

ECE1371 Advanced Analog Circuits

Lecture 2

MODN and the $\Delta\Sigma$ Toolbox

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Date	Lecture (M 13:00-15:00)	Ref	Homework
2015-01-05	RS 1 MOD1 & MOD2	ST 2, 3, A	1: Matlab MOD1&2
2015-01-12	RS 2 MODN + $\Delta\Sigma$ Toolbox	ST 4, B	2: $\Delta\Sigma$ Toolbox
2015-01-19	RS 3 Example Design: Part 1	ST 9.1, CCJM 14	3: Sw.-level MOD2
2015-01-26	RS 4 Example Design: Part 2	CCJM 18	
2015-02-02	TC 5 SC Circuits	R 12, CCJM 14	4: SC Integrator
2015-02-09	TC 6 Amplifier Design		
2015-02-16	Reading Week- No Lecture		
2015-02-23	TC 7 Amplifier Design		5: SC Int w/ Amp
2015-03-02	RS 8 Comparator & Flash ADC	CCJM 10	Project
2015-03-09	TC 9 Noise in SC Circuits	ST C	
2015-03-16	RS 10 Advanced $\Delta\Sigma$	ST 6.6, 9.4	
2015-03-23	TC 11 Matching & MM-Shaping	ST 6.3-6.5, +	
2015-03-30	TC 12 Pipeline and SAR ADCs	CCJM 15, 17	
2015-04-06	Exam	Proj. Report Due Friday April 10	
2015-04-13	Project Presentation		

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Course Goals

- Deepen understanding of CMOS analog circuit design through a top-down study of a modern analog system— a delta-sigma ADC
- Develop circuit insight through brief peeks at some nifty little circuits

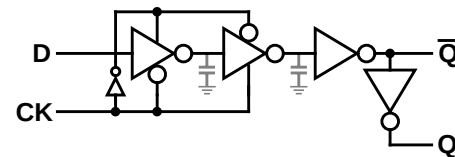
The circuit world is filled with many little gems that every competent designer ought to know.

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NLCOTD: Dynamic Flip-Flop

- Standard CMOS version



- Can the circuit be simplified?
Is a complementary clock necessary?

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Highlights

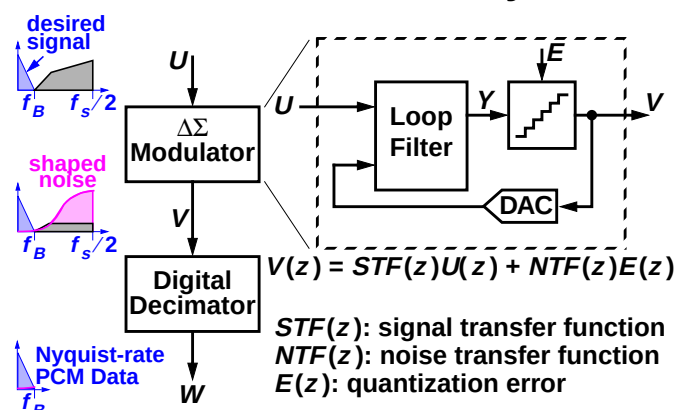
(i.e. What you will learn today)

- 1 N^{th} -order modulator (MODN)
- 2 High-level design with the $\Delta\Sigma$ Toolbox

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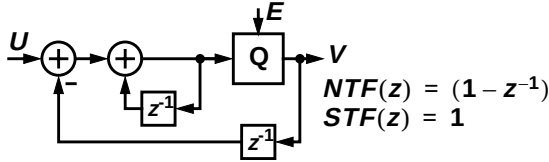
0. Review: A $\Delta\Sigma$ ADC System



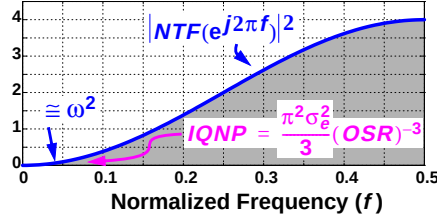
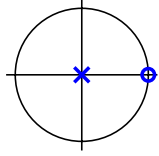
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Review: MOD1



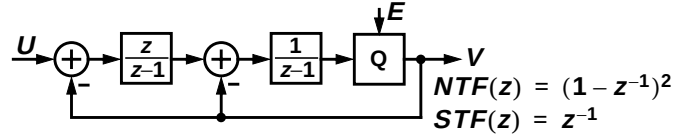
NTF poles & zeros:



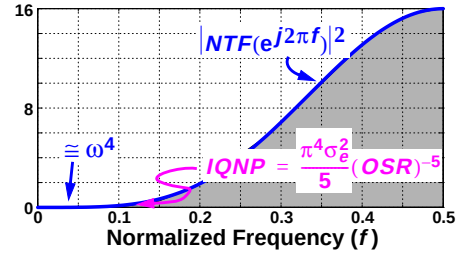
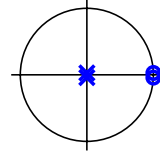
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Review: MOD2



NTF poles & zeros:



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Review Summary

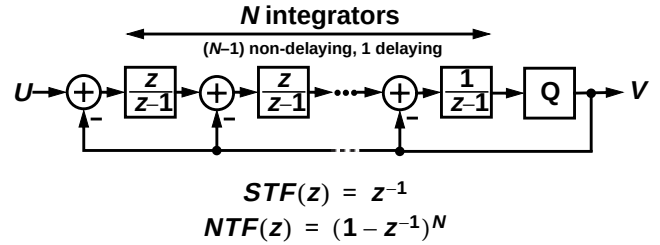
- $\Delta\Sigma$ works by spectrally separating the quantization noise from the signal
Requires oversampling. $OSR \equiv f_s / (2f_B)$.
Achieved by the use of *filtering* and *feedback*.
- A binary DAC is *inherently linear*, and thus a binary $\Delta\Sigma$ modulator is too
- MOD1-CT has *inherent anti-aliasing*
- MOD1 has $NTF(z) = 1 - z^{-1}$
 $\Rightarrow$ Arbitrary accuracy for DC inputs;
9 dB/octave SQNR-OSR trade-off.
- MOD2 has $NTF(z) = (1 - z^{-1})^2$
 $\Rightarrow$ 15 dB/octave SQNR-OSR trade-off.

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1. MODN

[Ch. 4 of Schreier & Temes]

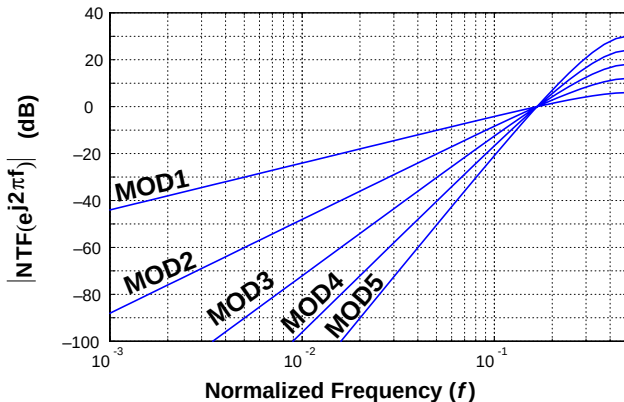


- MODN's NTF is the Nth power of MOD1's NTF

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NTF Comparison



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Predicted Performance

- In-band quantization noise power

$$\begin{aligned}
 IQNP &= \int_0^{0.5/OSR} |NTF(e^{j2\pi f})|^2 \cdot S_{ee}(f) df \\
 &\approx \int_0^{0.5/OSR} (2\pi f)^{2N} \cdot 2\sigma_e^2 df \\
 &= \frac{\pi^{2N}}{(2N+1)(OSR)^{2N+1}} \sigma_e^2
 \end{aligned}$$

- Quantization noise drops as the (2N+1)th power of OSR— (6N+3) dB/octave SQNR-OSR trade-off

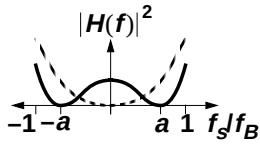
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Improving NTF Performance– NTF Zero Optimization

- Minimize the integral of $|NTF|^2$ over the passband

Normalize passband edge to 1 for ease of calculation:



Need to find the a_i which minimize the integral

$$\int_{-1}^1 (x^2 - a_1^2)^2 dx, \quad n = 2$$

$$\int_{-1}^1 x^2 (x^2 - a_1^2)^2 dx, \quad n = 3$$

$$\int_{-1}^1 (x^2 - a_1^2)^2 (x^2 - a_2^2)^2 dx, \quad n = 4$$

⋮

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Solutions Up to Order = 8

Order	Optimal Zero Placement Relative to f_B	SQNR Improvement
1	0	0 dB
2	$\pm 1/\sqrt{3}$	3.5 dB
3	0, $\pm \sqrt{3/5}$	8 dB
4	$\pm \sqrt{3/7} \pm \sqrt{(3/7)^2 - 3/35}$	13 dB
5	0, $\pm \sqrt{5/9} \pm \sqrt{(5/9)^2 - 5/21}$ [Y. Yang]	18 dB
6	$\pm 0.23862, \pm 0.66121, \pm 0.93247$	23 dB
7	0, $\pm 0.40585, \pm 0.74153, \pm 0.94911$	28 dB
8	$\pm 0.18343, \pm 0.52553, \pm 0.79667, \pm 0.96029$	34 dB

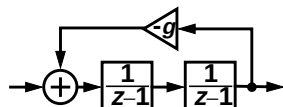
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Topological Implication

- Feedback around pairs integrators:

2 Delaying Integrators



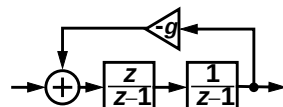
Poles are the roots of

$$1 + g \left(\frac{1}{z-1} \right)^2 = 0$$

$$\text{i.e. } z = 1 \pm j\sqrt{g}$$

Not quite on the unit circle, but fairly close if $g \ll 1$.

Non-delaying + Delaying Integrators (LDI Loop)



Poles are the roots of

$$1 + \frac{gz}{(z-1)^2} = 0$$

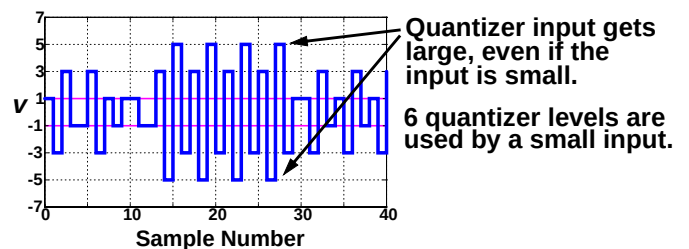
$$\text{i.e. } z = e^{\pm j\theta}, \cos \theta = 1 - g/2$$

Precisely on the unit circle, regardless of the value of g .

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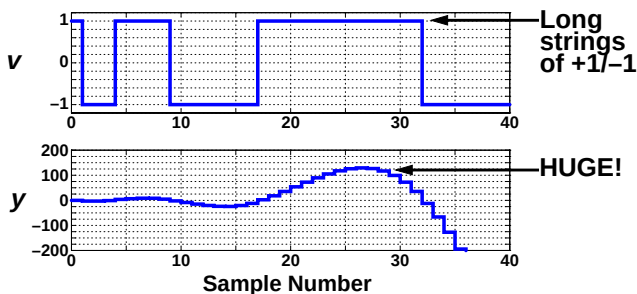
Problem: A High-Order Modulator Wants a Multi-bit Quantizer E.g. MOD3 with an Infinite Quantizer and Zero Input



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Simulation of MOD3-1b (MOD3 with a Binary Quantizer)



- MOD3-1b is unstable, even with zero input!

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Solutions to the Stability Problem Historical Order

1 Multi-bit quantization

Initially considered undesirable because we lose the inherent linearity of a 1-bit DAC.

2 More general NTF (not pure differentiation)

Lower the NTF gain so that quantization error is amplified less.

Unfortunately, reducing the NTF gain reduces the amount by which quantization noise is attenuated.

3 Multi-stage (MASH) architecture

- Combinations of the above are possible

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Multi-bit Quantization

A modulator with $NTF = H$ and $STF = 1$ is guaranteed to be stable if $|u| < u_{max}$ at all times, where $u_{max} = nlev + 1 - \|h\|_1$ and $\|h\|_1 = \sum_{i=0}^{\infty} |h(i)|$

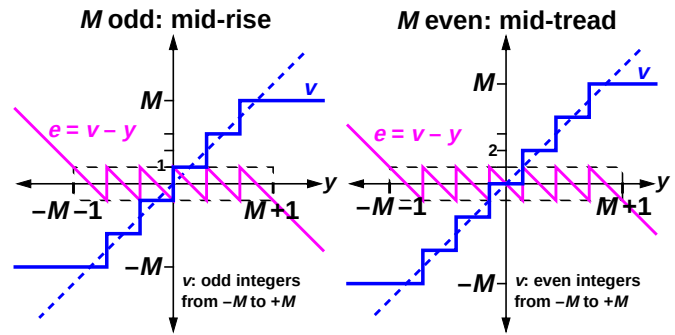
- In MODN $H(z) = (1 - z^{-1})^N$, so $h(n) = \{1, -a_1, a_2, -a_3, \dots, (-1)^N a_N, 0, \dots\}$, $a_i > 0$ and thus $\|h\|_1 = H(-1) = 2^N$
- $nlev = 2^N$ implies $u_{max} = nlev + 1 - \|h\|_1 = 1$
MODN is guaranteed to be stable with an N -bit quantizer if the input magnitude is less than $\Delta/2 = 1$. This result is quite conservative.
- Similarly, $nlev = 2^{N+1}$ guarantees that MODN is stable for inputs up to 50% of full-scale

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M-Step Symmetric Quantizer

$$\Delta = 2, (nlev = M + 1)$$



- No-overload range: $|y| \leq nlev \Rightarrow |e| \leq \Delta/2 = 1$

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Inductive Proof of $\|h\|_1$ Criterion

- Assume $STF = 1$ and $(\forall n)(|u(n)| \leq u_{max})$
- Assume $|e(i)| \leq 1$ for $i < n$. [Induction Hypothesis]

$$\begin{aligned} |y(n)| &= |u(n) + \sum_{i=1}^{\infty} h(i)e(n-i)| \\ &\leq u_{max} + \sum_{i=1}^{\infty} |h(i)| |e(n-i)| \\ &\leq u_{max} + \sum_{i=1}^{\infty} |h(i)| = u_{max} + \|h\|_1 - 1 \end{aligned}$$

$$\begin{aligned} \text{Then } u_{max} &= nlev + 1 - \|h\|_1 \\ \Rightarrow |y(n)| &\leq nlev \\ \Rightarrow |e(n)| &\leq 1 \end{aligned}$$

- So by induction $|e(i)| \leq 1$ for all $i > 0$

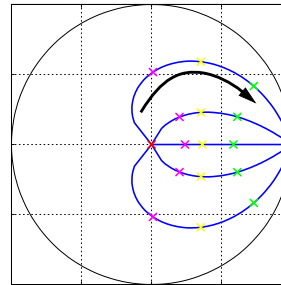
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More General NTF

- Instead of $NTF(z) = A(z)/B(z)$ with $B(z) = z^n$, use a more general $B(z)$

Roots of B are the poles of the NTF and must be inside the unit circle.



Moving the poles away from $z = 1$ toward $z = 0$ makes the gain of the NTF approach unity.

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The Lee Criterion for Stability in a 1-bit Modulator: $\|H\|_{\infty} \leq 2$ [Wai Lee, 1987]

- The measure of the "gain" of H is the maximum magnitude of H over frequency, aka the *infinity-norm* of H : $\|H\|_{\infty} \equiv \max_{\omega \in [0, 2\pi]} |H(e^{j\omega})|$

Q: Is the Lee criterion necessary for stability?

No. MOD2 is stable (for DC inputs less than FS) but $\|H\|_{\infty} = 4$.

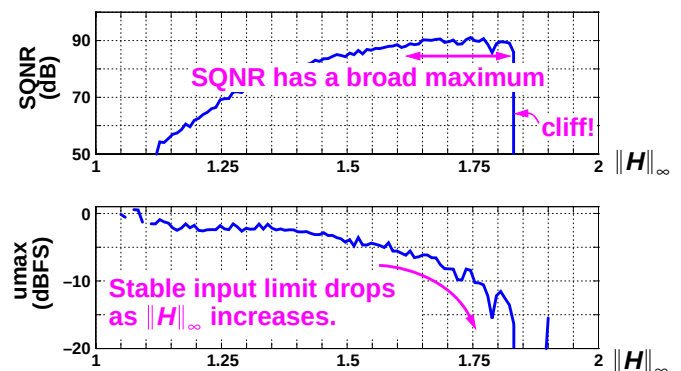
Q: Is the Lee criterion sufficient to ensure stability?

No. There are lots of counter-examples, but $\|H\|_{\infty} \leq 1.5$ often works.

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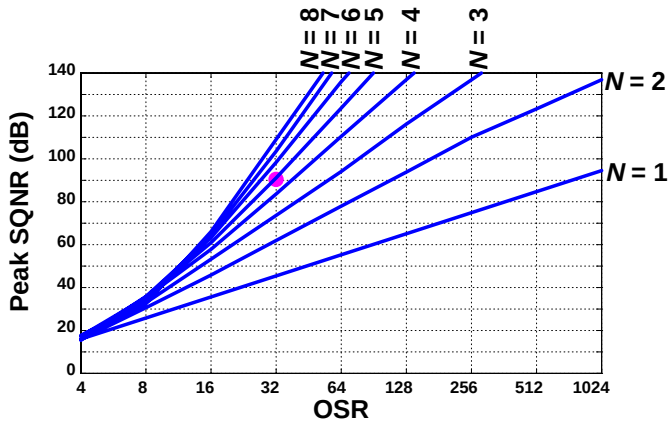
Simulated SQNR vs. $\|H\|_{\infty}$ 5th-order NTFs; 1-b Quant.; OSR = 32



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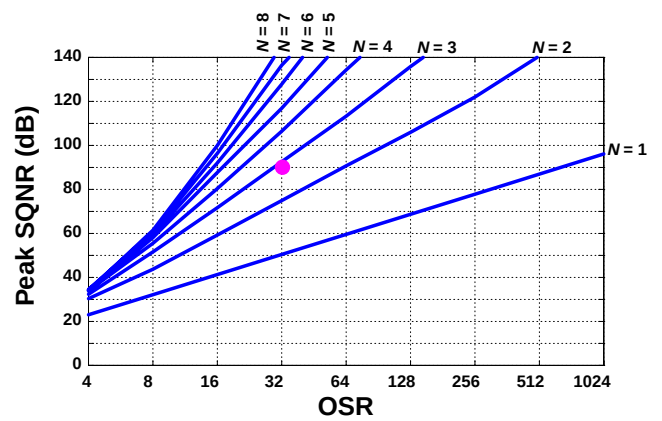
SQNR Limits— 1-bit Modulation



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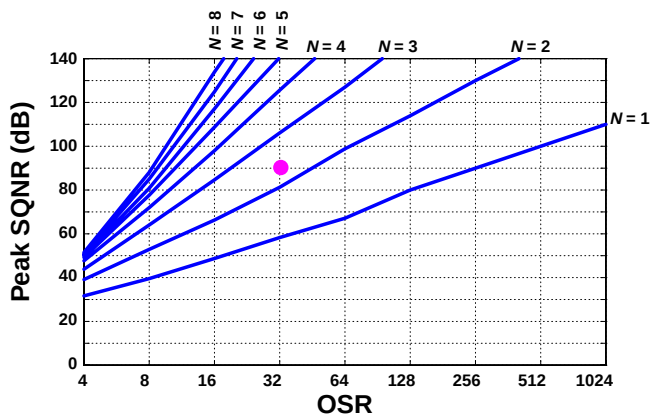
SQNR Limits for 2-bit Modulators



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SQNR Limits for 3-bit Modulators

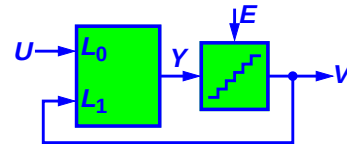


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Generic Single-Loop $\Delta\Sigma$ ADC

- Linear Loop Filter + Nonlinear Quantizer:



$$\begin{aligned} Y &= L_0 U + L_1 V \\ V &= Y + E \end{aligned} \Rightarrow \boxed{V = STF \cdot U + NTF \cdot E}, \text{ where}$$

$$NTF = \frac{1}{1 - L_1} \quad \& \quad STF = L_0 \cdot NTF$$

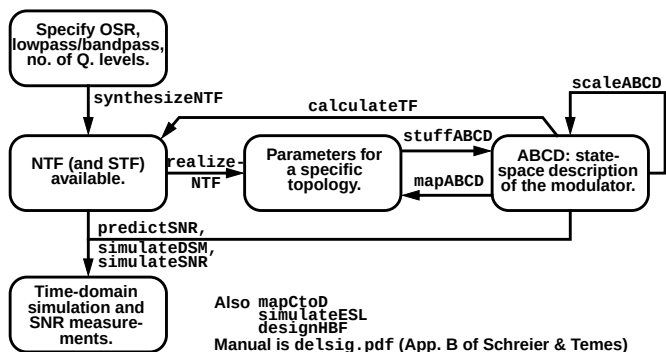
Inverse Relations:
 $L_1 = 1 - 1/NTF, L_0 = STF / NTF$

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$\Delta\Sigma$ Toolbox

<http://www.mathworks.com/matlabcentral/fileexchange>
 Search for "Delta Sigma Toolbox"

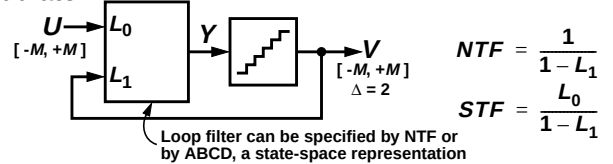


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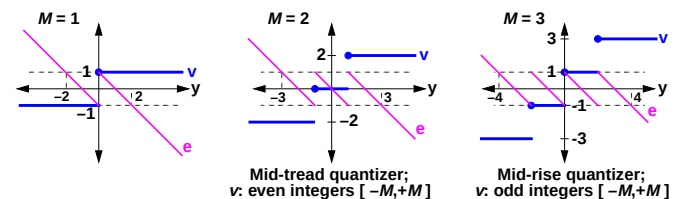
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$\Delta\Sigma$ Toolbox Modulator Model

Modulator:



Quantizer:



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NTF Synthesis

- Not all NTFs are realizable
Causality requires $h(0) = 1$, or, in the frequency domain, $H(\infty) = 1$. Recall $H(z) = h(0)z^0 + h(1)z^{-1} + \dots$
- Not all NTFs yield stable modulators
Rule of thumb for single-bit modulators:
 $\|H\|_{\infty} < 1.5$ [Lee].
- Can optimize NTF zeros to minimize the mean-square value of H in the passband
- The NTF and STF share poles, and in some modulator topologies the STF zeros are not arbitrary
Restrict the NTF such that an all-pole STF is maximally flat. (Almost the same as Butterworth poles.)

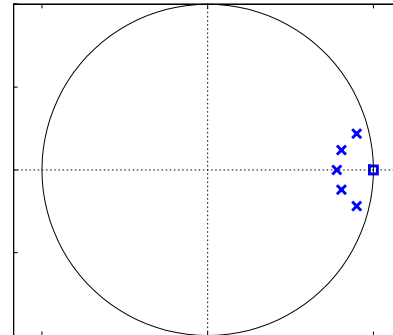
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Lowpass Example [dsdemo1]

5th-order NTF, all zeros at DC

- Pole/Zero diagram:



```
OSR = 32;
H = synthesizNTF(5);
plotPZ(H);

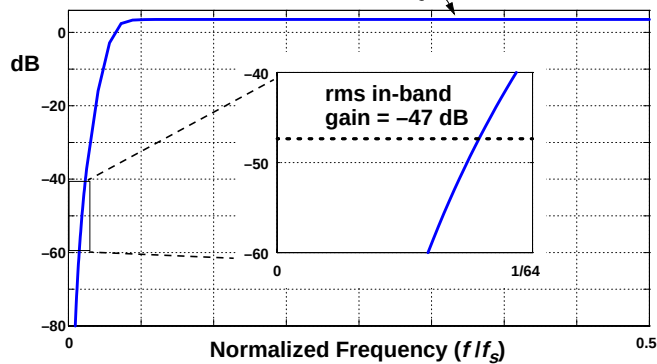
f = linspace(0,0.5);
z = exp(2i*pi*f);
H_z = evalTF(H,z);
plot(f,dbv(H_z));
g = rmsGain(H,0,0.5/OSR)
```

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Lowpass NTF

Out-of-band gain = 1.5

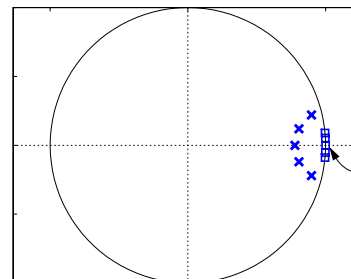


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Improved 5th-Order Lowpass NTF

Zeros optimized for OSR=32



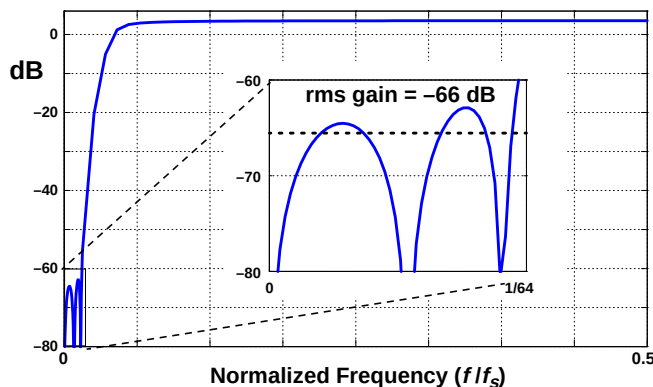
Zeros spread across the band-of-interest to minimize the rms value of the NTF.

```
OSR = 32;
H = synthesizNTF(5,OSR,1);
...
optimization flag
```

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Improved NTF



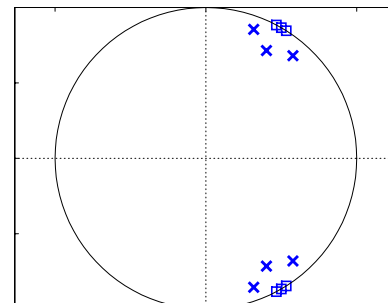
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Bandpass Example

```
OSR = 64;
f0 = 1/6;
H=synthesizNTF(6,OSR,1,[],f0);...
```

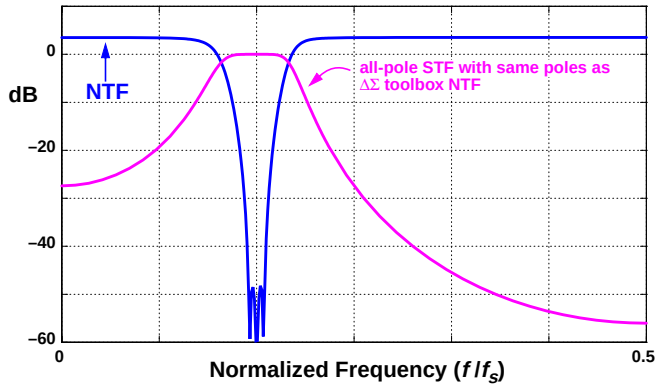
[] or NaN means Use default value, i.e. Hinf = 1.5



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Bandpass NTF and STF



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Summary: NTF Selection

- If OSR is high, a single-bit modulator may work
- To improve SQNR,
 - Optimize zeros,
 - Increase $\|H\|_{\infty}$, or
 - Increase order.
- If SQNR is insufficient, must use a multi-bit design
 - Can turn all the above knobs to enhance performance.
- Feedback DAC assumed to be ideal

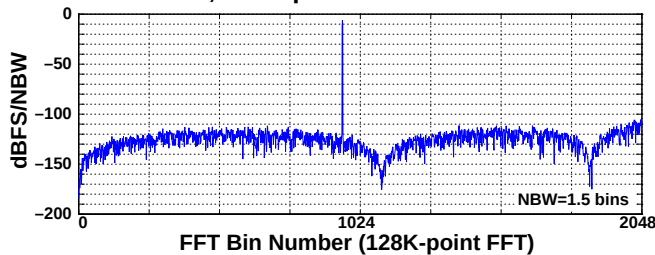
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NTF-Based Simulation [dsdemo2]

```
order=5; OSR=32;
ntf = synthesizNTF(order,OSR,1);
N=2^17; fbin=959; A=0.5; % 128K points
input = A*sin(2*pi*fbin/N*[0:N-1]);
output = simulateDSM(input,ntf);
spec = fft(output.*ds_hann(N)/(N/4));
plot(dB(spec(1:N/(2*OSR))));
```

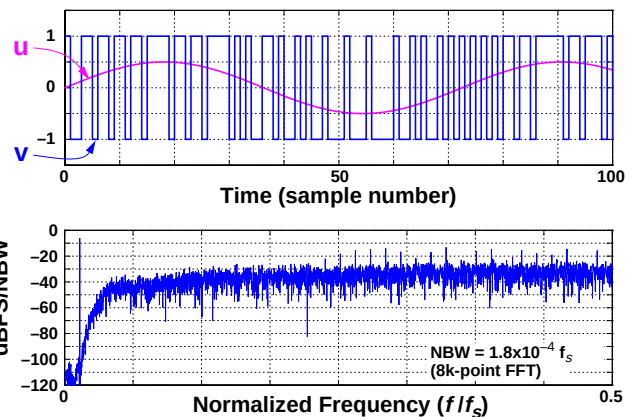
- In mex form; 128K points in < 0.1 sec



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Simulation Example Cont'd

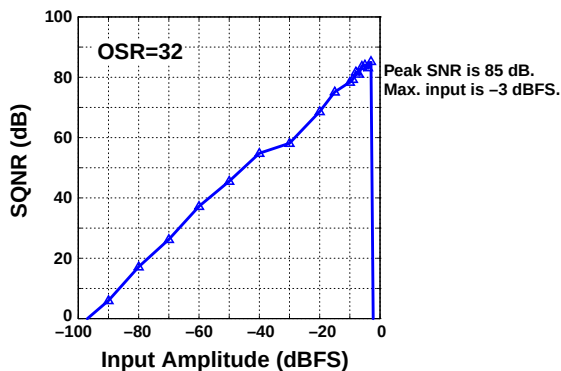


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SNR vs. Amplitude: simulateSNR

```
[snr amp] = simulateSNR(ntf,OSR);
plot(amp,snr,'b-^');
```



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Homework #2 (Due 2015-01-19)

A. Extract code from dsdemo1 & dsdemo2 to:

- 1 Create a 3rd-order NTF with zeros optimized for OSR = 32 and $\|NTF\|_{\infty} = 2$. Plot the poles/zeros and frequency response of your NTF.
- 2 Simulate an 8-step (9-level) $\Delta\Sigma$ modulator with this NTF. Plot example input and output waveforms. Plot a spectrum and the predicted noise curve.* Plot the SQNR vs. input amplitude curve and note the maximum stable input.

B. Compose your own short question and answer it.

*. Beware that with an M -step modulator the full-scale is M .

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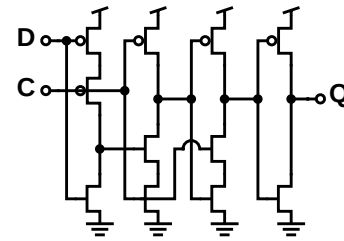
What You Learned Today And what the homework should solidify

- 1 N^{th} -order modulator (MODN)
- 2 High-level design with the $\Delta\Sigma$ Toolbox

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NLCOTD: True Single-Phase Dynamic FF

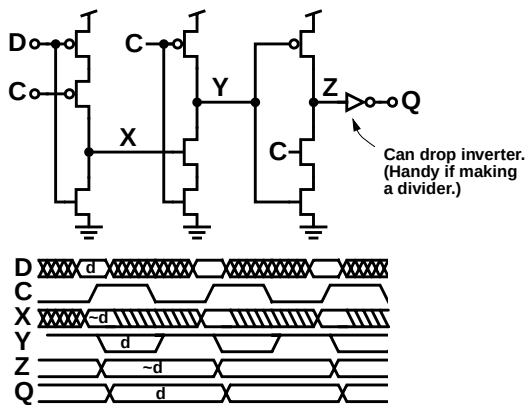


- + Clock not inverted anywhere
- + Small
- + Fast

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TSPFF Operation



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TSPFF Gotchas

- **Leakage:**
Won't work if clock is too slow.
Possible high current if clock is stopped.
Need to add devices that hold the dynamic nodes at a safe value.
- **No positive feedback**
Vulnerable to metastability.

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