ECE1371 Advanced Analog Circuits
Lecture 10

ADVANCED ΔΣ

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Course Goals

- **Deepen understanding of CMOS analog circuit design through a top-down study of a modern analog system— a delta-sigma ADC**
- **Develop circuit insight through brief peeks at some nifty little circuits**
  The circuit world is filled with many little gems that every competent designer ought to know.

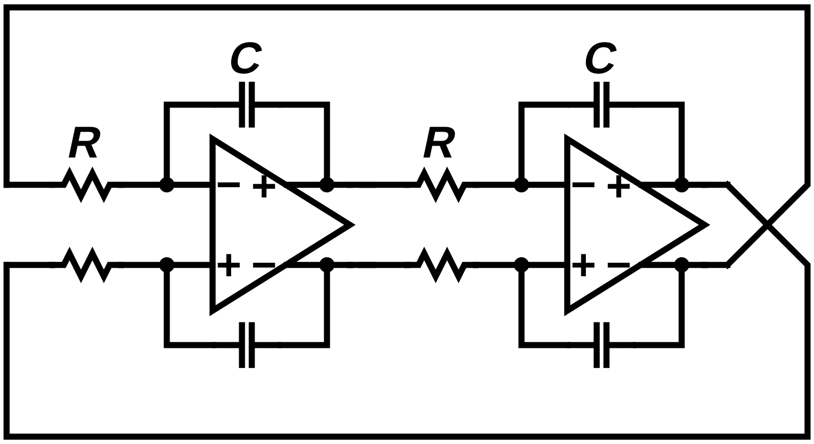
NLCOTD: High-Q Resonator

- **Want $Q \gg \sqrt{3}\frac{f_0}{BW}$ for small SQNR degradation**
- **In a TV tuner ADC f_0 = 44 MHz and BW = 8.5 MHz, so we needed $Q \gg 9$**

  In actuality the requirement was Q > 20.
- **How can Q be kept high despite finite amplifier gain and bandwidth?**

Date	Lecture (M 13:00-15:00)			Ref	Homework
2015-01-05	RS	1	MOD1 & MOD2	ST 2, 3, A	1: Matlab MOD1&2
2015-01-12	RS	2	MOD*N* + ΔΣ Toolbox	ST 4, B	2: ΔΣ Toolbox
2015-01-19	RS	3	Example Design: Part 1	ST 9.1, CCJM 14	3: Sw.-level MOD2
2015-01-26	RS	4	Example Design: Part 2	CCJM 18	
2015-02-02	TC	5	SC Circuits	R 12, CCJM 14	4: SC circuit
2015-02-09	TC	6	Amplifier Design		
2015-02-16	Reading Week– No Lecture				
2015-02-23	TC	7	Amplifier Design		5: SC Int w/ Amp
2015-03-02	RS	8	Comparator & Flash ADC	CCJM 10	Project
2015-03-09	TC	9	Noise in SC Circuits	ST C	
2015-03-16	RS	10	Advanced ΔΣ	ST 6.6, 9.4	
2015-03-23	TC	11	Matching & MM-Shaping	ST 6.3-6.5, +	
2015-03-30	TC	12	Pipeline and SAR ADCs	CCJM 15, 17	
2015-04-06	Exam			Proj. Report Due Friday April 10	
2015-04-13	Project Presentation				

Active-RC Resonator Structure



$$f_0 = \frac{1}{2\pi RC}$$

- Frequency-tuning: adjust C until the desired resonant frequency is achieved

  No Q-tuning.

– Amplifier drives both R and C ⇒ Q trouble?

Highlights
(i.e. What you will learn today)

1 Feedback vs. Feedforward topology

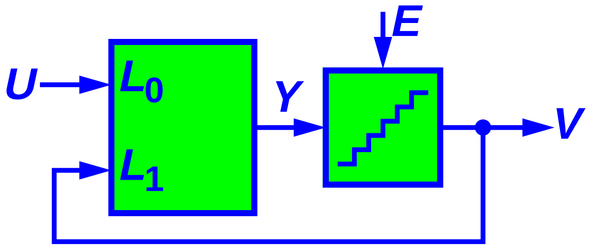
2 State-space (ABCD) representation of the loop filter in the ΔΣ Toolbox

3 MASH Modulators

4 Continuous-Time Modulators

5 Bandpass and Quadrature Bandpass ΔΣ

Review:
Generic Single-Loop ΔΣ ADC



$$\begin{aligned} Y &= L_0 U + L_1 V \\ V &= Y + E \end{aligned} \Rightarrow \boxed{V = STF \cdot U + NTF \cdot E}\text{, where}$$

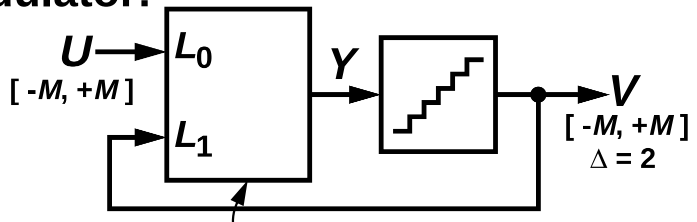
$$NTF = \frac{1}{1 - L_1} \quad \& \quad STF = L_0 \cdot NTF$$

Inverse Relations:

$L_1 = 1 - 1/NTF$, $L_0 = STF / NTF$

Review: ΔΣ Toolbox Model

Modulator:

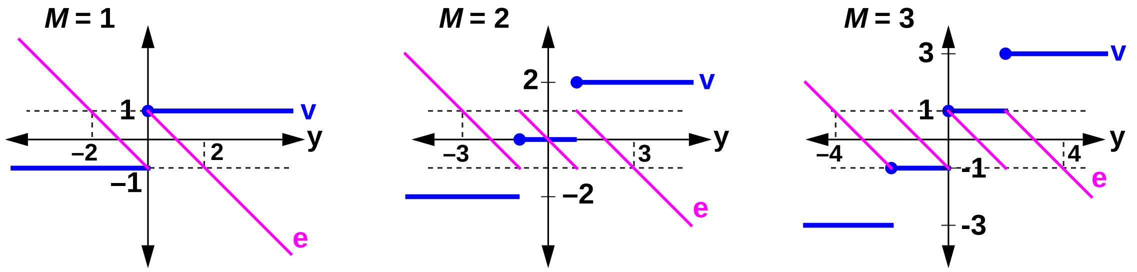


Loop filter can be specified by NTF or by ABCD, a state-space representation

$$NTF = \frac{1}{1 - L_1}$$

$$STF = \frac{L_0}{1 - L_1}$$

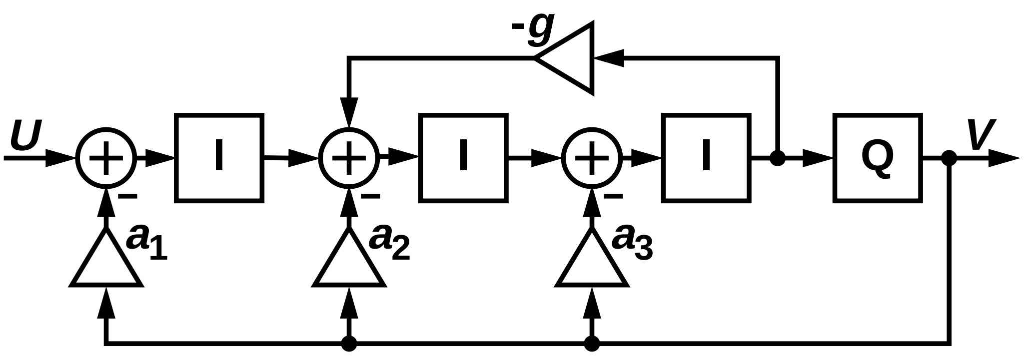
Quantizer:



Mid-tread quantizer; *v*: even integers [–*M*,+*M*]

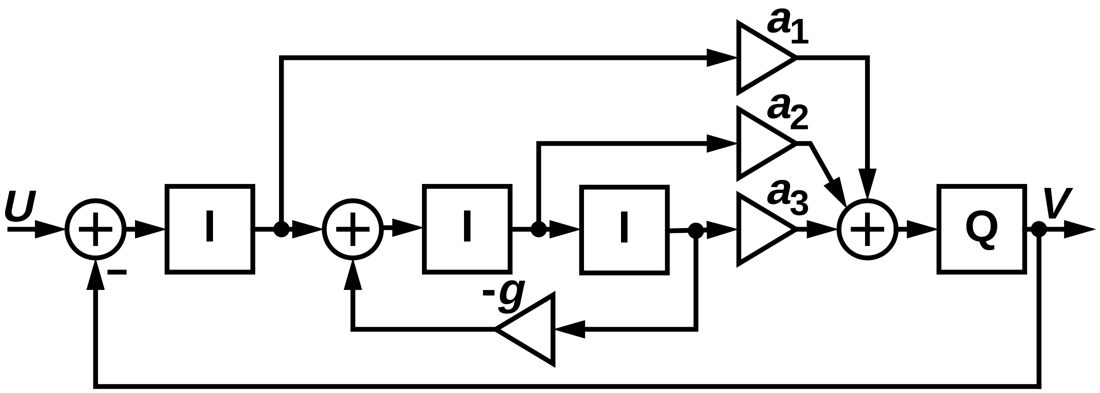
Mid-rise quantizer; *v*: odd integers [–*M*,+*M*]

Feedback Topology



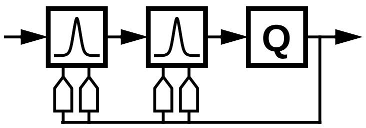
- *N* integrators precede the quantizer
- Feedback from the quantizer to the input of each integrator (via a DAC)
- Local feedback around pairs of integrators to set the NTF's zeros
- Multiple input feed-in branches are possible

Feedforward Topology



- *N* integrators in a row
- Each integrator output is fed forward to the quantizer
- Local feedback around pairs of integrators to control NTF zeros
- Multiple input feed-in branches also possible

Feedback vs. Feedforward STF



$$L_0(z) = \frac{N_0(z)}{D(z)} \qquad L_1(z) = \frac{N_1(z)}{D(z)}$$

$$NTF(z) = \frac{1}{1 - L_1(z)} = \frac{D(z)}{N_1(z) - D(z)}$$

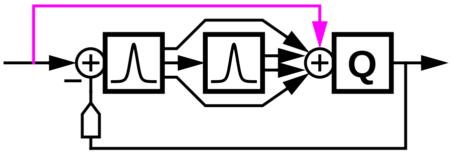
poles of LF are zeros of NTF

$$STF(z) = \frac{L_0(z)}{1 - L_1(z)} = \frac{N_0(z)}{N_1(z) - D(z)}$$

same poles as NTF

zeros = zeros of L_0

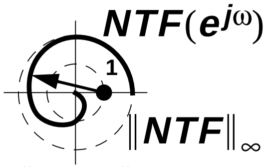
STF often has no zeros, only poles.



$$L_0(z) = -L_1(z) = L(z)$$

$$NTF(z) = \frac{1}{1 - L_1(z)} = \frac{1}{1 + L(z)}$$

$$STF(z) = \frac{L(z)}{1 + L(z)} = 1 - NTF(z)$$

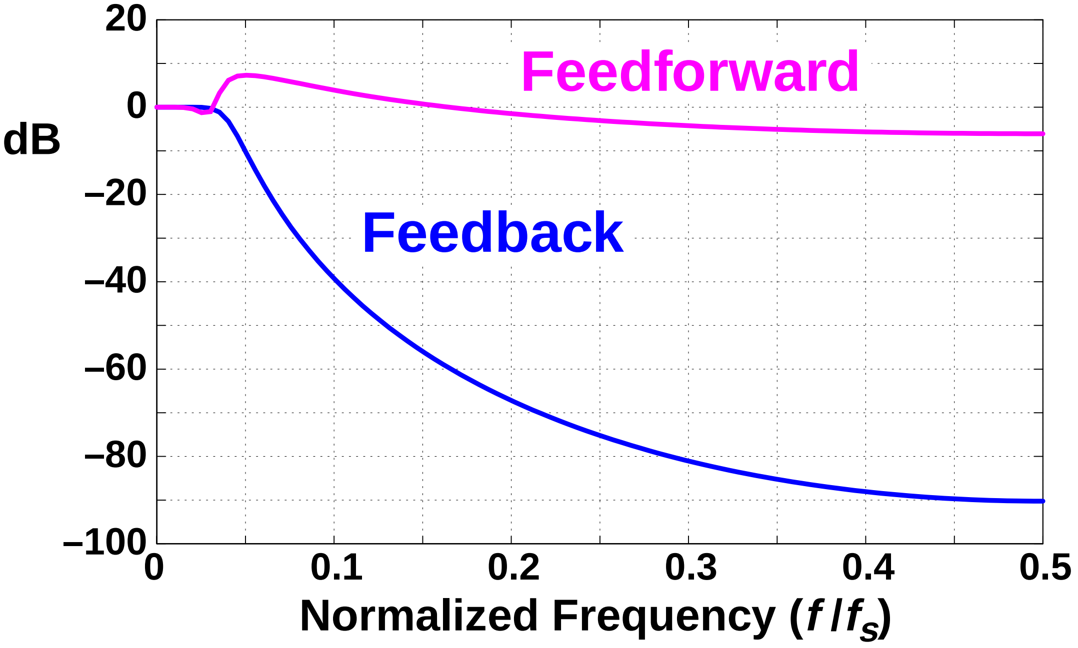


$$\|STF\|_\infty \approx \|NTF\|_\infty + 1$$

With extra feed-in to Q, *STF*(*z*) = 1.

STF Comparison

5^{th}-Order; Single Feed-In



Feedforward vs. Feedback

- **FF has relaxed dynamic range requirements**

  "All stages except the last have attenuated signal components."

- **FB has better STF and, for CT modulators, a better AAF**

  In a discrete-time modulator, the STF of FF can be made unity by adding a signal feedforward term to the input of the quantizer.

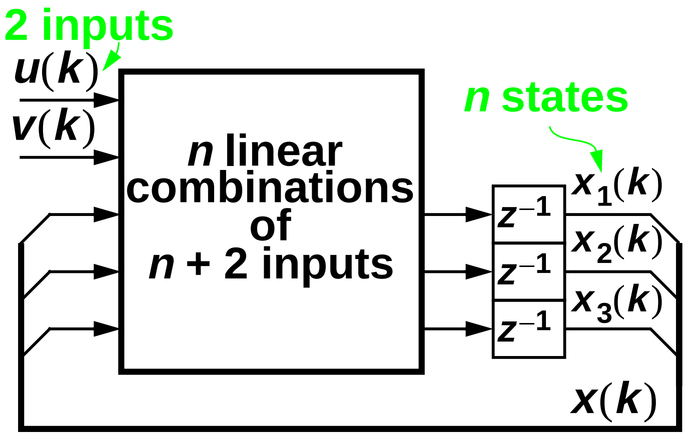
- **FB needs many DACs;**
  FF needs a summation block

  Can do partial summation before the last integrator.

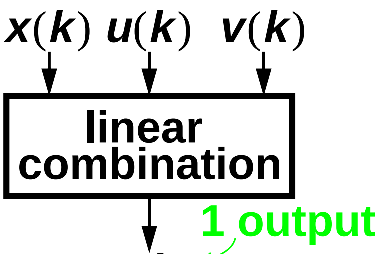
- **FF with signal to quantizer: timing can be tricky**

  Need to quantize *u* and feed it back in zero time.

ABCD: A *State-Space* Representation of the Loop Filter



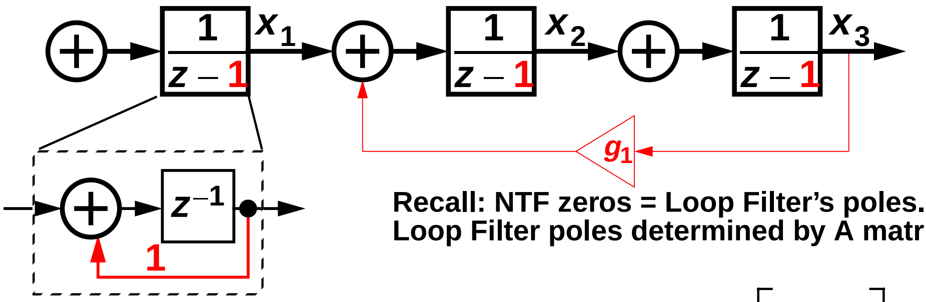
$$x(k+1) = \overset{n\times n}{A}x(k) + \overset{n\times 2}{B}\begin{bmatrix}u(k)\\ v(k)\end{bmatrix}$$



$$y(k) = \underset{1\times n}{C}x(k) + \underset{1\times 2}{D}\begin{bmatrix}u(k)\\ v(k)\end{bmatrix}$$

$$ABCD = \begin{array}{c} \\ n \\ 1 \end{array}\begin{array}{c|c} n & 2 \\ A & B \\ \hline C & D \end{array}$$

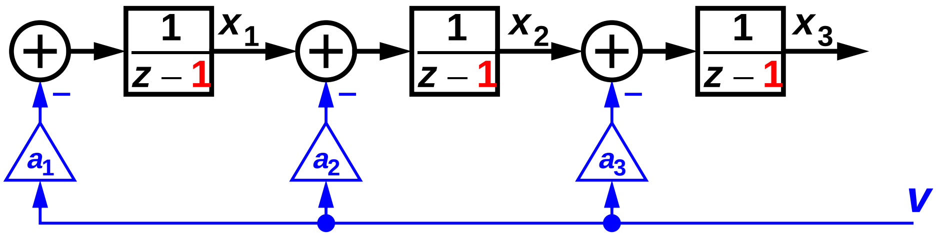
Ex.: Cascade of Integrators Feedback (CIFB) Topology



Recall: NTF zeros = Loop Filter's poles.
Loop Filter poles determined by A matrix.

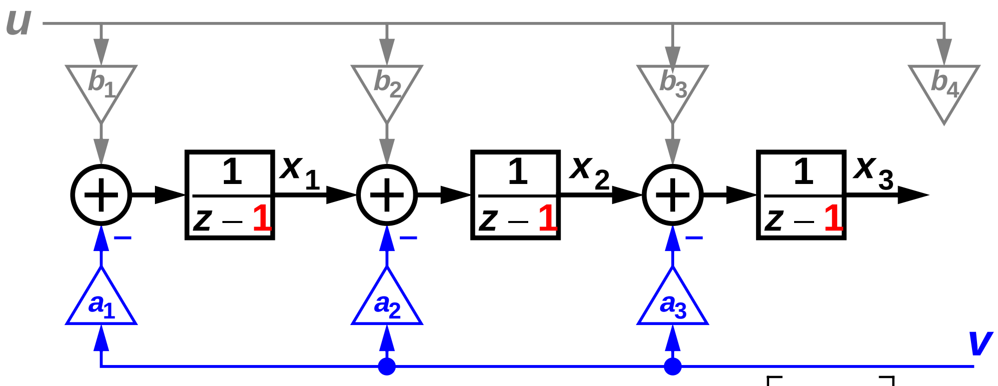
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ y(k) \end{bmatrix} = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & - & - \\ 1 & 1 & g_1 & - & - \\ 0 & 1 & 1 & - & - \\ \hline 0 & 0 & 1 & - & - \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ u(k) \\ v(k) \end{bmatrix}$$

CIFB cont'd:
a_i Control NTF & STF Poles



$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & - & -a_1 \\ 1 & 1 & 0 & - & -a_2 \\ 0 & 1 & 1 & - & -a_3 \\ 0 & 0 & 1 & - & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ u(k) \\ v(k) \end{bmatrix}$$

b_i Control STF Zeros



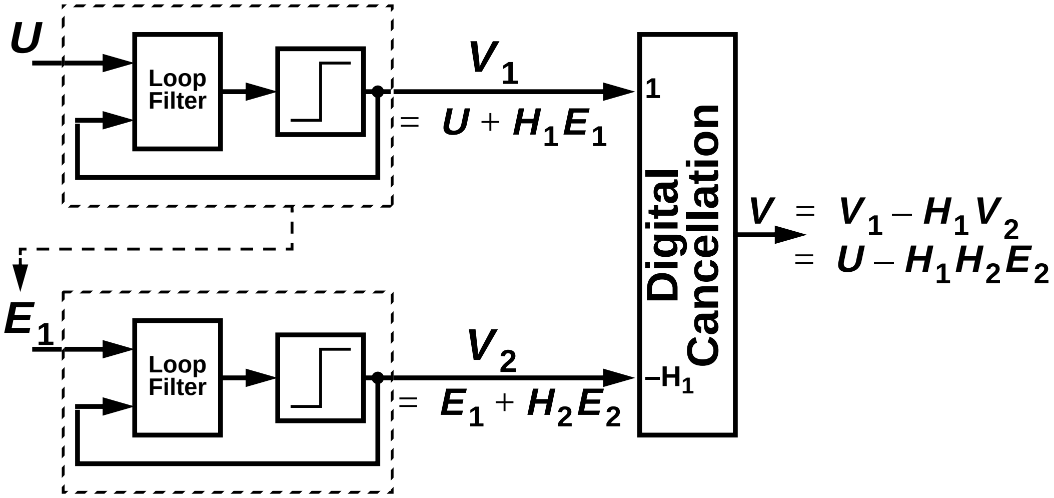
$$\begin{bmatrix} x_1(k+1)\\ x_2(k+1)\\ x_3(k+1)\\ y(k)\end{bmatrix} = \left[\begin{array}{ccc|cc} 1&0&0&b_1&-a_1\\ 1&1&0&b_2&-a_2\\ 0&1&1&b_3&-a_3\\ \hline 0&0&1&b_4&0\end{array}\right]\begin{bmatrix}x_1(k)\\ x_2(k)\\ x_3(k)\\ u(k)\\ v(k)\end{bmatrix}$$

ABCD and the Toolbox

- **`simulateDSM` simulates a modulator given an ABCD description of its loop filter**
- **`realizeNTF` gives (unscaled) coefficients for any of the supported topologies**
- **`stuffABCD` produces an ABCD matrix given the coefficients for one of the supported topologies**
  `mapABCD` performs the inverse operation.
- **`scaleABCD` does dynamic range scaling on any ABCD matrix**
- **`calculateTF` calculates the NTF and STF from ABCD**
  Useful for checking implementation of new topologies.

Cascade (MASH) Modulators

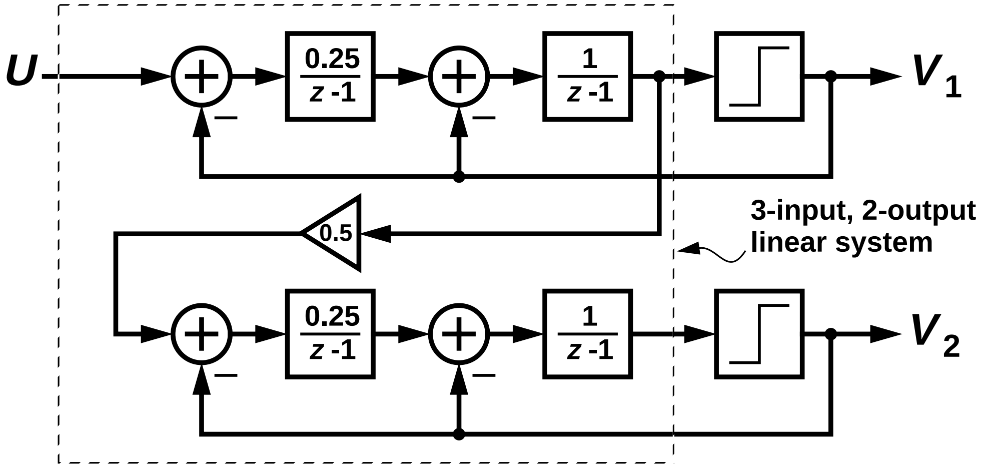
- **Put two (or more) modulators in "series"**



- **The resulting NTF is the *product* of the individual NTFs**

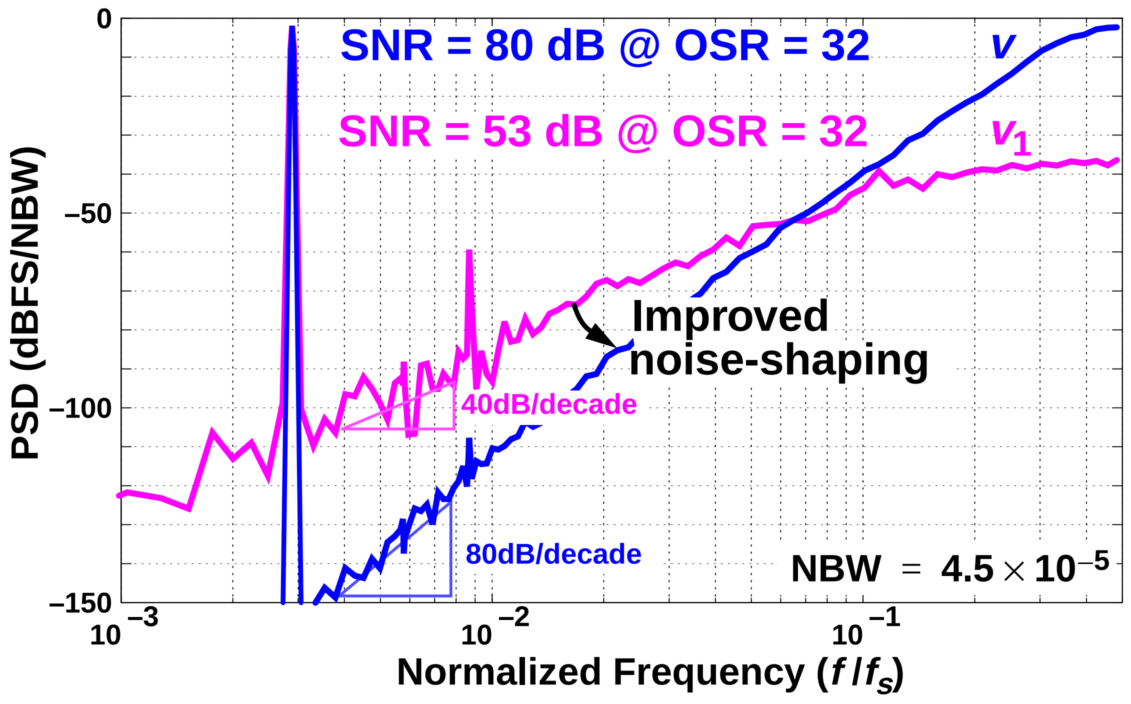
Example: 2-2 Cascade

- **Use Two MOD2b:** $H(z) = \left(\frac{z-1}{z-0.5}\right)^2$

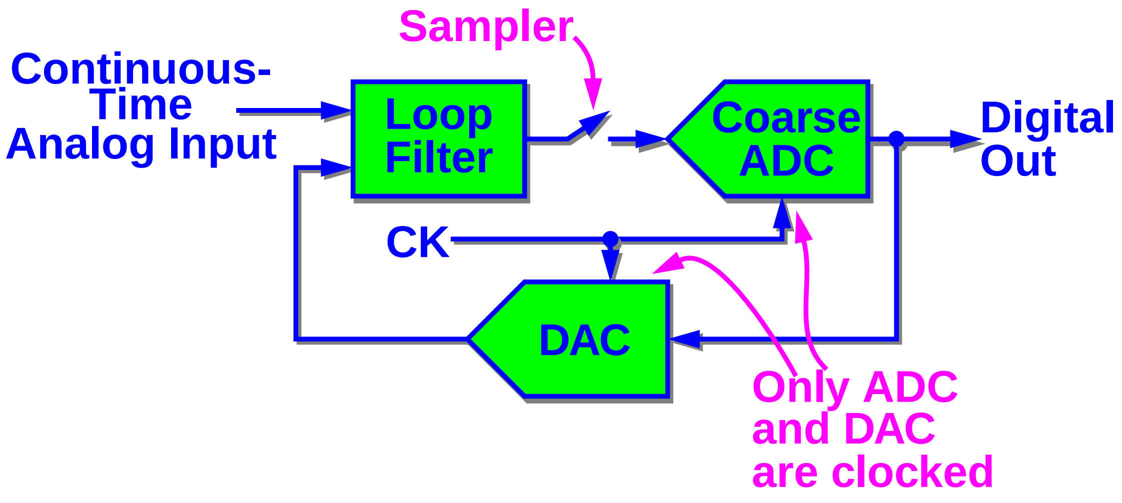


$$\left(V = \frac{1}{z^3}V_1 + \frac{8(z-1)^2(z-0.5)^2}{z^3(z-0.75)}V_2\right) \Rightarrow \left(H(z) = \frac{8(z-1)^4}{(z-0.75)}\right)$$

Example MASH Spectra



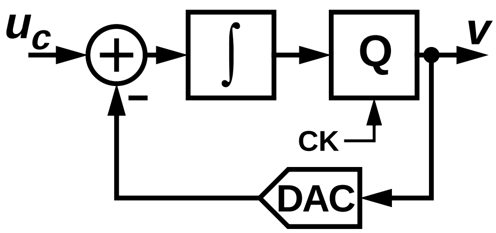
A Continuous-Time ΔΣ ADC



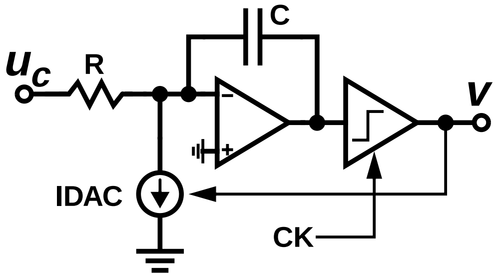
- **Loop filter implemented with continuous-time circuitry**
- **Sampling occurs after the loop filter**

Example: MOD1-CT

Block Diagram



Schematic

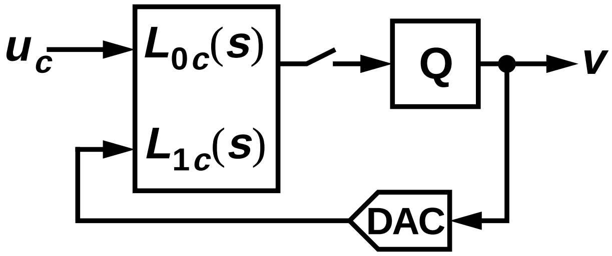


- Note: Input is a simple resistor, *not* a switched capacitor

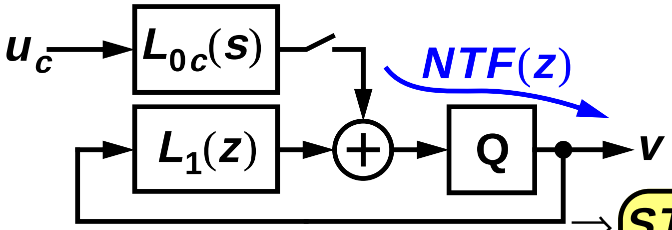
  CT ADCs are easier to drive than DT ADCs.

Inherent Anti-Aliasing

- ΔΣ ADC with CT Loop Filter



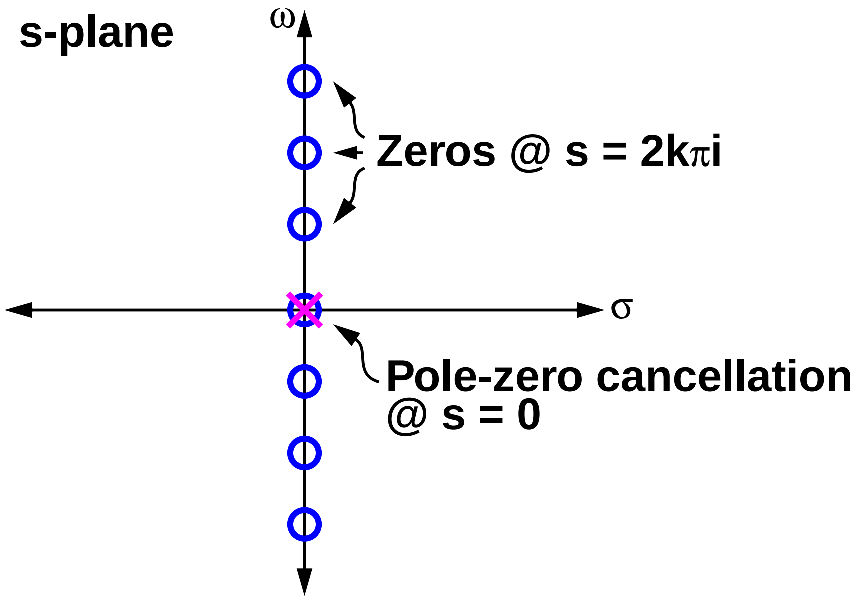
- Equivalent system with DT feedback path



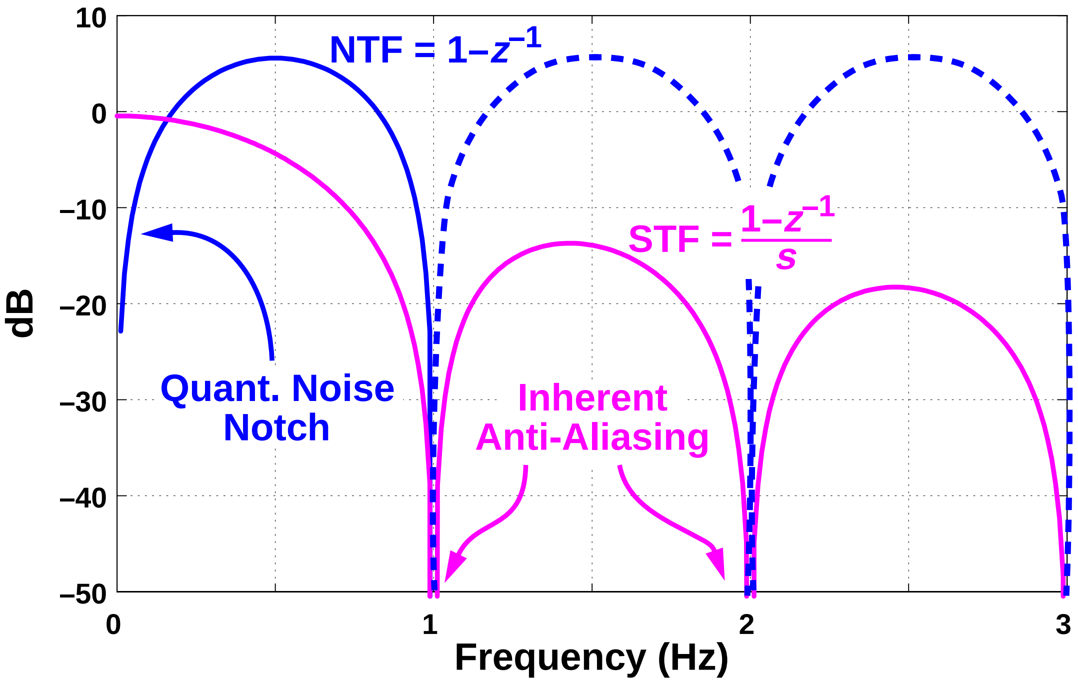
$$\Rightarrow \boxed{STF = L_{0c}(s)NTF(z)}$$

Example: MOD1-CT $STF = \frac{1 - z^{-1}}{s}$

Recall $z = e^{s}$



MOD1-CT Frequency Responses



Inherent AAF Summary

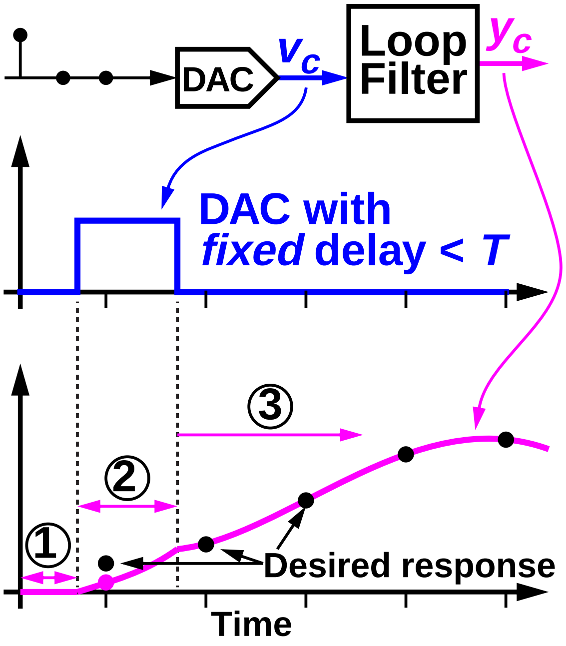
$$STF = L_{0c}(s)NTF(z)$$

- STF contains the zeros of the NTF
- Any frequency which aliases to the passband is attenuated by at least as much as the quantization noise

  Anti-alias performance tracks modulator order.
  Also true for MASH systems.
- The effective anti-alias filter is

$$AAF(f) = \frac{STF(f)}{STF(f_{alias})} = \frac{L_{0c}(f)}{L_{0c}(f_{alias})}$$

Effect of Quantizer/DAC Delay



1 Loop outputs zero until DAC pulse begins

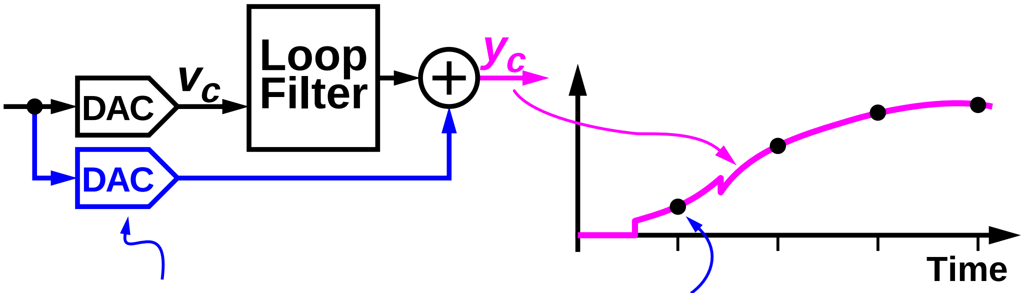
2 Loop responds as if input were a step

3 Loop follows trajectory of an n^{th}-order linear system with zero input but non-zero initial conditions

⇒ At best, samples of the pulse response will match the desired impulse response except at the first point.
The NTF will be wrong.

Compensating for Feedback Delay

- To fix the first sample, add a *direct feedback* DAC

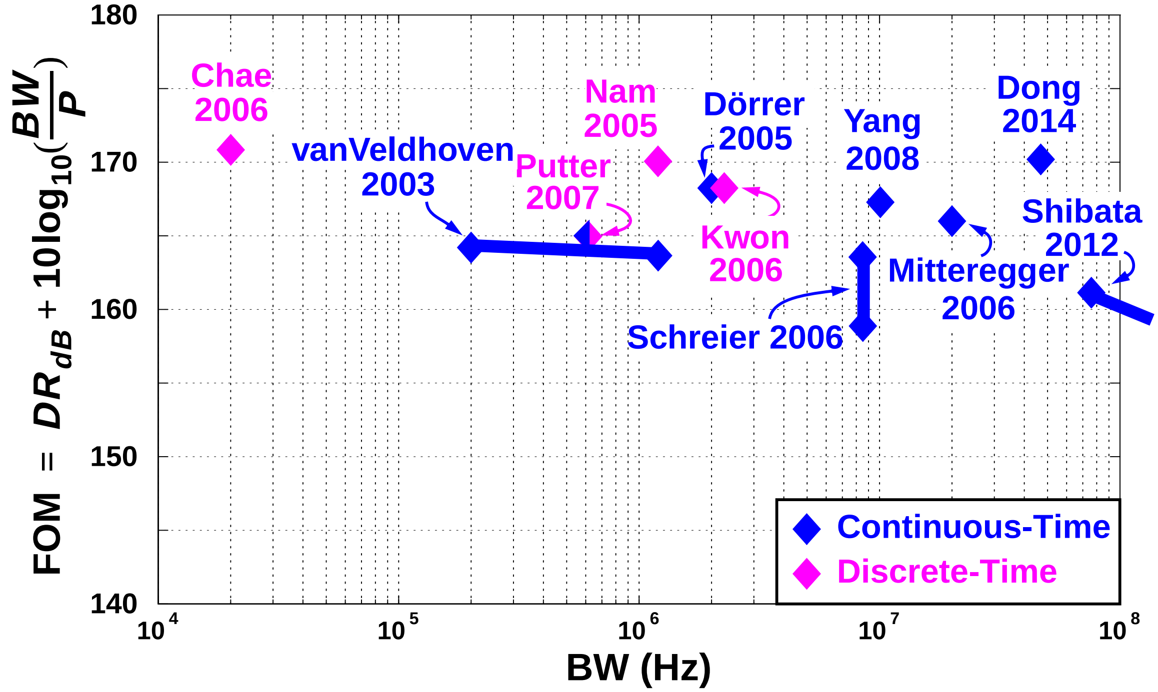


Direct Feedback DAC corrects the first sample

- With enough DACs, one for each errant sample, any finite number of points can be repaired

  In principle, the delay of the main feedback path can be anything, but the system becomes sensitive to coefficient errors.

DT ΔΣ vs. CT ΔΣ



References— DT vs. CT

BW (Hz)	DR (dB)	P (mW)	FOM	Reference	Architecture *N*=order (*M*=#steps)
20k	85	0.036	172	Chae, ISSCC 2008:27.2	ΔΣ SC 3(1)
614k	82	3.1	165	Putter, ISSCC 2007:13.4	ΔΣ A-RC+SC 6(1)
1.2M	82	8	164	vanVeldhoven, ISSCC 2003:3.4	QΔΣ gm-C 5(1)
1.2M	96	44	171	Nam, JSSC 2005-09	ΔΣ SC 2(32)-2(8)
2M	80	3.0	168	Dörrer, ISSCC 2005:27.1	ΔΣ A-RC 3(15)
2.2M	86	14	168	Kwon, ISSCC 2006:3.4	ΔΣ SC 2(4)
8.5M	88	375	162	Schreier, ISSCC 2006:3.2	QBPΔΣ A-RC 4(16)
10M	87	100	167	Yang, ISSCC 2008:27.6	ΔΣ A-RC 5(7)
20M	76	20	166	Mitteregger, ISSCC 2006:3.1	ΔΣ A-RC 3(15)
53M	88	235	172	Dong, JSSC 2014-12	ΔΣ A-RC 0(16)-3(16)
75M	80	550	161	Shibata, JSSC 2012-12	BPΔΣ LC/A-RC 6(16)

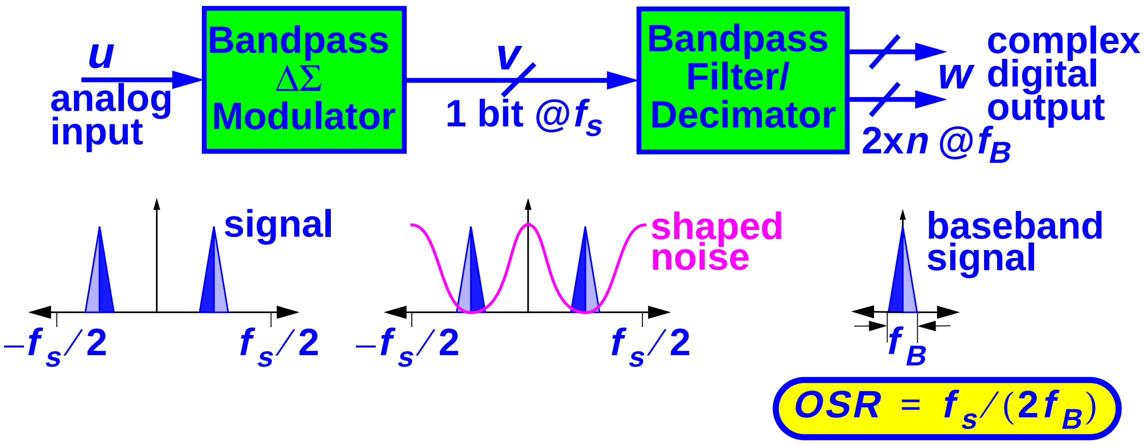
Advantages of Discrete-Time

1. Less sensitive to jitter
2. Accurate transfer functions regardless of f_{CK}
3. FF topology with no STF-peaking is possible
4. DAC dynamics are non-critical
5. Math is simpler

Advantages of Continuous-Time

1. Higher speed
2. Inherent anti-aliasing
3. Easier to drive (well-defined Z_{in})
4. Sampling is non-critical
5. Lower power (?)

A Bandpass ΔΣ ADC System



- ADC converts its analog input into a noise-shaped digital output
- DSP removes out-of-band noise (and signals) and translates the signal to baseband

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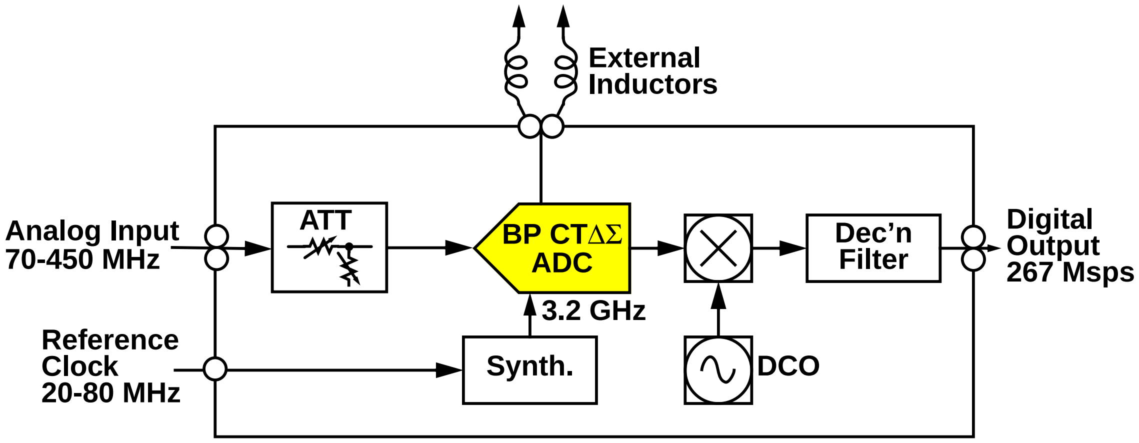
Example CT Bandpass ΔΣ ADC: The AD6676 [Shibata 2012]

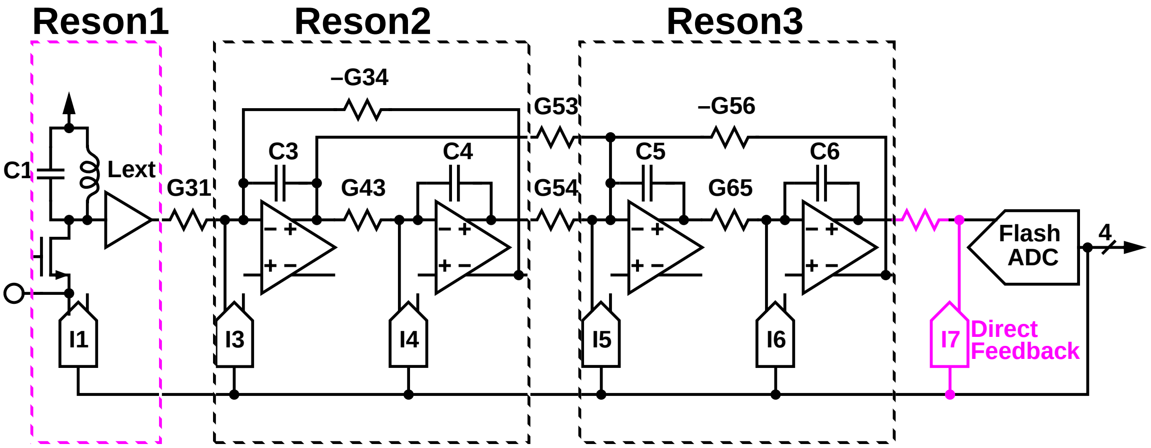


- IF subsystem containing attenuator, synthesizer, CT BP ΔΣ ADC and digital filter

AD6676 Specs

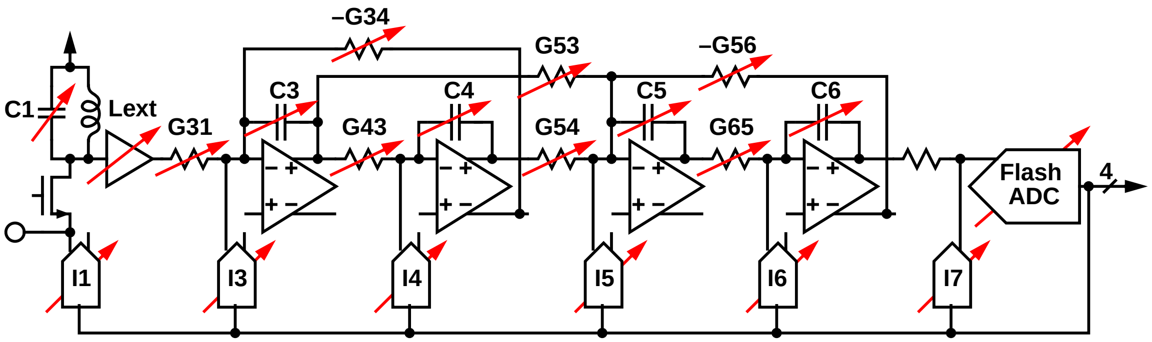
Parameter	Value	Notes
Z_{in}	50 Ω	
ADC FS	–16 to –4 dBm	
Attenuation	0 to +27 dB	
NSD	< –157 dBFS/Hz	BW = 75 MHz; 12-dB att.
IF	70-450 MHz	
BW	up to 100 MHz	<3-dB NSD degradation
F_{ck}	2-3.2 GHz	
Power	1.25 W	Includes digital filter & JESD204B interface

ADC Schematic



- 6^{th}-order FB-style loop filter
  One LC resonator plus two active-RC resonators.
  G53 makes up for missing DAC2.
- 16-step quantization, [1 2] DAC timing

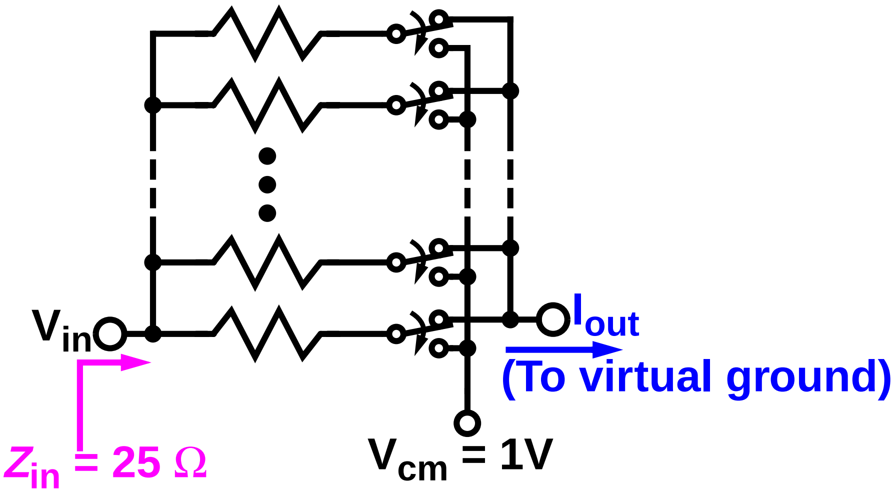
Programmable Everything



- **Feedback currents, integrating capacitors, inter-stage conductances, flash LSB**

  The inductors were also selectable via the cascode (2 choices).

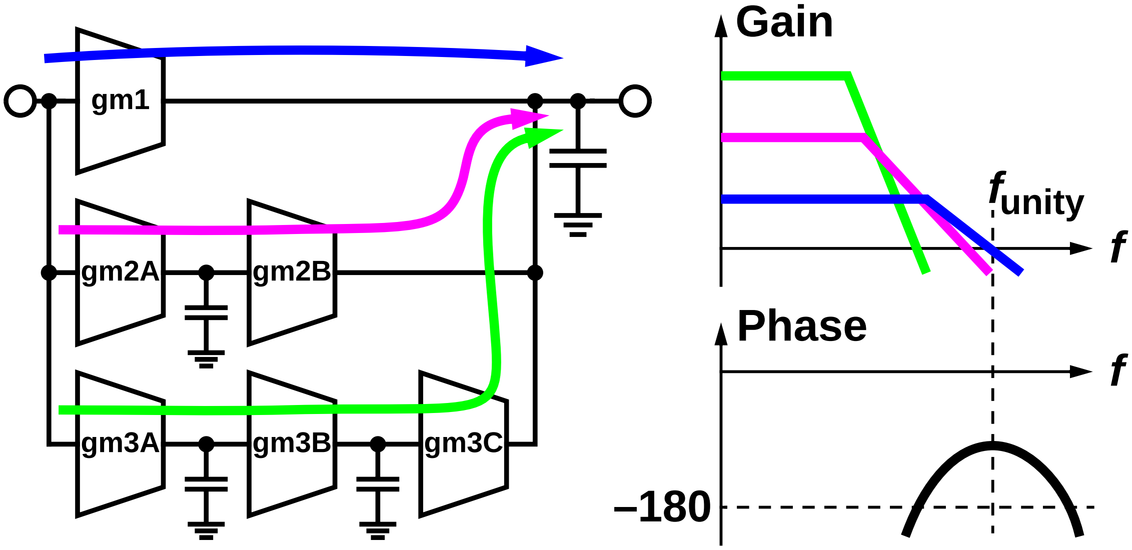
Attenuator Schematic



- **Resistors switched between virtual ground provided by cascode and V_{cm}**

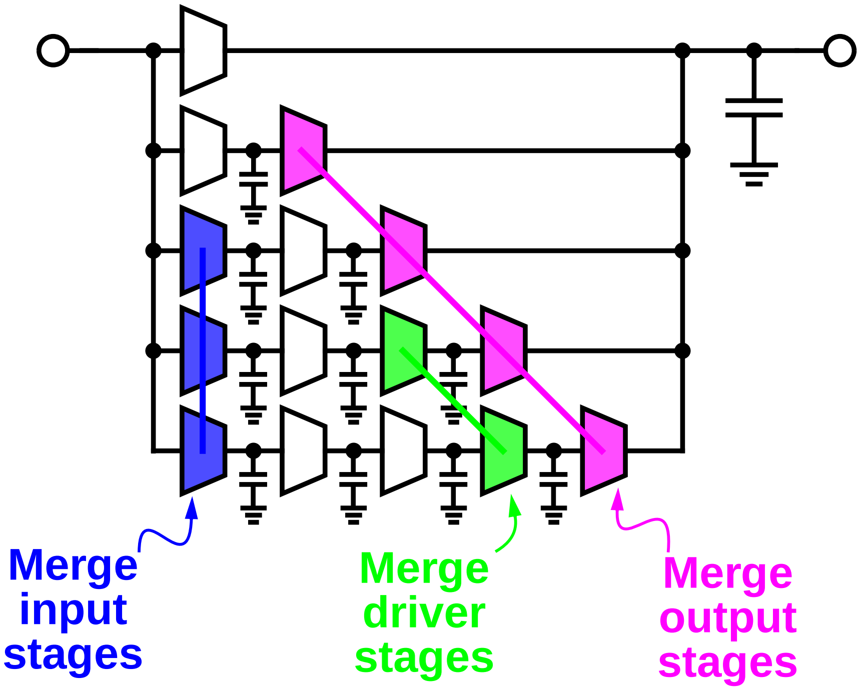
  Maintains matching independent of attenuation.

Feed-Forward Amplifier Concept



- High gain provided by longest path
- Stability by shorter, higher-bandwidth paths

FF Amp— Stage-Sharing



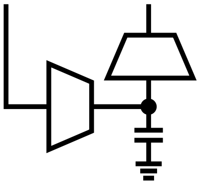
Merge input stages

Merge driver stages

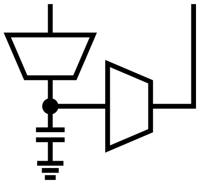
Merge output stages

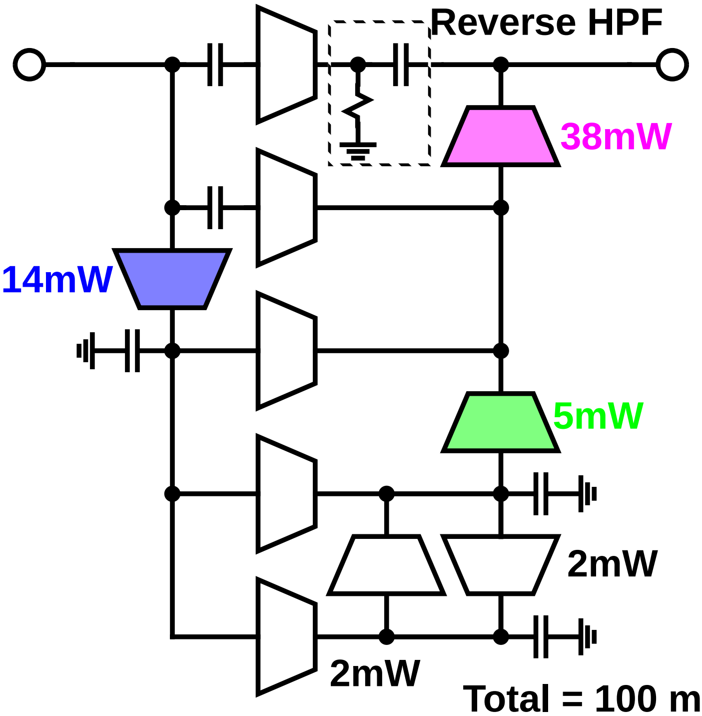
FF Amp— A1 Architecture

Built up by adding L-sections:



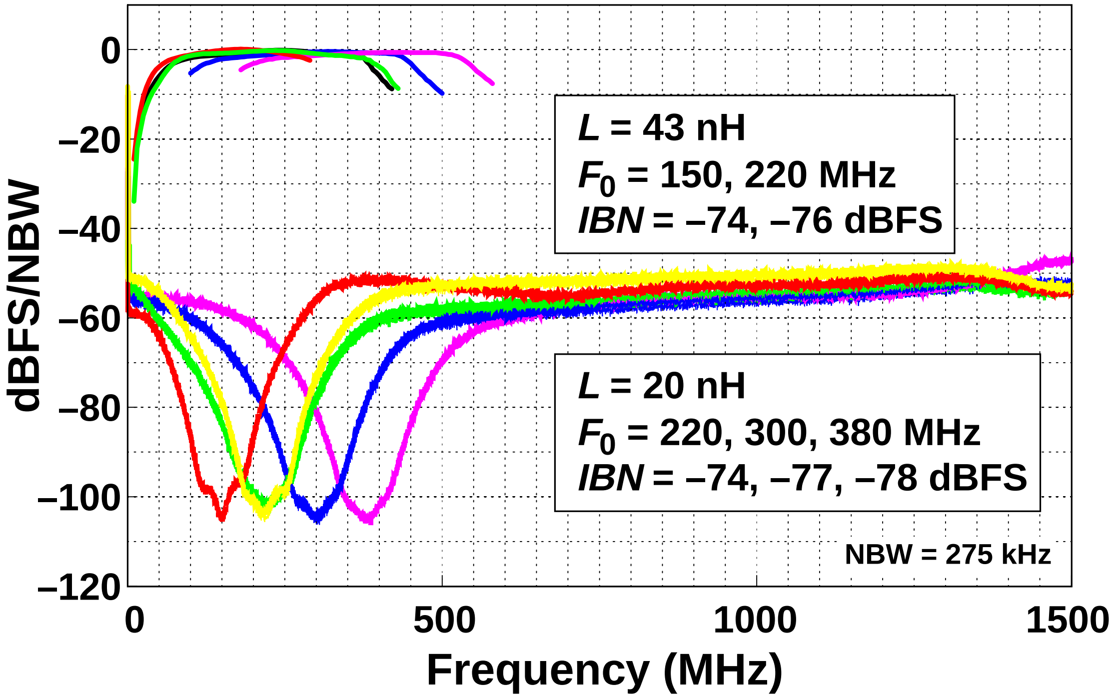
or





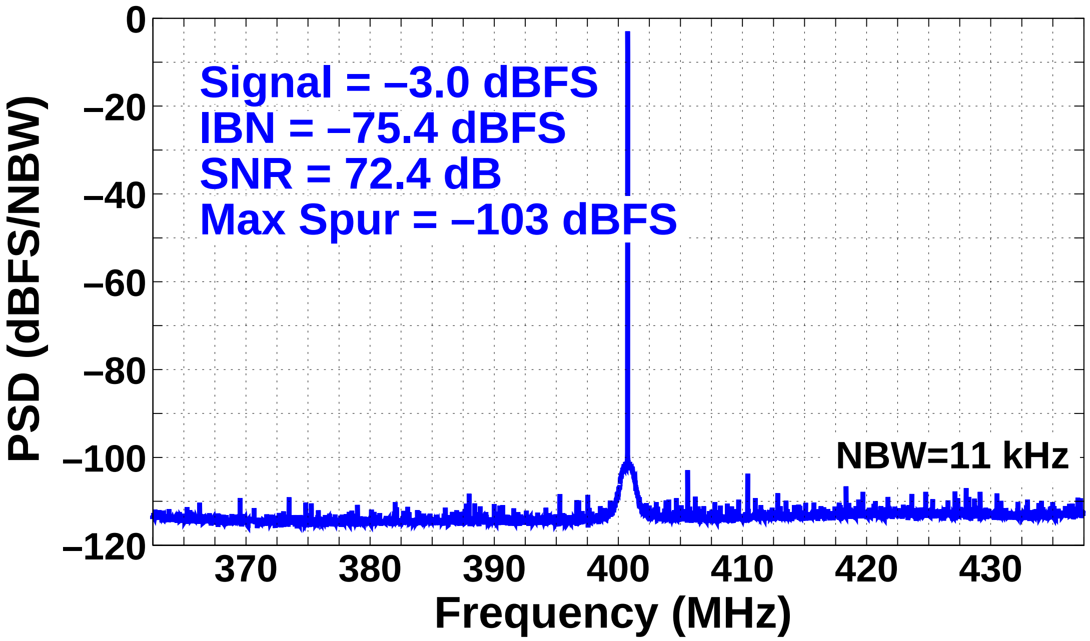
Total = 100 mW

F_0 = 150-400 MHz; F_s = 3 GHz

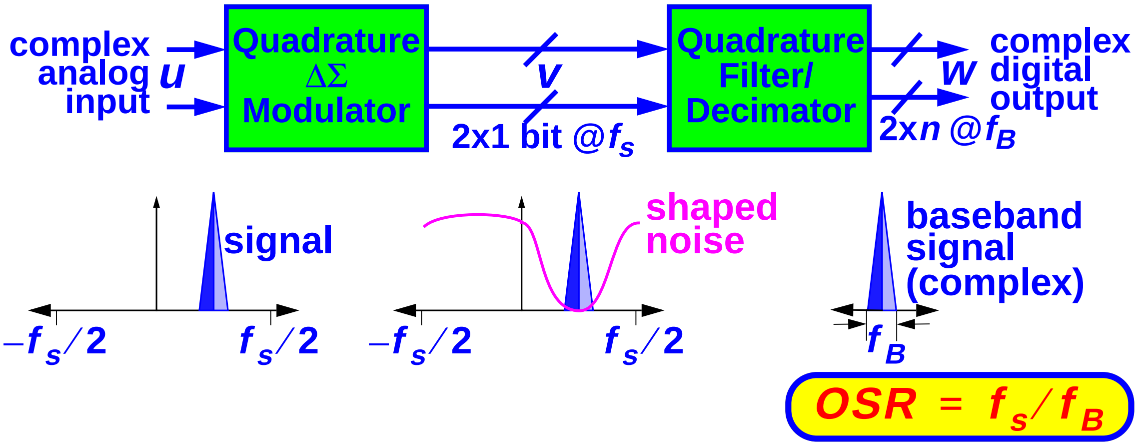


Example In-Band Spectrum

IF = 400 MHz, BW = 75 MHz

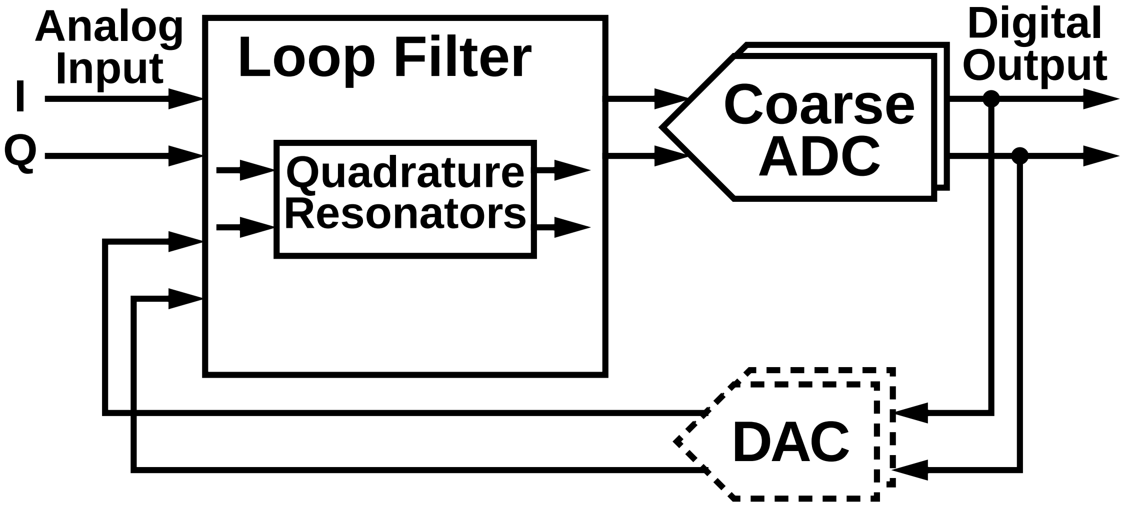


A Quadrature ΔΣ ADC System



- Modulator converts its quadrature analog input into a pair of bit-stream outputs
- DSP removes out-of-band noise and translates the signal to baseband

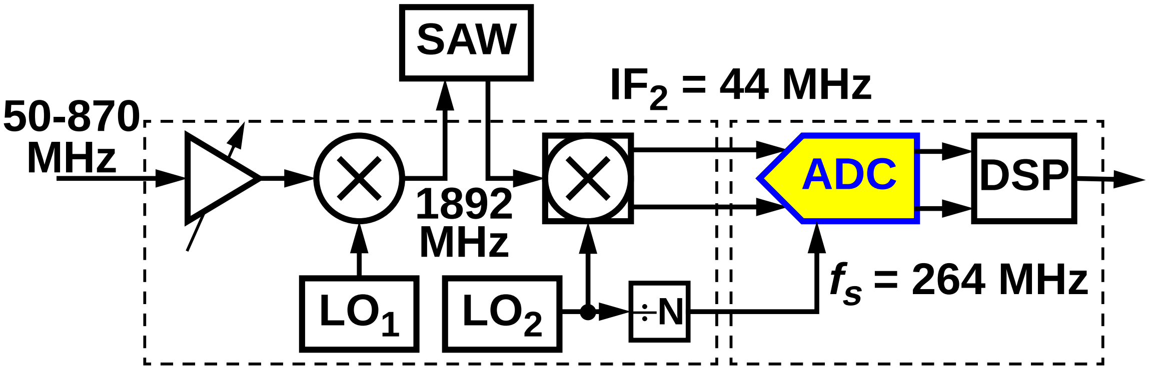
A Quadrature ΔΣ Modulator



- **A ΔΣ ADC with quadrature everything**

  NTF and STF are complex.

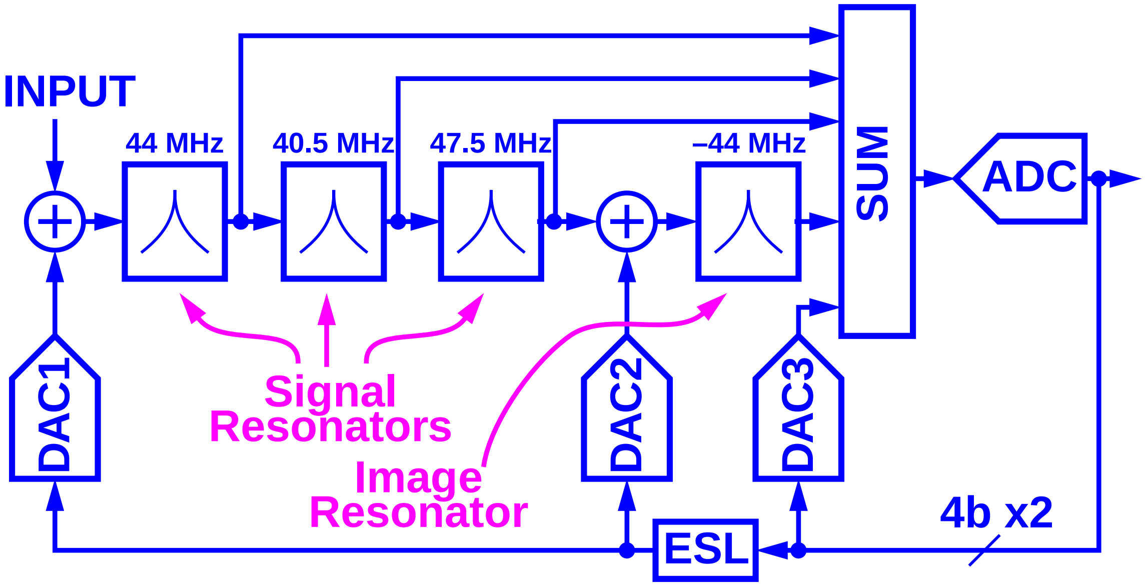
Example: A Quadrature ΔΣ ADC for a TV Tuner System



RF Front End **IF Backend**

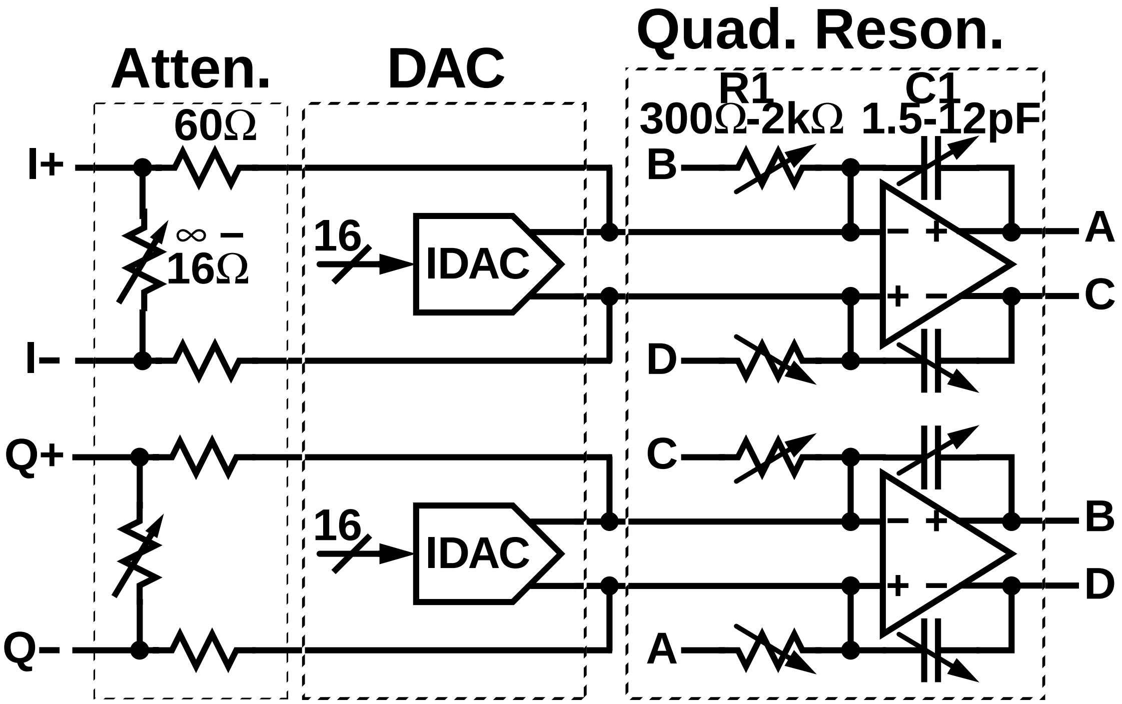
- **Dual-conversion super-heterodyne receiver containing a quadrature bandpass ΔΣ ADC**

ADC Architecture

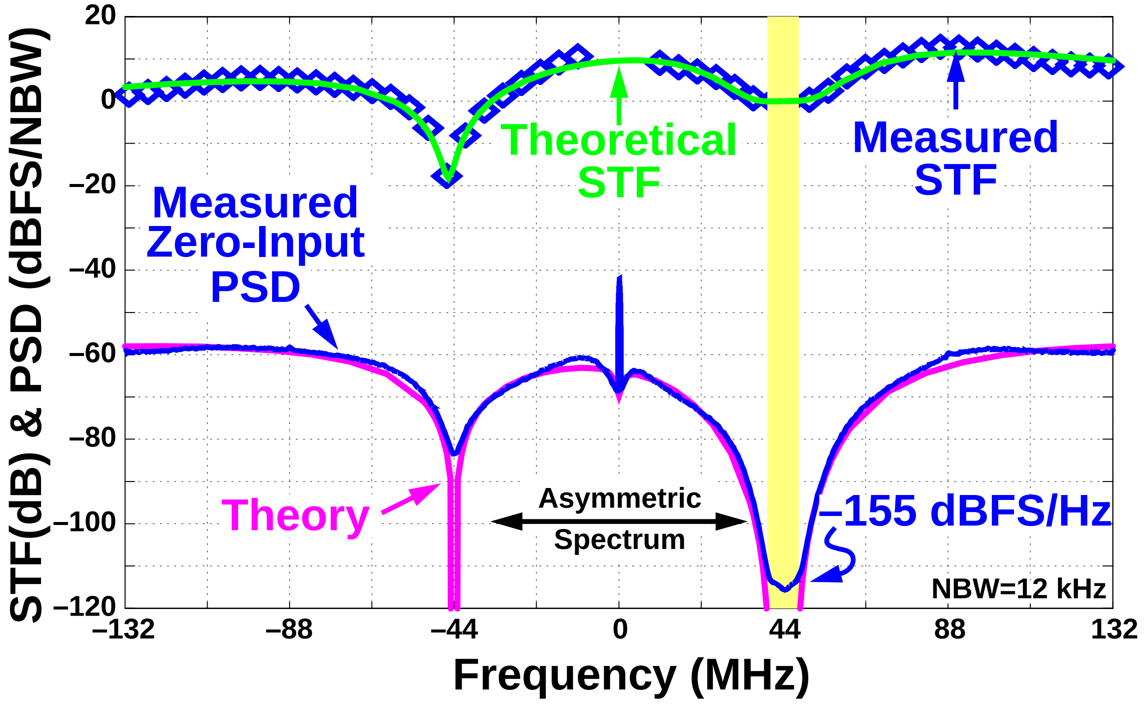


- (3+1)th-order, 4-b, feedforward A-RC modulator

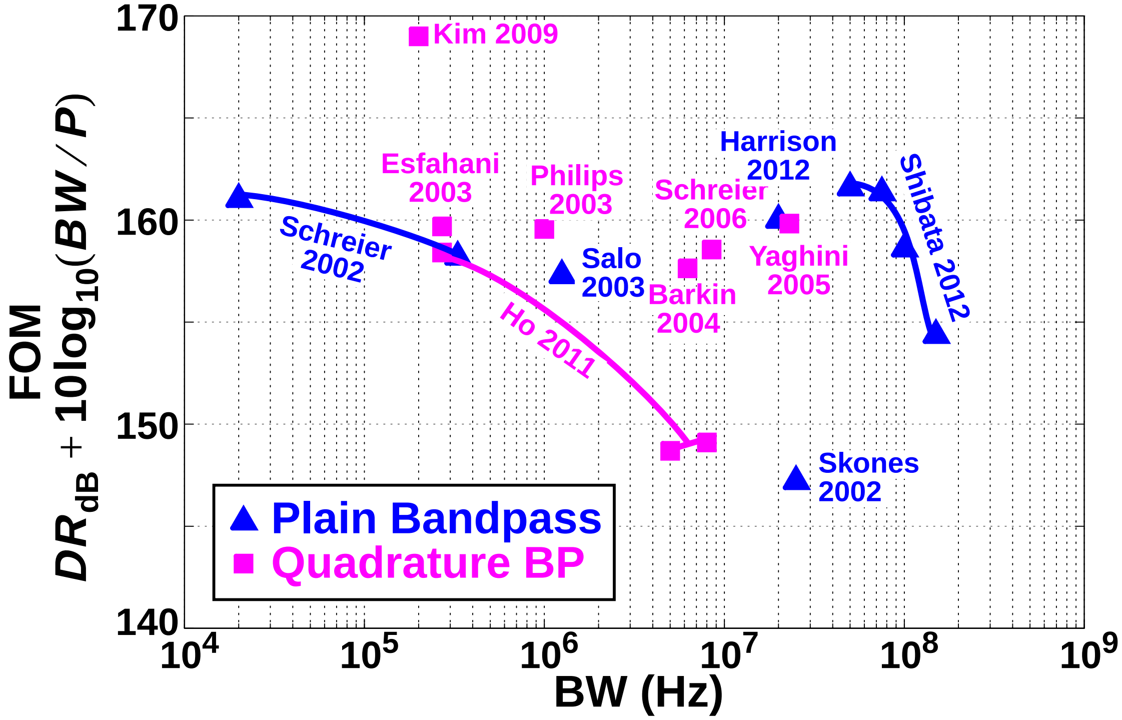
Quadrature ADC Front-End



STF & NTF



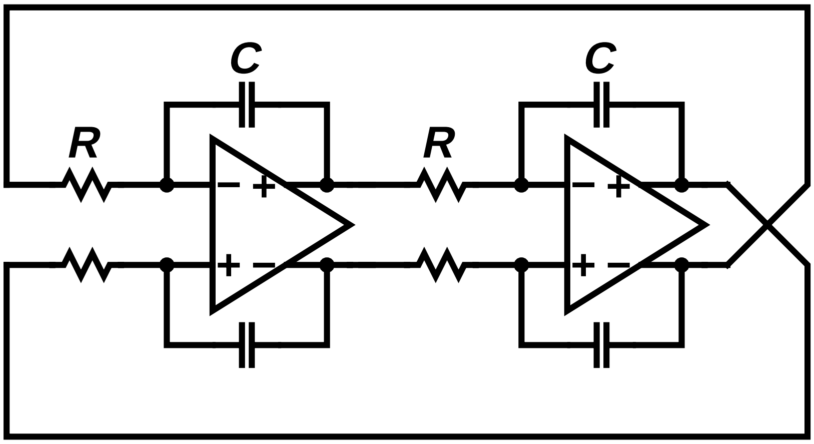
FOM Comparison



NLCOTD: High-Q Resonator

- **Want $Q \gg \sqrt{3}\dfrac{f_0}{BW}$ for small SQNR degradation**
- **In a TV tuner ADC f_0 = 44 MHz and BW = 8.5 MHz, so we needed $Q \gg 9$**
  - **Actual requirement was Q > 20.**
  - **How can Q be kept high despite finite amplifier gain and bandwidth?**

Active-RC Resonator Structure



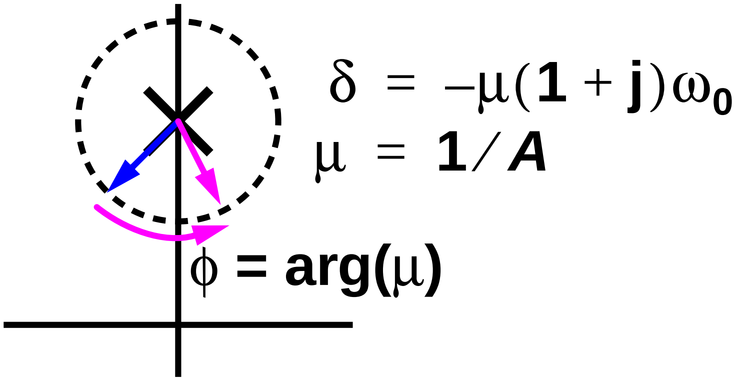
$$f_0 = \frac{1}{2\pi RC}$$

- **Tuned by adding positive feedback to make an oscillator and adjusting C until the desired resonance is achieved**

– **Amplifier drives both R and C ⇒ trouble?**

Amplifier Gain and Phase

- **Finite gain degrades Q**
- **Phase lag enhances Q**
- **Analysis shows $\phi = 45^\circ$ yields high Q, regardless of amplifier gain**

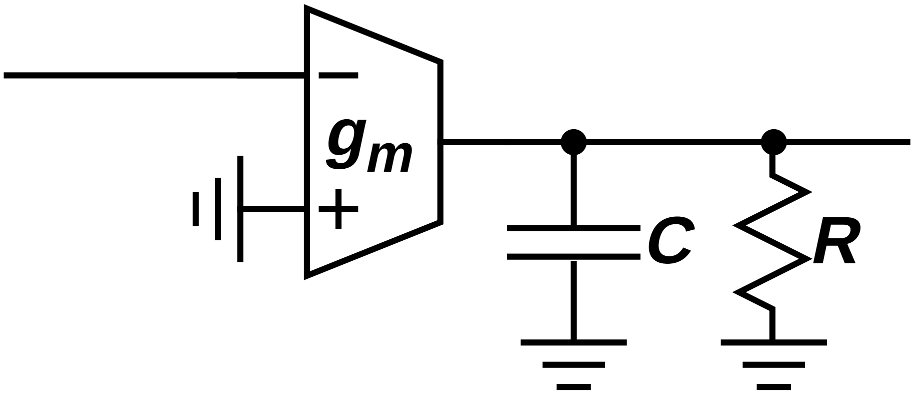


$$\delta = -\mu(\mathbf{1} + \mathbf{j})\omega_0$$

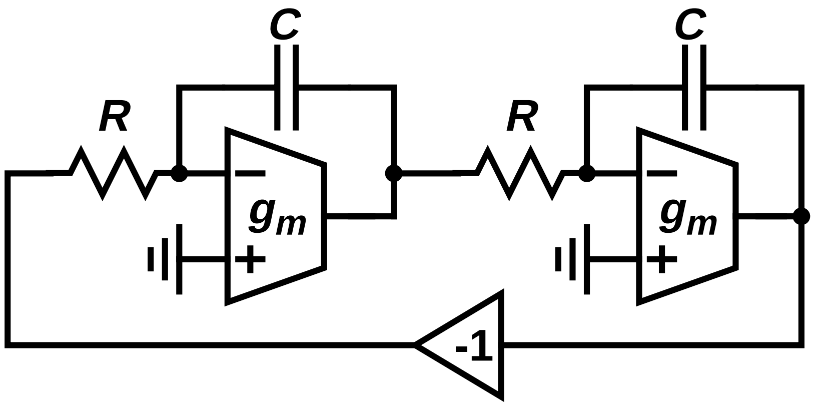
$$\mu = \mathbf{1}/\boldsymbol{A}$$

$$\phi = \mathbf{arg}(\mu)$$

An Amplifier with $\phi = 45^\circ$ @ f_0:

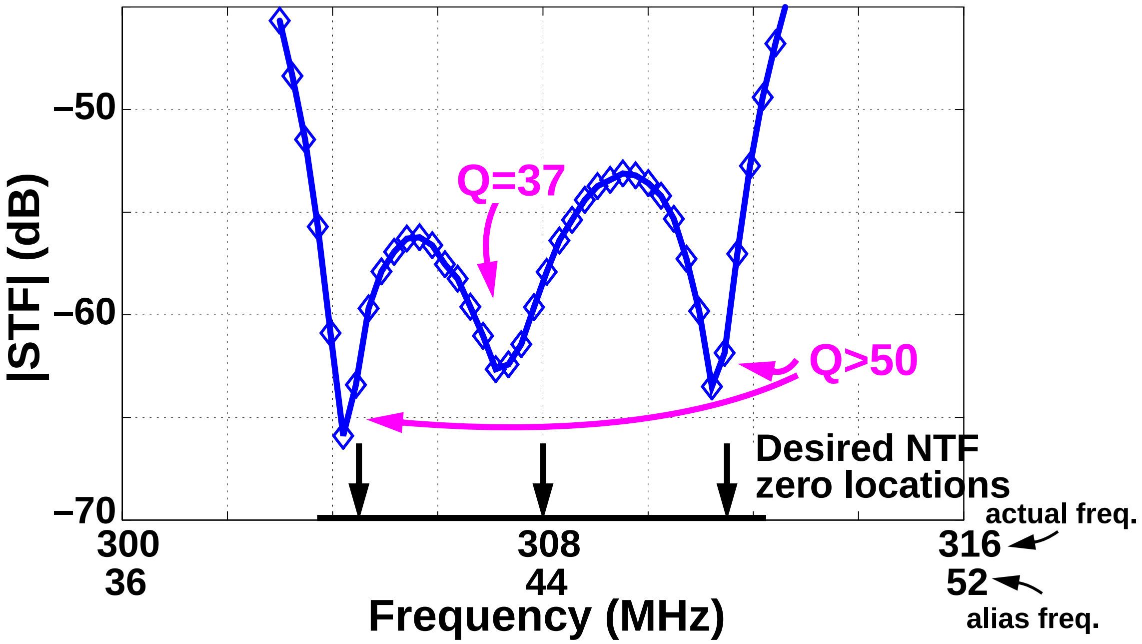


Resulting High-Q Resonator



- **Amplifier load yields $\phi = 45^\circ$ @ f_0**
- **Finite g_m shifts the pole frequency, but does not degrade Q!**

Measured STF in an Alias Band



- **Resonator Q is well above the design target!**

What You Learned Today

1. Feedback vs. Feedforward topology
2. State-space (ABCD) representation of the loop filter in the ΔΣ Toolbox
3. MASH Modulators
4. Continuous-Time Modulators
5. Bandpass and Quadrature Bandpass ΔΣ