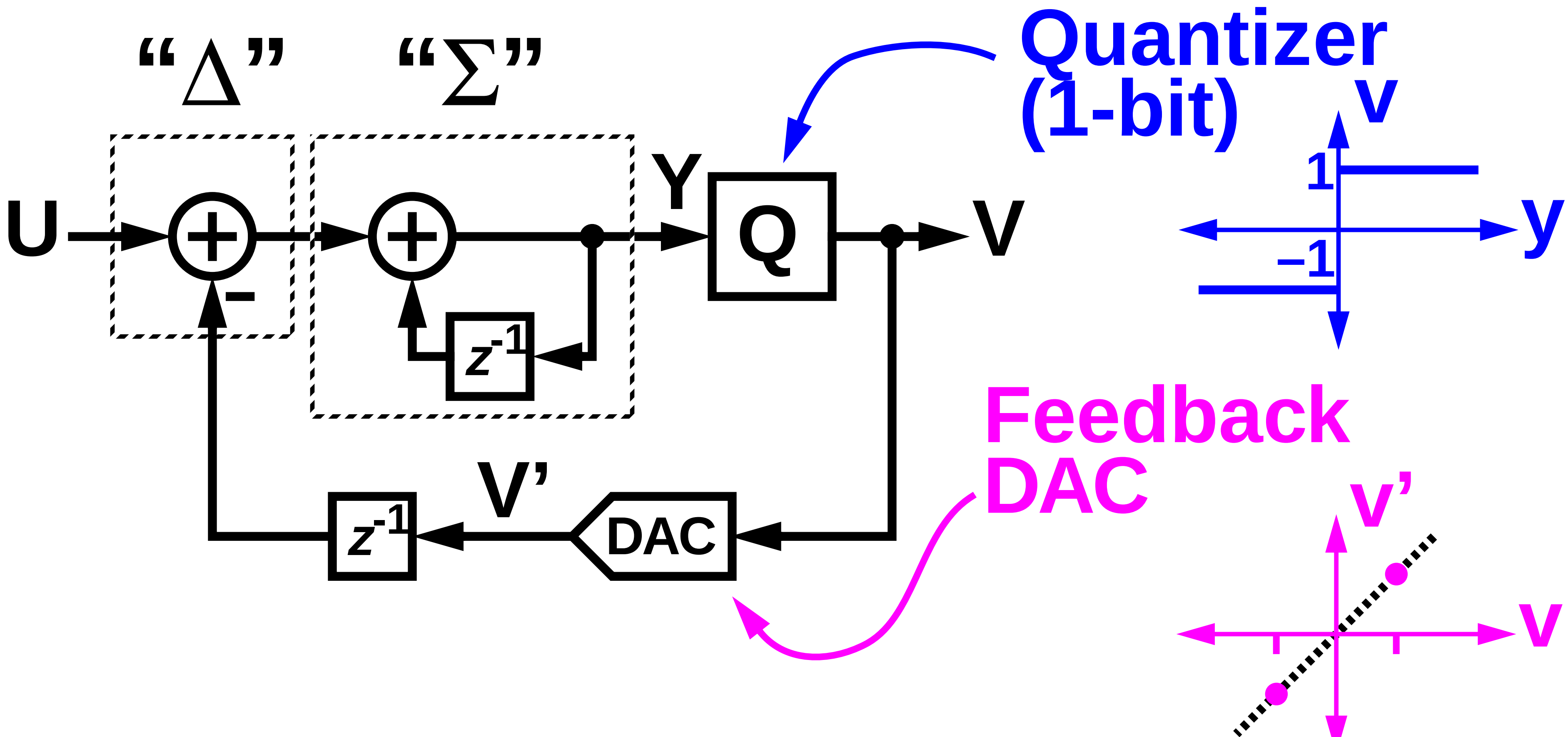
\+ **Inherent Linearity**

  Simple structures can yield very high SNR.

\+ **Robust Implementation**

  ΔΣ tolerates sizable component errors.

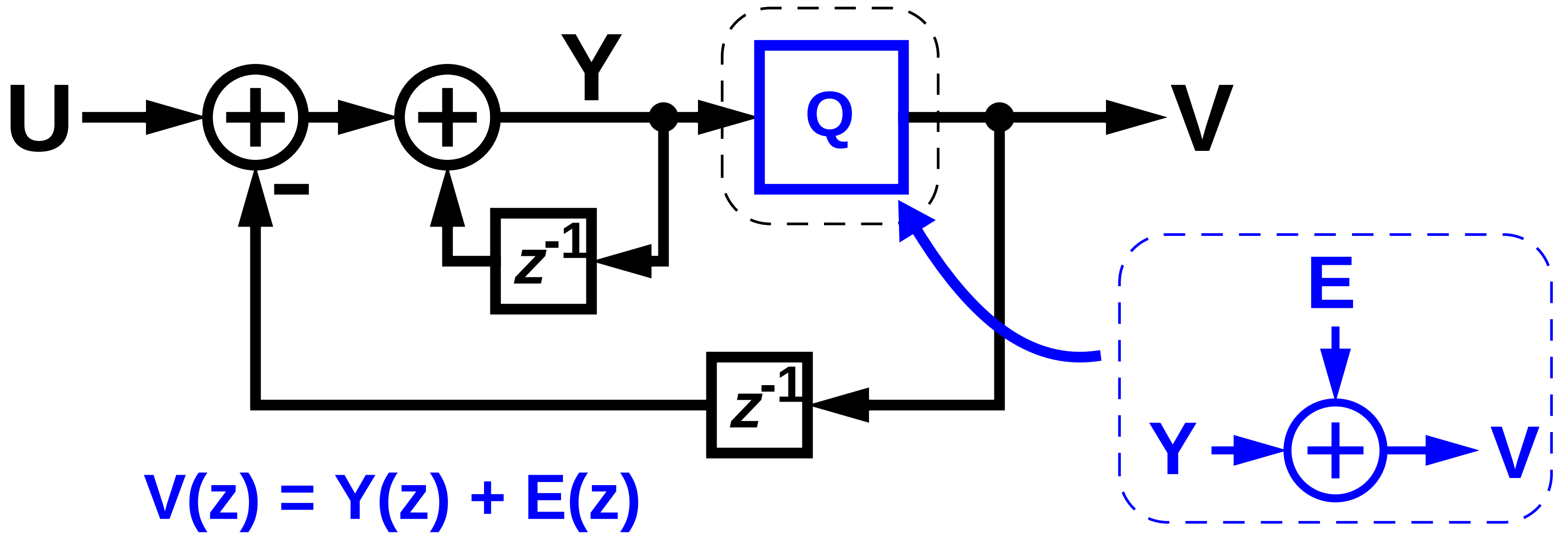
2. MOD1: 1st-Order ΔΣ Modulator
[Ch. 2 of Schreier & Temes]



Since two points define a line,
a binary DAC is inherently linear.

MOD1 Analysis

- **Exact analysis is intractable for all but the simplest inputs, so treat the quantizer as an additive noise source:**



$V(z) = Y(z) + E(z)$

$Y(z) = (U(z) - z^{-1}V(z)) / (1-z^{-1})$

$\Rightarrow (1-z^{-1})\, V(z) = U(z) - z^{-1}V(z) + (1-z^{-1})E(z)$

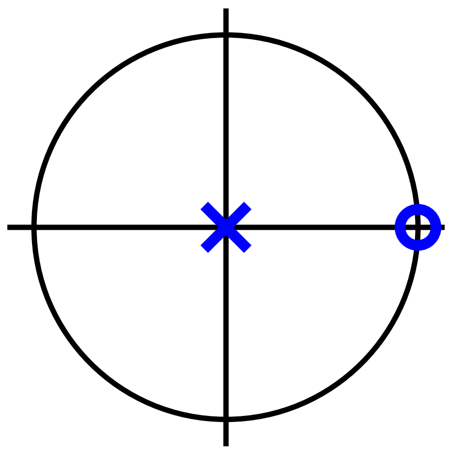
$$\boxed{V(z) = U(z) + (1-z^{-1})E(z)}$$

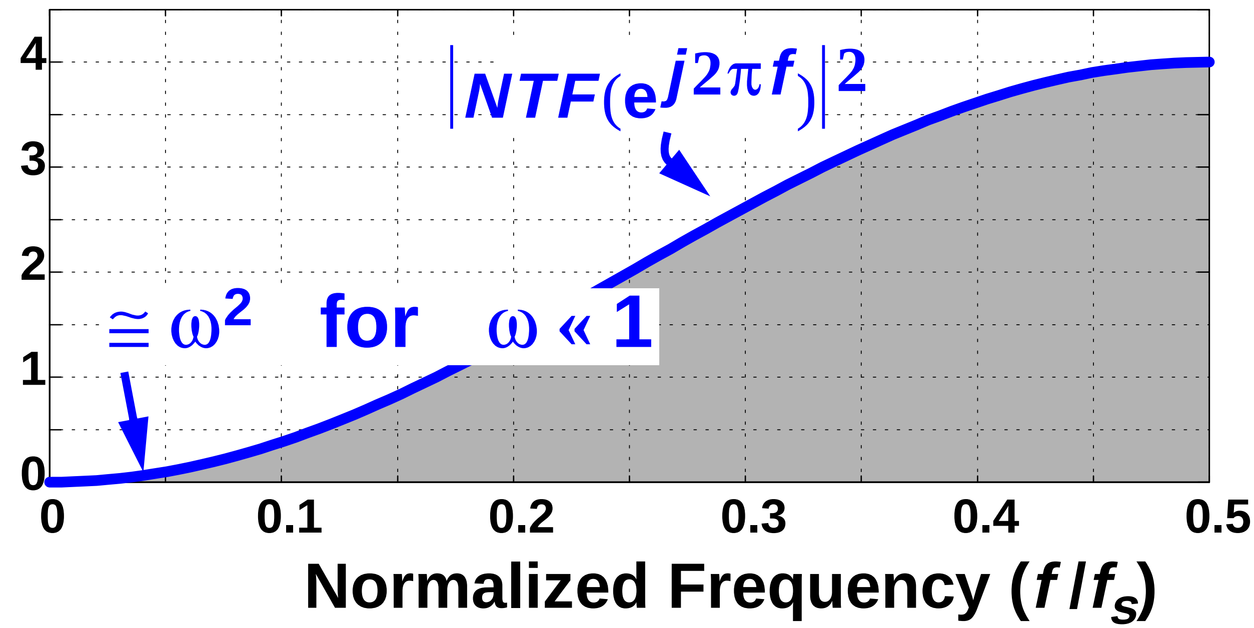
The Noise Transfer Function (NTF)

- **In general, $V(z) = STF(z) \cdot U(z) + NTF(z) \cdot E(z)$**
- **For MOD1, $NTF(z) = 1 - z^{-1}$**

  The quantization noise has spectral shape!

Poles & zeros:





- **The total noise power increases, but the noise power at low frequencies is reduced**

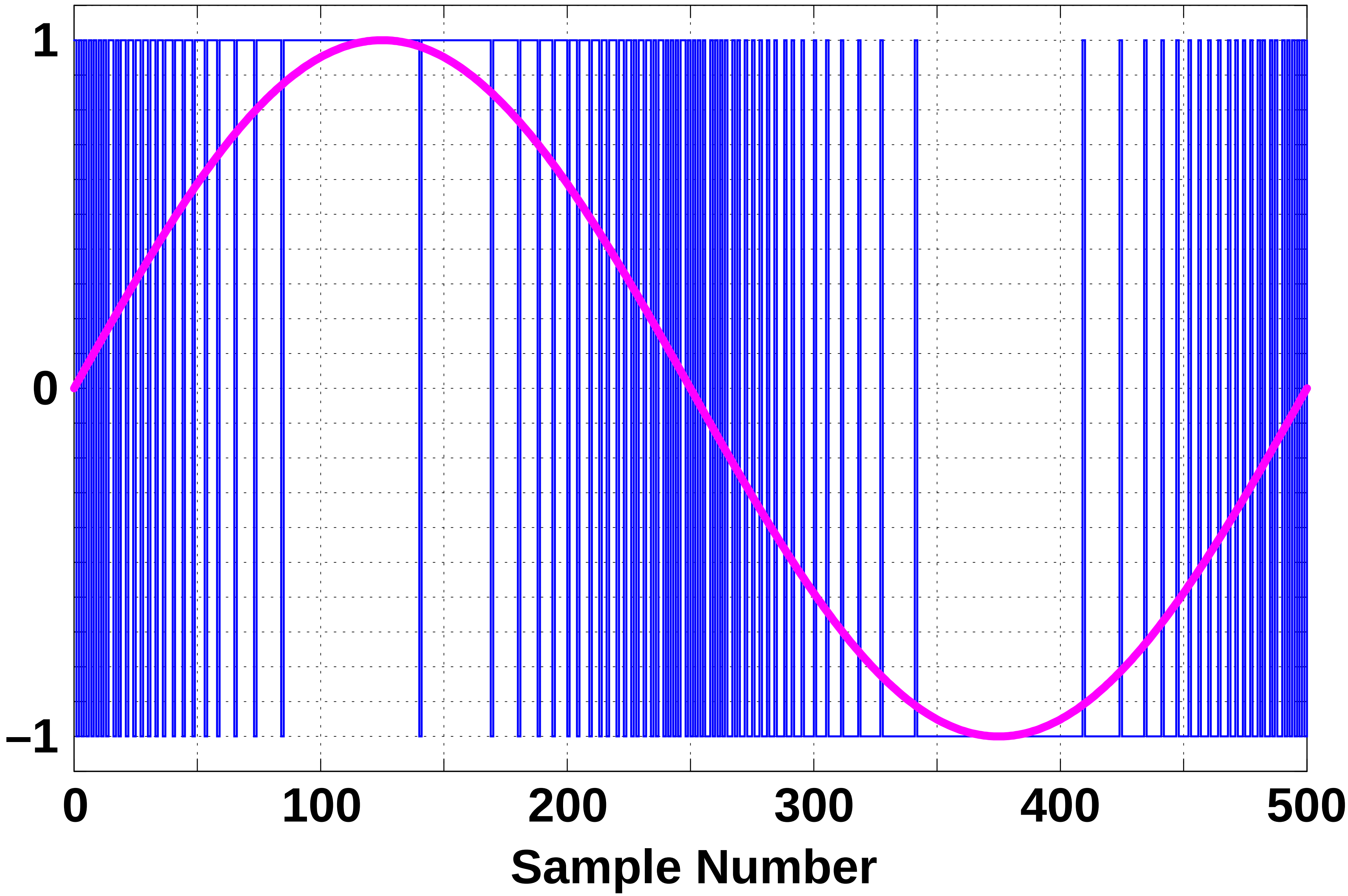
In-band Quant. Noise Power

- **Assume that e is white with power σ_e^2**
  i.e. $S_{ee}(\omega) = \sigma_e^2 / \pi$
- **The in-band quantization noise power is**

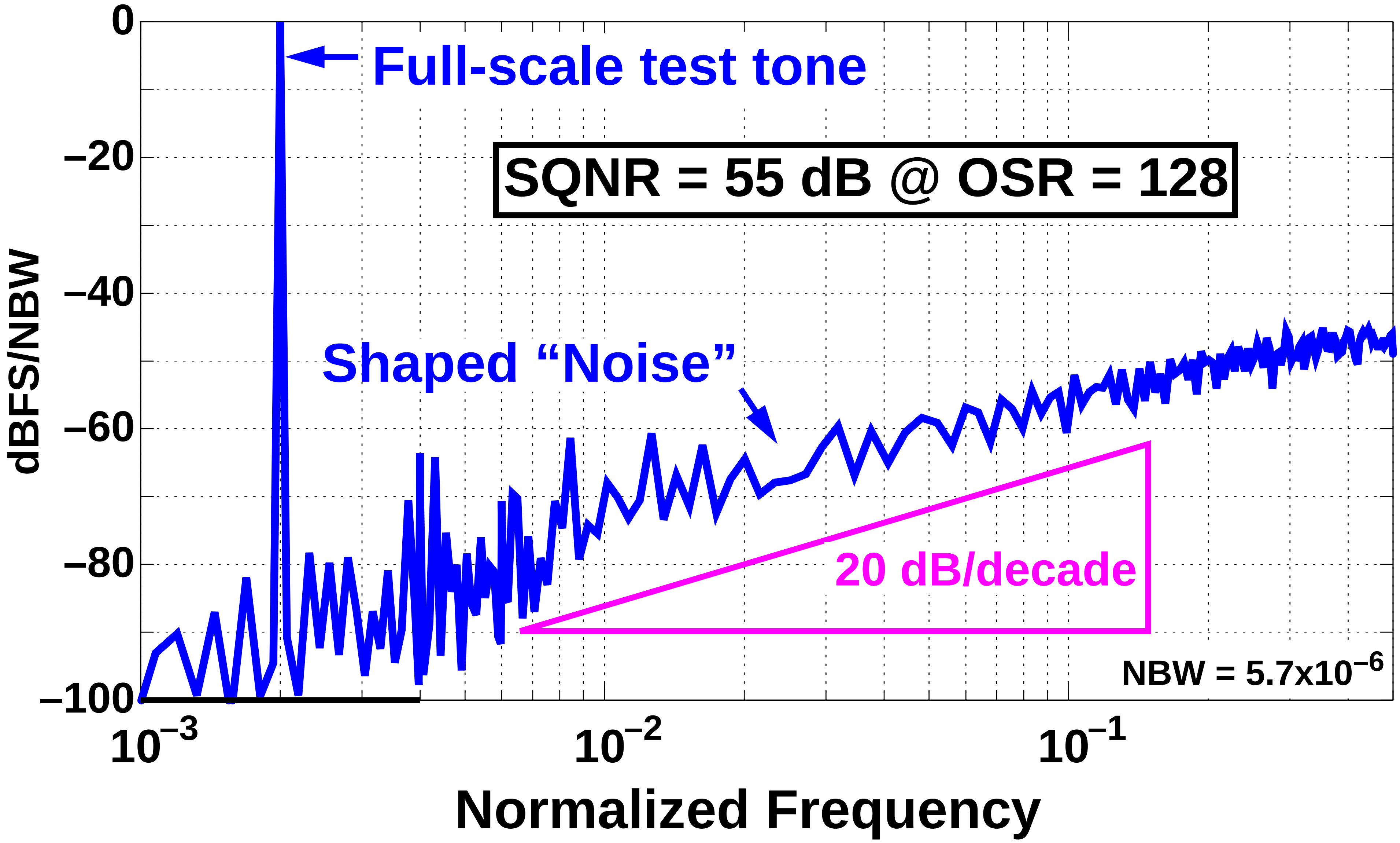
$$IQNP = \int_0^{\omega_B} \left|H(e^{j\omega})\right|^2 S_{ee}(\omega)\, d\omega \cong \frac{\sigma_e^2}{\pi} \int_0^{\omega_B} \omega^2\, d\omega$$

- **Since $OSR \equiv \dfrac{\pi}{\omega_B}$, $IQNP = \dfrac{\pi^2 \sigma_e^2}{3} (OSR)^{-3}$**
- **For MOD1, an octave increase in *OSR* increases SQNR by 9 dB**
  "1.5-bit/octave SQNR-OSR trade-off."

A Simulation of MOD1— Time



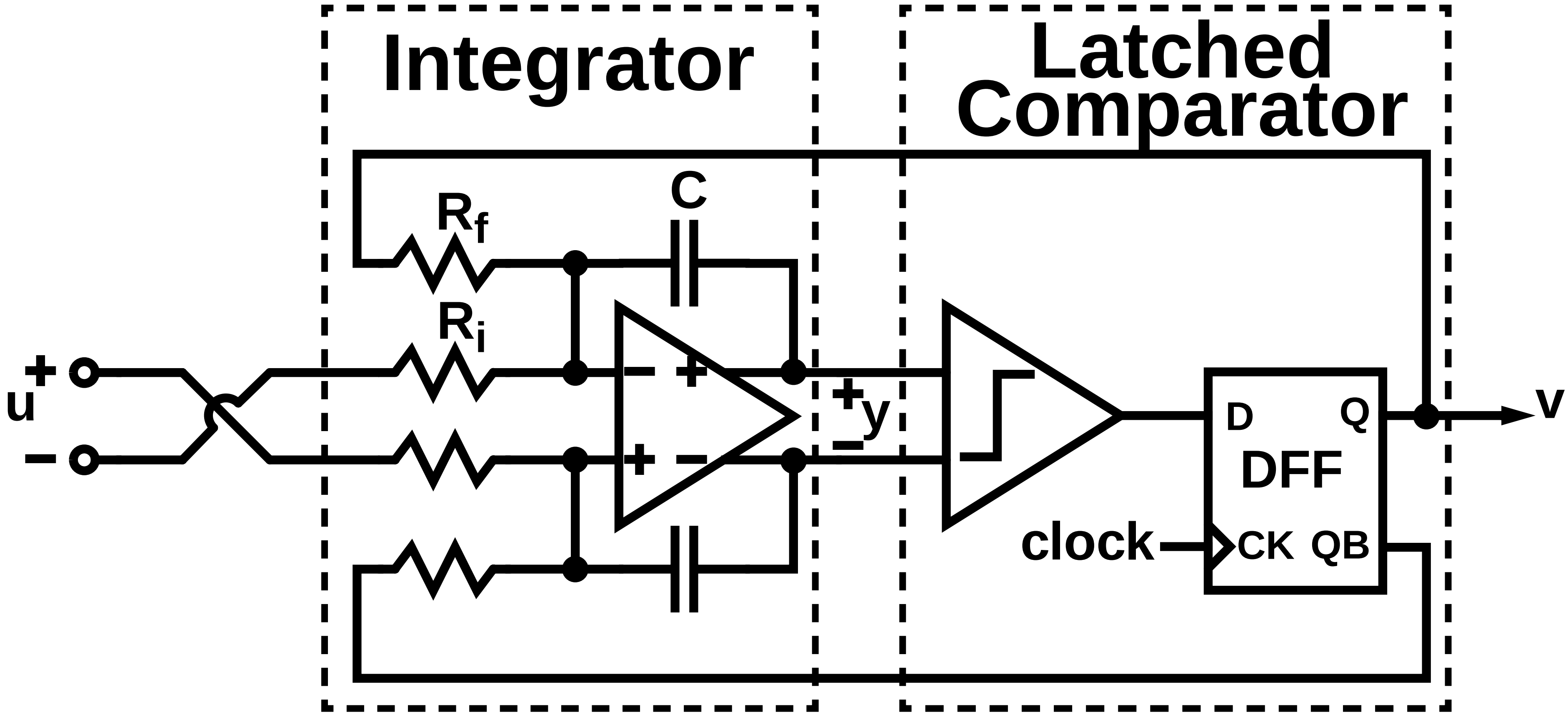
A Simulation of MOD1— Freq.



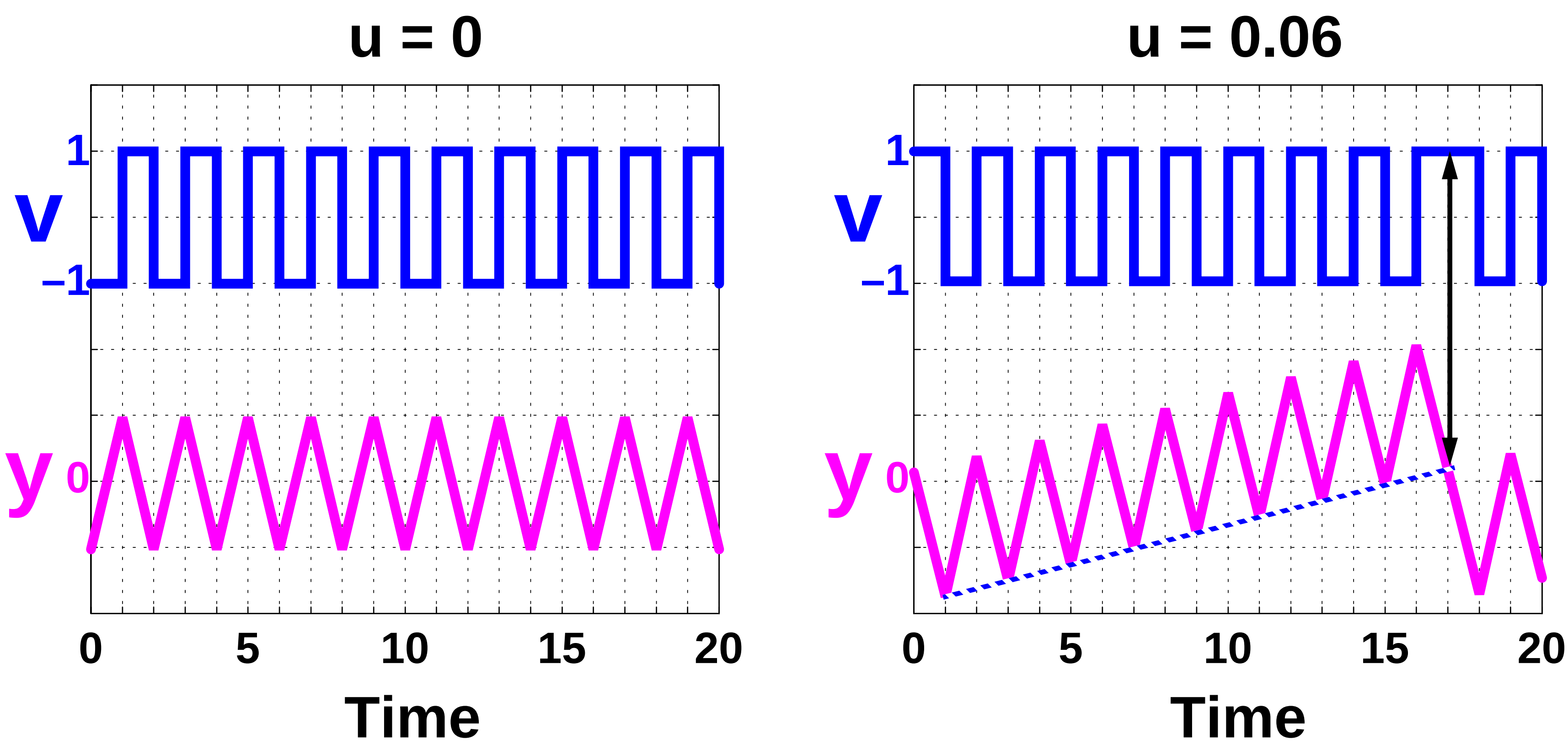
CT Implementation of MOD1

- R_i/R_f sets the full-scale; C is arbitrary

  Also observe that an input at f_s is rejected by the integrator— *inherent anti-aliasing*



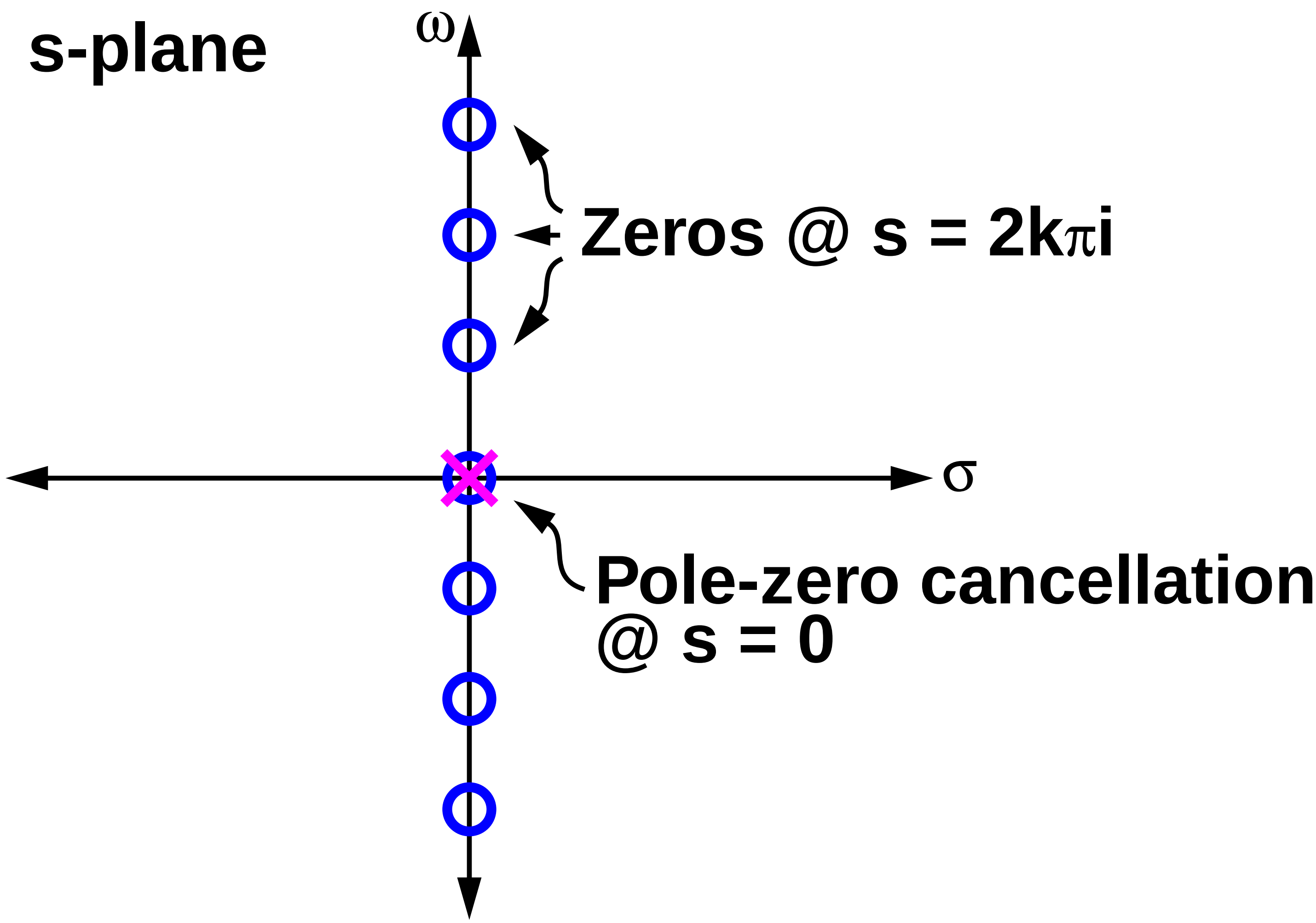
MOD1-CT Waveforms



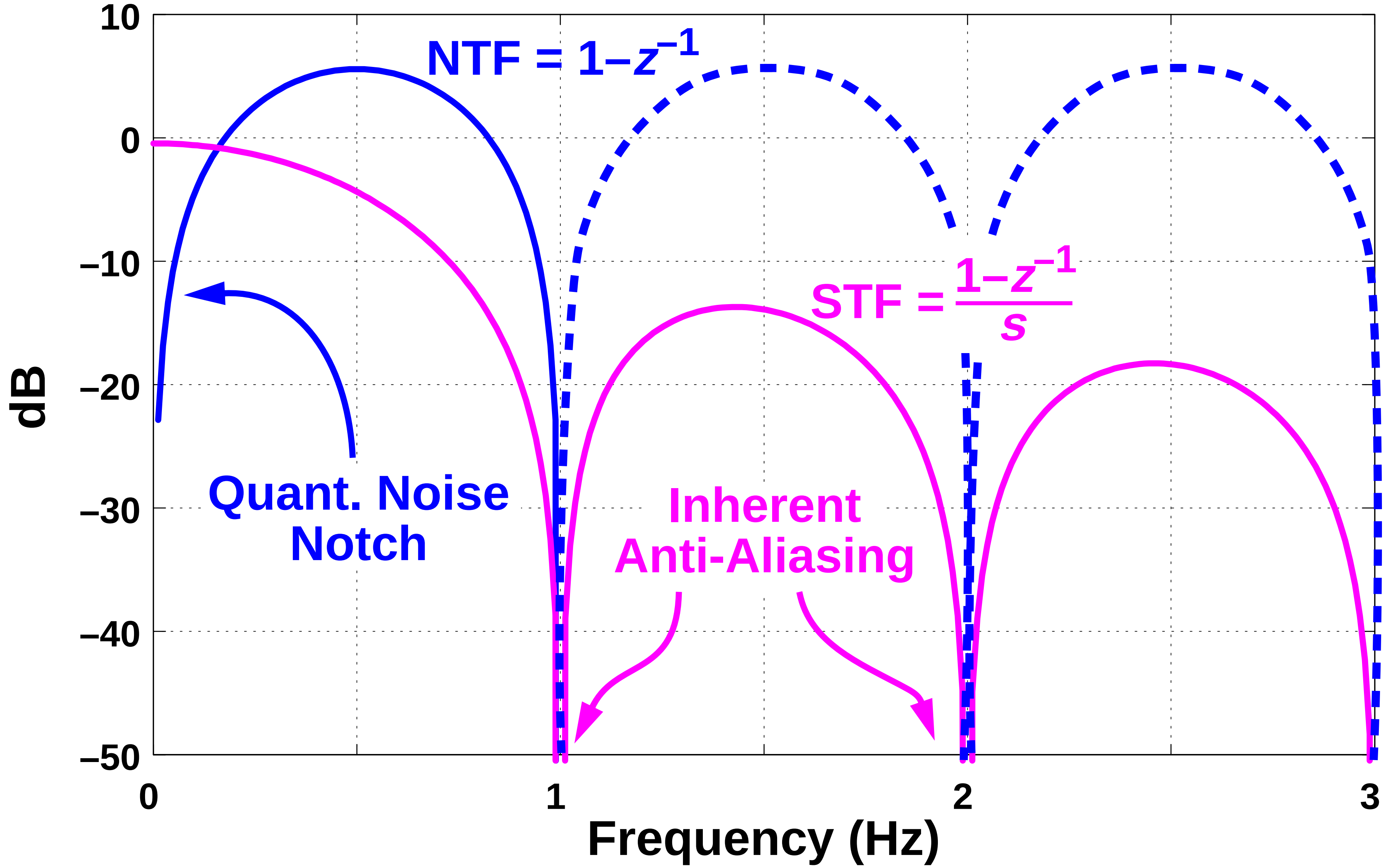
- **With *u*=0, *v* alternates between +1 and –1**
- **With *u*>0, *y* drifts upwards; *v* contains consecutive +1s to counteract this drift**

MOD1-CT STF = $\frac{1 - z^{-1}}{s}$

Recall $z = e^{s}$



MOD1-CT Frequency Responses



Summary

- **$\Delta\Sigma$ works by spectrally separating the quantization noise from the signal**

  Requires oversampling. $OSR \equiv f_s / (2 f_B)$.

- **Noise-shaping is achieved by the use of *filtering* and *feedback***

- **A binary DAC is *inherently linear,* and thus a binary $\Delta\Sigma$ modulator is too**

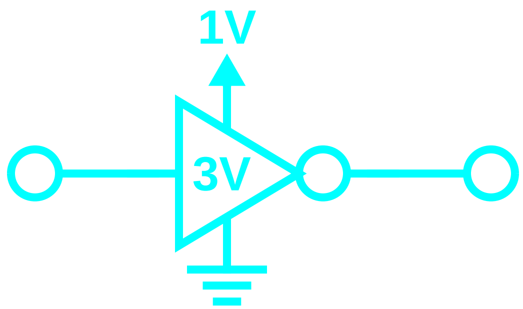
- **MOD1 has $NTF(z) = 1 - z^{-1}$**

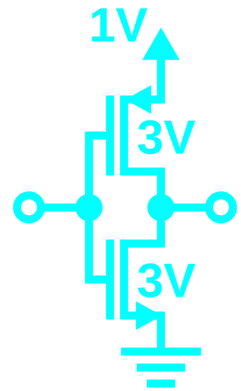
  $\Rightarrow$ **Arbitrary accuracy for DC inputs.**
  1.5 bit/octave SQNR-OSR trade-off.

- **MOD1-CT has *inherent anti-aliasing***

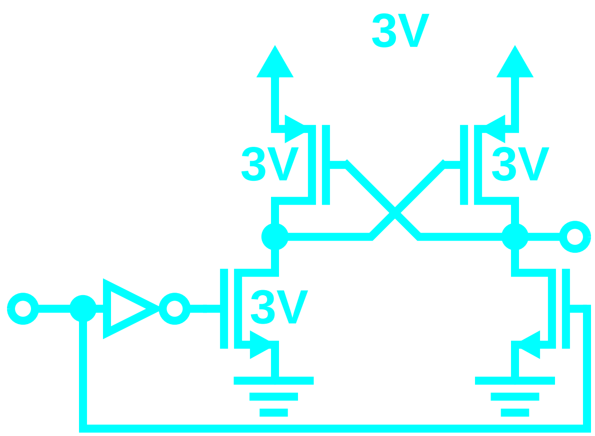
NLCOTD

3V → 1V:



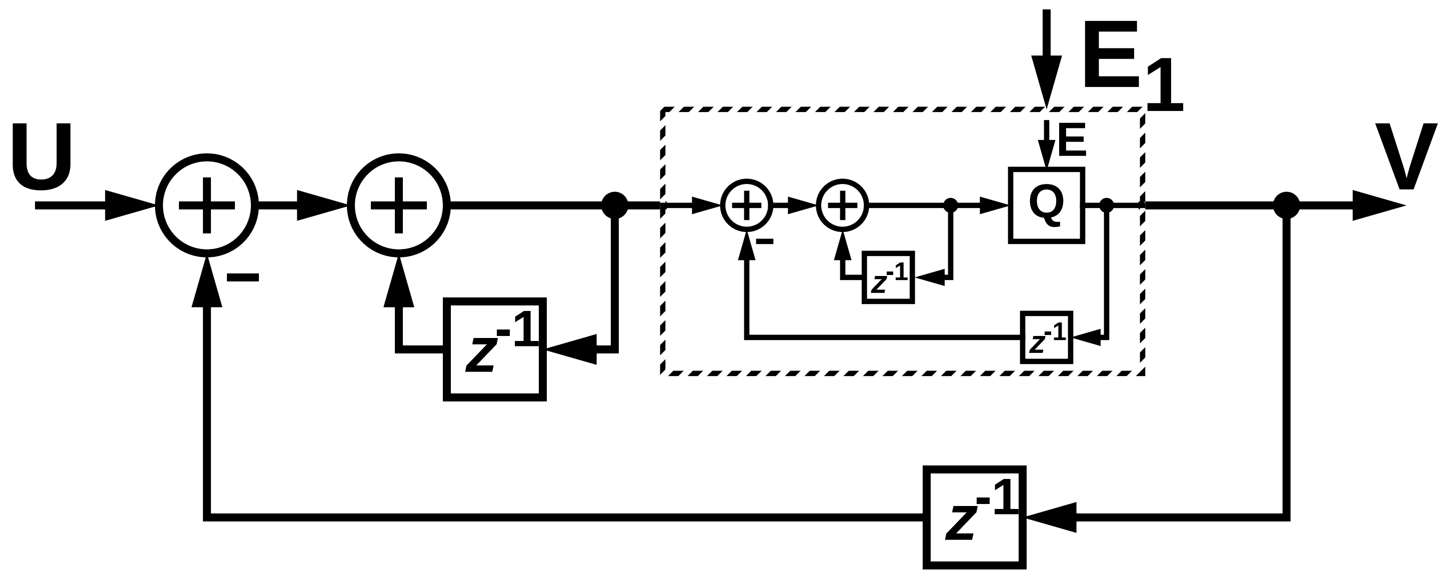


1V → 3V:



3. MOD2: 2^{nd}-Order ΔΣ Modulator
[Ch. 3 of Schreier & Temes]

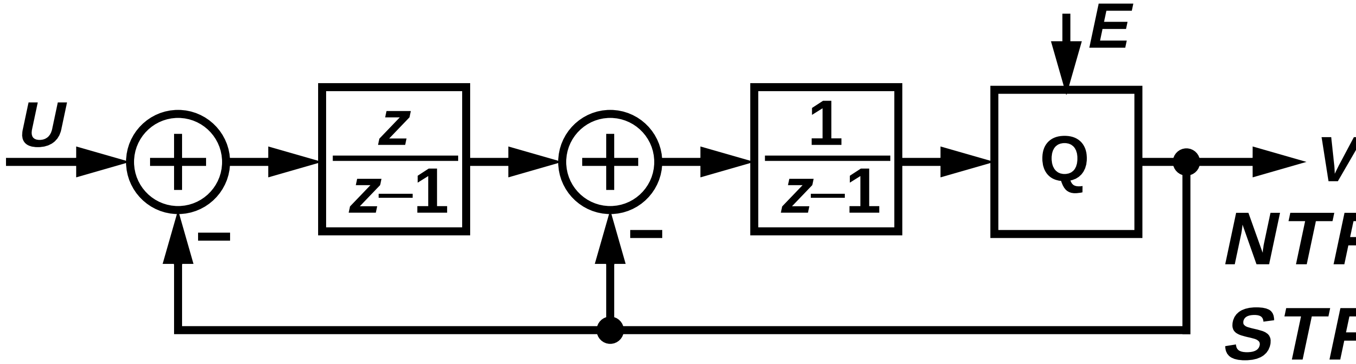
- **Replace the quantizer in MOD1 with another copy of MOD1 in a recursive fashion:**



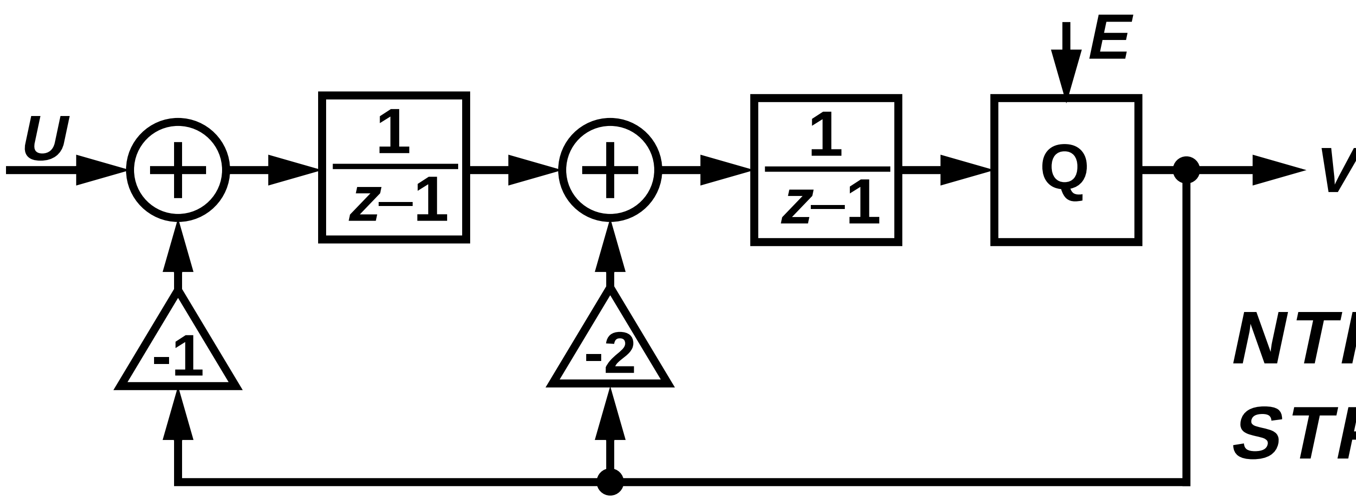
$$V(z) = U(z) + (1 - z^{-1})E_1(z),\ E_1(z) = (1 - z^{-1})E(z)$$

$$\Rightarrow V(z) = U(z) + (1 - z^{-1})^2 E(z)$$

Simplified Block Diagrams

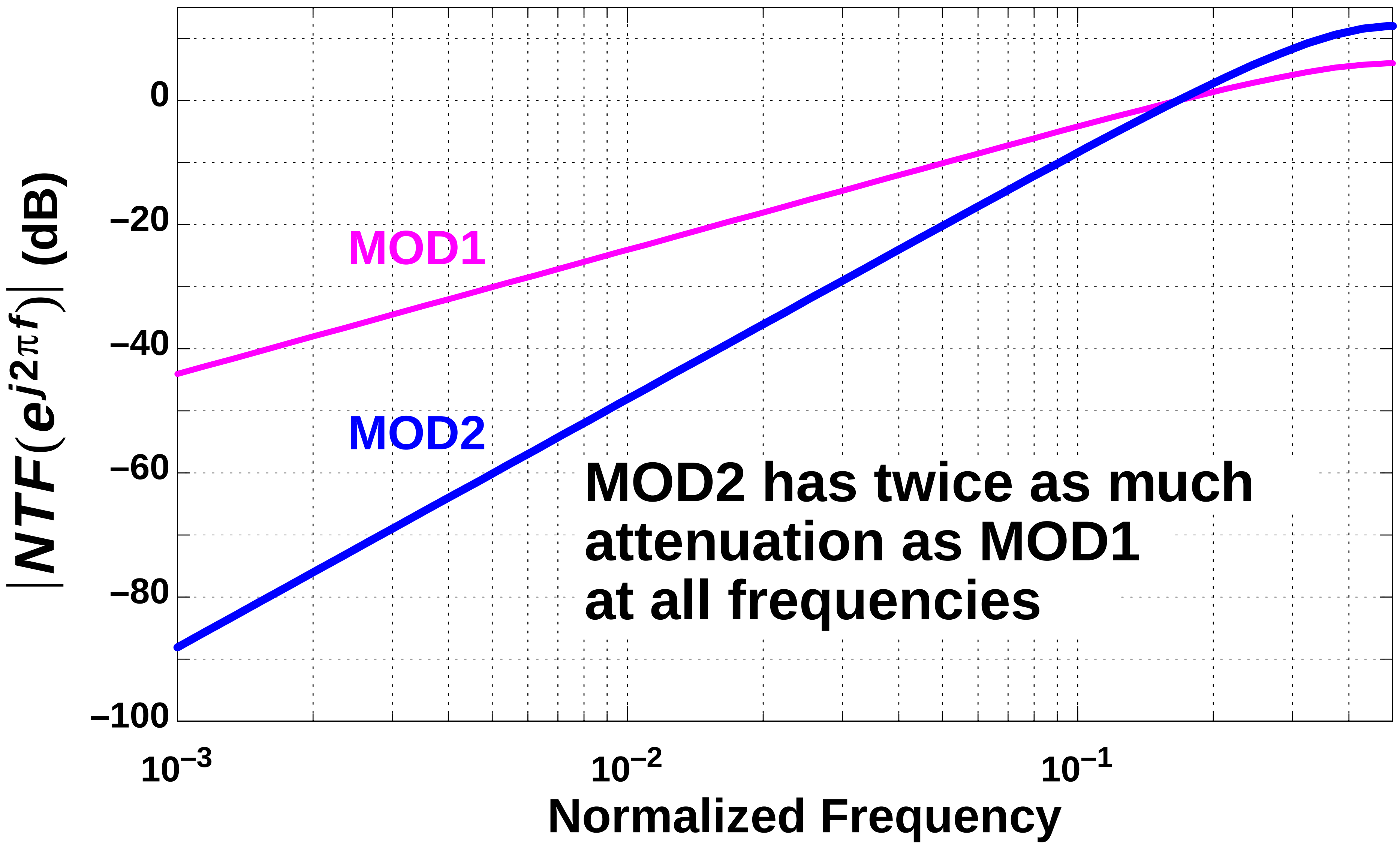


$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-1}$$



$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-2}$$

NTF Comparison



In-band Quant. Noise Power

- **For MOD2,** $\left|H(e^{j\omega})\right|^2 \approx \omega^4$

- **As before,** $IQNP = \int_0^{\omega_B} \left|H(e^{j\omega})\right|^2 S_{ee}(\omega)\, d\omega$ **and** $S_{ee}(\omega) = \sigma_e^2 / \pi$

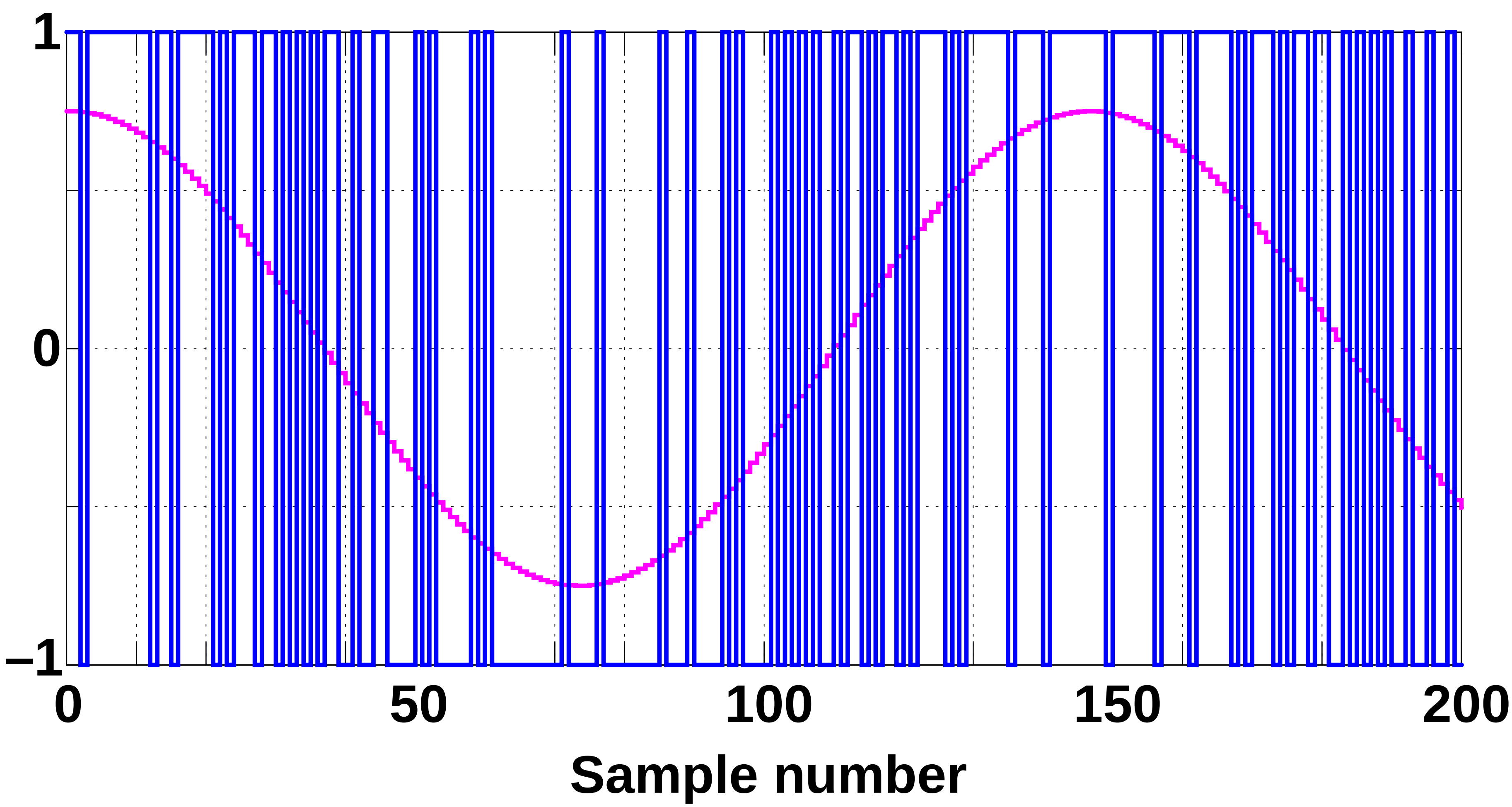
- **So now** $IQNP = \dfrac{\pi^4 \sigma_e^2}{5} (OSR)^{-5}$ $\ast$

  With binary quantization to $\pm$**1,**
  $\Delta = 2$ **and thus** $\sigma_e^2 = \Delta^2 / 12 = 1/3$**.**

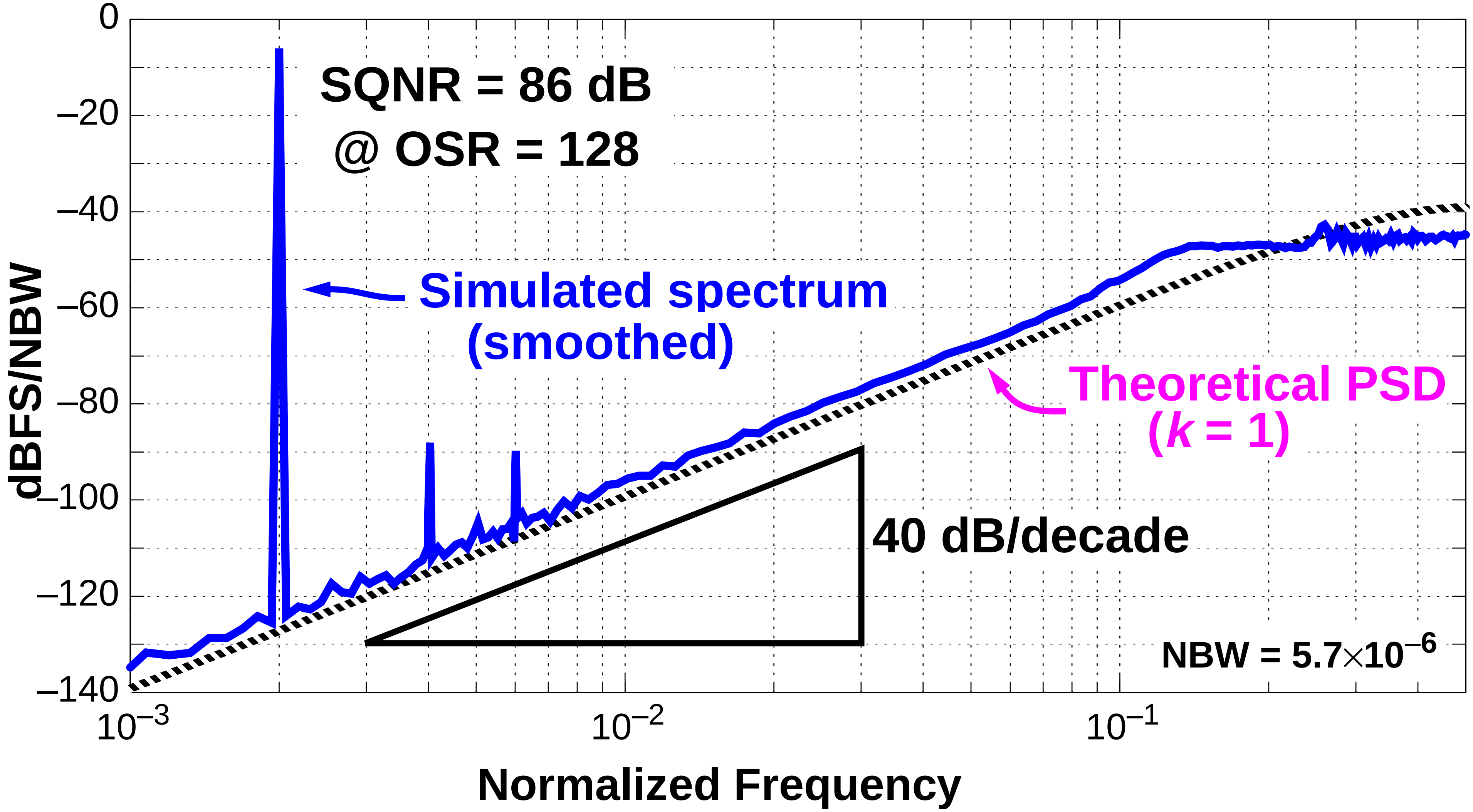
- **"An octave increase in *OSR* increases MOD2's SQNR by 15 dB (2.5 bits)"**

Simulation Example

Input at 75% of FullScale

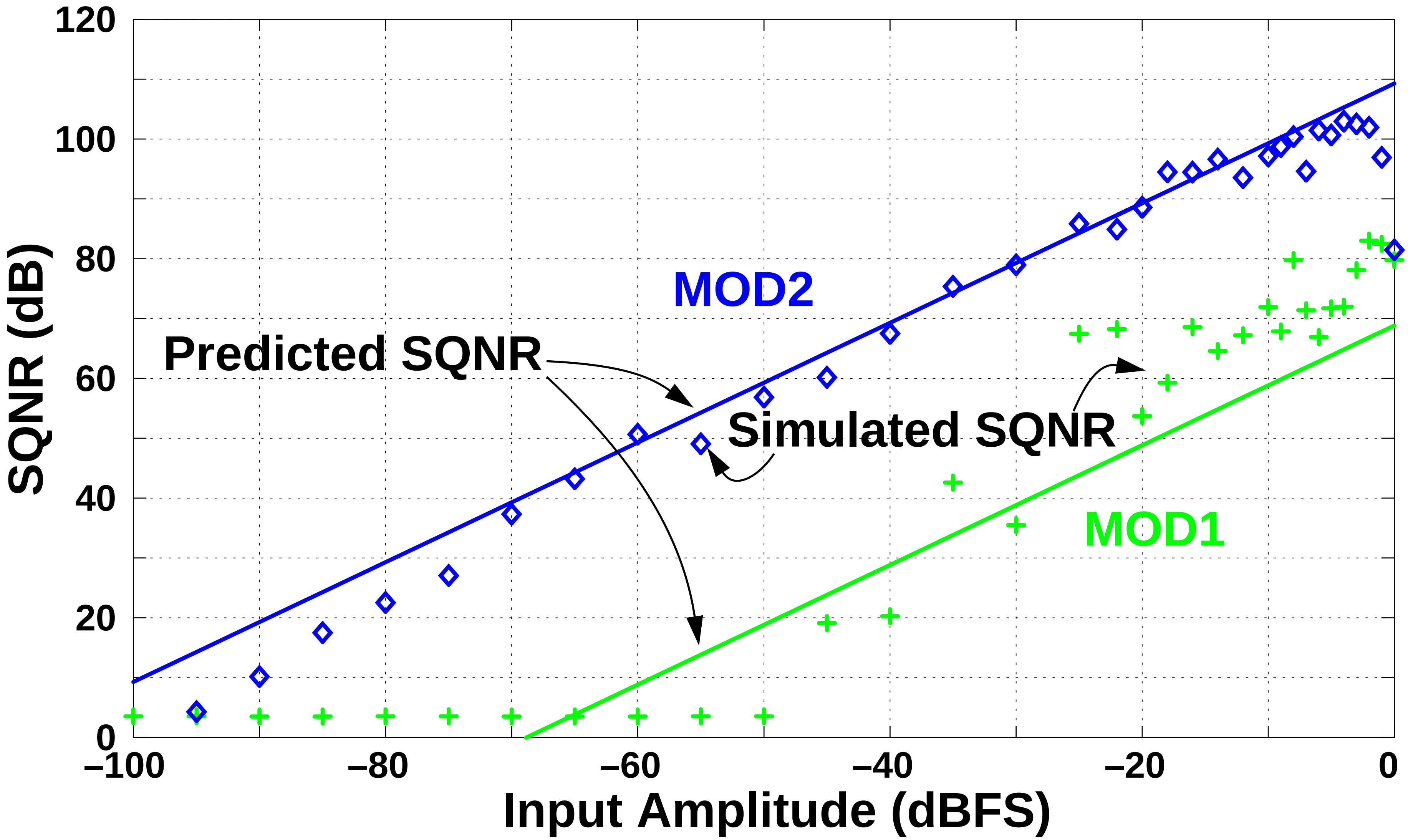


Simulated MOD2 PSD
Input at 50% of FullScale

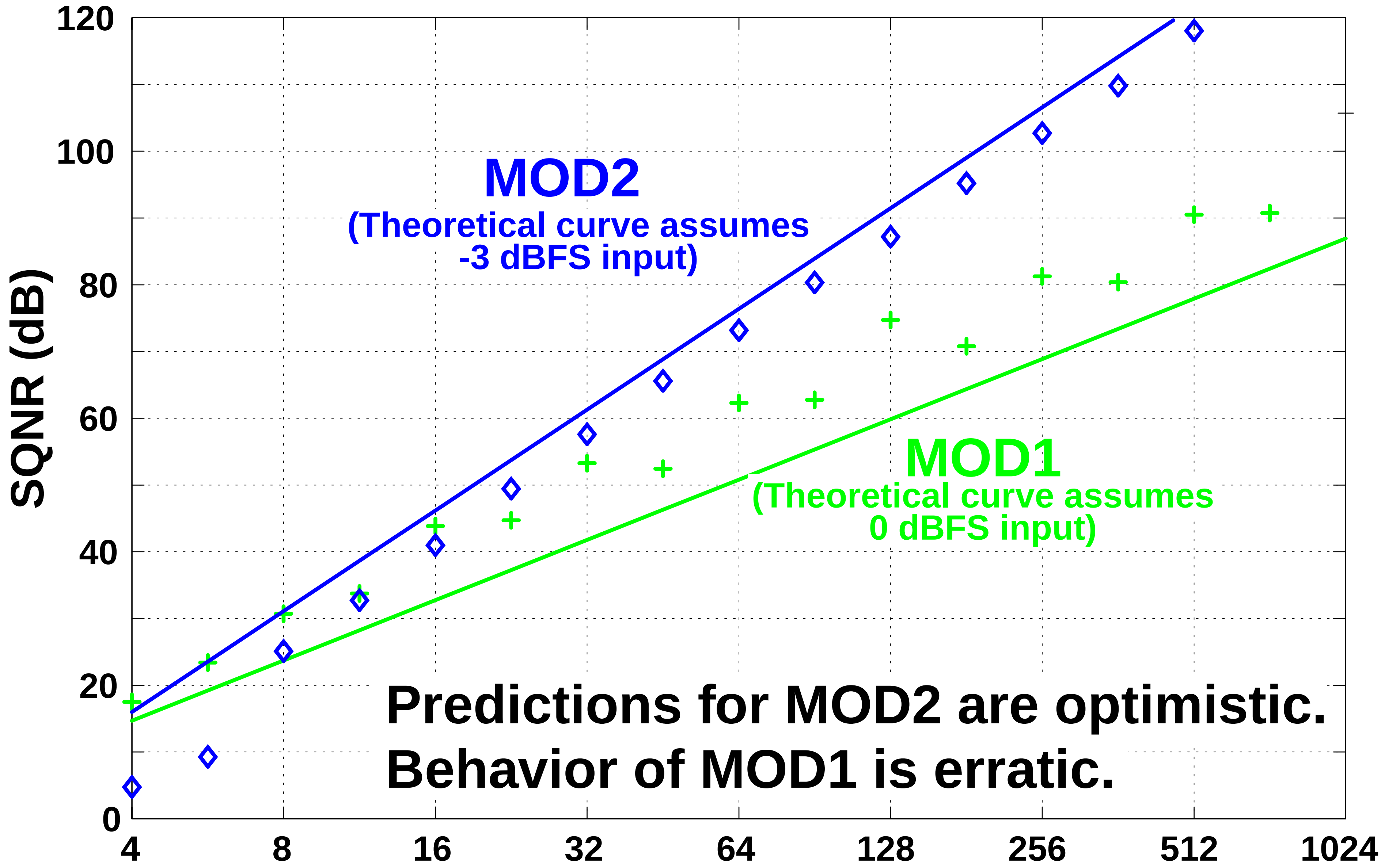


SQNR vs. Input Amplitude

MOD1 & MOD2 @ OSR = 256

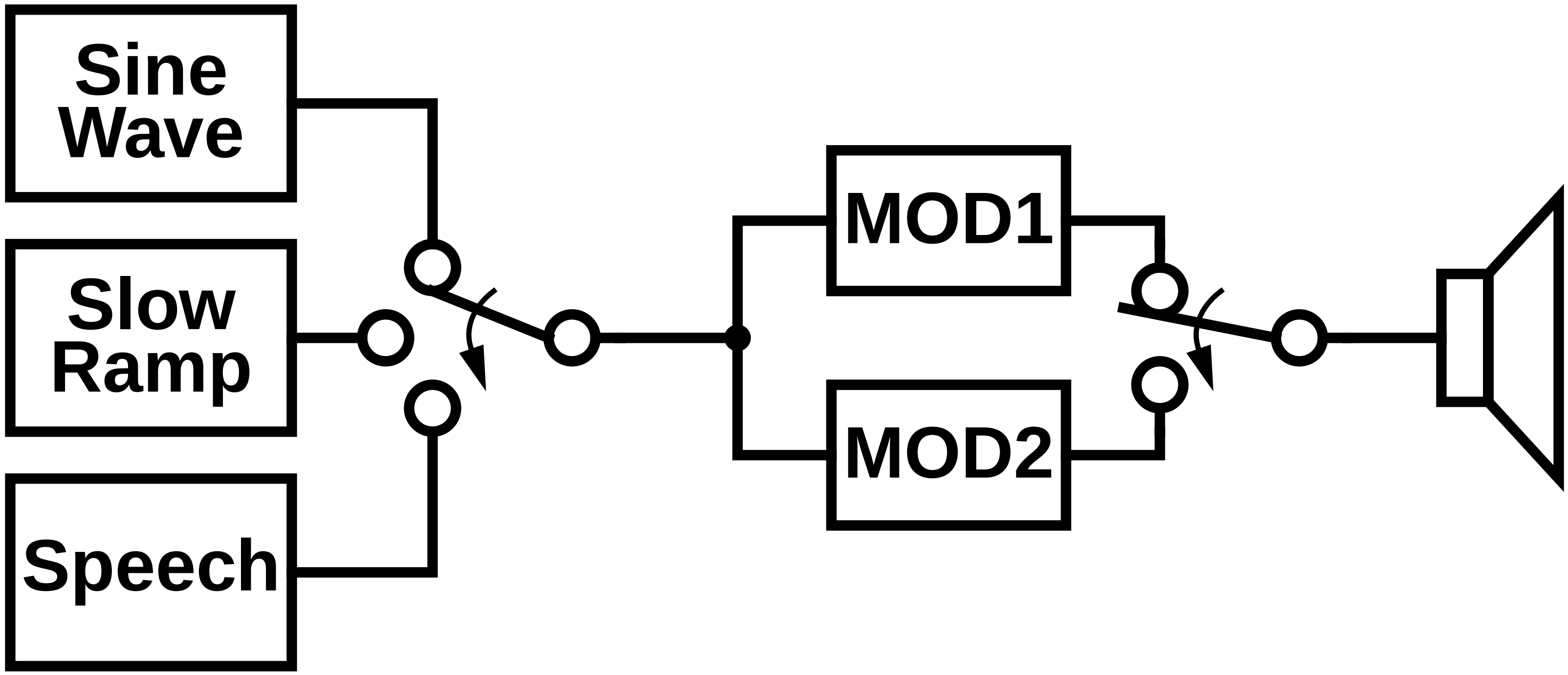


SQNR vs. OSR



120
100
80
60
40
20
0
SQNR (dB)
4
8
16
32
64
128
256
512
1024
MOD2
(Theoretical curve assumes
-3 dBFS input)
MOD1
(Theoretical curve assumes
0 dBFS input)
Predictions for MOD2 are optimistic.
Behavior of MOD1 is erratic.

Audio Demo: MOD1 vs. MOD2
[dsdemo4]

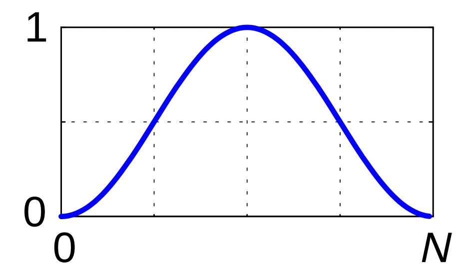


MOD1 + MOD2 Summary

- **$\Delta\Sigma$ ADCs rely on filtering and feedback to achieve high SNR despite coarse quantization**
  - They also rely on digital signal processing.
  - $\Delta\Sigma$ ADCs need to be followed by a digital decimation filter and $\Delta\Sigma$ DACs need to be preceded by a digital interpolation filter.
- **Oversampling eases analog filtering requirements**
  - Anti-alias filter in an ADC; image filter in a DAC.
- **Binary quantization yields inherent linearity**
- **MOD2 is better than MOD1**
  - 15 dB/octave vs. 9 dB/octave SQNR-OSR trade-off.
  - Quantization noise more white.
  - Higher-order modulators are even better.

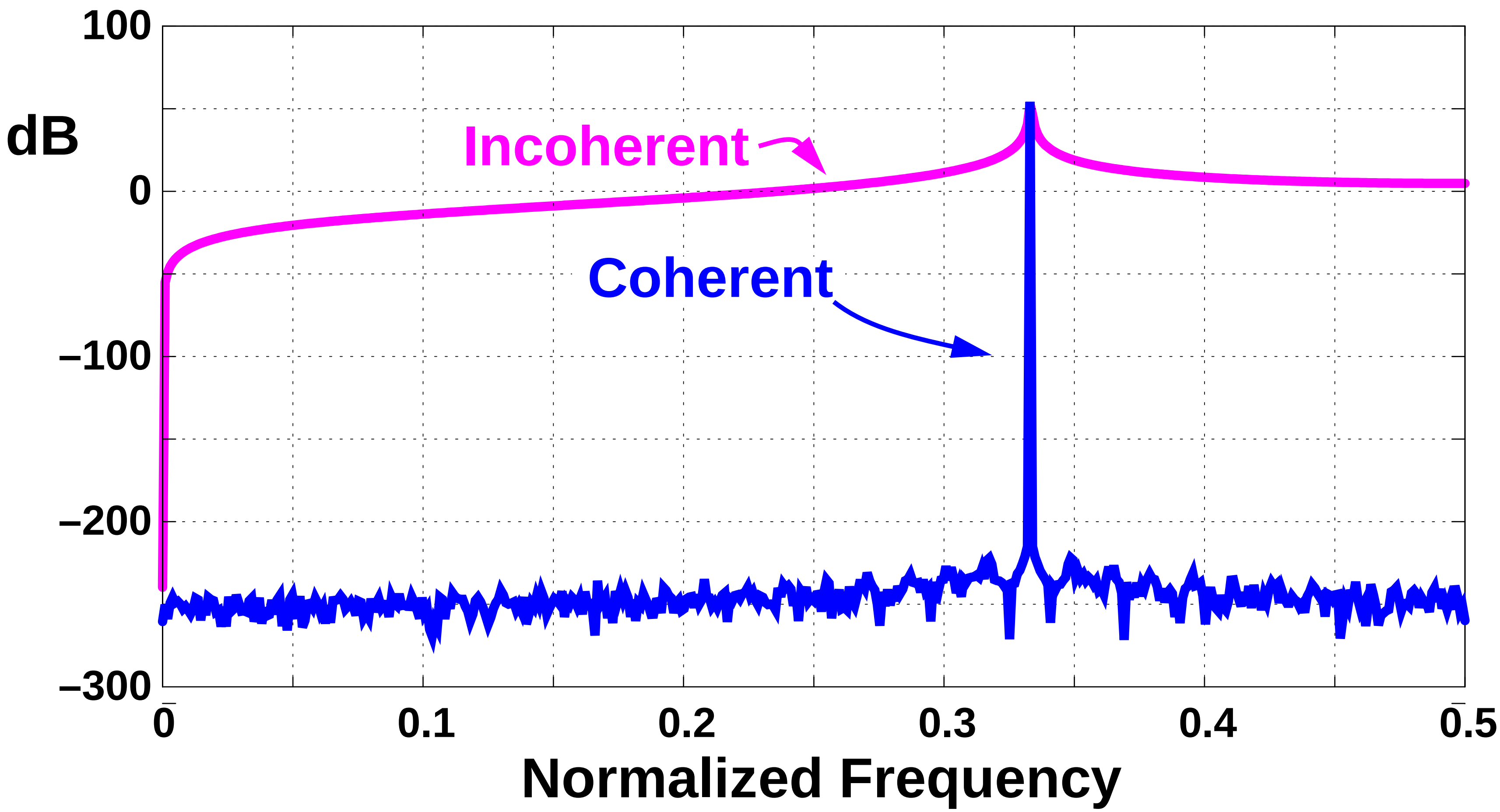
4. Good FFT Practice
[Appendix A of Schreier & Temes]

- **Use coherent sampling**
  I.e. have an integer number of cycles in the record.
- **Use windowing**
  A Hann window $w(n) = (1 - \cos(2\pi n / N)) / 2$ works well.



- **Use enough points**
  Recommend $N = 64 \cdot OSR$.
- **Scale (and smooth) the spectrum**
  A full-scale sine wave should yield a 0-dBFS peak.
- **State the noise bandwidth**
  For a Hann window, $NBW = 1.5 / N$.

Coherent vs. Incoherent Sampling

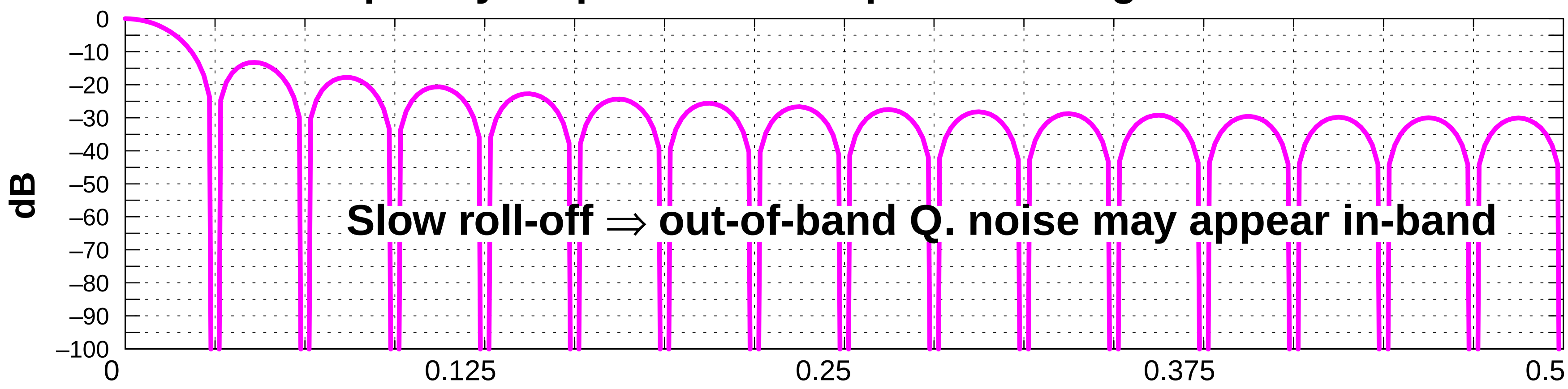


- **Coherent sampling: only one non-zero FFT bin**
- **Incoherent sampling: “spectral leakage”**

Windowing

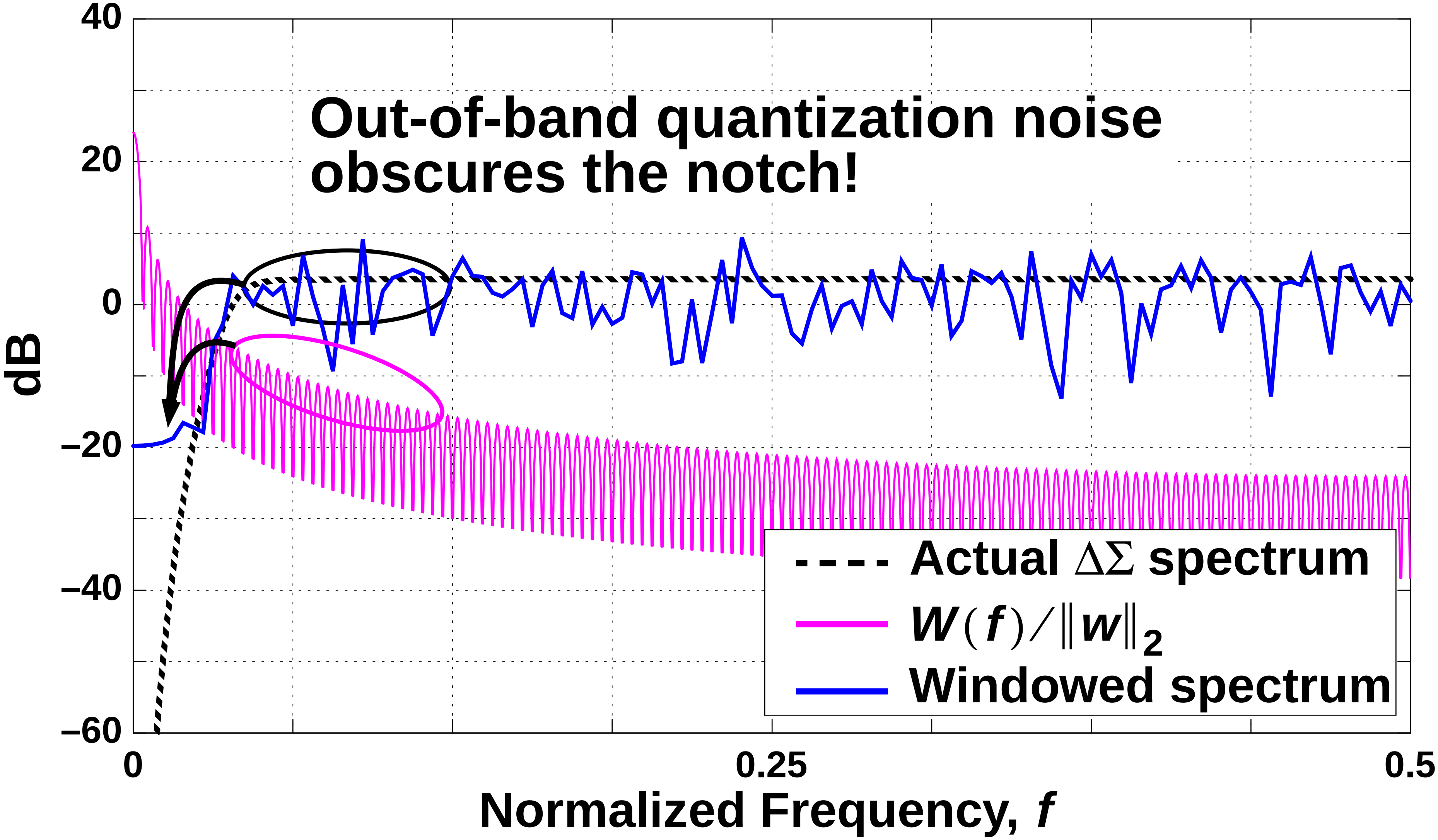
- **$\Delta\Sigma$ data is usually not periodic**
  Just because the input repeats does not mean that the output does too!
- **A finite-length data record = an infinite record multiplied by a *rectangular window*:**
  $$w(n) = 1,\ 0 \le n < N$$
  Windowing is unavoidable.
- **"Multiplication in time is convolution in frequency"**

Frequency response of a 32-point rectangular window:

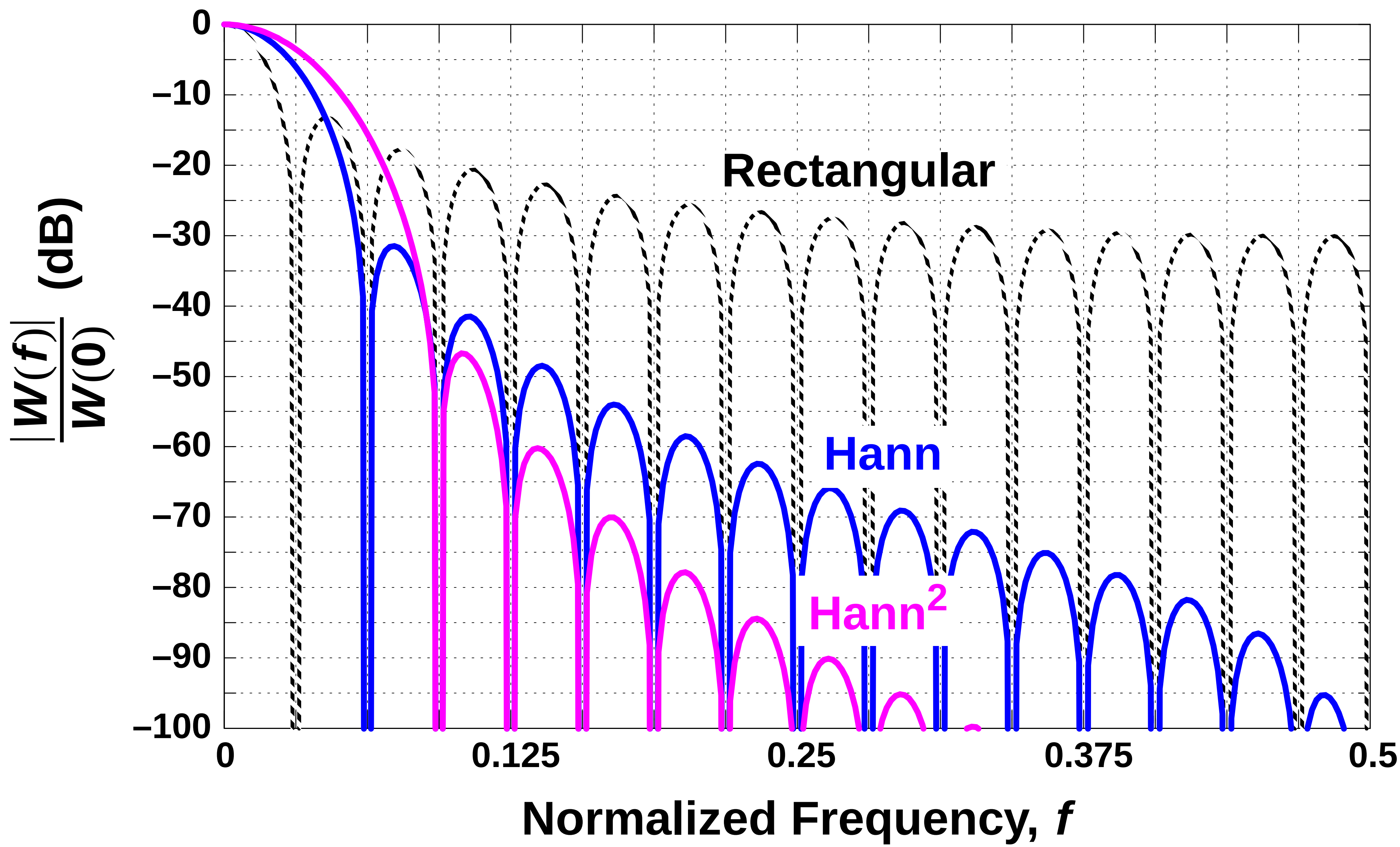


Example Spectral Disaster

Rectangular window, $N = 256$



Window Comparison ($N = 16$)



Window Properties

Window	Rectangular	Hann†	Hann^2
$w(n)$, $n = 0, 1, \ldots, N-1$ ($w(n) = 0$ otherwise)	1	$\dfrac{1 - \cos\frac{2\pi n}{N}}{2}$	$\left(\dfrac{1 - \cos\frac{2\pi n}{N}}{2}\right)^2$
Number of non-zero FFT bins	1	3	5
$\|w\|_2^2 = \sum w(n)^2$	N	$3N/8$	$35N/128$
$W(0) = \sum w(n)$	N	$N/2$	$3N/8$
$NBW = \dfrac{\|w\|_2^2}{W(0)^2}$	$1/N$	$1.5/N$	$35/18N$

†. MATLAB's "hann" function causes spectral leakage of tones located in FFT bins unless you add the optional argument "periodic."

Window Length, N

- **Need to have enough in-band noise bins to**

1 Make the number of signal bins a small fraction of the total number of in-band bins

<20% signal bins $\Rightarrow$ >15 in-band bins $\Rightarrow$ $N > 30 \cdot OSR$

2 Make the SNR repeatable

$N = 30 \cdot OSR$ yields std. dev. ~1.4 dB.

$N = 64 \cdot OSR$ yields std. dev. ~1.0 dB.

$N = 256 \cdot OSR$ yields std. dev. ~0.5 dB.

- **$N = 64 \cdot OSR$ is recommended**

FFT Scaling

- The FFT implemented in MATLAB is

$$X_M(k+1) = \sum_{n=0}^{N-1} x_M(n+1)\, e^{-j\frac{2\pi kn}{N}}$$

- If $x(n) = A\sin(2\pi fn/N)$[†], then

$$|X(k)| = \begin{cases} \dfrac{AN}{2} & ,\ k = f \text{ or } N-f \\ 0 & ,\ \text{otherwise} \end{cases}$$

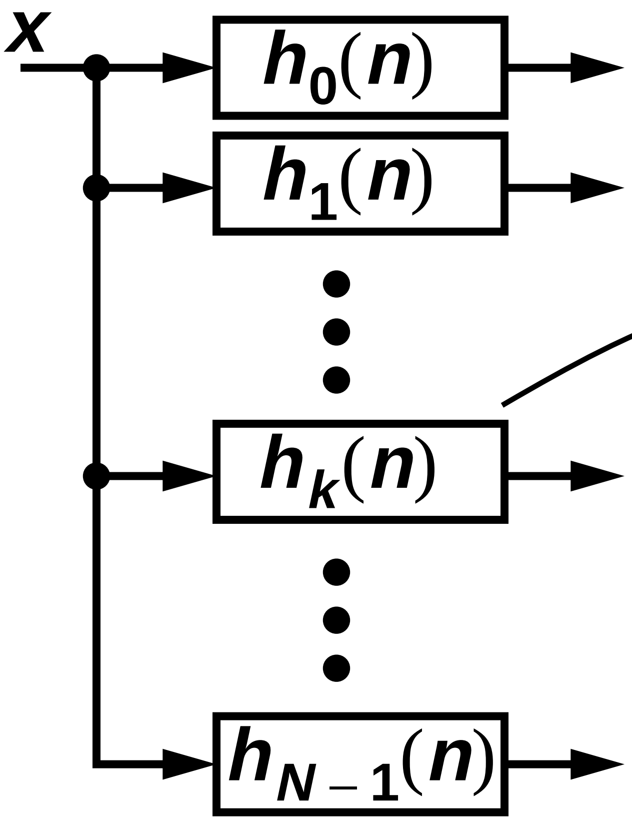
$\Rightarrow$ Need to divide FFT by $(N/2)$ to get A.

---

†. f is an integer in $(0, N/2)$. I've defined $X(k) \equiv X_M(k+1)$, $x(n) \equiv x_M(n+1)$ since Matlab indexes from 1 rather than 0.

The Need For Smoothing

- **The FFT can be interpreted as taking 1 sample from the outputs of N complex FIR filters:**



$$y_0(N) = X(0)$$

$$y_1(N) = X(1)$$

$$h_k(n) = \begin{cases} e^{j\frac{2\pi k}{N}n}, & 0 \le n < N \\ 0, & \text{otherwise} \end{cases}$$

$$y_k(N) = X(k)$$

$$y_{N-1}(N) = X(N-1)$$

$\Rightarrow$ **an FFT yields a high-variance spectral estimate**

How To Do Smoothing

1. **Average multiple FFTs**

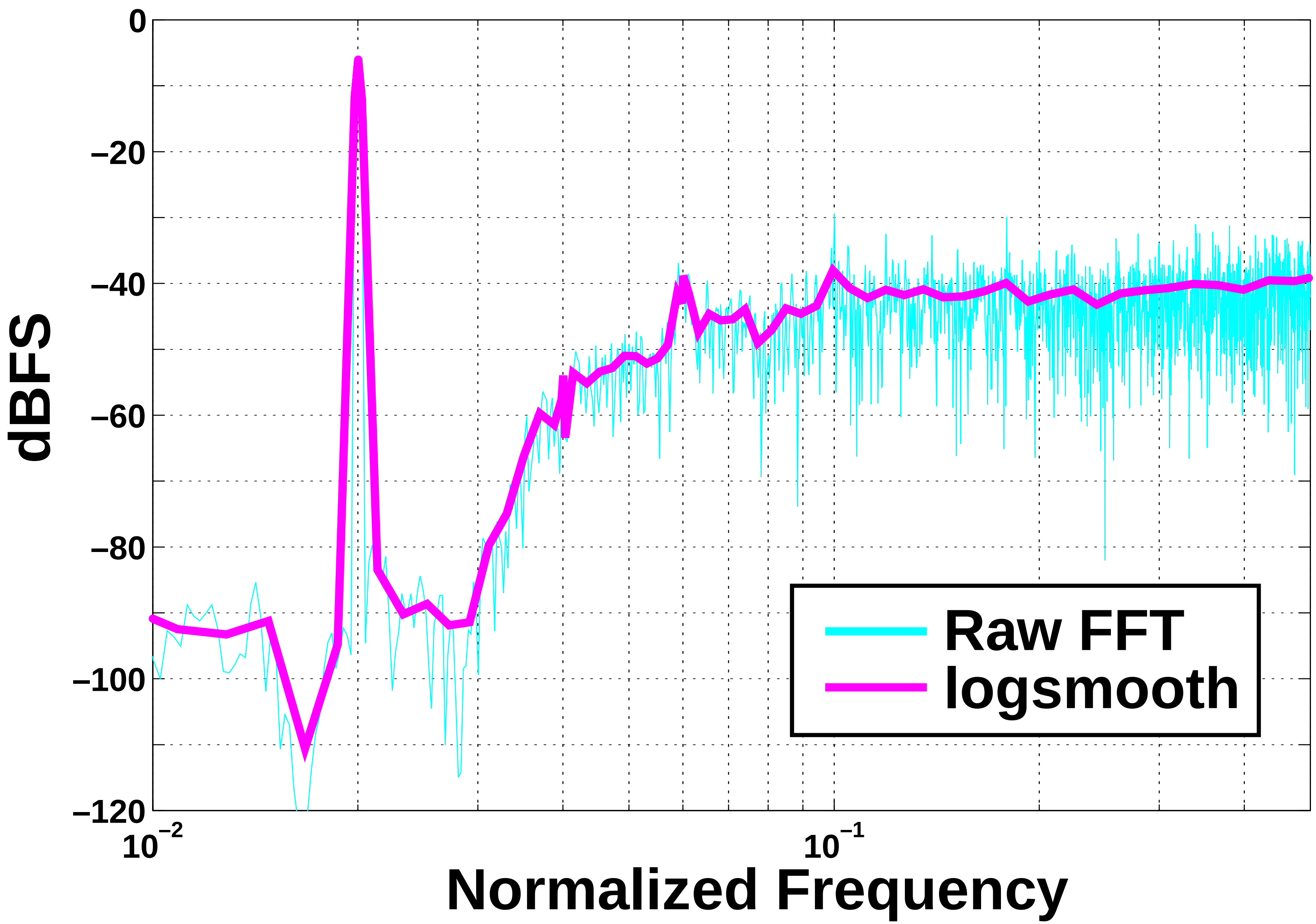
    Implemented by MATLAB's `psd()` function

2. **Take one big FFT and "filter" the spectrum**

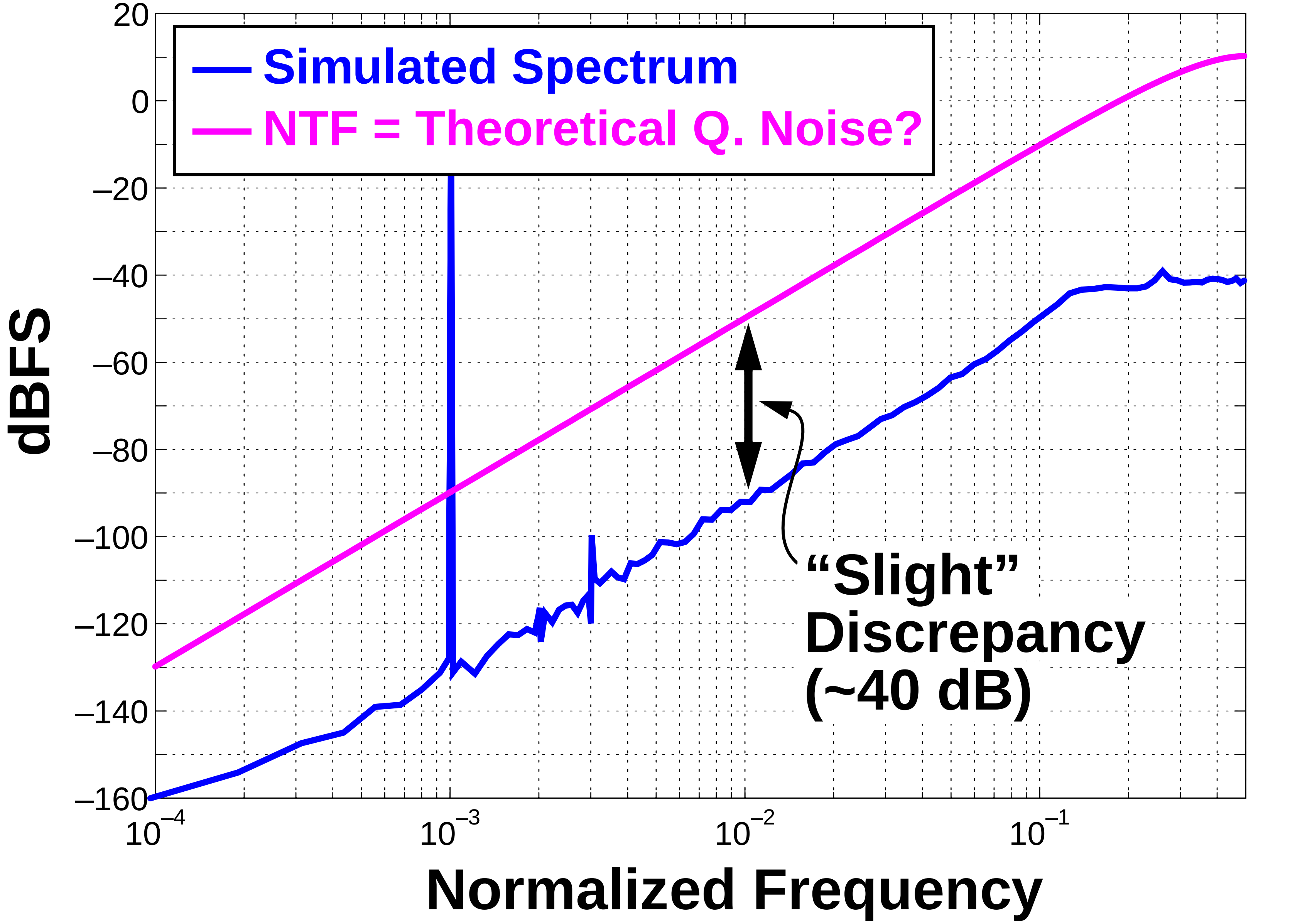
    Implemented by the ΔΣ Toolbox's `logsmooth()` function

    `logsmooth()` averages an exponentially-increasing number of bins in order to reduce the density of points in the high-frequency regime and make a nice log-frequency plot

Raw and Smoothed Spectra



Simulation vs. Theory (MOD2)



What Went Wrong?

1. We normalized the spectrum so that a full-scale sine wave (which has a power of 0.5) comes out at 0 dB (whence the “dBFS” units)

   $\Rightarrow$ We need to do the same for the error signal.

   i.e. use $S_{ee}(f) = 4/3$.

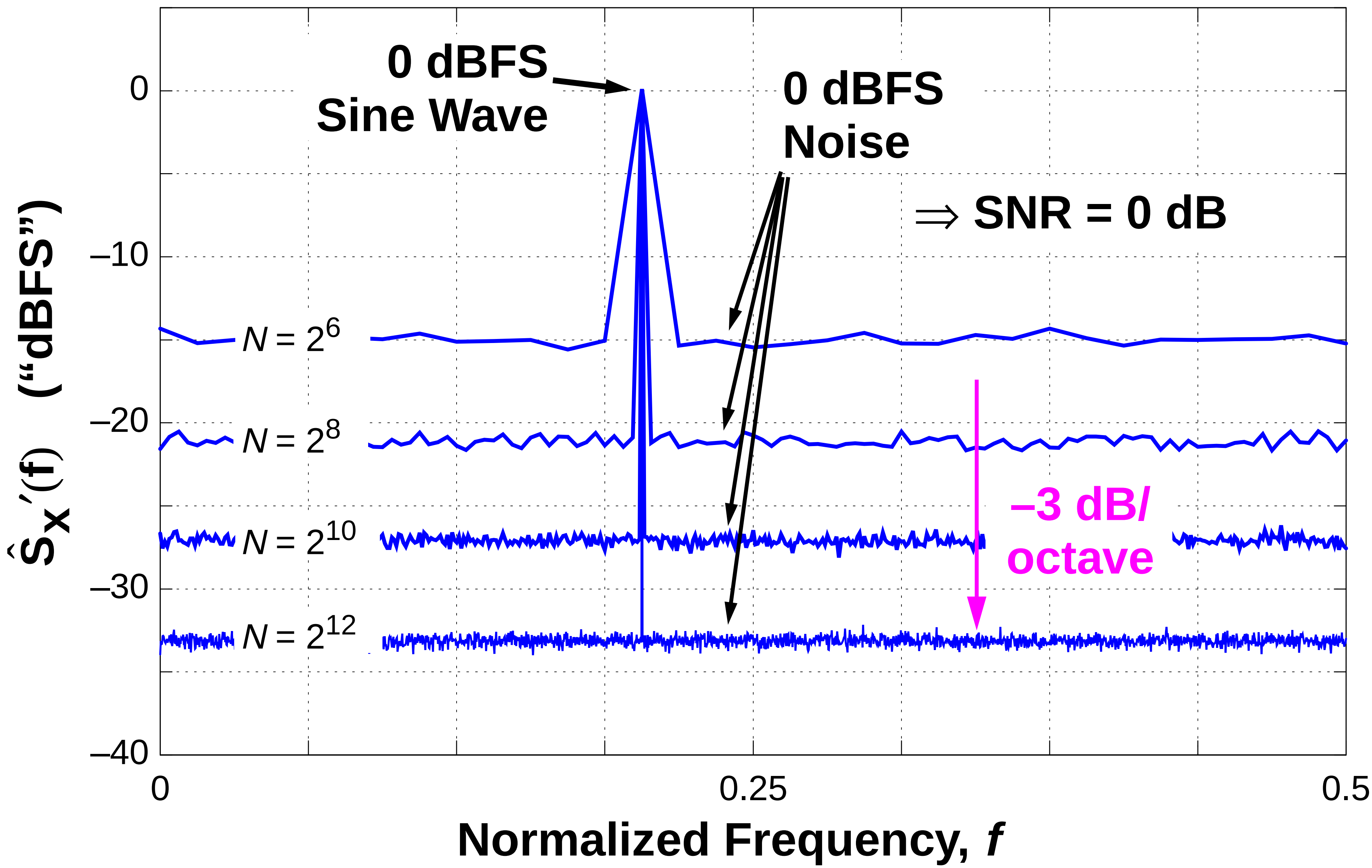
   But this makes the discrepancy 3 dB worse.

2. We tried to plot a *power spectral density* together with something that we want to interpret as a *power spectrum*

- Sine-wave components are located in individual FFT bins, but broadband signals like noise have their power spread over all FFT bins!

  The “noise floor” depends on the length of the FFT.

Spectrum of a Sine Wave + Noise



Observations

- **The power of the sine wave is given by the height of its spectral peak**
- **The power of the noise is spread over all bins**

  The greater the number of bins, the less power there is in any one bin.

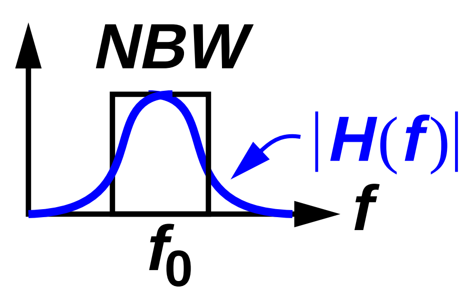
- **Doubling *N* reduces the power per bin by a factor of 2 (i.e. 3 dB)**

  But the total integrated noise power does *not* change.

So How Do We Handle Noise?

- **Recall that an FFT is like a filter bank**
- **The longer the FFT, the narrower the bandwidth of each filter and thus the lower the power at each output**
- **We need to know the *noise bandwidth* (NBW) of the filters in order to convert the power in each bin (filter output) to a power density**
- **For a filter with frequency response $H(f)$,**

$$NBW = \frac{\int |H(f)|^2 df}{H(f_0)^2}$$



FFT Noise Bandwidth

Rectangular Window

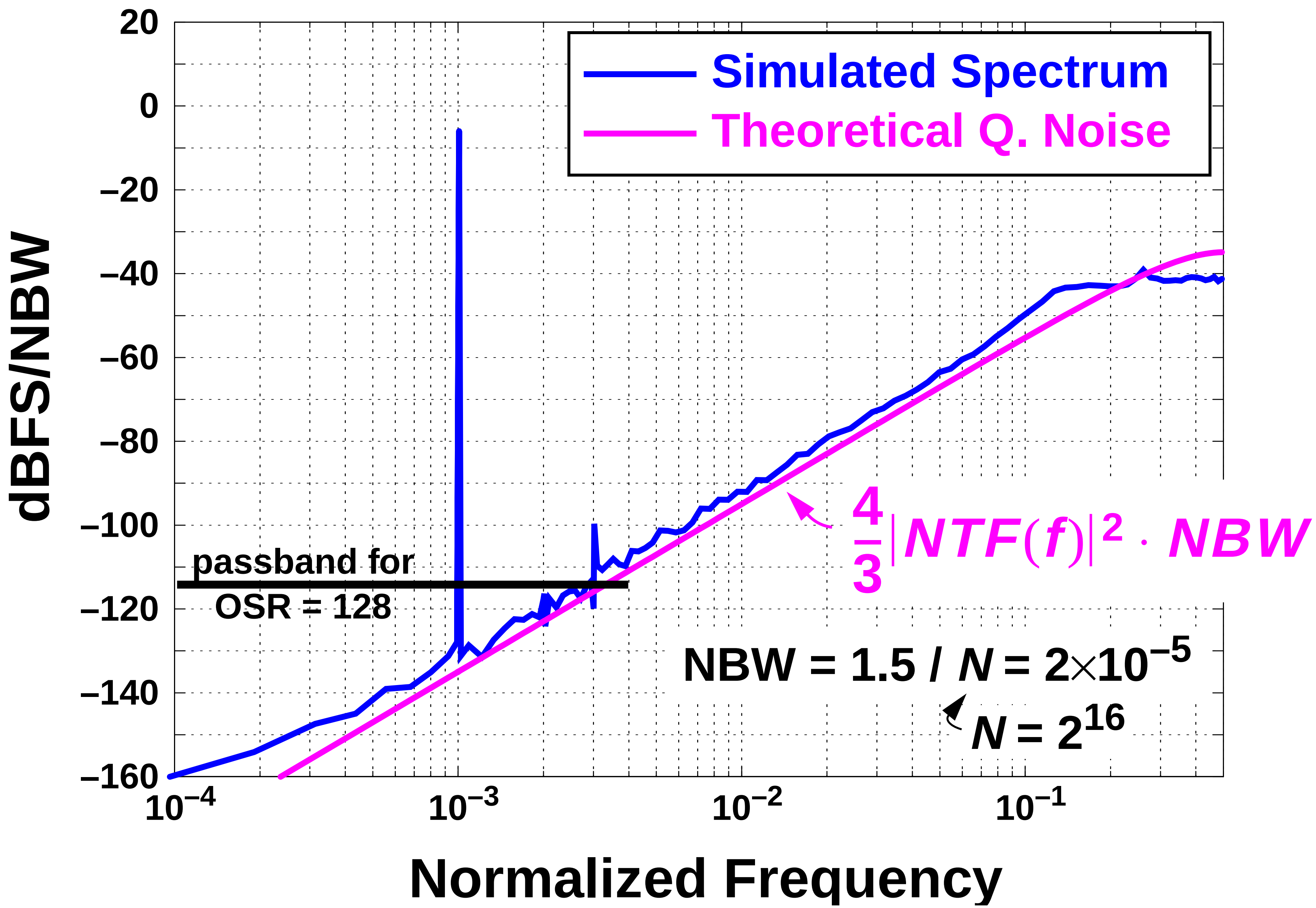
$$h_k(n) = \exp\left(j\frac{2\pi k}{N}n\right),\ H_k(f) = \sum_{n=0}^{N-1} h_k(n)\exp(-j2\pi fn)$$

$$f_0 = \frac{k}{N},\ H_k(f_0) = \sum_{n=0}^{N-1} 1 = N$$

$$\int |H_k(f)|^2 = \sum |h_k(n)|^2 = N \text{ [Parseval]}$$

$$\therefore NBW = \frac{\int |H_k(f)|^2 df}{H_k(f_0)^2} = \frac{N}{N^2} = \frac{1}{N}$$

Better Spectral Plot



Homework #1 (Due 2015-01-12)

A. **Create a Matlab function that computes MOD1's output sequence given a vector of input samples and exercise your function in the following ways.**

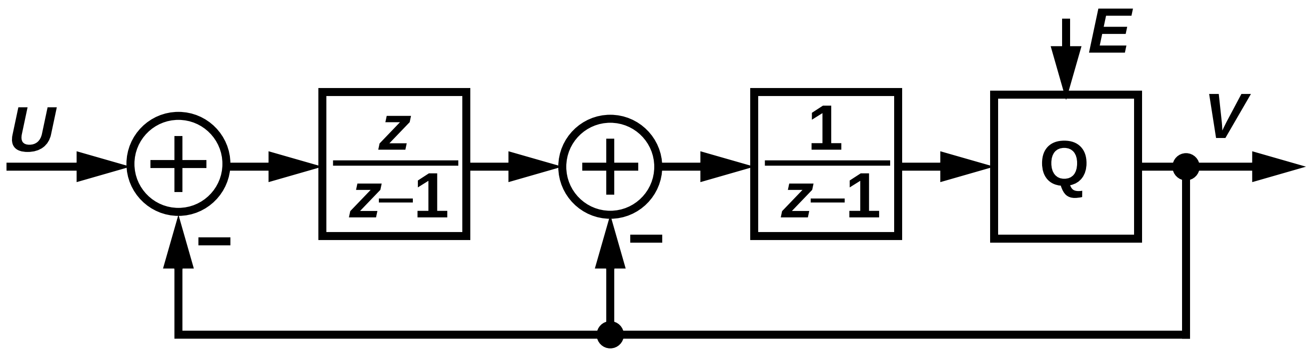
   1 **Verify that the average of the output equals the input for a few random DC inputs in [–1,1].**

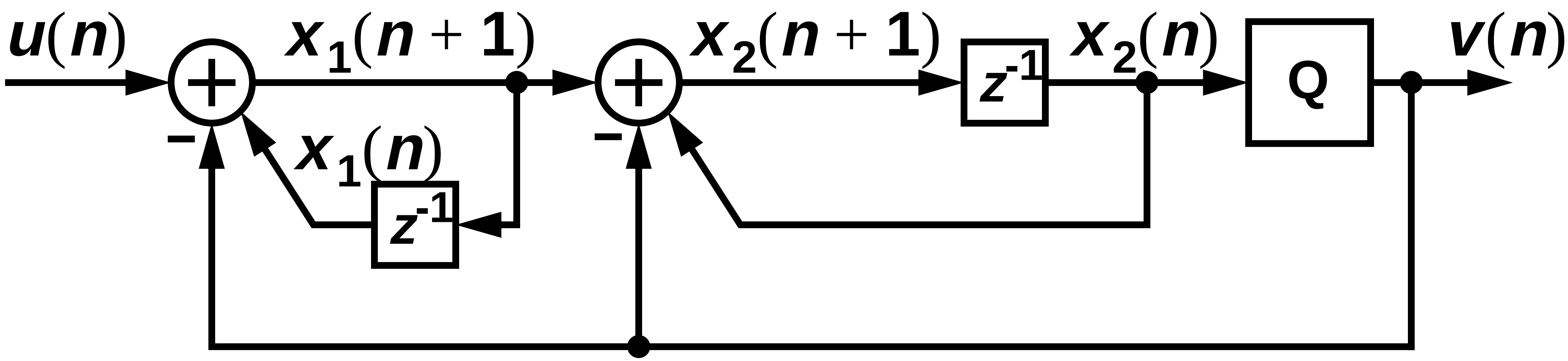
   2 **Plot the output spectrum with a half-scale sine-wave input. Use good FFT practice. Include the theoretical quantization noise curve and list the theoretical and simulated SQNR for OSR = 128.**

B. **Repeat with MOD2.**

C. **Compose your own question and answer it.**

MOD2 Expanded





Difference Equations:

$$v(n) = Q(x_2(n))$$

$$x_1(n+1) = x_1(n) - v(n) + u(n)$$

$$x_2(n+1) = x_2(n) - v(n) + x_1(n+1)$$

Example Matlab Code

```
function [v] = simulateMOD2(u)
    x1 = 0;
    x2 = 0;
    for i = 1:length(u)
        v(i) = quantize( x2 );
        x1 = x1 + u(i) - v(i);
        x2 = x2 + x1 - v(i);
    end
return

function v = quantize( y )
    if y>=0
        v = 1;
    else
        v = -1;
    end
return
```

What You Learned Today

And what the homework should solidify

1 **MOD1: 1st-order ΔΣ modulator**
   Structure and theory of operation

2 **Inherent linearity of binary modulators**

3 **Inherent anti-aliasing of continuous-time modulators**

4 **MOD2: 2nd-order ΔΣ modulator**

5 **Good FFT practice**