ECE1371 Advanced Analog Circuits
Lecture 8

COMPARATOR & FLASH ADC DESIGN

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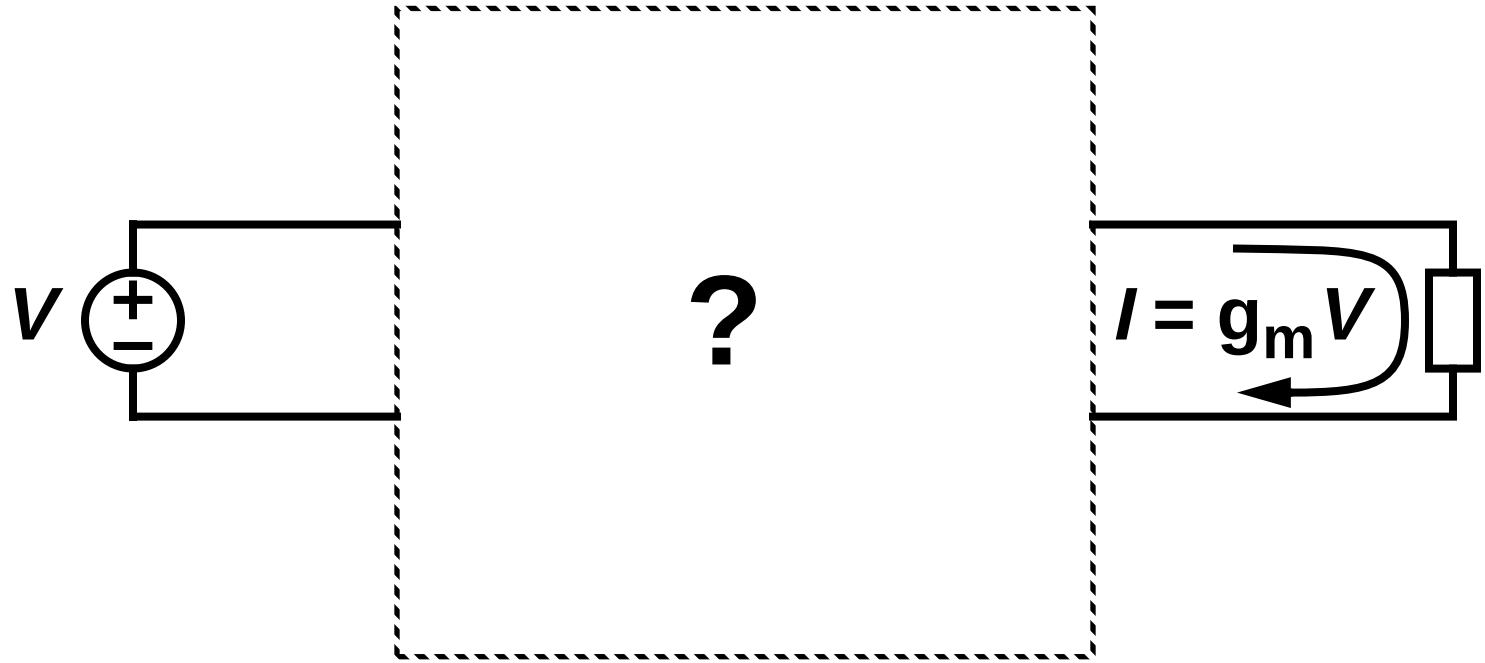
Course Goals

- **Deepen understanding of CMOS analog circuit design through a top-down study of a modern analog system— a delta-sigma ADC**
- **Develop circuit insight through brief peeks at some nifty little circuits**

  The circuit world is filled with many little gems that every competent designer ought to know.

Date	Lecture (M 13:00-15:00)			Ref	Homework
2015-01-05	RS	1	MOD1 & MOD2	ST 2, 3, A	1: Matlab MOD1&2
2015-01-12	RS	2	MOD*N* + $\Delta\Sigma$ Toolbox	ST 4, B	2: $\Delta\Sigma$ Toolbox
2015-01-19	RS	3	Example Design: Part 1	ST 9.1, CCJM 14	3: Sw.-level MOD2
2015-01-26	RS	4	Example Design: Part 2	CCJM 18	
2015-02-02	TC	5	SC Circuits	R 12, CCJM 14	4: SC circuit
2015-02-09	TC	6	Amplifier Design		
2015-02-16	Reading Week– No Lecture				
2015-02-23	TC	7	Amplifier Design		5: SC Int w/ Amp
2015-03-02	RS	8	Comparator & Flash ADC	CCJM 10	Project
2015-03-09	TC	9	Noise in SC Circuits	ST C	
2015-03-16	RS	10	Advanced $\Delta\Sigma$	ST 6.6, 9.4	
2015-03-23	TC	11	Matching & MM-Shaping	ST 6.3-6.5, +	
2015-03-30	TC	12	Pipeline and SAR ADCs	CCJM 15, 17	
2015-04-06	Exam			Proj. Report Due Friday April 10	
2015-04-13	Project Presentation				

NLCOTD: Linear Transconductor

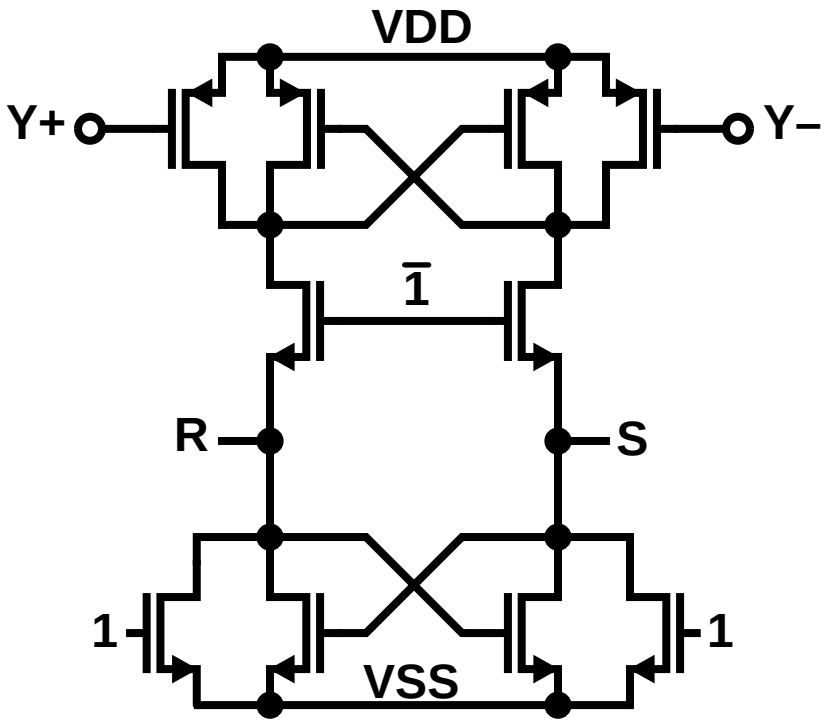


- **Useful in**
  1. **gm-C filter**
  2. **LNA**
  3. **mixer**
  4. **CT $\Delta\Sigma$ ADC**

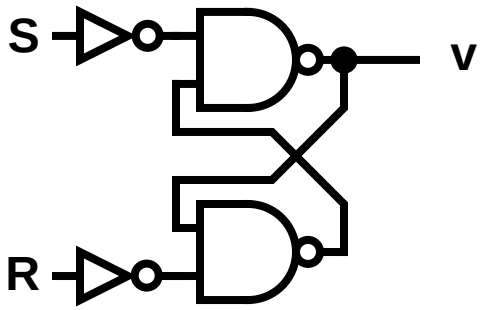
Highlights
(i.e. What you will learn today)

1. Operation of Example Comparator Circuit
2. Regeneration Time Constant τ
3. Metastability, Probability of Error
4. Offset, Dynamic Offset, Auto-zeroing
5. Flash ADC
6. A Bunch of Transconductor Circuits

Review: Latched Comparator
From Lecture #4's 1-MHz MOD2



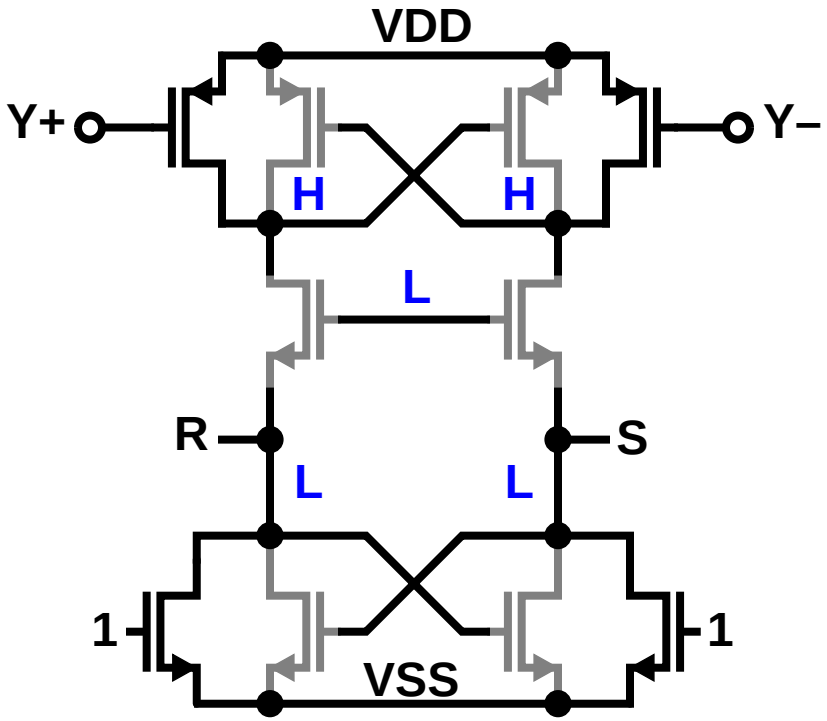
Set/Reset Latch:

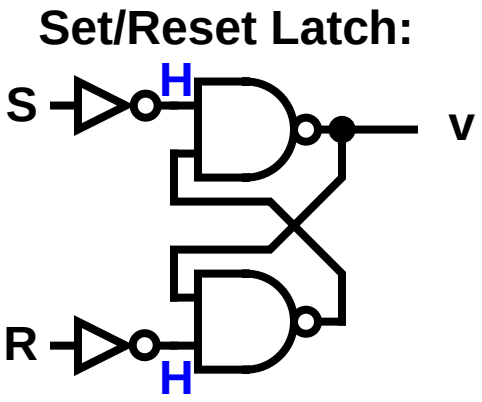


Inverter thresholds are chosen so that the inverters respond only after R/S have resolved.

- **Falling phase 1 initiates regenerative action**
  S and R connected to a Set/Reset latch.

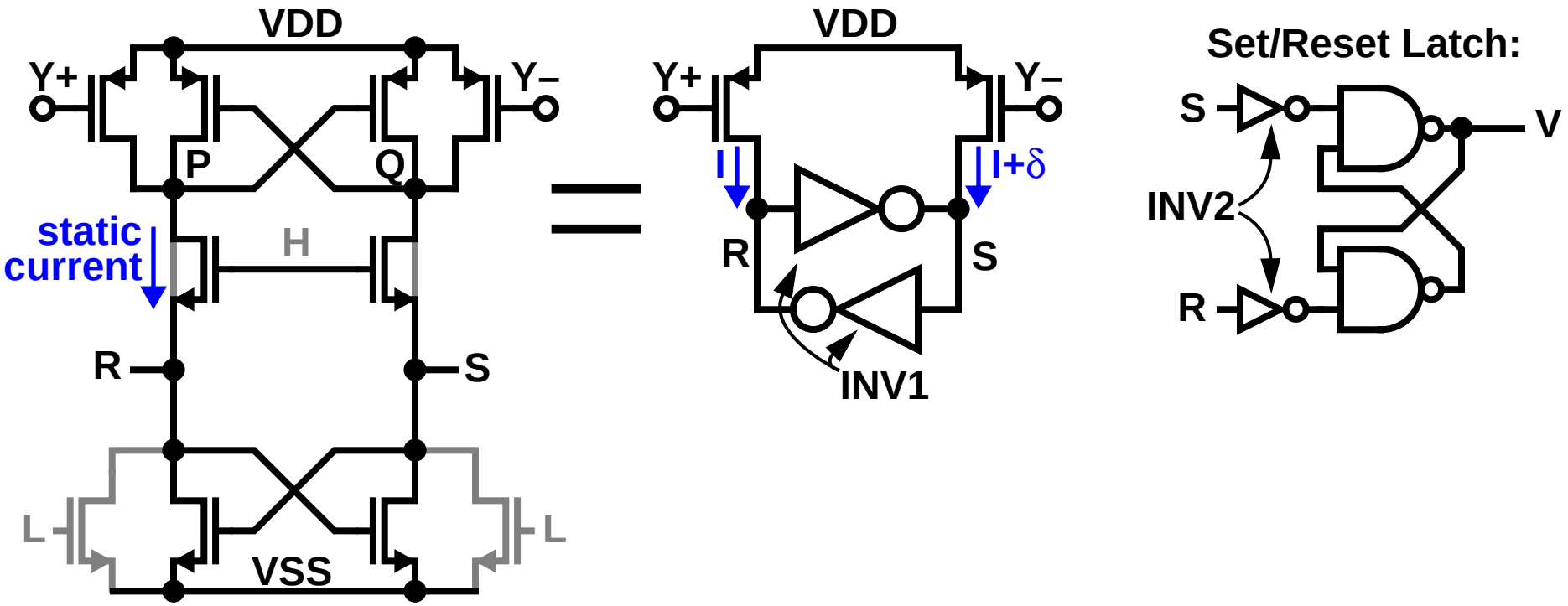
Phase 1 = High: “Reset” Mode

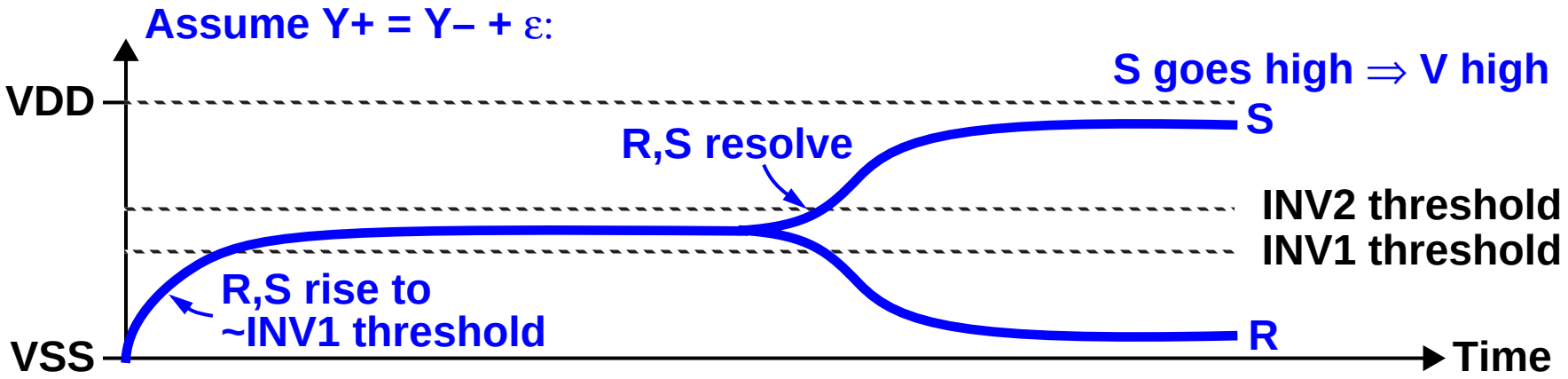




- **Grayed-out devices are off**
  $\Rightarrow$ the active part of the comparator is reset
- **R and S are low $\Rightarrow$ the SR latch is in hold mode**

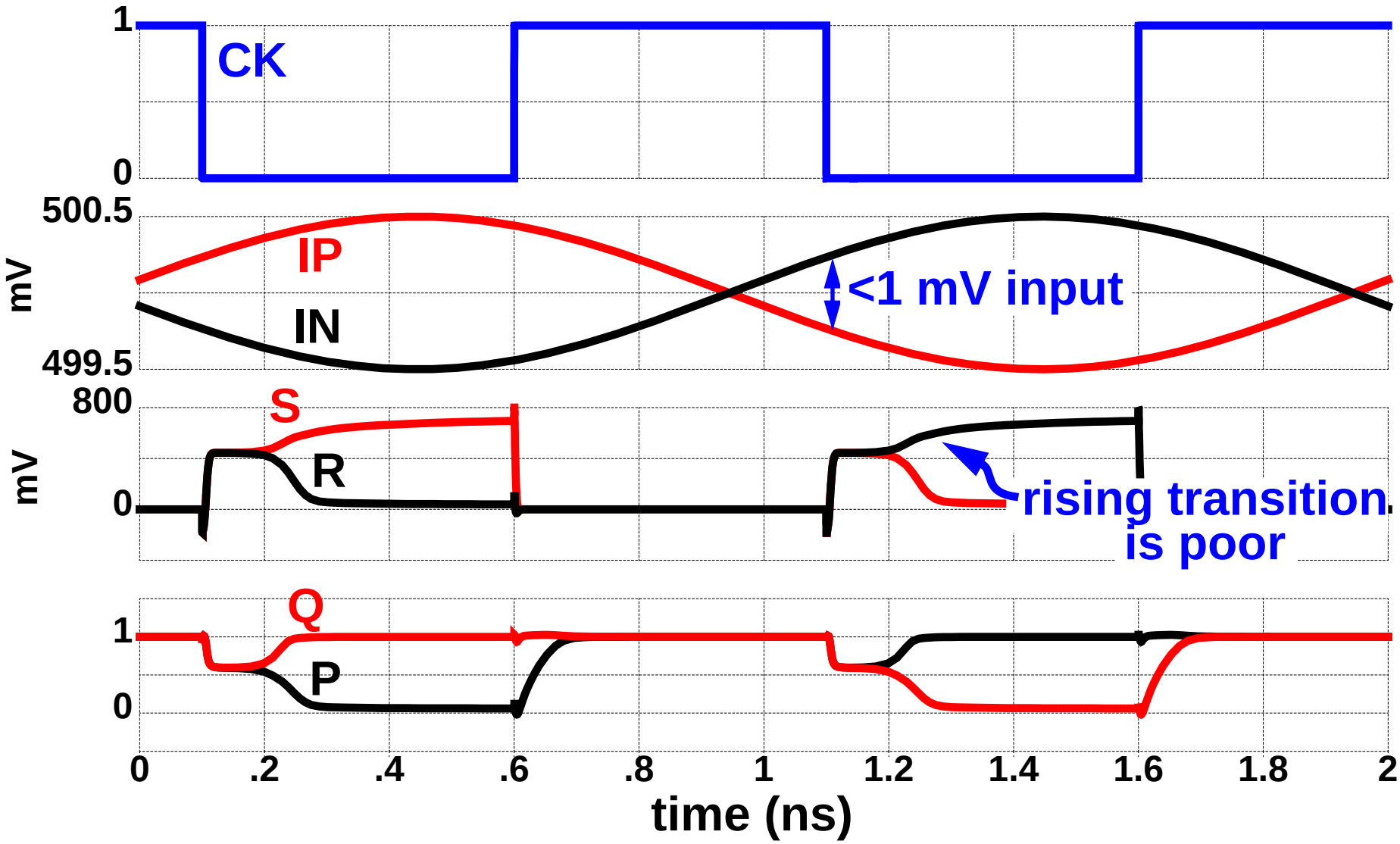
Phase 1 Goes Low: “Latch” Mode



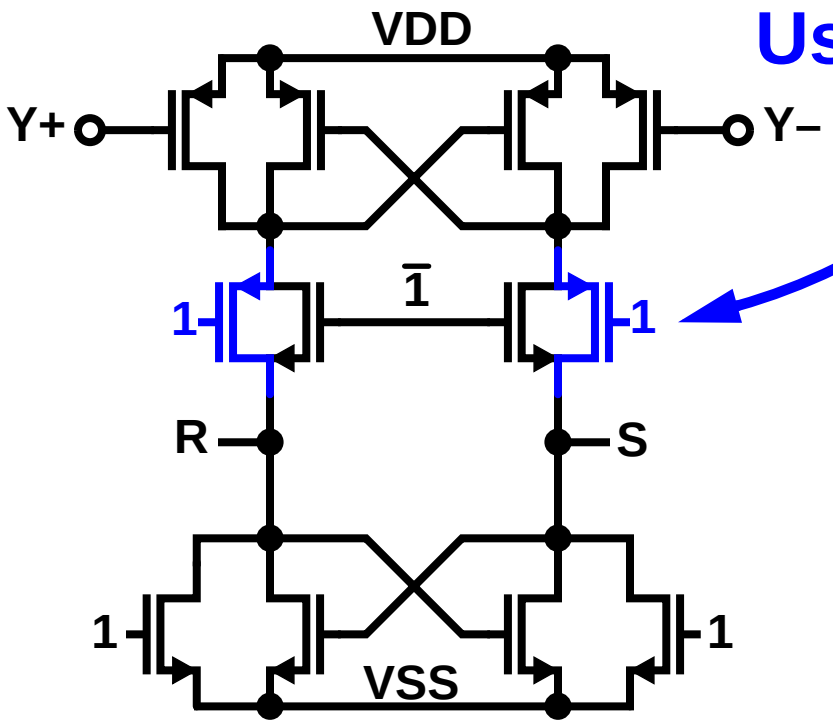


Example Waveforms

Quick Design in 65nm (VDD = 1V)

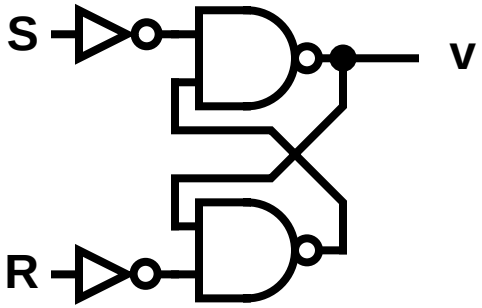


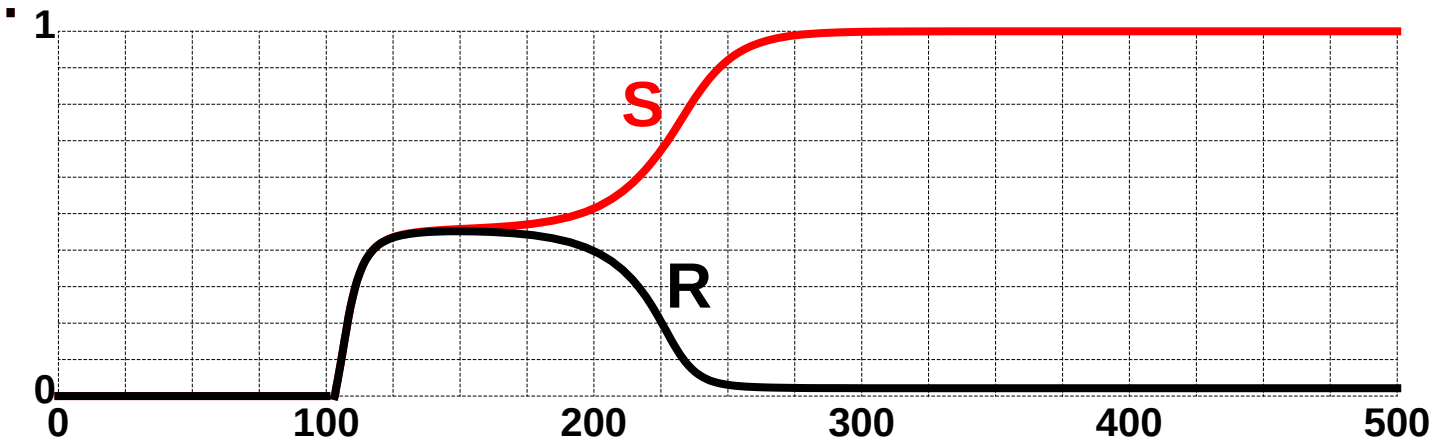
Better Design



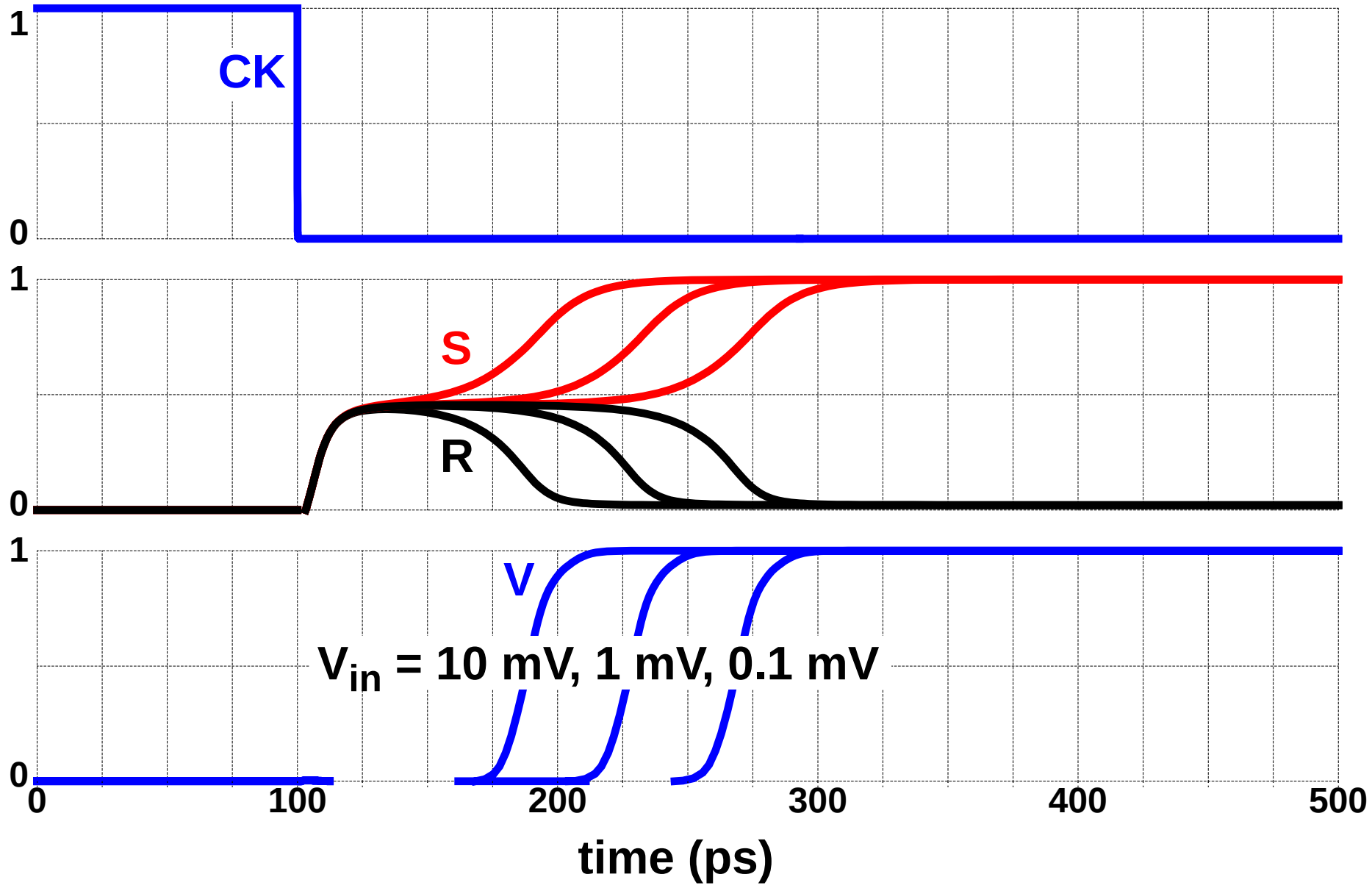
Use a transmission gate

Set/Reset Latch:

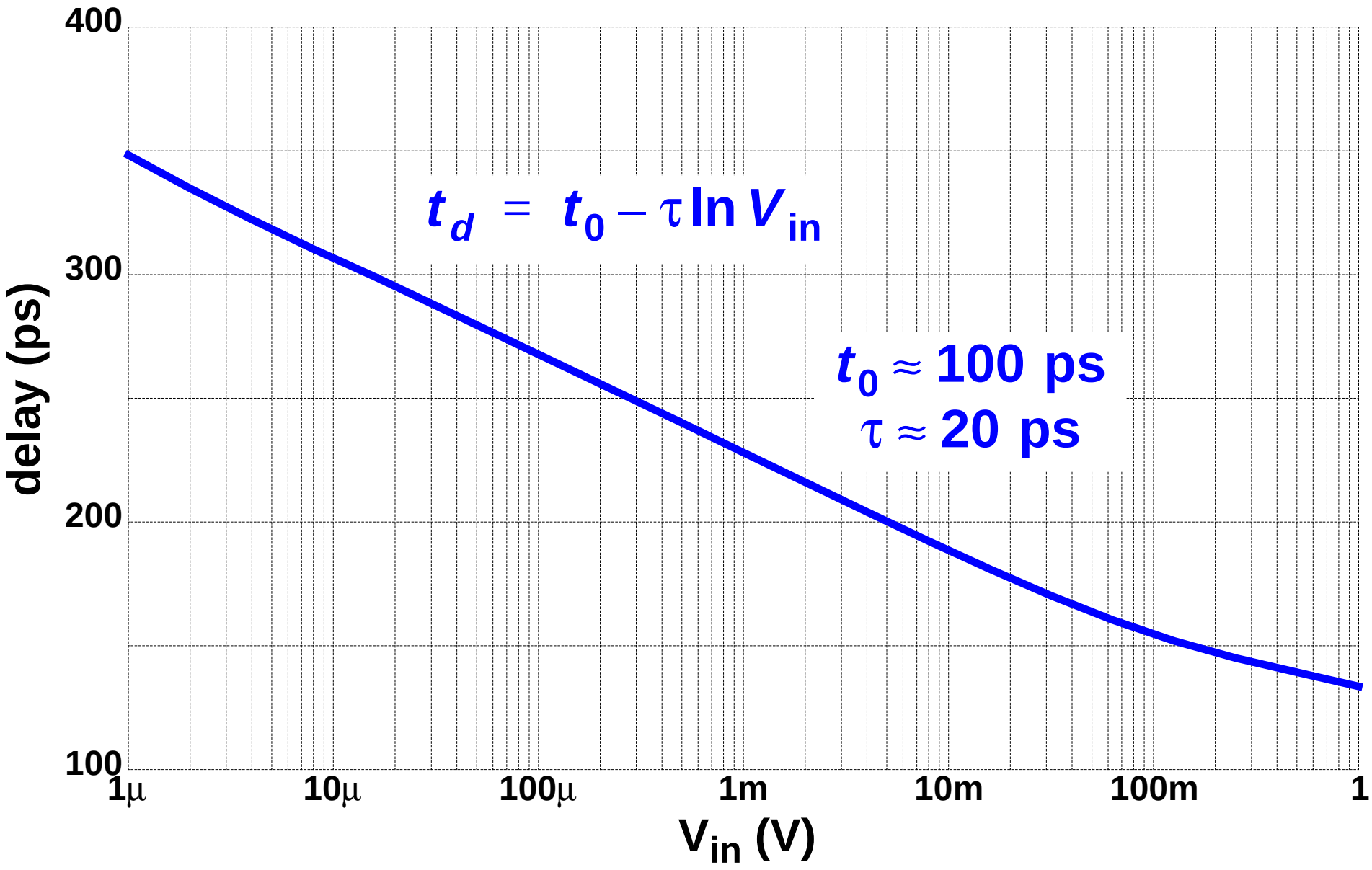




Responses for Various V_{in}



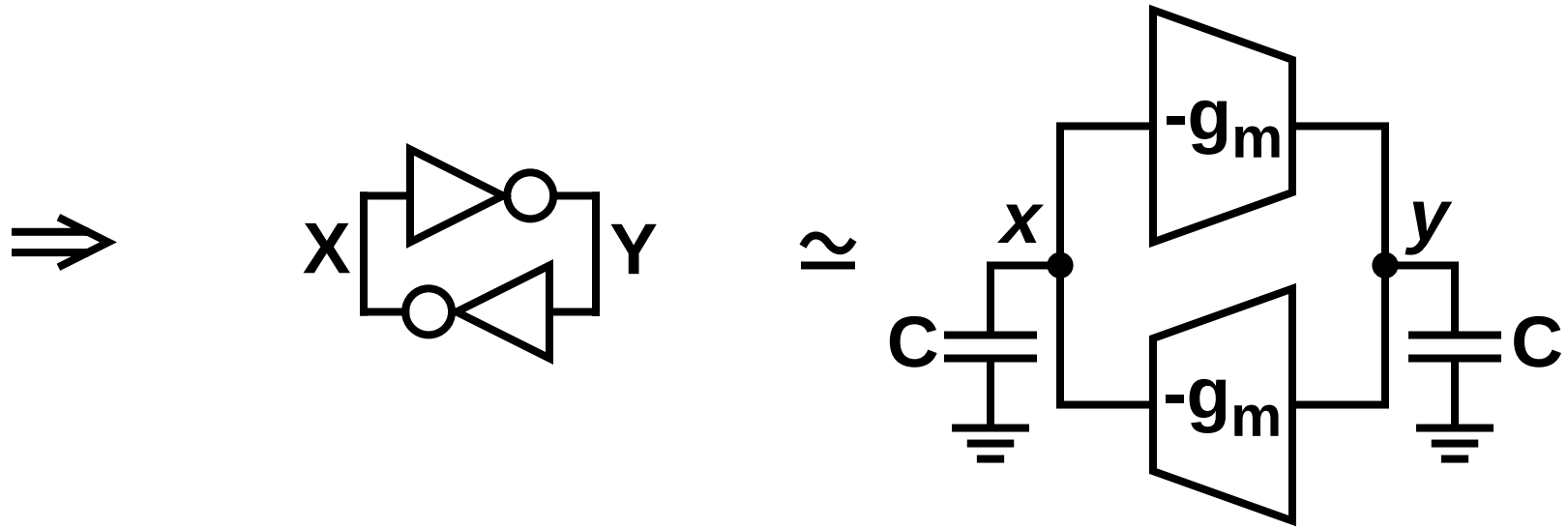
Delay vs. V_{in}


400
300
200
100
delay (ps)
$t_d = t_0 - \tau \ln V_{in}$
$t_0 \approx 100$ ps
$\tau \approx 20$ ps
1µ
10µ
100µ
1m
10m
100m
1
V_{in} (V)

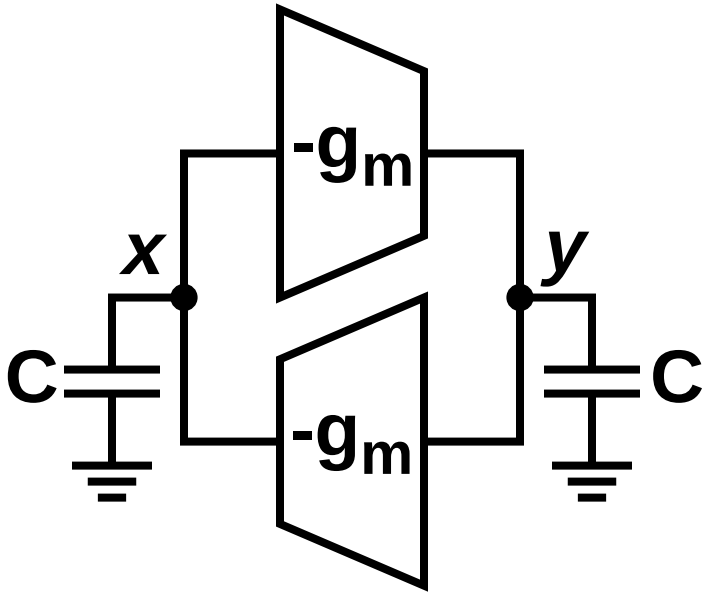
Latch Mode Dynamics

- **For V_{in} near the trip point, an inverter is essentially just a transconductor:**

- **So near balance the comparator looks like this:**



Small-Signal Analysis



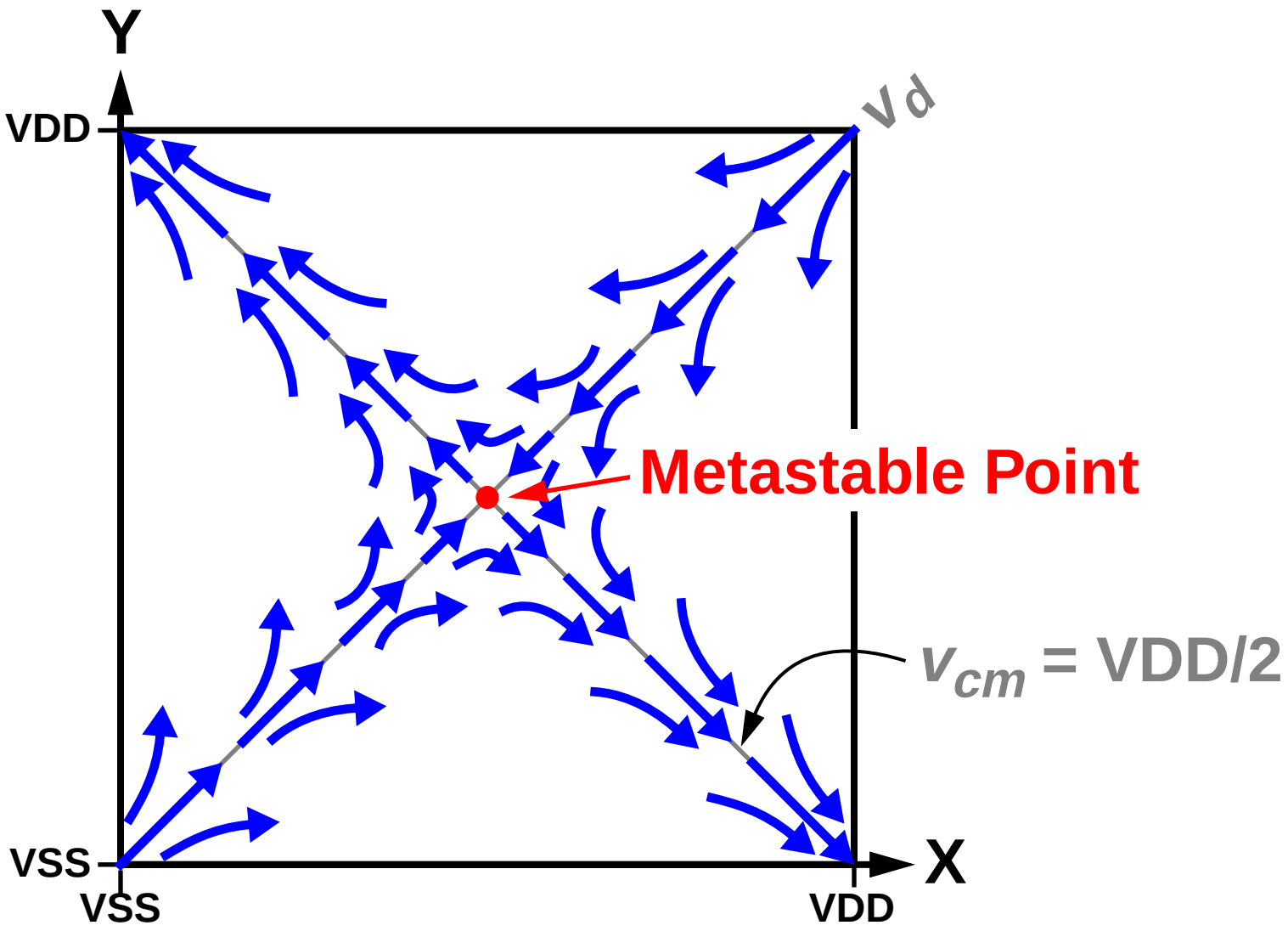
$$C\dot{x} = -g_m y$$
$$C\dot{y} = -g_m x$$

$$v_d = x - y$$
$$v_{cm} = \frac{x + y}{2}$$

$$C\dot{v}_d = g_m v_d \qquad C\dot{v}_{cm} = -g_m v_{cm}$$
$$v_d(t) = v_{d0} e^{t/\tau} \qquad v_{cm}(t) = v_{cm0} e^{-t/\tau}$$
$$\tau = \frac{C}{g_m}$$

- **Differential component grows exponentially**
- **CM component decays exponentially**

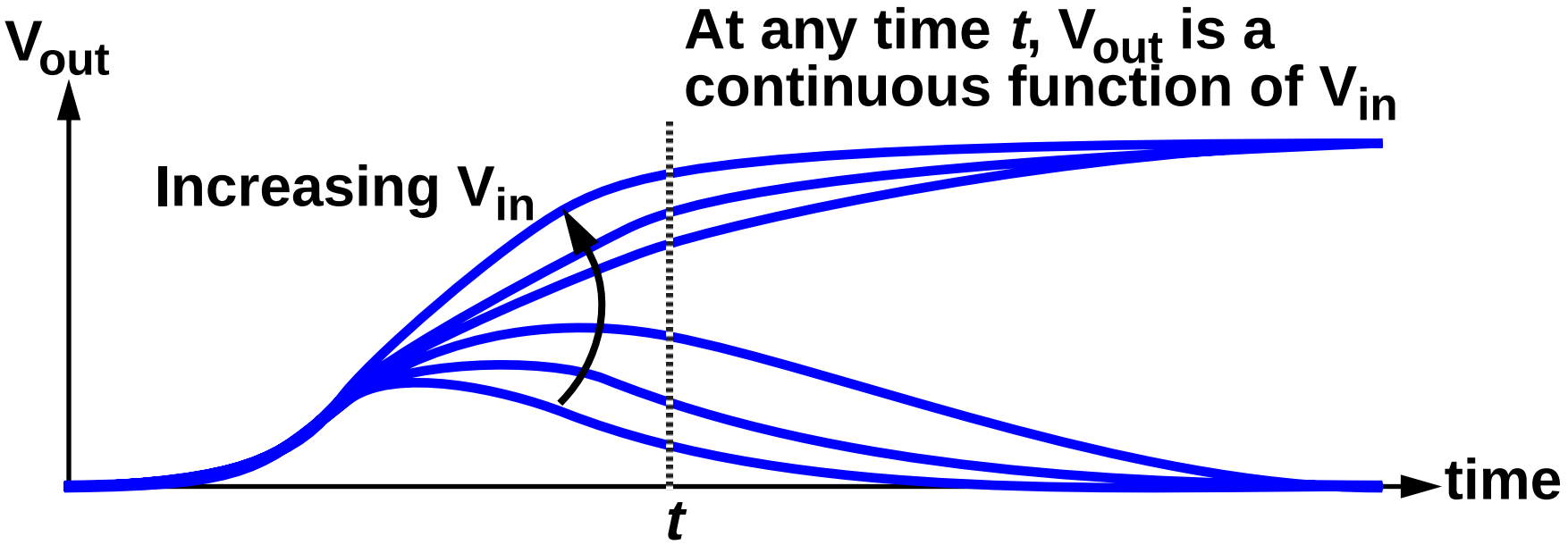
State-Space



- $v_{cm} \rightarrow$ VDD/2, $v_d \rightarrow \pm$VDD

Metastability

- **Metastability is fundamentally unavoidable**
- **Assuming the universe is continuous and deterministic, a comparator can be unresolved for any length of time**



Probability of Error, P_E

$$P_E = P\{\text{not resolved by time } t\}$$

$$= P\left\{ V_{in} < \exp -\left(\frac{t - t_0}{\tau} \right) \right\}$$

- Take $t_0 = 100$ ps and $\tau = 20$ ps
- Then for $t = 500$ ps (1 GHz clock with a half-cycle between the comparator's clock and the clock of the subsequent latch),

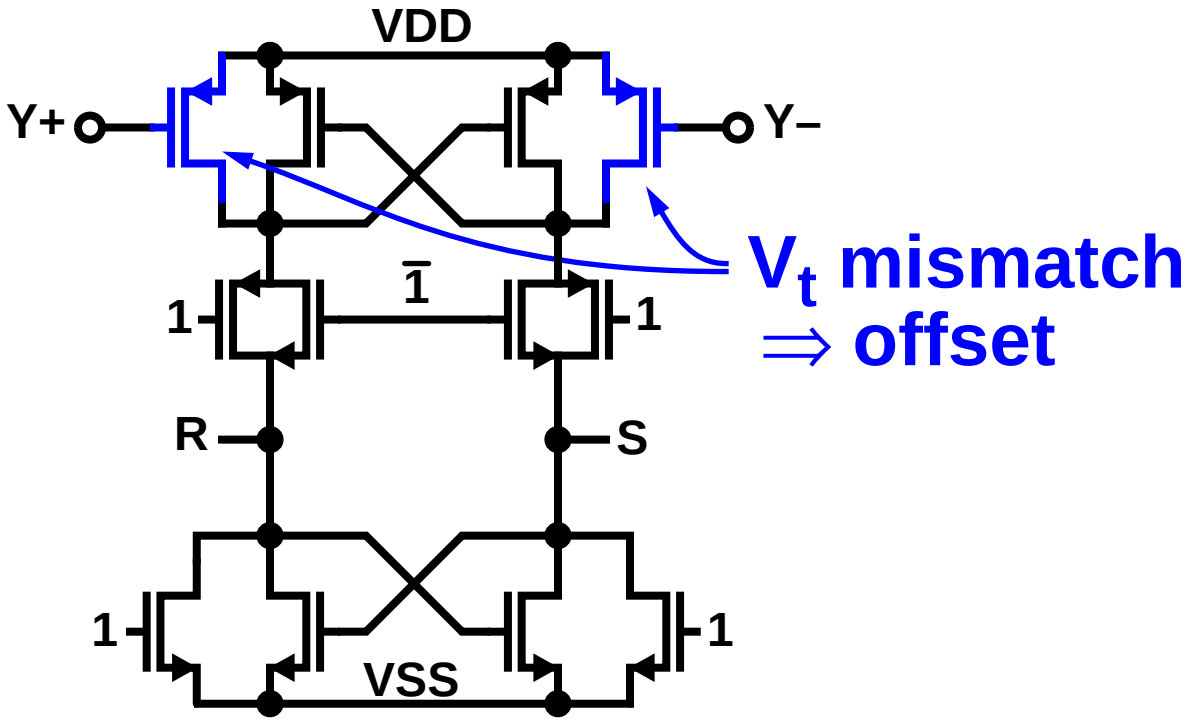
$$P_E = P\{|V_{in}| < 2 \text{ nV}\}$$

- Assuming V_{in} is uniformly distributed in [–0.5, +0.5] V, $P_E = 2 \times 10^{-9}$
  Metastability occurs twice a second!

Metastability Summary

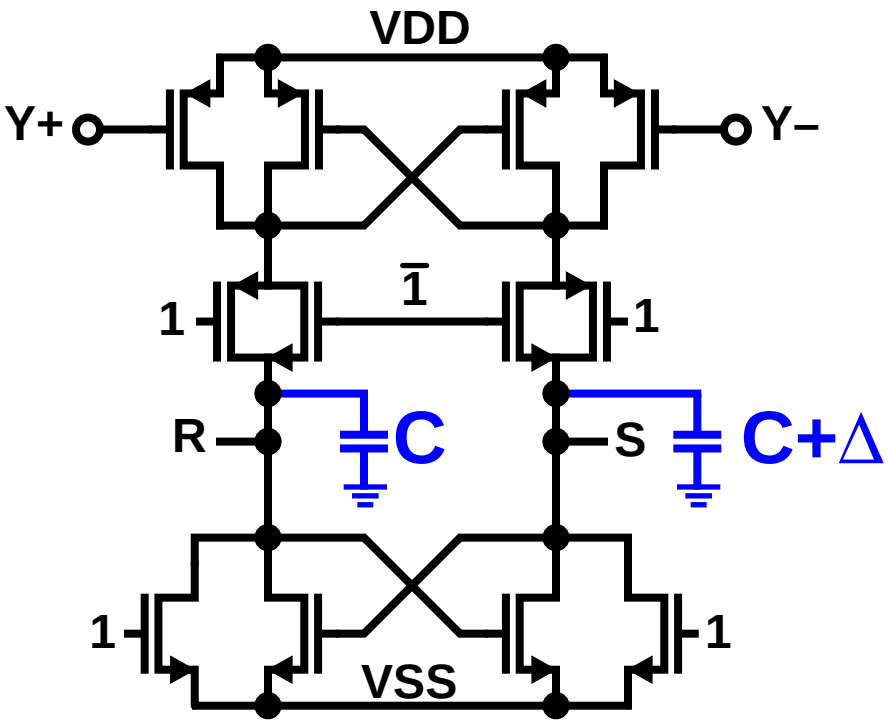
- **Metastability is unavoidable**
  - **All you can do is make τ small and give enough time for regeneration to make P_E small.**
- **Supplementary Slides examine the effects of quantized charge and noise**
  - **Noise helps a comparator resolve when $V_{in} = 0$ but usually does not reduce P_E**
- **At best, metastability causes SNR degradation**
  - **At worst, it causes a system failure.**

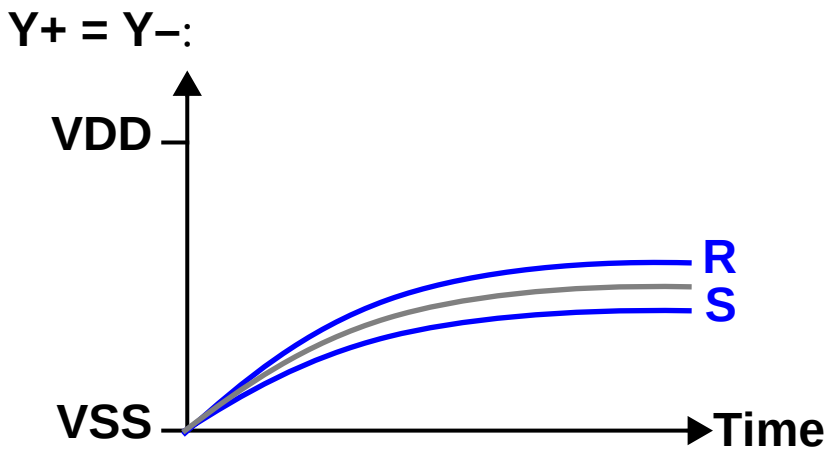
Offset



- **Obvious sources of offset include mismatch in the input differential pair as well as mismatch in the regenerating devices**

Dynamic Offset

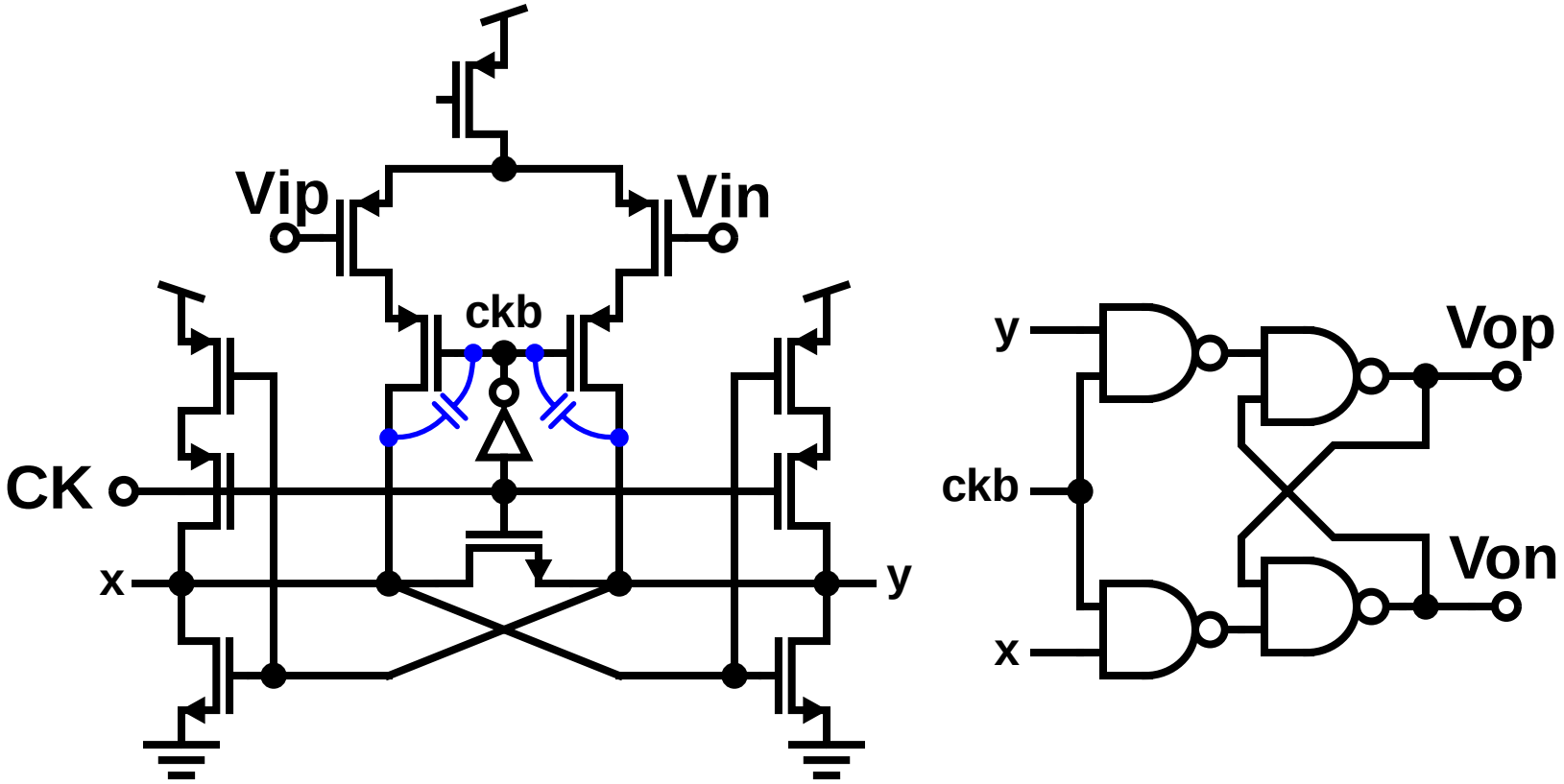




S rises more slowly than R
⇒ offset

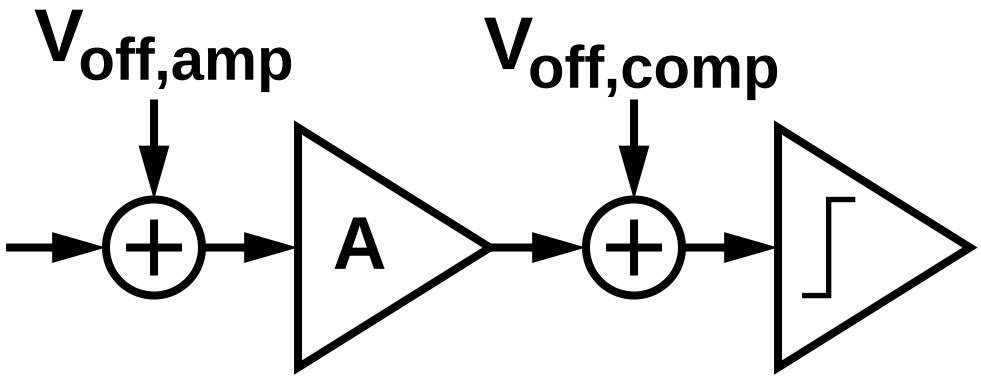
- **Mismatched parasitic capacitance also causes offset**
  - **20 mV/fF for this comparator!**
- **Bad design– Can fix this!**

Improved Comparator
[S&T Fig. 9.36]



- **Reset when CK = 1; regenerates when CK = 0**
- **x & y don't step if biased properly**
  Mismatch in overlap capacitance still a problem.

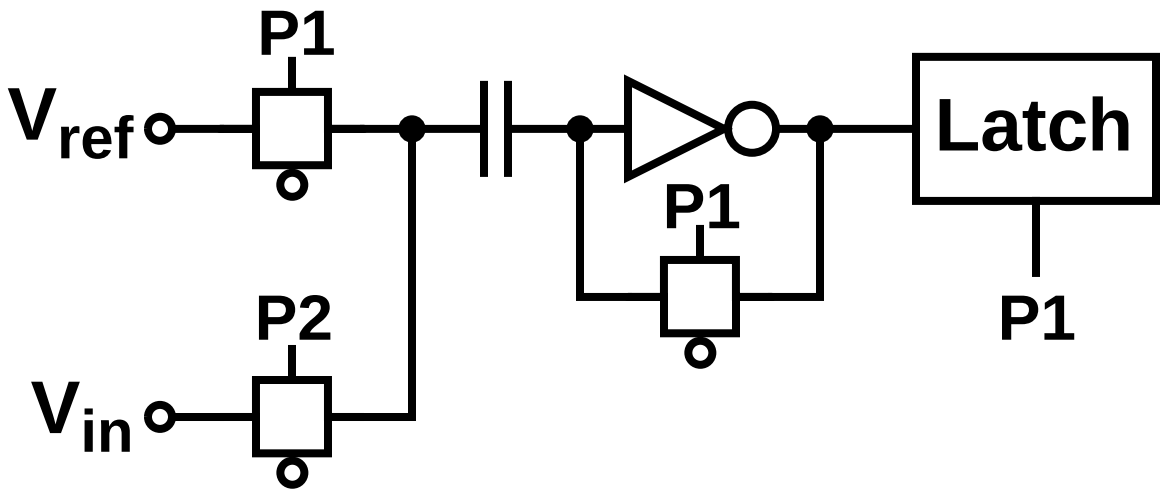
Reducing Offset with a Preamp



$$V_{off,tot} = V_{off,amp} + V_{off,comp}/A$$

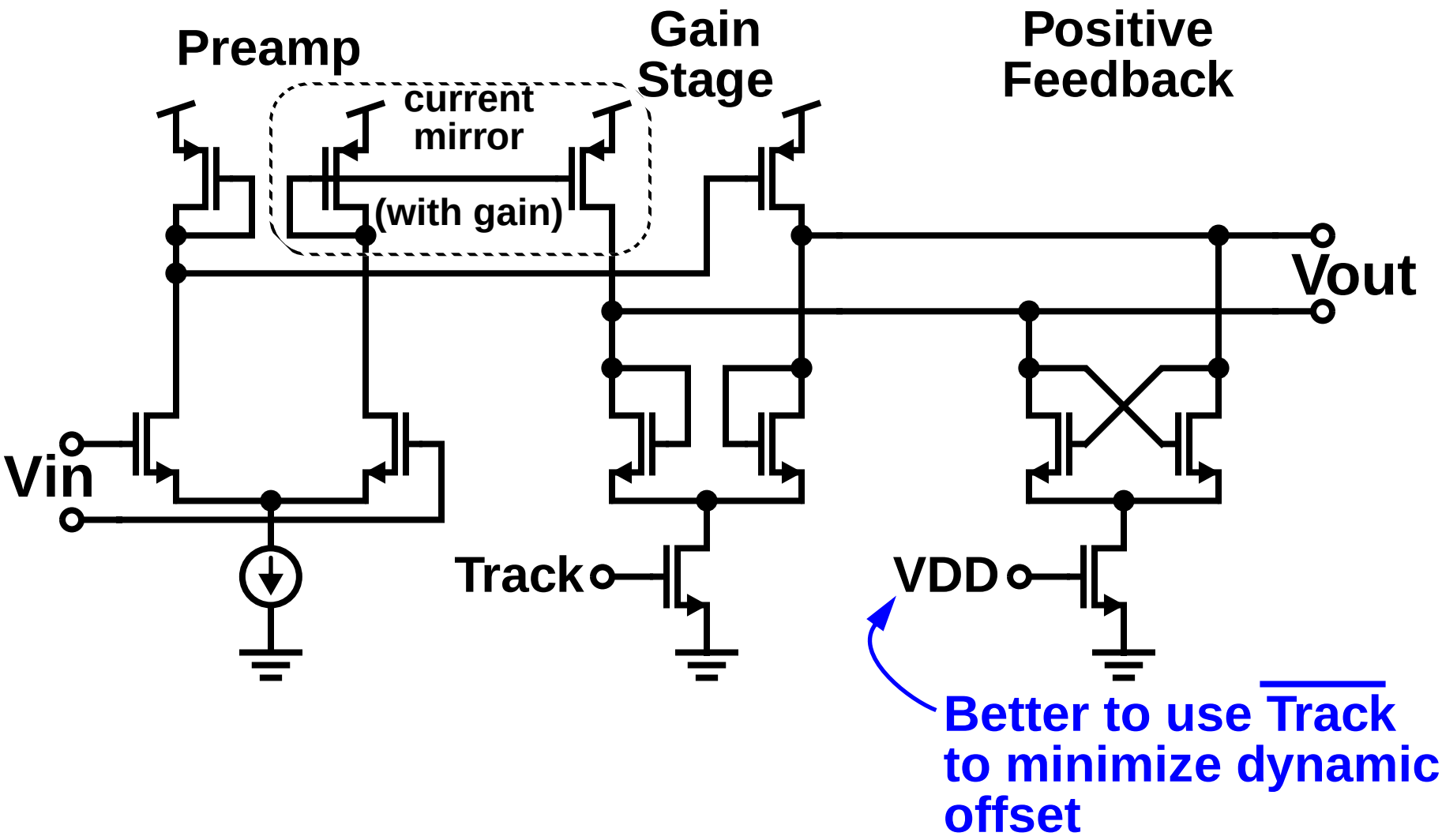
+ **Comparator offset is reduced by preamp gain**
  Amplifier offset dominates.
+ **Amplifier also isolates driving stage from "charge kickback"**
– **Amplifier bandwidth limits speed, especially recovery from overload**

Auto-zeroed SC Comparator
[J&M Fig 13.17]



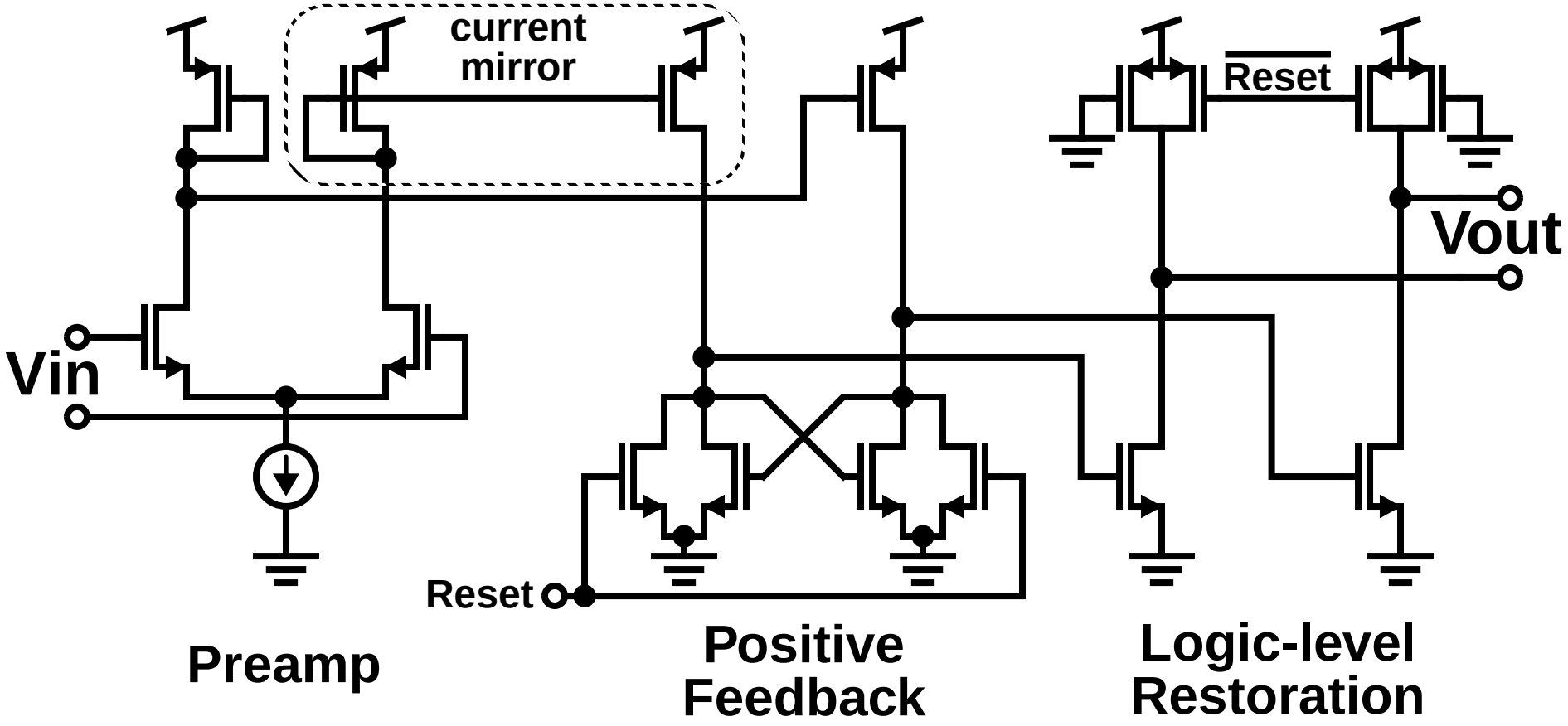
- During P1, the inverter/amplifier is biased at its threshold/offset voltage
- During P2, the difference between V_{in} and V_{ref} is amplified

Comparator with Preamp
[J&M Fig. 7.16]



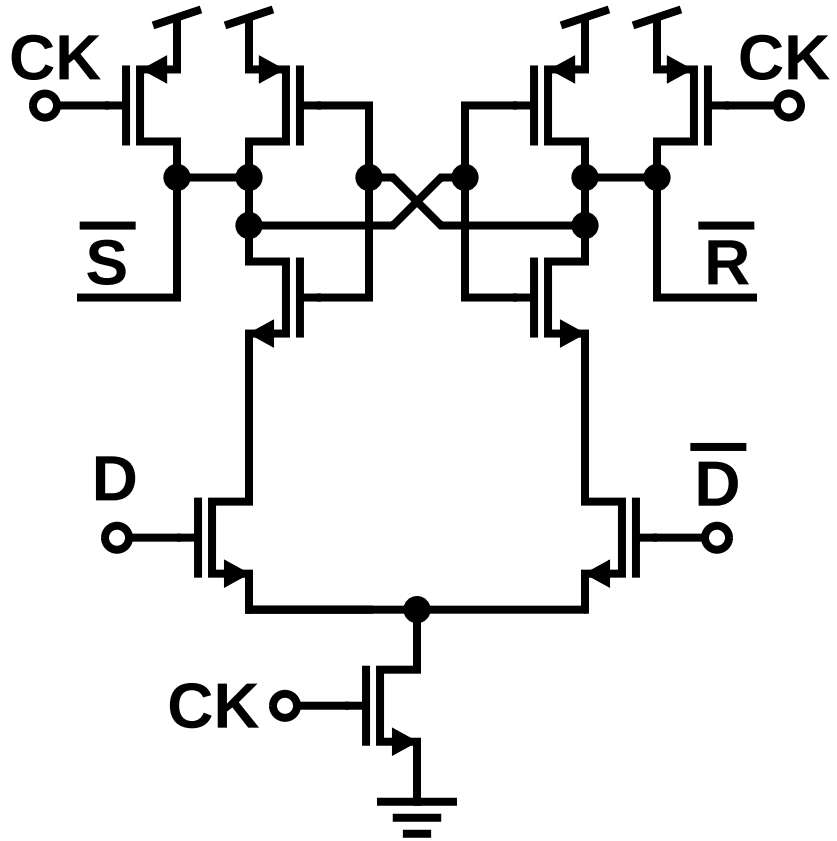
Two-Stage Comparator

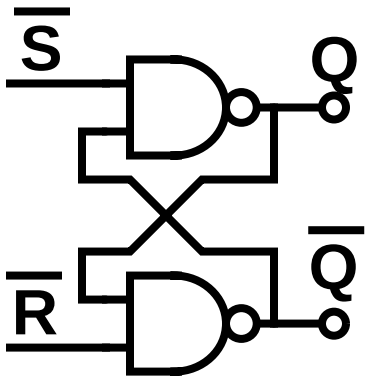
[J&M Fig 7.17]



- **Precharges regeneration nodes low & digital output nodes high**

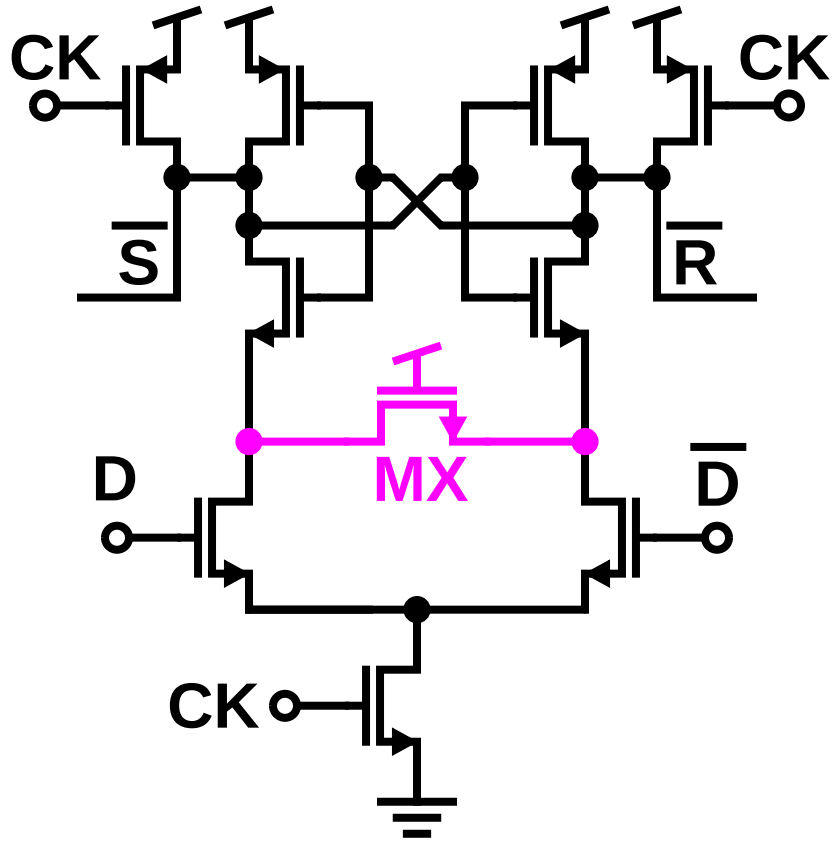
StrongArm Flip Flop

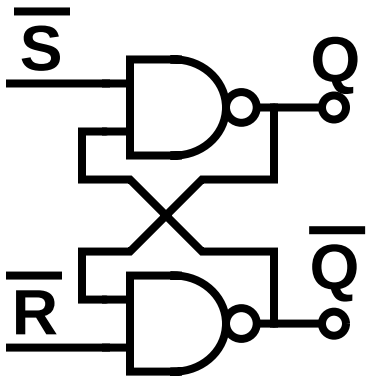




- + **No static current**
- + **Can be used as a comparator**
  - **Note that input CM voltage defines the bias point during regeneration.**

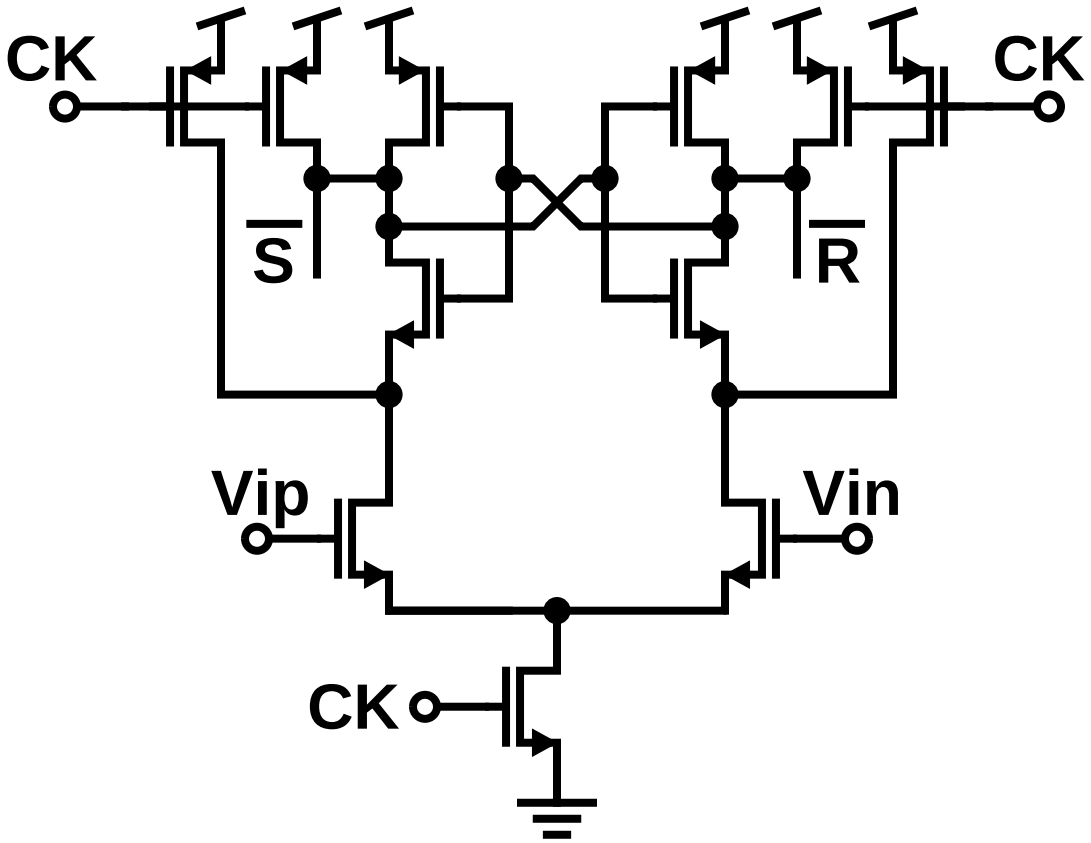
Improved StrongArm Design

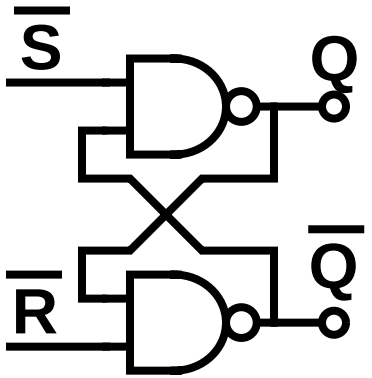




- **MX provides DC path to ground for leakage currents in case D changes after CK goes high**
  - **MX is needed to make latch static.**

Alternative Arrangement

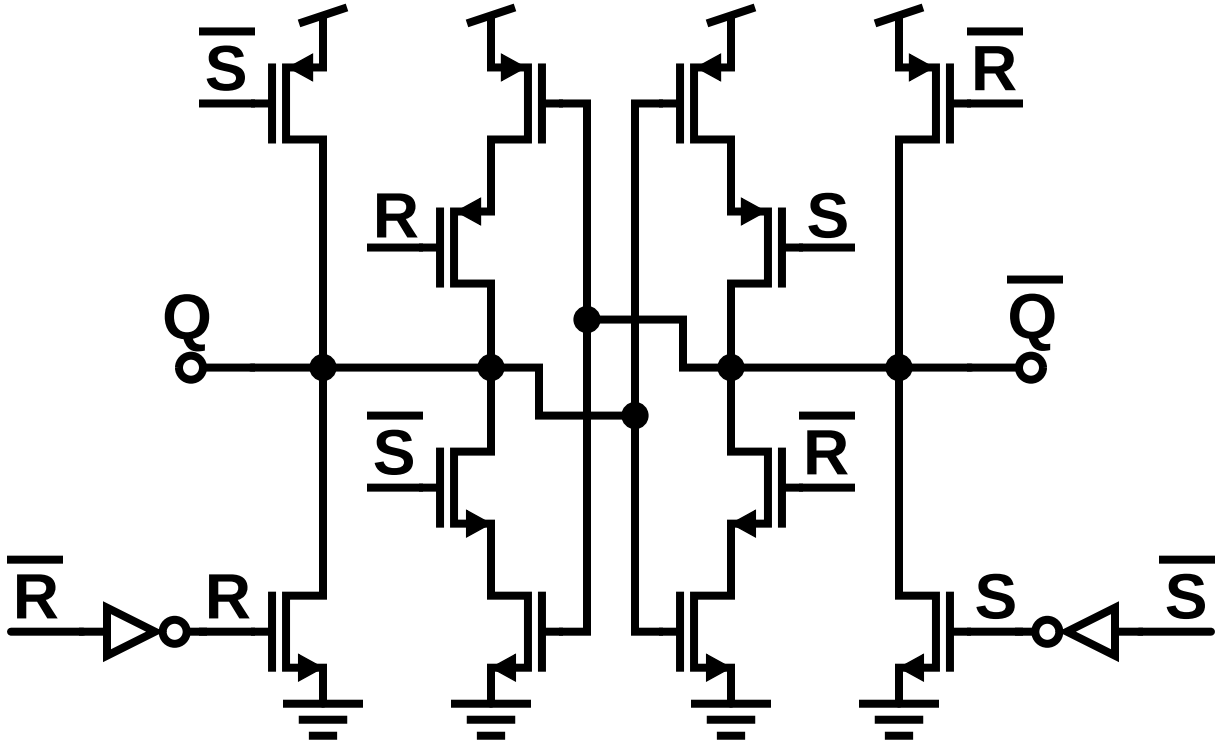




- **Well-defined initial state ⇒ less hysteresis**

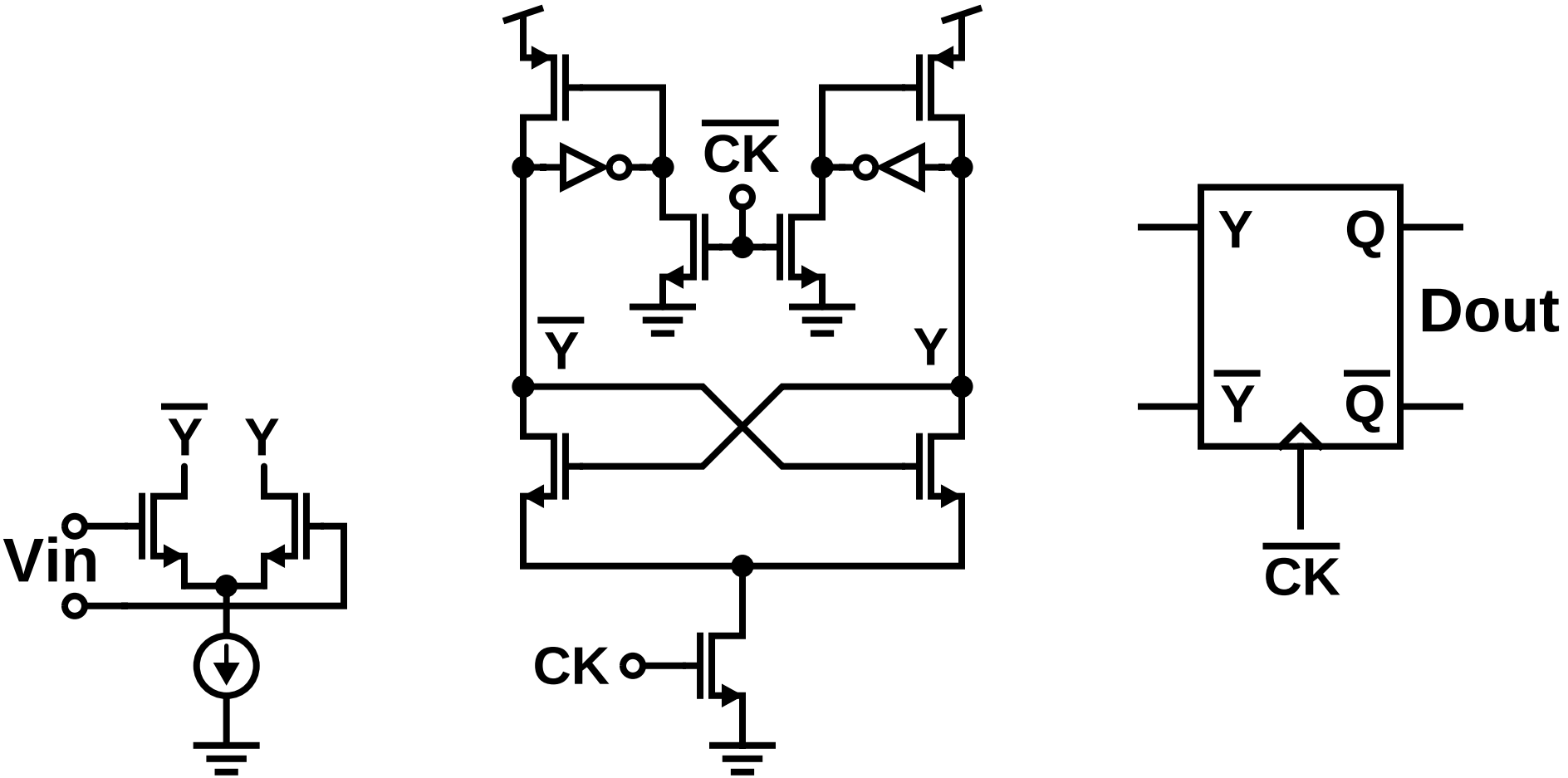
  [Abidi 2014CICC] "Understanding the Regenerative Comparator Circuit"

Symmetric Latch



- **Faster than cross-coupled NAND gates**

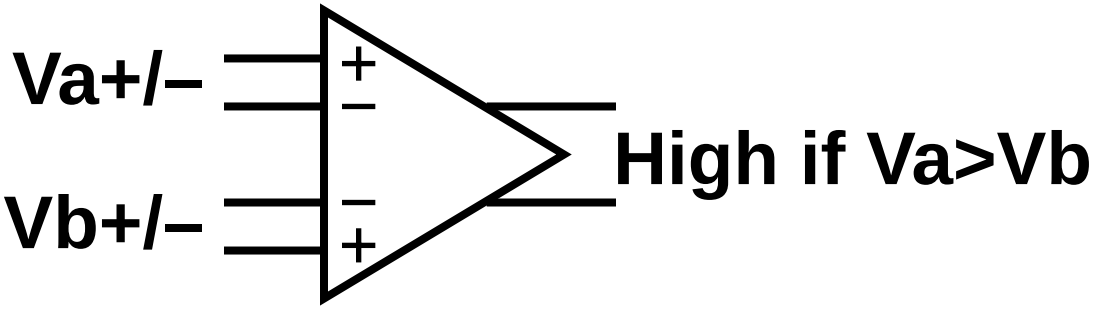
Yang's High-Speed Comparator



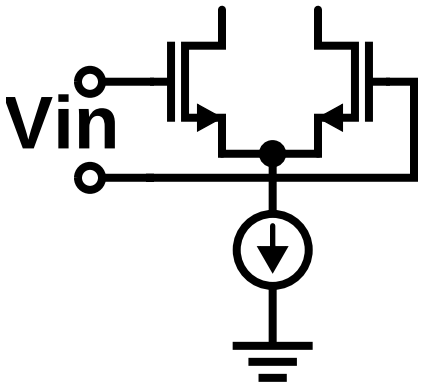
- τ_{nom} = **6 ps (!) in 65nm CMOS**

  Used in a CT$\Delta\Sigma$ ADC clocked at 4 GHz [Shibata2012].

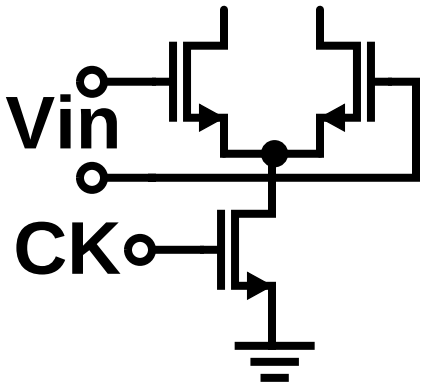
Dual-Difference Comparator



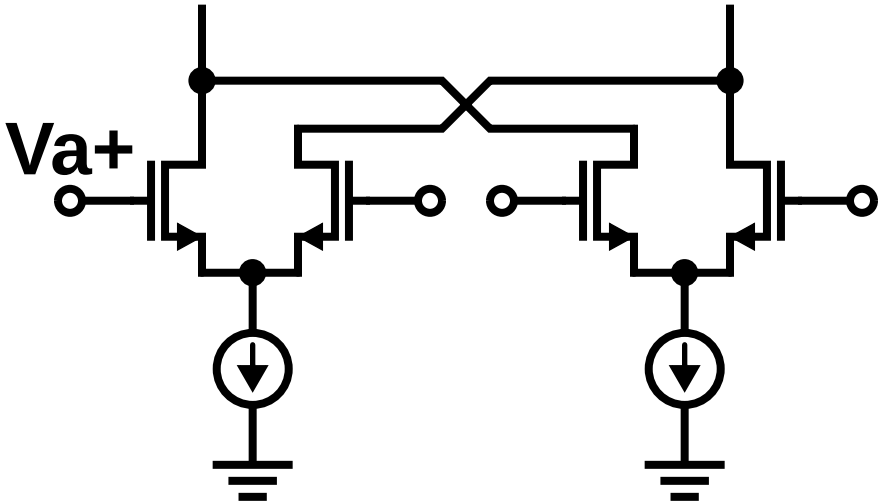
In any of our comparator circuits, replace:



or

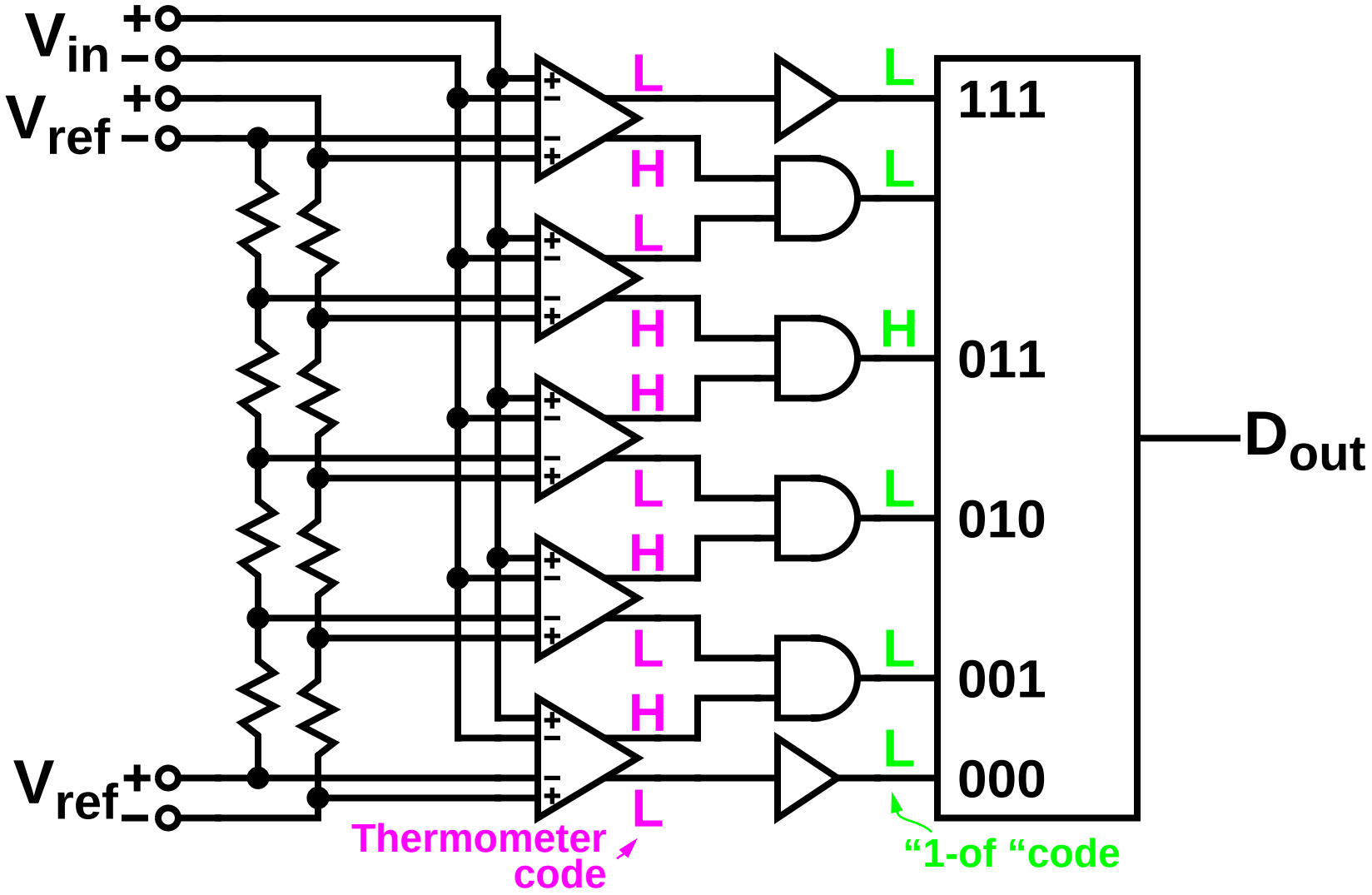


with:

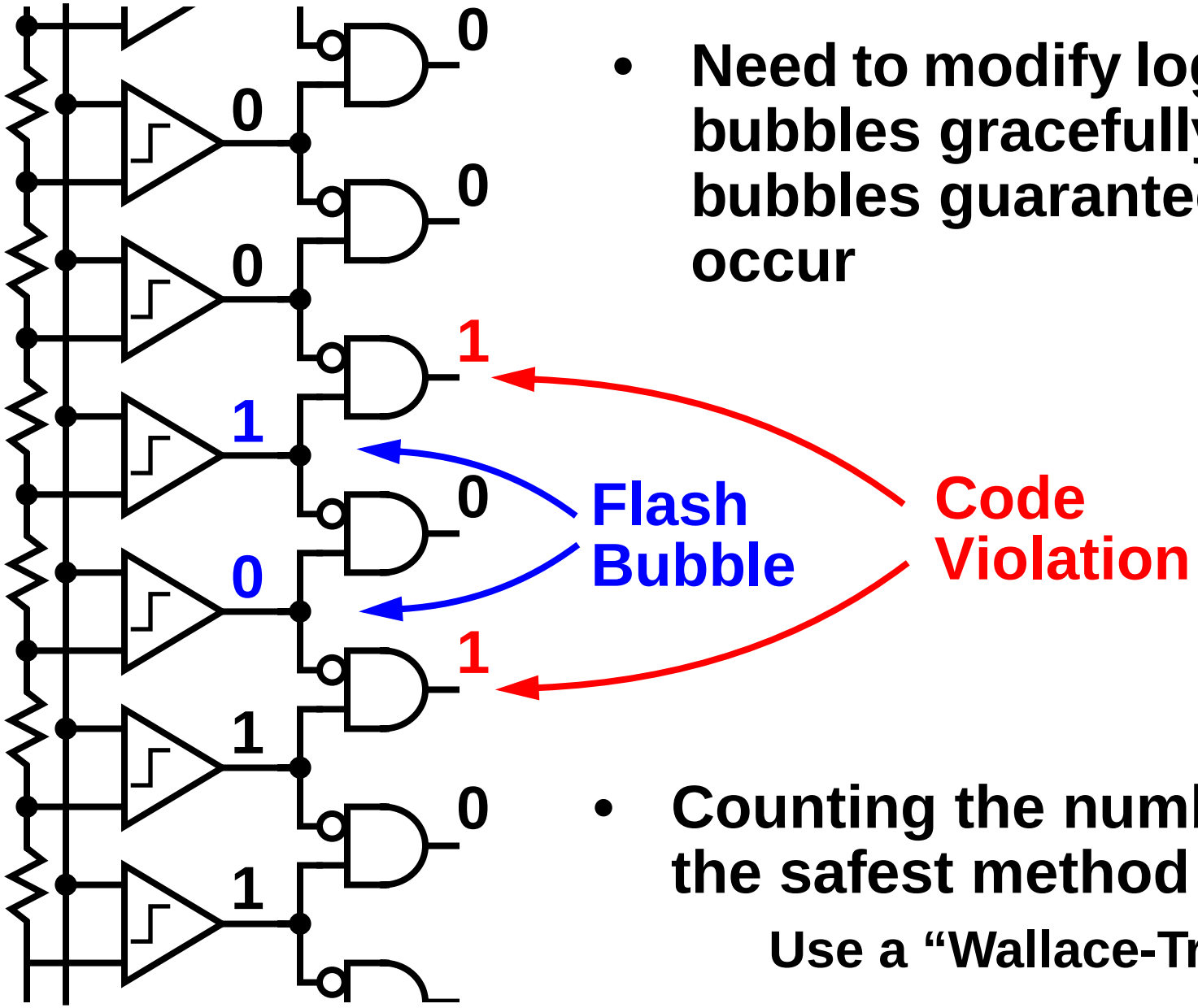


Q: Where should Va– go?

Flash ADC

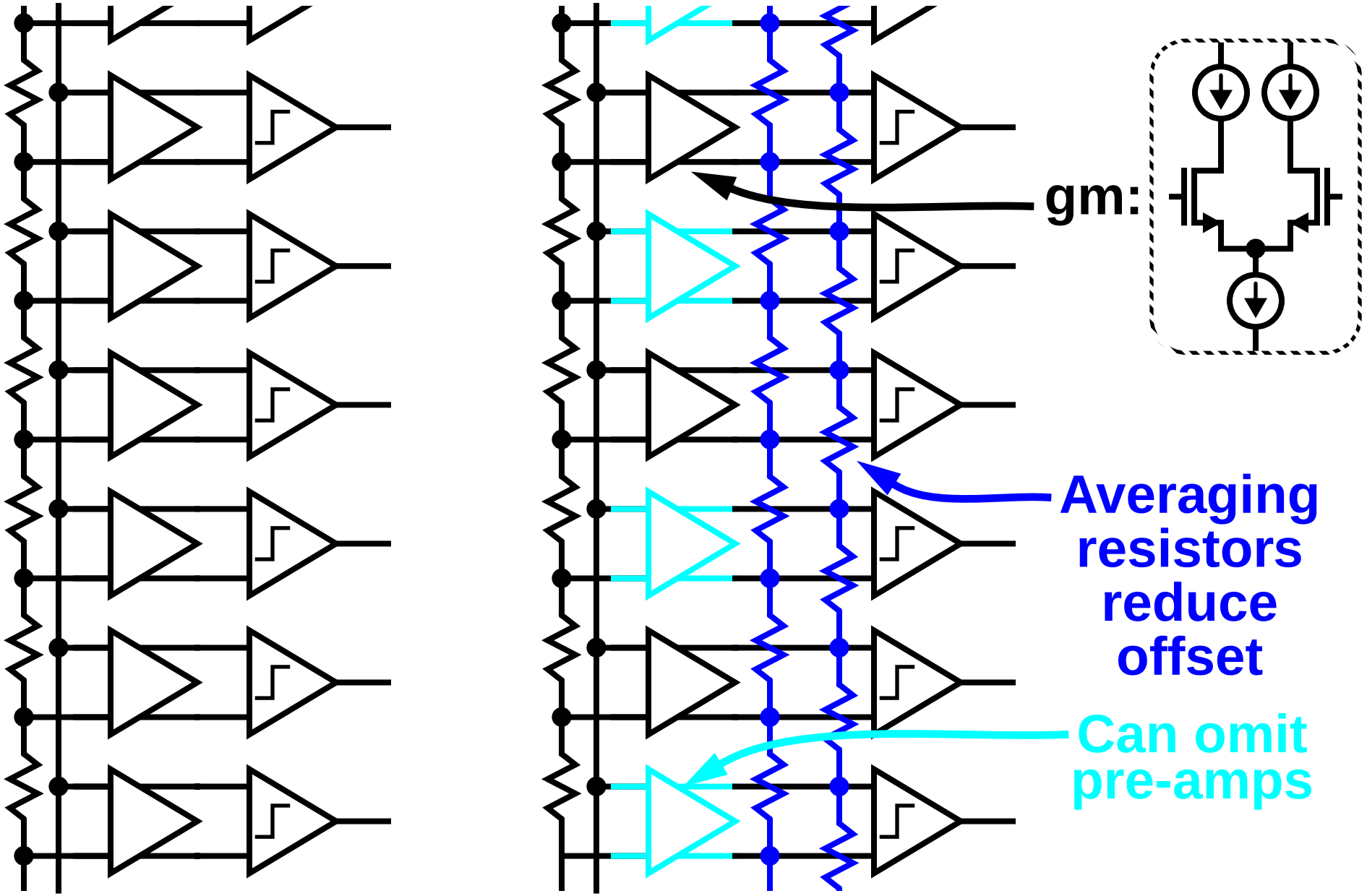

V_{in}
V_{ref}
V_{ref}
L
H
L
H
H
L
H
L
H
L
111
011
010
001
000
D_{out}
Thermometer code
"1-of "code
Resistor String
Comparator Bank
Decode Logic

Flash Bubbles

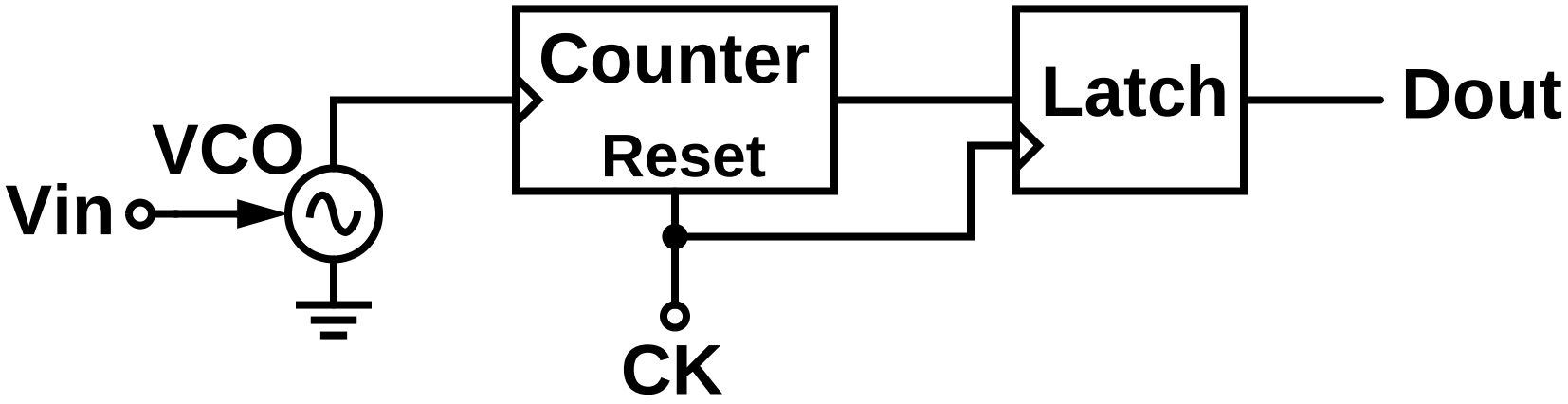


- Need to modify logic to handle bubbles gracefully unless bubbles guaranteed to not occur
- Counting the number of 1s is the safest method
  - Use a “Wallace-Tree Adder”

Averaging and Interpolation

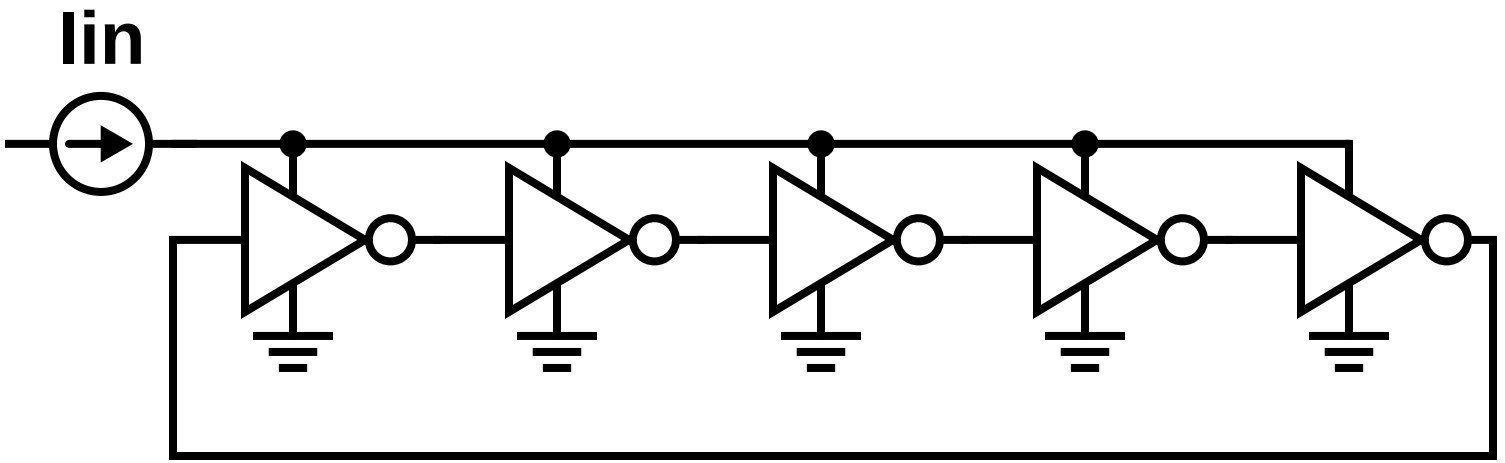


VCO as a Quantizer— Concept



+ Simple
+ Monotonic
– Not linear
– Ill-defined full-scale

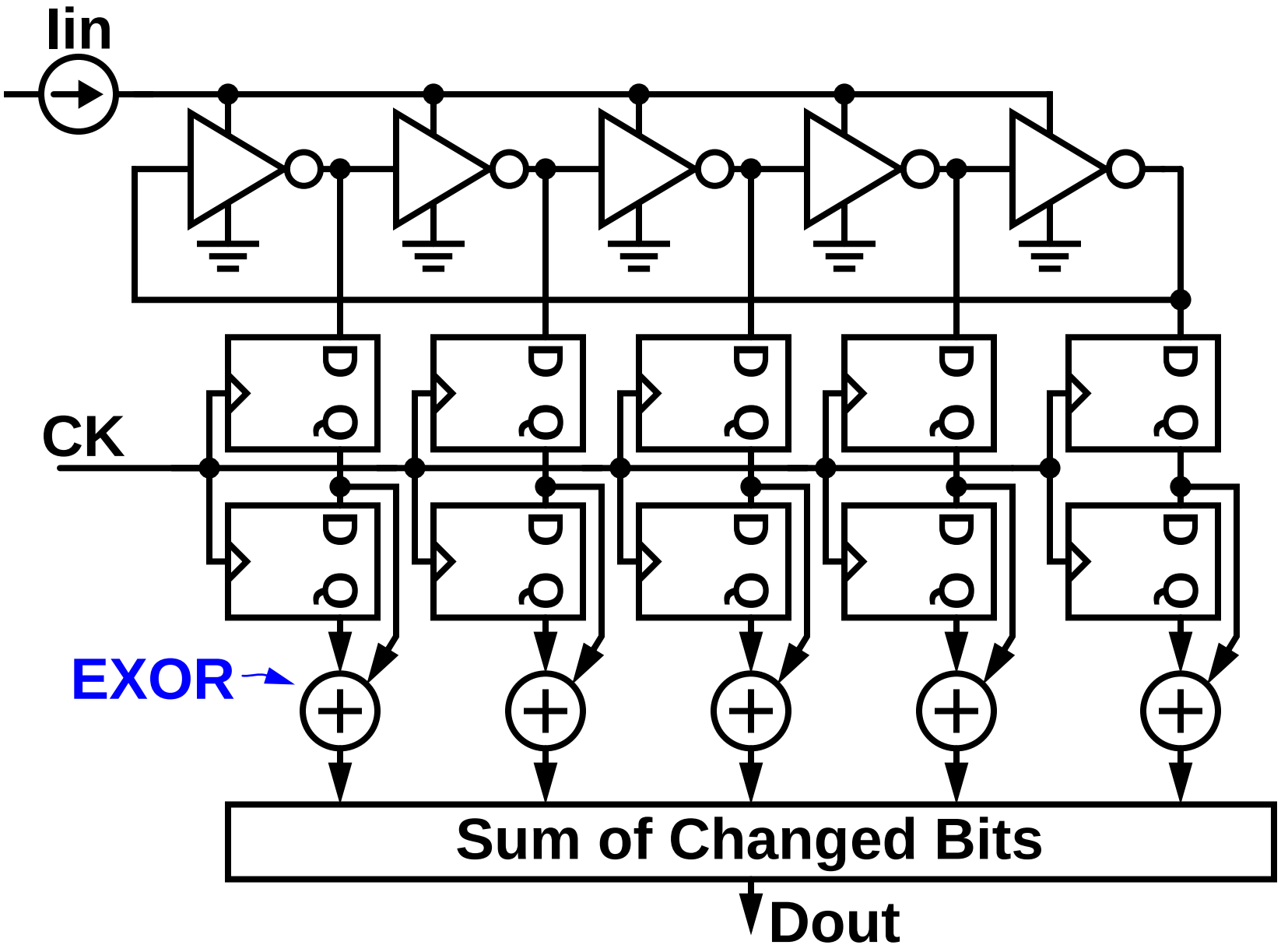
Current-Controlled Ring Oscillator (CCRO)



- **Oscillation frequency is proportional to current**

$\Rightarrow$ **Can make a fairly linear quantizer**

CCRO + Phase latch + EXOR = Q!

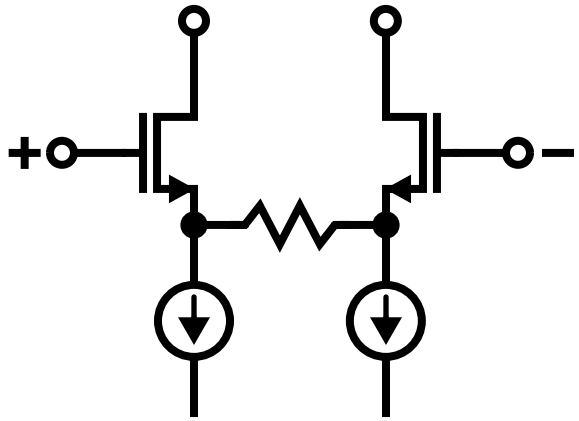


Improvements to CCRO Quantizer

- **Put EXOR before 1st FF and remove 2nd FF**
  - **To eliminate unnecessary delay.**
- **Add inverters to increase resolution**
  - **Must keep f_{osc} constant to maintain full-scale.**
- **Use 2 rings in a pseudo-differential arrangement**
  - **To cancel even-order distortion and make mid-scale self-referenced.**
- **Use multiple rings with phase interpolation to increase resolution without hitting f_{osc} ceiling**
- **Driving the elements of the DACs with the EXOR outputs directly results in *mismatch-shaping***

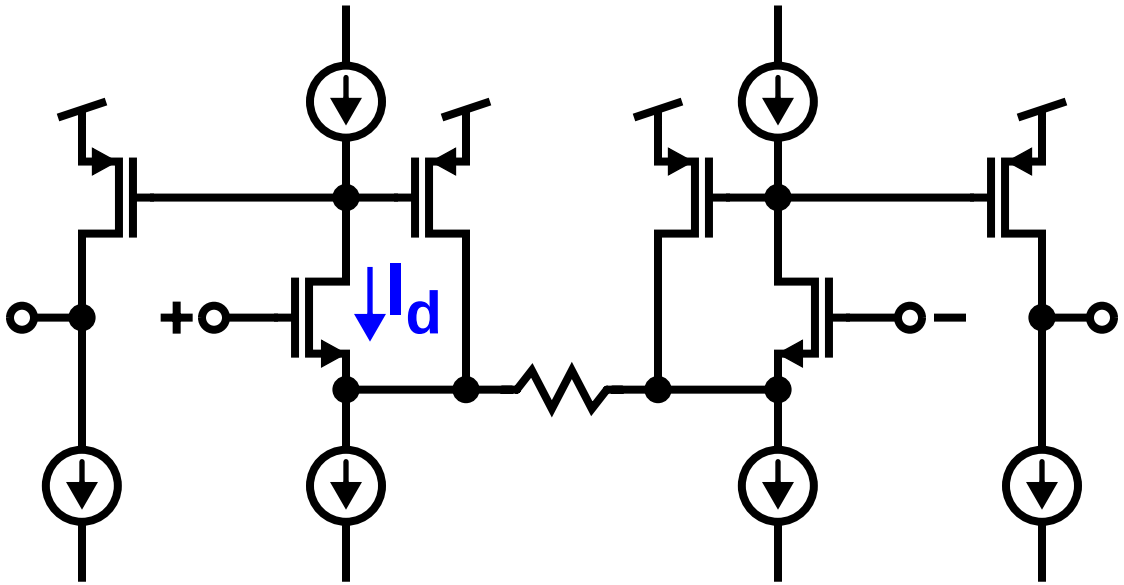
NLCOTD: Linear Transconductors

Degenerated Differential Pair



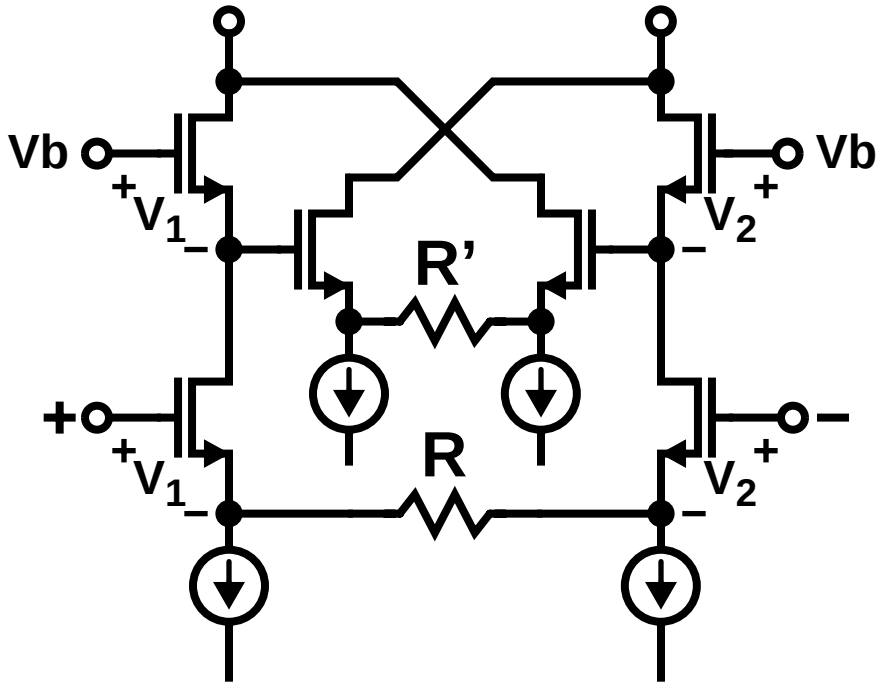
- \+ **Simple!**
- – **V_{gs} varies nonlinearly with $I_{out} \Rightarrow g_m$ is nonlinear**

Force Constant V_{gs}



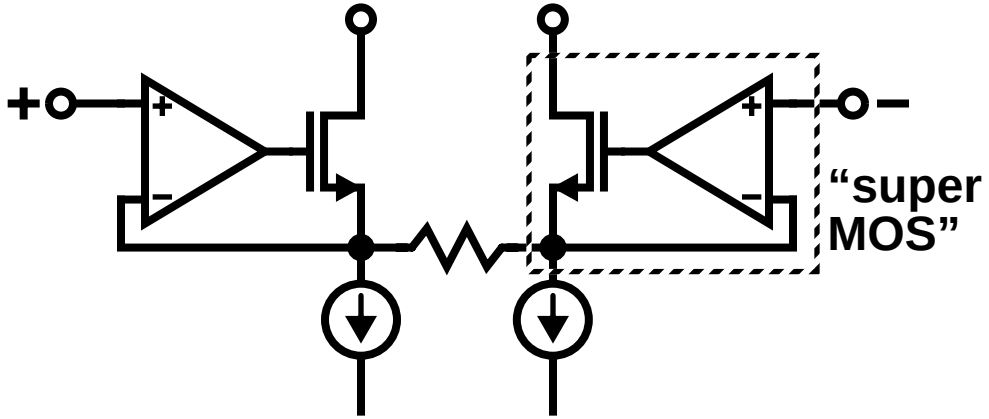
- I_d constant $\Rightarrow$ V_{gs} constant
  - Linearity dependent on current-mirror linearity

Cascomp



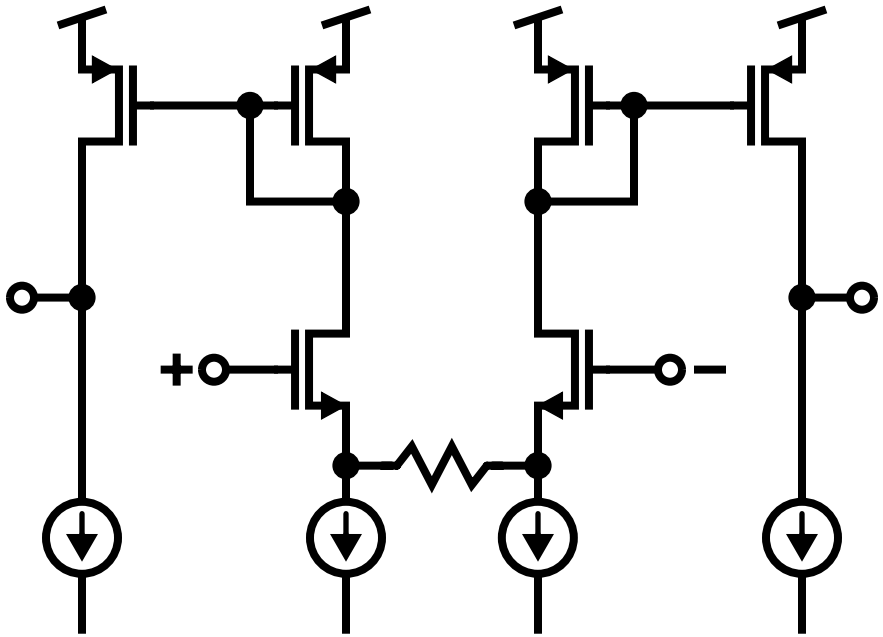
- **V_{gs} of input devices replicated in cascodes and distortion-cancelling current injected into output**
- **+ All NMOS $\Rightarrow$ fast**
- **– Cancellation depends on matching**
  - **Should tie bulk to source?**

Add Op Amps



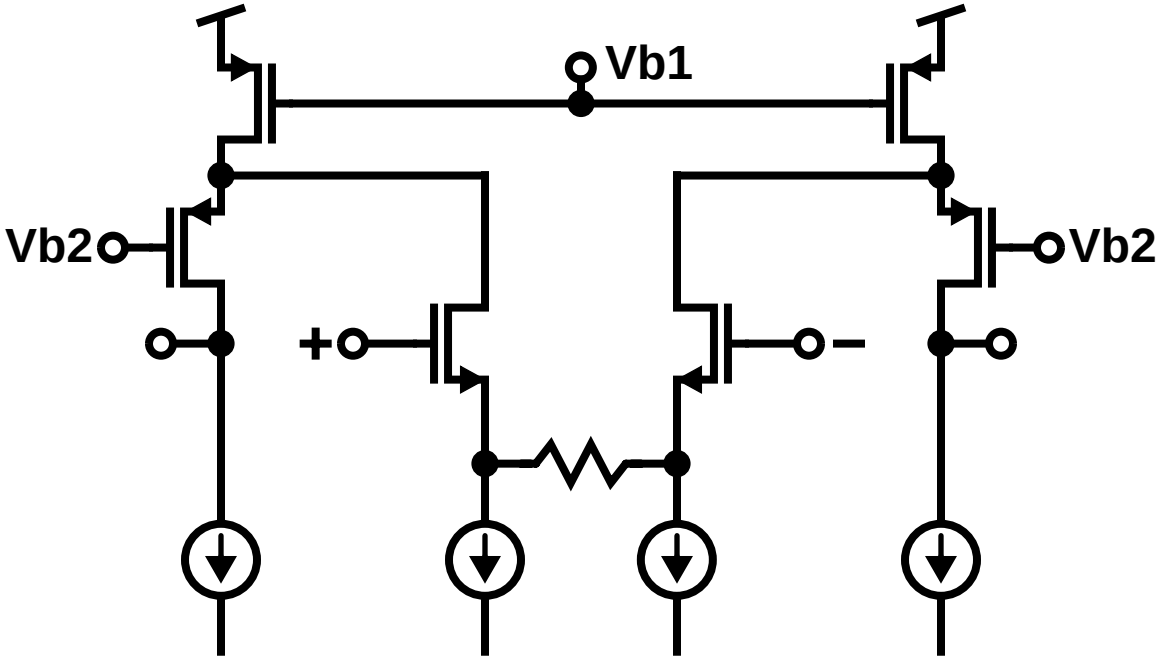
+ **Linearity limited only by op amp gain and BW**
+ **High output resistance**
– **Output compliance depends on input swing**

Mirror the Output Current



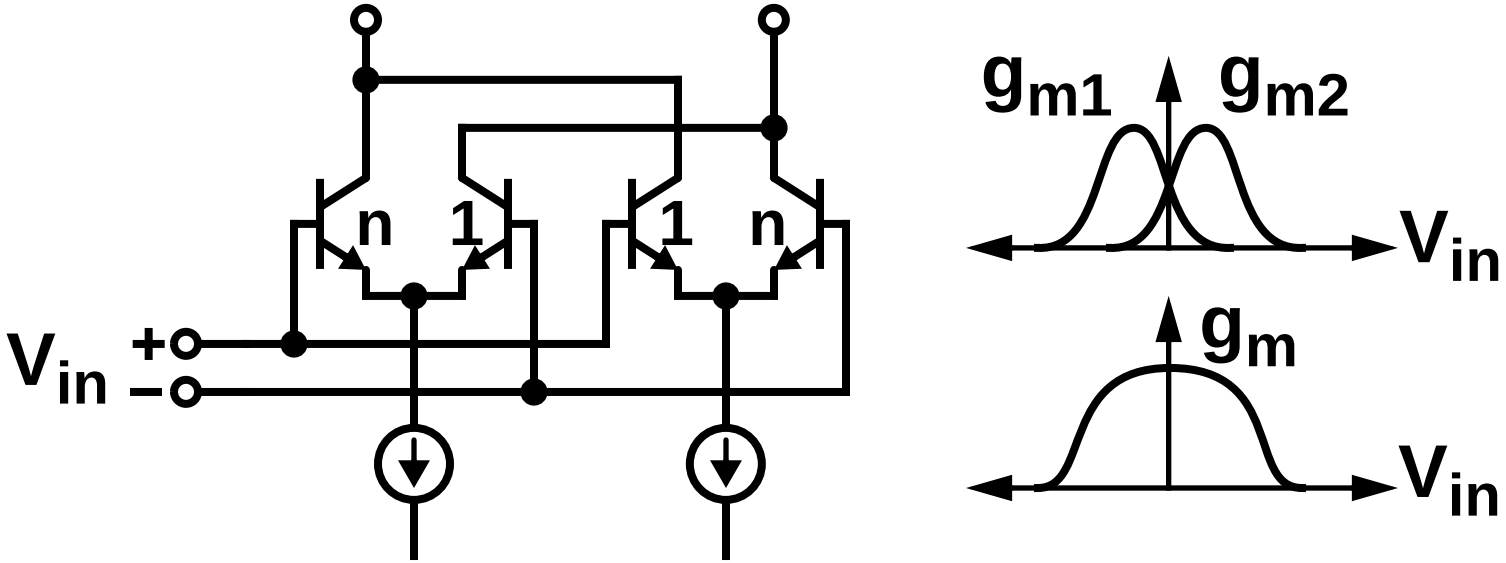
- **Output compliance is VDD – 2 V_{dsat}**
- **Top of differential pair at VDD – V_{gs}**

Fold the Output Current



+ **Increased headroom for differential pair**
+ **Increased output resistance**

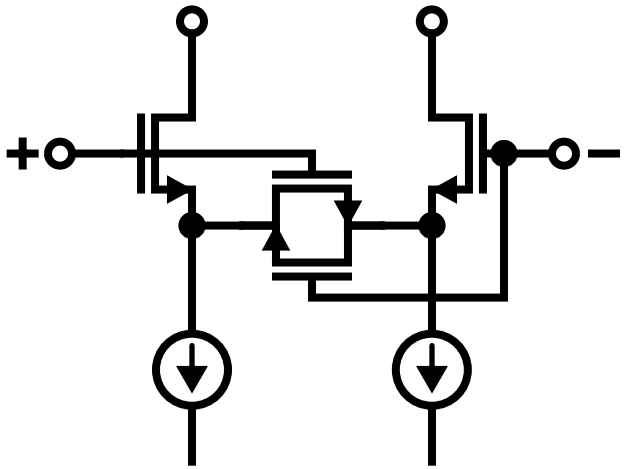
Multi-Tanh Doublet
[Gilbert JSSC Dec. 1998]



- **With BJTs, ratioing the emitter areas creates a well-controlled offset**
- **With the right offset, the cubic term in the nonlinearity is zero!**

$$n = 2 + \sqrt{3} = 3.73 \approx 15/4$$

MOS Quad



- **Supposedly can get less distortion than a degenerated differential pair by fiddling with W/L**

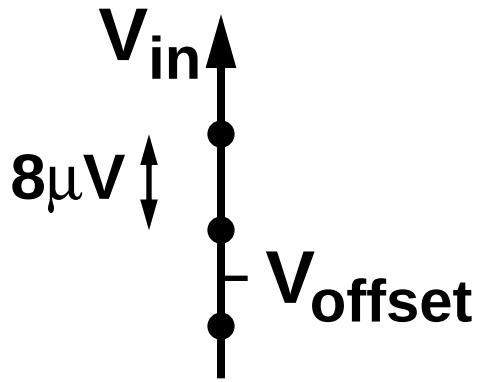
What You Learned Today

1. Operation of Example Comparator Circuit
2. Regeneration Time Constant τ
3. Metastability, Probability of Error
4. Offset, Dynamic Offset, Auto-zeroing
5. Flash ADC
6. A Bunch of Transconductor Circuits

Supplementary Slides
(Regarding Metastability)

Quantized Charge Helps?

- If C = 20 fF, then 1 electron yields 8 μV
- V_{in} = 2 nV and hence metastability impossible?

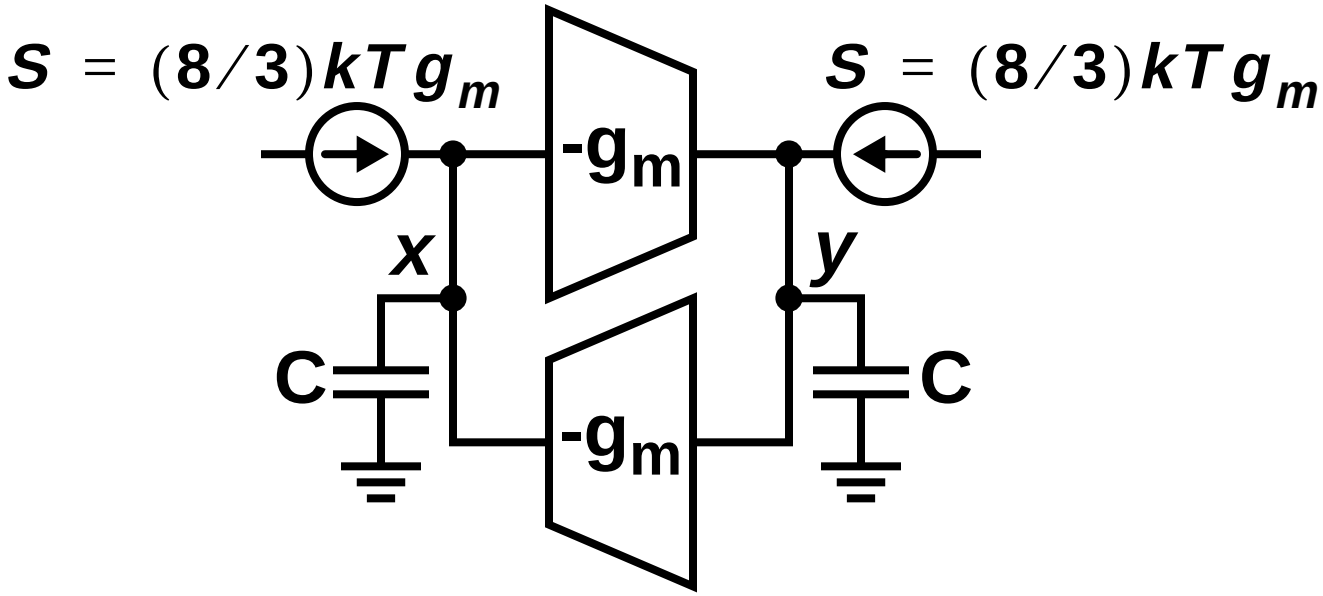


+ Unless V_{offset} is within 2 nV of one of the discrete V_{in} levels, metastability can't happen
– But if V_{offset} is within 2 nV of an allowed V_{in} level, metastability will be abnormally frequent
- Offset drift will tend to make metastability appear/disappear sporadically (?)

Noise Helps?

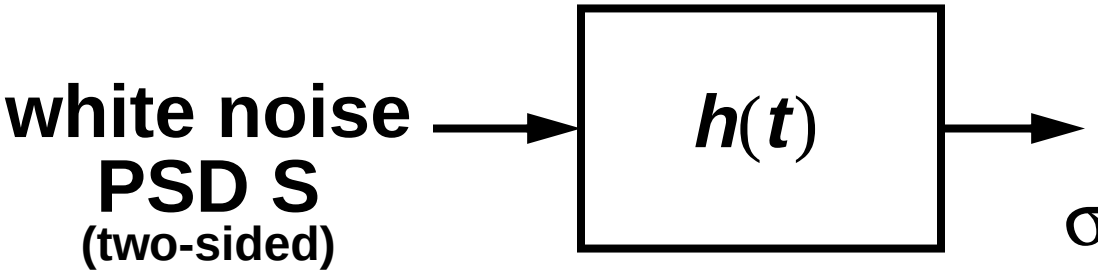
- **kT/C = 500 μV, so it is impossible to guarantee that metastability will result even if $V_{in} = 0$**
- **+ Noise does help a comparator resolve if it is metastable**
- **– But for any given noise (random initial condition), there is always an input which results in metastability**
- **Noise makes it hard to set initial conditions that will result in metastability, but does not reduce the probability of error**

What About Noise During Regeneration?



- Noise from the g_ms prevents metastability?

Math Break: Filtered White Noise

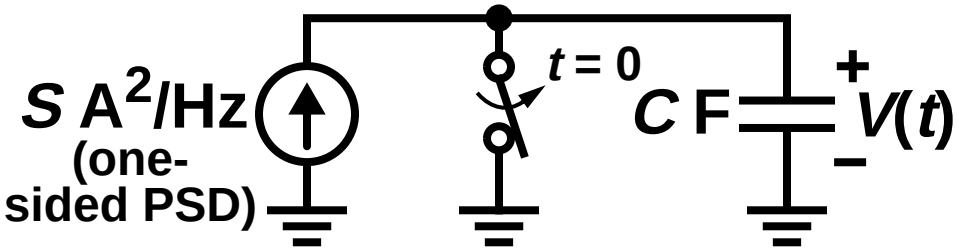


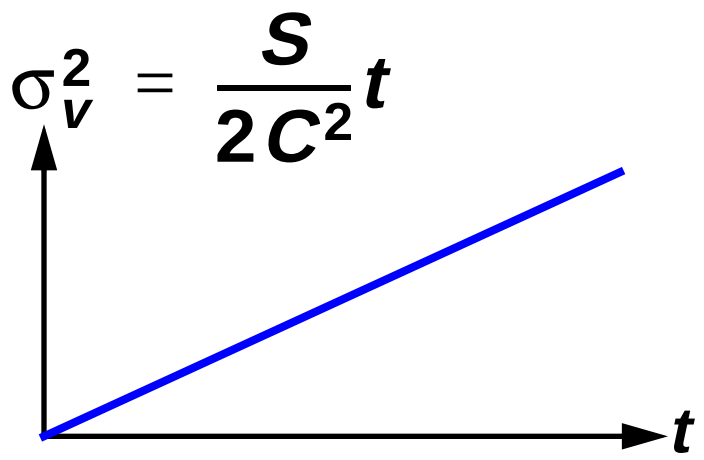
$$\sigma_y^2 = S\int_{-\infty}^{\infty} |h(t)|^2\,dt$$

- The power of the output is the product of the PSD and the power gain of the filter
- Example: $h(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$

$$\Rightarrow \sigma_y^2 = ST$$

Circuits Application: Integrated White Noise

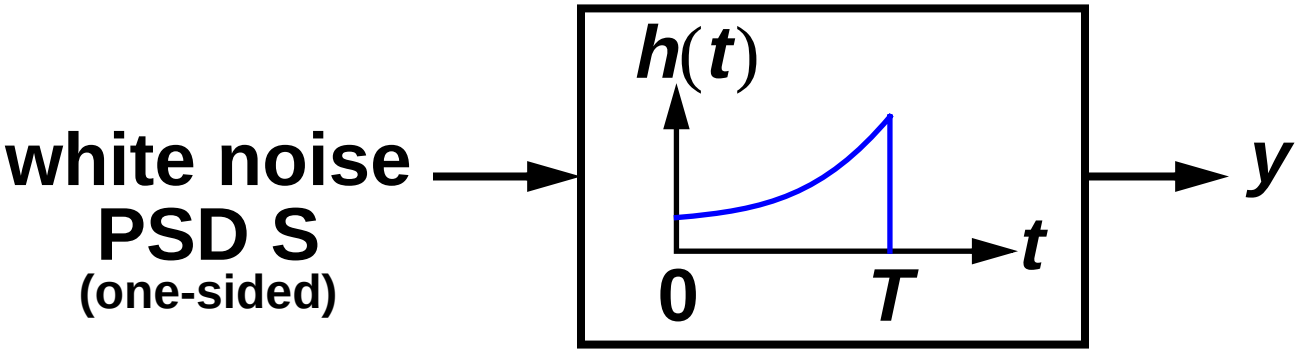




- **Variance of V increases linearly with time**
- **"Random Walk" or "Brownian Motion"**
  For any given increment of time Δt, the change in V is a random variable with constant variance

$$\delta(t) \equiv V(t) - V(t - \Delta t) \Rightarrow \sigma_\delta^2 = \text{constant}$$

Today's Application



$$h(t) = \begin{cases} e^{t/\tau} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

$$\sigma_y^2 = \frac{S\tau}{4}\left(e^{\frac{2T}{\tau}} - 1\right)$$

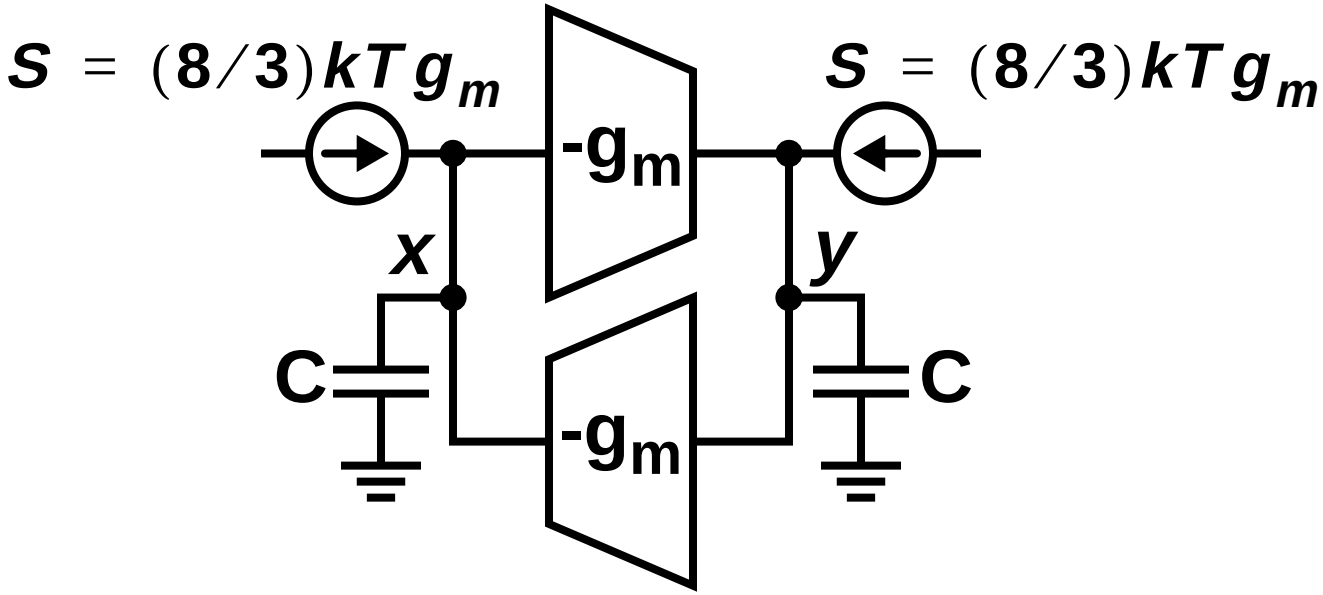
Another Math Fact: PDF of a Sum

- **Suppose *x* and *y* are two independent random variables with probability density functions (PDFs)** $\rho_x(x)$ **and** $\rho_y(y)$

- **Then the PDF of their sum is the convolution of the individual PDFs**

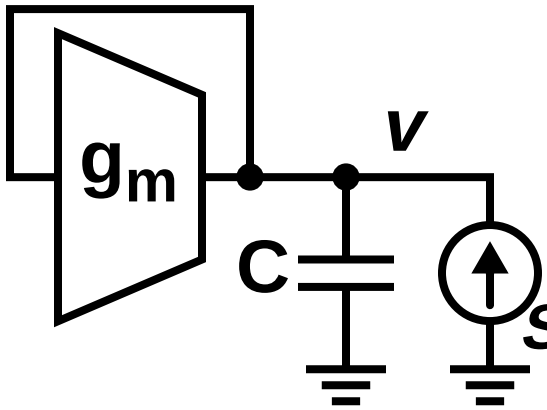
$$z = x + y \Rightarrow \rho_z(z) = \int_{-\infty}^{\infty} \rho_x(x)\rho_y(z-x)\,dx$$

Back to Circuits



- **Noise from the g_ms prevents metastability?**

Differential Circuit



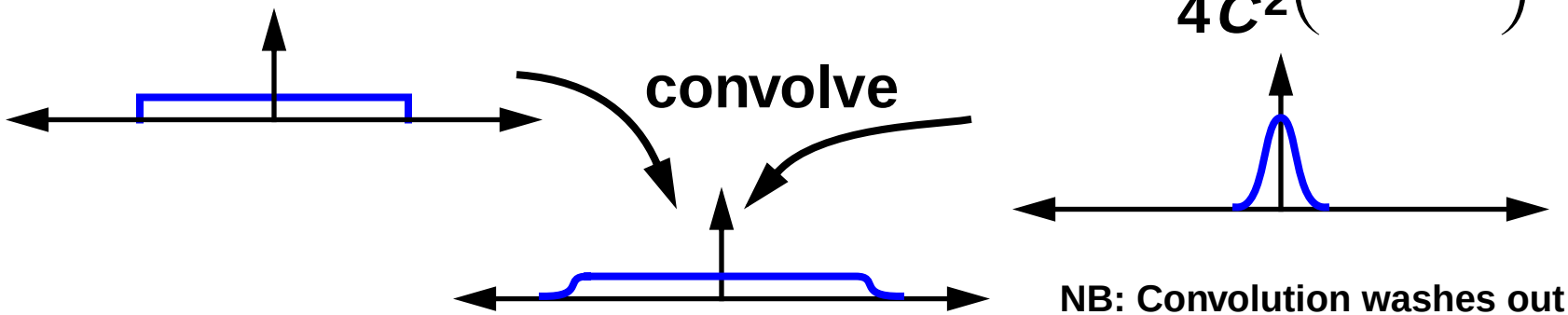
$$S_{ii} = (16/3)kTg_m$$

$$v(t) = v_0 e^{t/\tau} + \frac{1}{C}\int_0^t i(\xi) e^{\frac{t-\xi}{\tau}} d\xi$$

Random variable with the same PDF as v_0, just scaled by $e^{t/\tau}$

r.v. with variance

$$\sigma^2 = \frac{S_{ii}\tau}{4C^2}\left(e^{\frac{2t}{\tau}} - 1\right)$$



NB: Convolution washes out discreteness in v_0's PDF

Making Numbers…

- Assume g_m = 1 mA/V, C = 20 fF, t = 400 ps
  $\Rightarrow$ τ = 20 ps; 20τ to resolve
- If v_0 uniformly distributed in [–2,+2] mV, then 1st term is uniformly distributed in [–1,+1] MV
- Standard deviation of 2nd term is 250 kV
  - Equivalent to a 0.5-mV initial condition
- Noise during regeneration helps when the input is known to be small, but is usually negligible compared to the exponential growth of the initial conditions