

ECE1371 Advanced Analog Circuits

Lecture 5

ADVANCED $\Delta\Sigma$

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Course Goals

- **Deepen understanding of CMOS analog circuit design through a top-down study of a modern analog system**
 - The lectures will focus on Delta-Sigma ADCs, but you may do your project on another analog system.
- **Develop circuit insight through brief peeks at some nifty little circuits**
 - The circuit world is filled with many little gems that every competent designer ought to know.

Date	Lecture		Ref	Homework
2008-01-07	RS	1	Introduction: MOD1 & MOD2	S&T 2-3, A Matlab MOD2
2008-01-14	RS	2	Example Design: Part 1	S&T 9.1, J&M 10 Switch-level sim
2008-01-21	RS	3	Example Design: Part 2	J&M 14 Q-level sim
2008-01-28	TC	4	Pipeline and SAR ADCs	Arch. Comp.
2008-02-04	ISSCC– No Lecture			
2008-02-11	RS	5	Advanced $\Delta\Sigma$	S&T 4, 6.6, 9.4, B $\Delta\Sigma$ Toolbox; Proj.
2008-02-18	Reading Week– No Lecture			
2008-02-25	RS	6	Comparator & Flash ADC	J&M 7
2008-03-03	TC	7	SC Circuits	J&M 10
2008-03-10	TC	8	Amplifier Design	
2008-03-17	TC	9	Amplifier Design	
2008-03-24	TC	10	Noise in SC Circuits	S&T C
2008-03-31	Project Presentation			
2008-04-07	TC	11	Matching & MM-Shaping	Project Report
2008-04-14	RS	12	Switching Regulator	Q-level sim

Highlights

(i.e. What you will learn today)

1 High-Order $\Delta\Sigma$ Modulation

NTF, SQNR, instability, $\|H\|_{\infty}$, $\|h\|_1$

2 Feedback, Feedforward and Generic $\Delta\Sigma$ Topologies

NTF selection

3 $\Delta\Sigma$ Toolbox

Demos: NTF synthesis and simulation

Examples: Lowpass and Bandpass, MASH

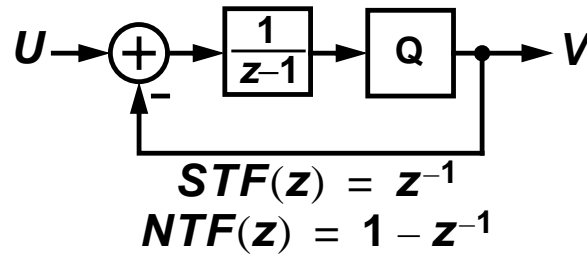
ABCD: State-space representation

4 Continuous-Time $\Delta\Sigma$

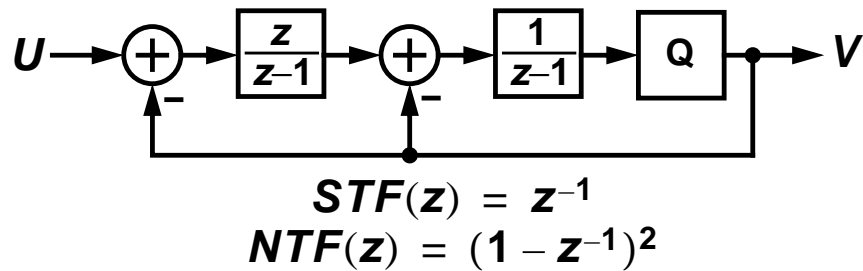
Inherent anti-aliasing

Review: MOD1 & MOD2

MOD1:



MOD2:



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Review: SQNR vs. OSR

- The in-band quantization noise power is:

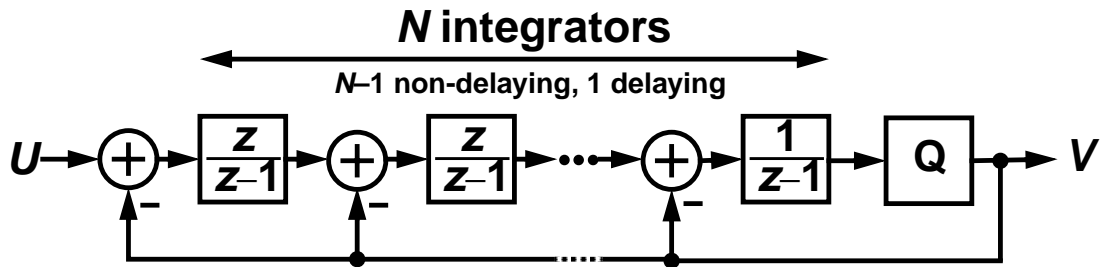
$$\begin{aligned}
 IQNP &= \int_0^{\omega_B} |H(e^{j\omega})|^2 S_{ee}(\omega) d\omega \\
 &= \begin{cases} \frac{\pi^2 \sigma_e^2}{3} (OSR)^{-3} & \text{for MOD1} \\ \frac{\pi^4 \sigma_e^2}{5} (OSR)^{-5} & \text{for MOD2} \end{cases}
 \end{aligned}$$

⇒ 9 dB/octave SQNR-OSR trade-off for MOD1;
15 dB/octave for MOD2

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1. MODN



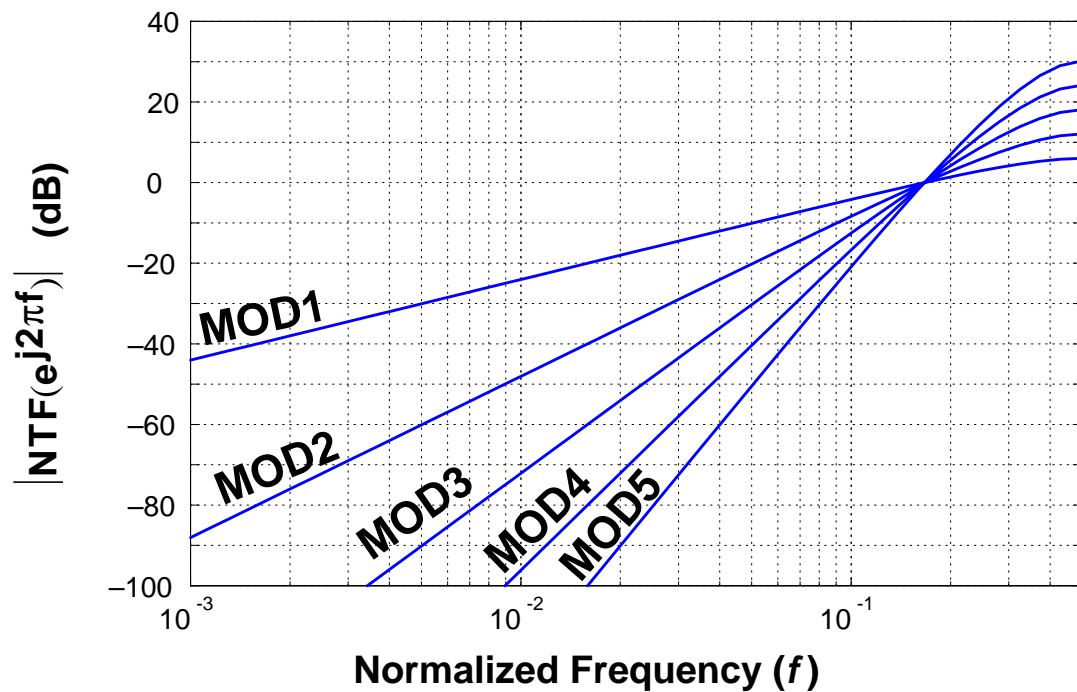
$$STF(z) = z^{-1}$$
$$NTF(z) = (1 - z^{-1})^N$$

- MODN's NTF is the N^{th} power of MOD1's NTF

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NTF Comparison



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Predicted Performance

- In-band quantization noise power

$$\begin{aligned}
 IQNP &= \int_0^{0.5/OSR} |NTF(e^{j2\pi f})|^2 \cdot S_{ee}(f) df \\
 &\approx \int_0^{0.5/OSR} (2\pi f)^{2N} \cdot 2\sigma_e^2 df \\
 &= \frac{\pi^{2N}}{(2N+1)(OSR)^{2N+1}} \sigma_e^2
 \end{aligned}$$

- Quantization noise drops as the $(2N+1)^{\text{th}}$ power of OSR— $(6N+3)$ dB/octave SQNR-OSR trade-off

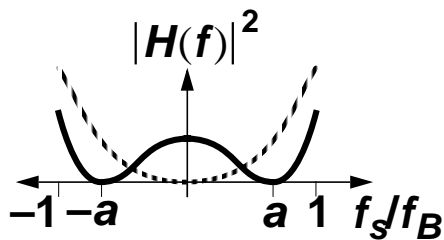
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Improving NTF Performance— NTF Zero Optimization

- Minimize the integral of $|NTF|^2$ over the passband

Normalize passband edge to 1 for ease of calculation:



Need to find the a_i which minimize the integral

$$\begin{aligned}
 &\int_{-1}^1 (x^2 - a_1^2)^2 dx, & n = 2 \\
 &\int_{-1}^1 x^2 (x^2 - a_1^2)^2 dx, & n = 3 \\
 &\int_{-1}^1 (x^2 - a_1^2)^2 (x^2 - a_2^2)^2 dx, & n = 4 \\
 &\vdots
 \end{aligned}$$

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Solutions Up to Order = 8

Order	Optimal Zero Placement Relative to f_B	SQNR Improvement
1	0	0 dB
2	$\pm 1/\sqrt{3}$	3.5 dB
3	0, $\pm\sqrt{3/5}$	8 dB
4	$\pm\sqrt{3/7} \pm \sqrt{(3/7)^2 - 3/35}$	13 dB
5	0, $\pm\sqrt{5/9} \pm \sqrt{(5/9)^2 - 5/21}$ [Y. Yang]	18 dB
6	$\pm 0.23862, \pm 0.66121, \pm 0.93247$	23 dB
7	0, $\pm 0.40585, \pm 0.74153, \pm 0.94911$	28 dB
8	$\pm 0.18343, \pm 0.52553, \pm 0.79667, \pm 0.96029$	34 dB

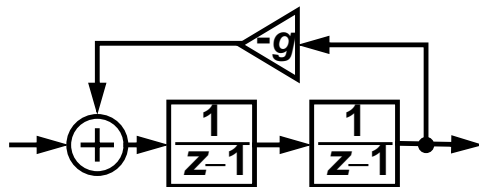
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Topological Implication

- Feedback around pairs integrators:

2 Delaying Integrators



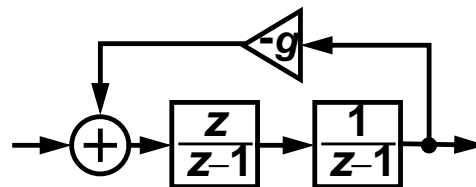
Poles are the roots of

$$1 + g\left(\frac{1}{z-1}\right)^2 = 0$$

i.e. $z = 1 \pm j\sqrt{g}$

Not quite on the unit circle,
but fairly close if $g \ll 1$.

Non-delaying + Delaying Integrators (LDI Loop)



Poles are the roots of

$$1 + \frac{gz}{(z-1)^2} = 0$$

i.e. $z = e^{\pm j\theta}$, $\cos \theta = 1 - g/2$

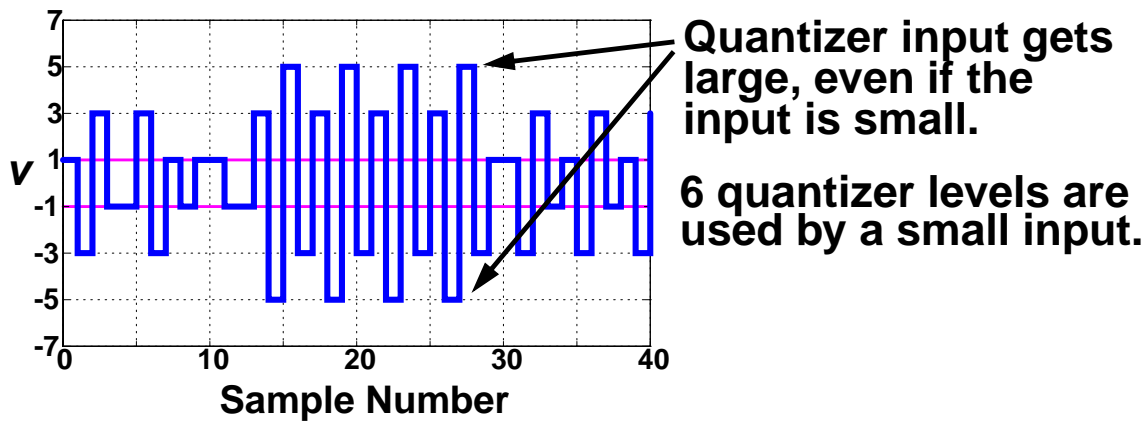
Precisely on the unit circle,
regardless of the value of g .

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Problem: A High-Order Modulator Wants a Multi-bit Quantizer

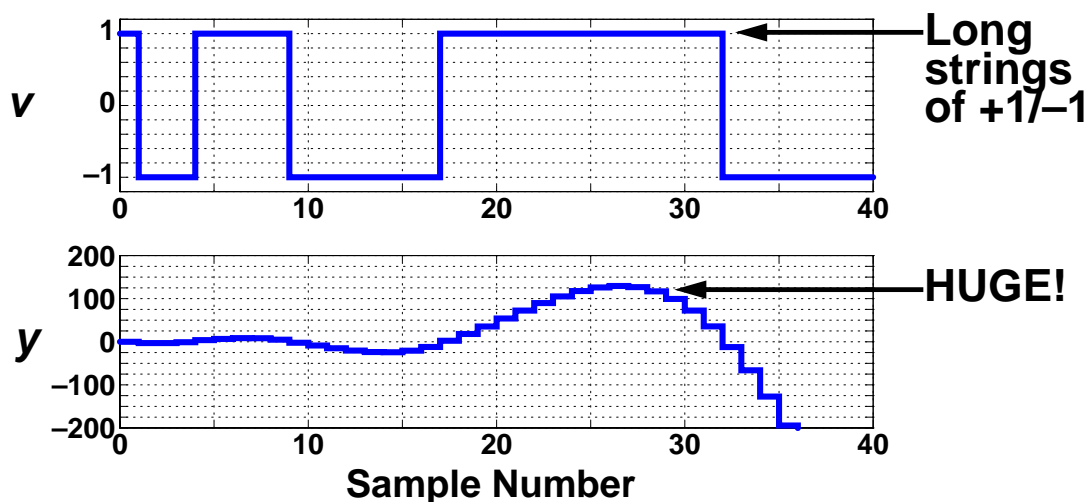
E.g. MOD3 with an Infinite Quantizer
and Zero Input



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Simulation of MOD3-1b (MOD3 with a Binary Quantizer)



- MOD3-1b is unstable, even with zero input

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Solutions to the Stability Problem

Historical Order

1 Multi-bit quantization

Initially considered undesirable because we lose the inherent linearity of a 1-bit DAC.

2 More general NTF (not pure differentiation)

Lower the NTF gain so that quantization error is amplified less.

Unfortunately, reducing the NTF gain reduces the amount by which quantization noise is attenuated.

3 Multi-stage (MASH) architecture

- Combinations of the above are possible

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Multi-bit Quantization

A modulator with $NTF = H$ and $STF = 1$ is guaranteed to be stable if $|u| < u_{max}$ at all times, where $u_{max} = nlev + 1 - \|h\|_1$ and $\|h\|_1 = \sum_{i=0}^{\infty} |h(i)|$

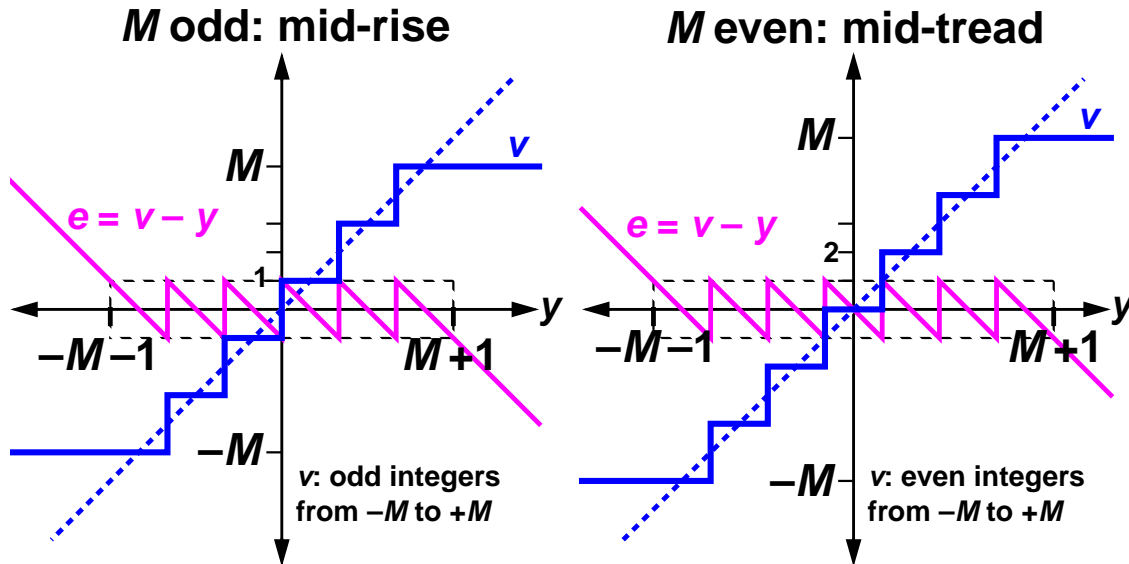
- In MODN $H(z) = (1 - z^{-1})^N$, so $h(n) = \{1, -a_1, a_2, -a_3, \dots, (-1)^N a_N, 0 \dots\}$, $a_i > 0$ and thus $\|h\|_1 = H(-1) = 2^N$
- $nlev = 2^N$ implies $u_{max} = nlev + 1 - \|h\|_1 = 1$
MODN is guaranteed to be stable with an N -bit quantizer if the input magnitude is less than $\Delta/2 = 1$.
This result is quite conservative.
- Similarly, $nlev = 2^{N+1}$ guarantees that MODN is stable for inputs up to 50% of full-scale

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M-Step Symmetric Quantizer

$$\Delta = 2, (nlev = M + 1)$$



- **No-overload range: $|y| \leq nlev \Rightarrow |e| \leq \Delta/2 = 1$**

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Inductive Proof of $\|h\|_1$ Criterion

- **Assume STF = 1 and $(\forall n)(|u(n)| \leq u_{max})$**
- **Assume $|e(i)| \leq 1$ for $i < n$. [Induction Hypothesis]**

$$\begin{aligned} |y(n)| &= \left| u(n) + \sum_{i=1}^{\infty} h(i) e(n-i) \right| \\ &\leq u_{max} + \sum_{i=1}^{\infty} |h(i)| |e(n-i)| \\ &\leq u_{max} + \sum_{i=1}^{\infty} |h(i)| = u_{max} + \|h\|_1 - 1 \end{aligned}$$

$$\text{Then } u_{max} = nlev + 1 - \|h\|_1$$

$$\Rightarrow |y(n)| \leq nlev$$

$$\Rightarrow |e(n)| \leq 1$$

- **So by induction $|e(i)| \leq 1$ for all $i > 0$**

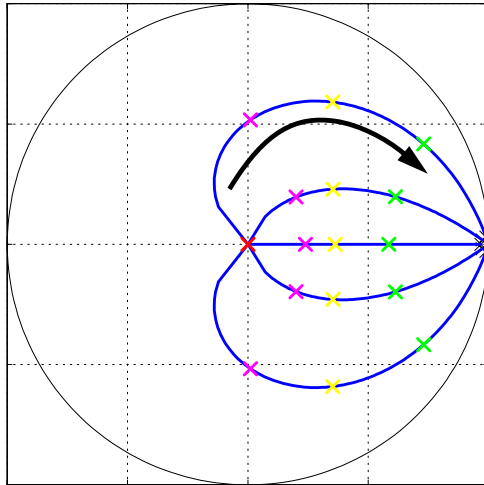
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More General NTF

- Instead of $NTF(z) = A(z)/B(z)$ with $B(z) = z^n$, use a more general $B(z)$

Roots of B are the poles of the NTF and must be inside the unit circle.



Moving the poles away from $z = 1$ toward $z = 0$ makes the gain of the NTF approach unity.

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The Lee Criterion for Stability in a 1-bit Modulator: $\|H\|_{\infty} \leq 2$ [Wai Lee, 1987]

- The measure of the “gain” of H is the maximum magnitude of H over frequency, aka the *infinity-norm* of H : $\|H\|_{\infty} \equiv \max_{\omega \in [0, 2\pi]} (|H(e^{j\omega})|)$

Q: Is the Lee criterion necessary for stability?

No. MOD2 is stable (for DC inputs less than FS) but for MOD2 $\|H\|_{\infty} = 4$.

Q: Is the Lee criterion sufficient to ensure stability?

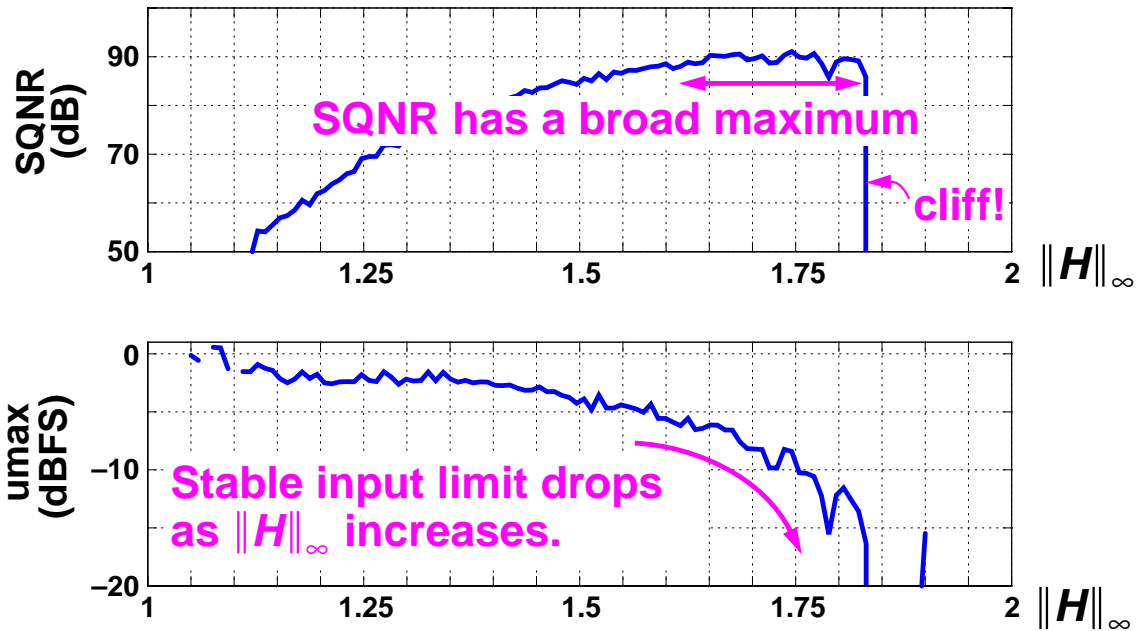
No. There are lots of counter-examples, but $\|H\|_{\infty} \leq 1.5$ often works.

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Simulated SQNR vs. $\|H\|_\infty$

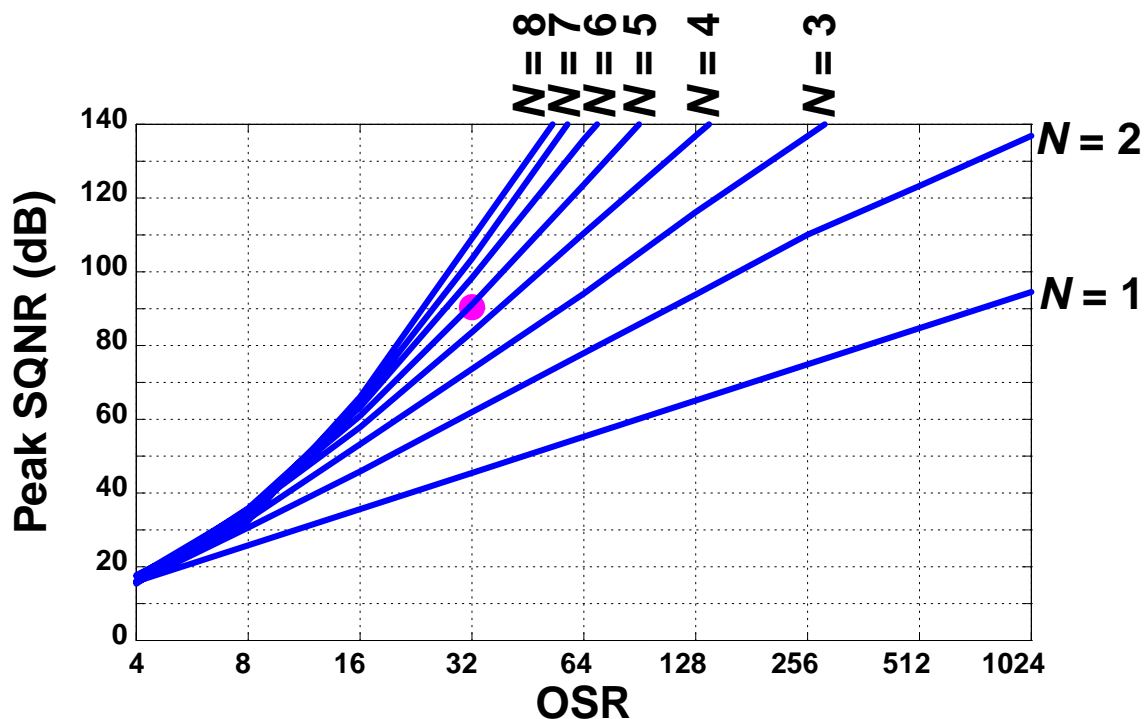
5th-Order NTFs, 1-b Quant., OSR = 32



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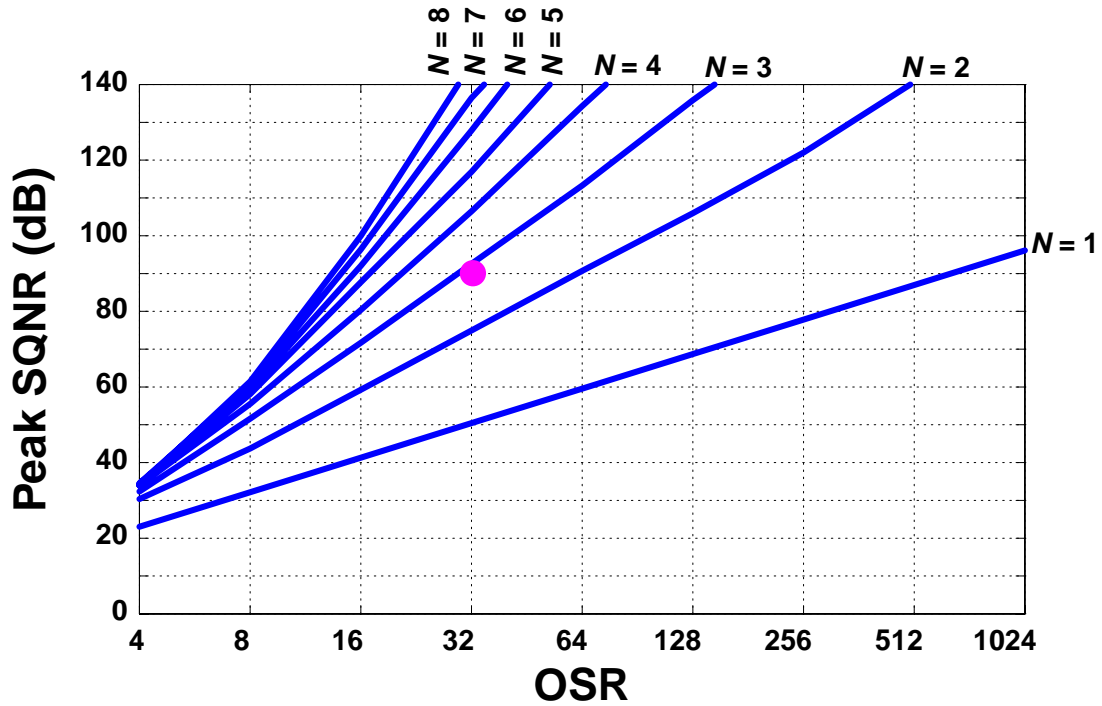
SQNR Limits— 1-bit Modulation



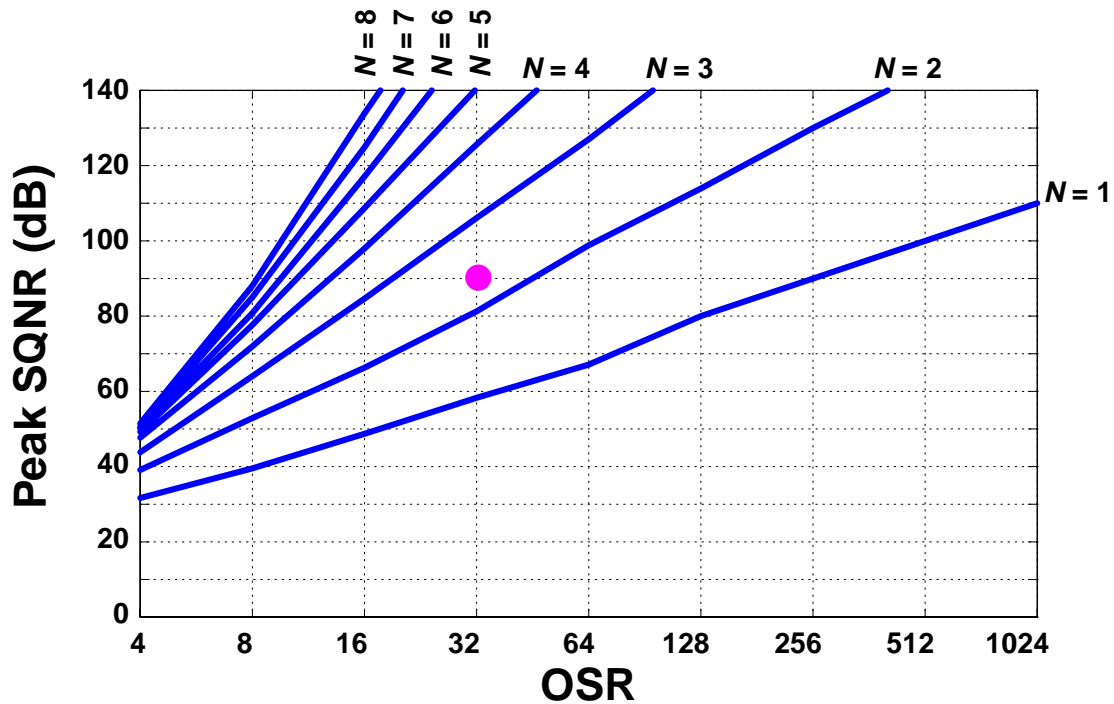
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SQNR Limits for 2-bit Modulators

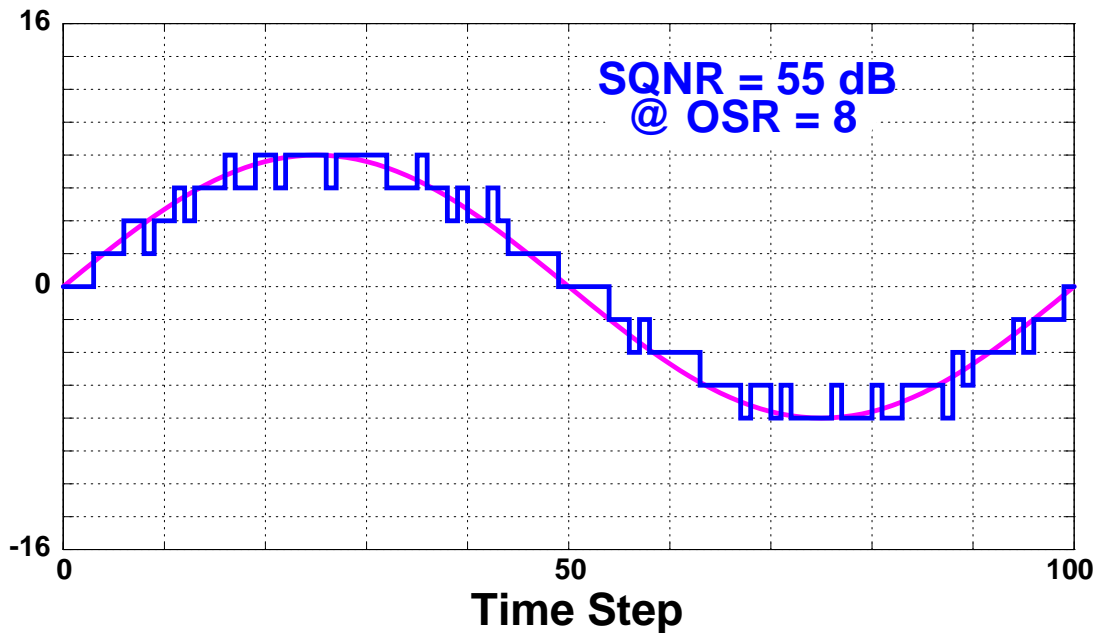


SQNR Limits for 3-bit Modulators



Example Waveforms

7th-order 17-level modulator, $\|H\|_{\infty} = 2$

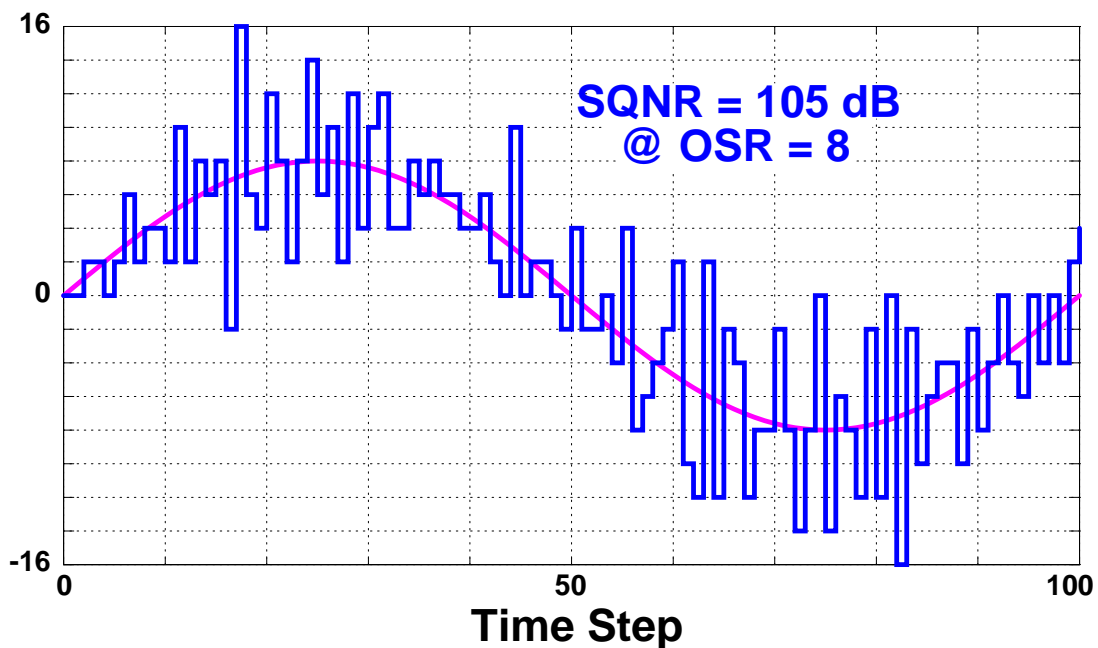


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Example Waveforms

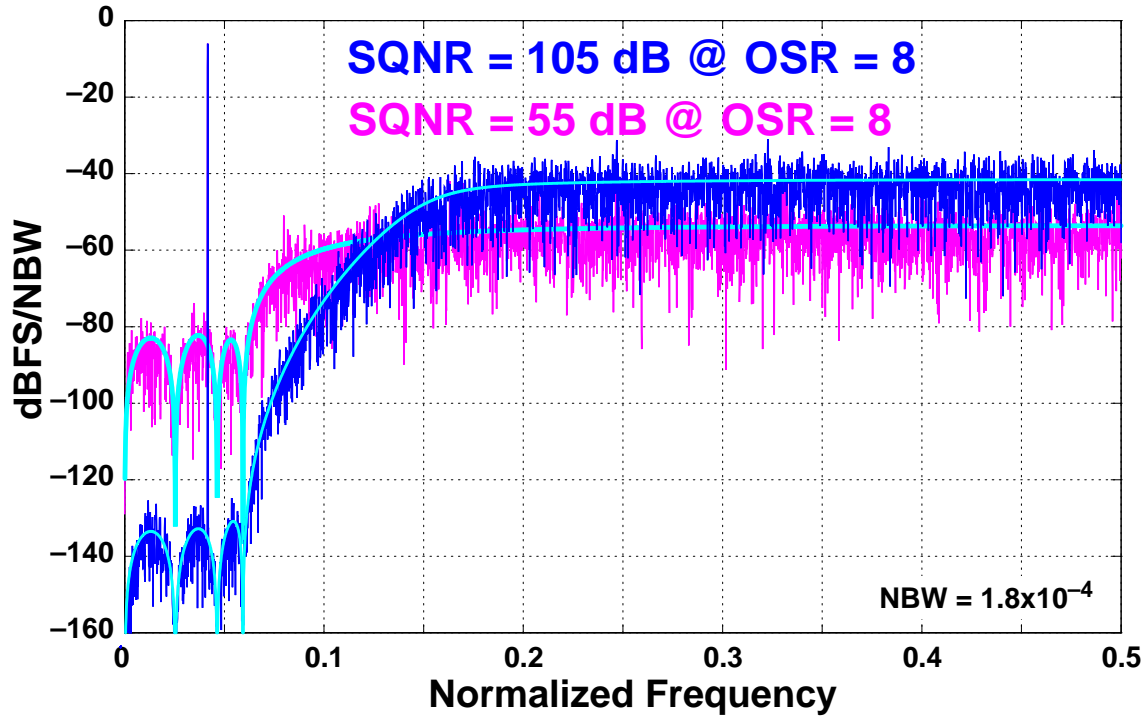
7th-order 17-level modulator, $\|H\|_{\infty} = 8$



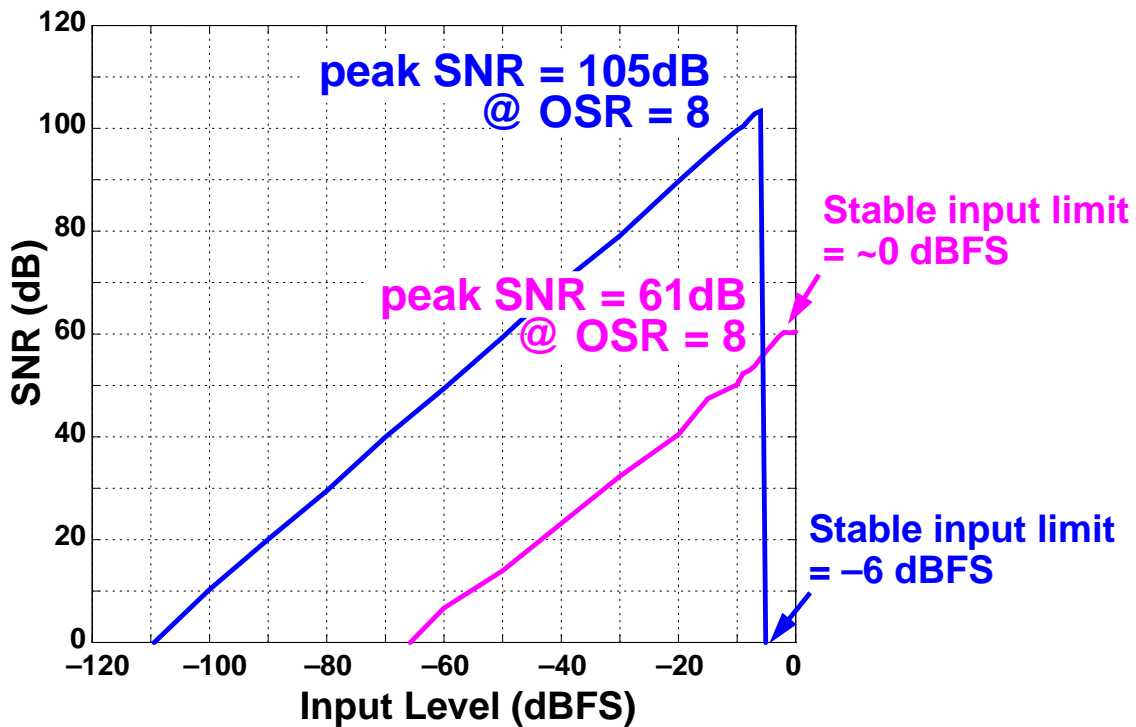
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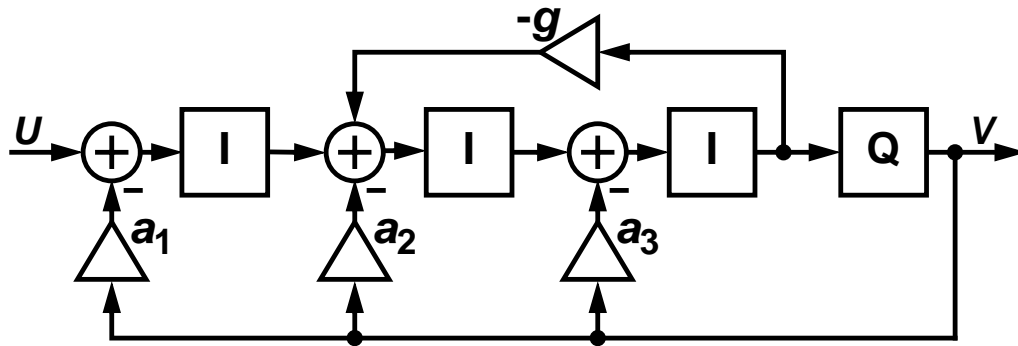
Simulated Spectra



SQNR Curves



2. Feedback Topology

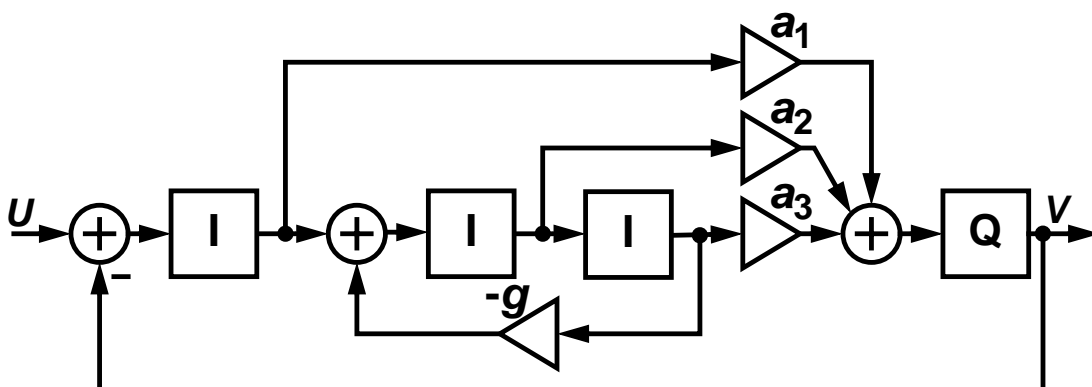


- N integrators precede the quantizer
- Feedback from the quantizer to the input of each integrator (via a DAC)
- Local feedback around pairs of integrators to control NTF zeros
- Multiple input feed-in branches are possible

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Feedforward Topology



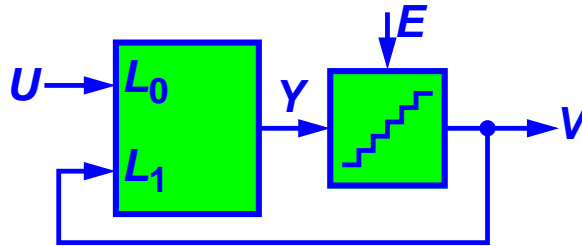
- N integrators in a row
- Each integrator output is fed forward to the quantizer
- Local feedback around pairs of integrators to control NTF zeros
- Multiple input feed-in branches also possible

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Generic Single-Loop $\Delta\Sigma$ ADC

- Linear Loop Filter + Nonlinear Quantizer:



$$\begin{aligned}
 Y &= L_0 U + L_1 V \\
 V &= Y + E
 \end{aligned}
 \Rightarrow V = \text{STF} \cdot U + \text{NTF} \cdot E, \text{ where}$$

$$\text{NTF} = \frac{1}{1 - L_1} \quad \& \quad \text{STF} = L_0 \cdot \text{NTF}$$

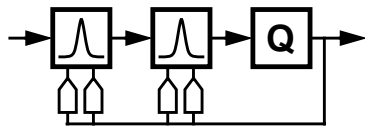
Inverse Relations:

$$L_1 = 1 - 1/\text{NTF}, \quad L_0 = \text{STF} / \text{NTF}$$

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Feedback vs. Feedforward STF



$$L_0(z) = \frac{N_0(z)}{D(z)} \quad L_1(z) = \frac{N_1(z)}{D(z)}$$

$$\text{NTF}(z) = \frac{1}{1 - L_1(z)} = \frac{D(z)}{N_1(z) - D(z)}$$

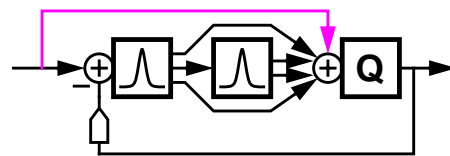
poles of LF are zeros of NTF

$$\text{STF}(z) = \frac{L_0(z)}{1 - L_1(z)} = \frac{N_0(z)}{N_1(z) - D(z)}$$

same poles as NTF

zeros = zeros of L_0

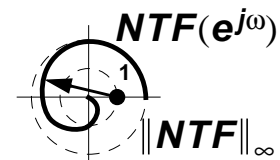
STF often has no zeros, only poles.



$$L_0(z) = -L_1(z) = L(z)$$

$$\text{NTF}(z) = \frac{1}{1 - L_1(z)} = \frac{1}{1 + L(z)}$$

$$\text{STF}(z) = \frac{L(z)}{1 + L(z)} = 1 - H(z)$$



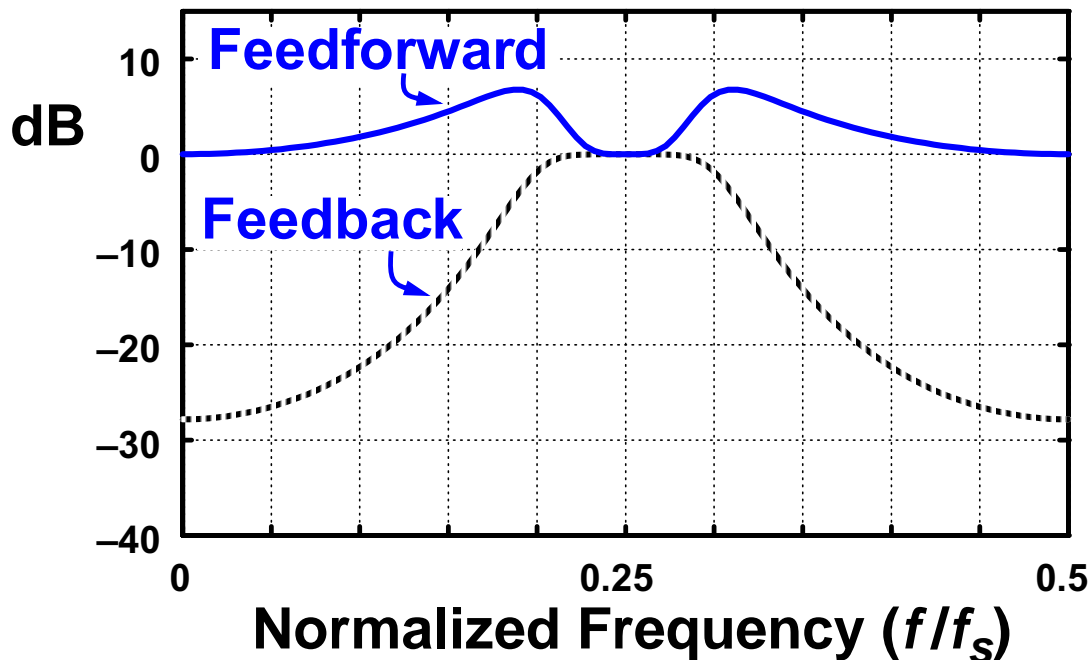
$$\| \text{STF} \|_{\infty} \approx \| \text{NTF} \|_{\infty} + 1$$

With extra feed-in to Q, $G(z) = 1$.

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STF Comparison



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Feedforward vs. Feedback

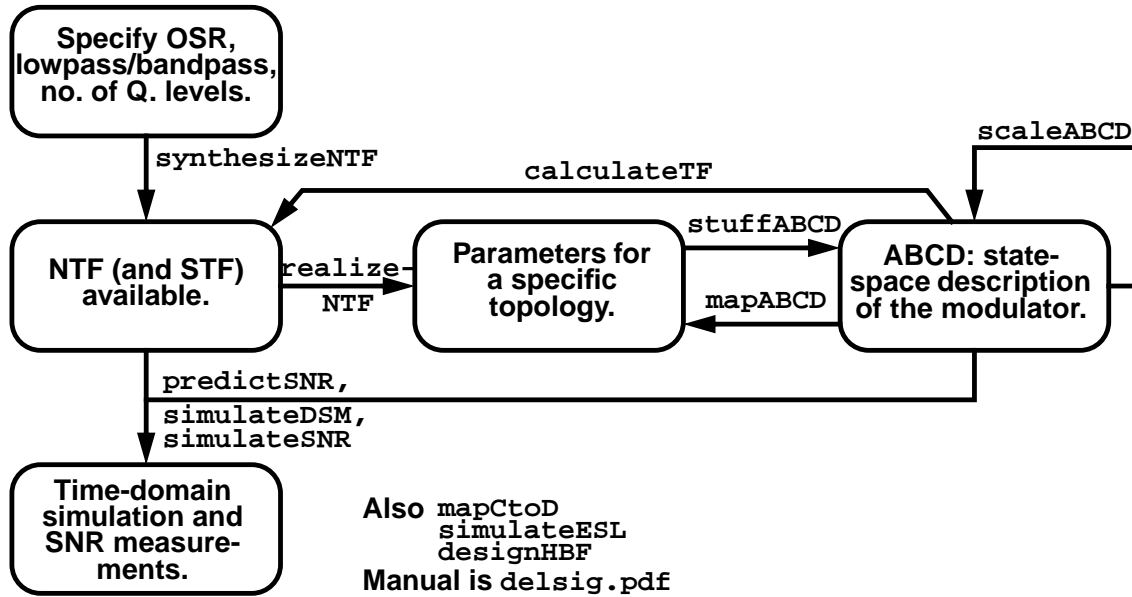
- **FF has relaxed dynamic range requirements**
“All stages except the last have attenuated signal components.”
- **FB has better STF and, in CT modulators, a better AAF**
In a discrete-time modulator, the STF of FF can be made unity by adding a signal feedforward term to the input of the quantizer.
- **FB needs many DACs;
FF needs a summation block**
Can do partial summation before the last integrator.
- **FF timing can be tricky**
Need to quantize u and feed it back in zero time.

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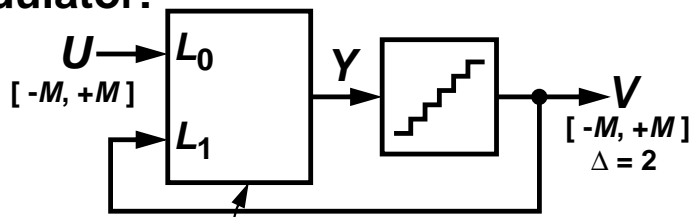
3. $\Delta\Sigma$ Toolbox

<http://www.mathworks.com/matlabcentral/fileexchange>
 Search for “delsig” or “Delta Sigma Toolbox”



$\Delta\Sigma$ Toolbox Modulator Model

Modulator:

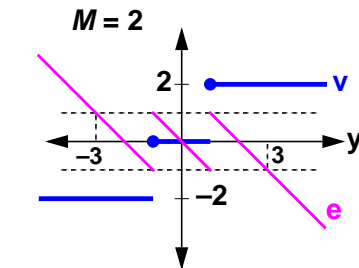
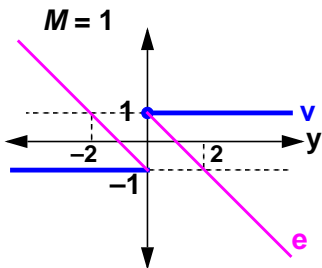


$$NTF = \frac{1}{1 - L_1}$$

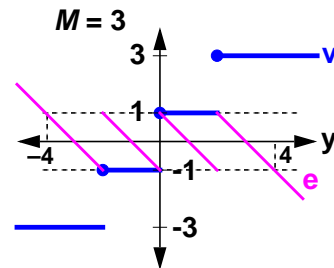
$$STF = \frac{L_0}{1 - L_1}$$

Loop filter can be specified by NTF or by ABCD, a state-space representation

Quantizer:



Mid-tread quantizer;
v: even integers $[-M, +M]$



Mid-rise quantizer;
v: odd integers $[-M, +M]$

NTF Synthesis

`synthesizeNTF`

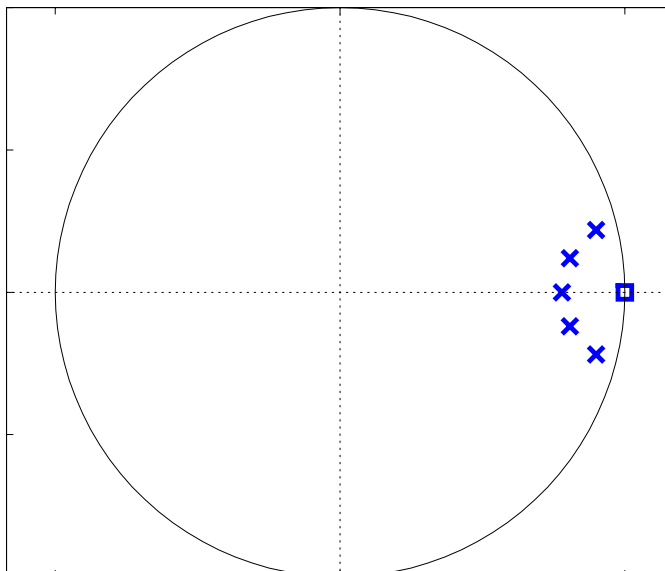
- **Not all NTFs are realizable**
Causality requires $h(0) = 1$, or, in the frequency domain, $H(\infty) = 1$. Recall $H(z) = h(0)z^0 + h(1)z^{-1} + \dots$
- **Not all NTFs yield stable modulators**
Rule of thumb for single-bit modulators: $\|H\|_{\infty} < 1.5$ [Lee].
- **Can optimize NTF zeros to minimize the mean-square value of H in the passband**
- **The NTF and STF share poles, and in some modulator topologies the STF zeros are not arbitrary**
Restrict the NTF such that an all-pole STF is maximally flat. (Almost the same as Butterworth poles.)

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Lowpass Example [dsdemo1] 5th-order NTF, all zeros at DC

- **Pole/Zero diagram:**



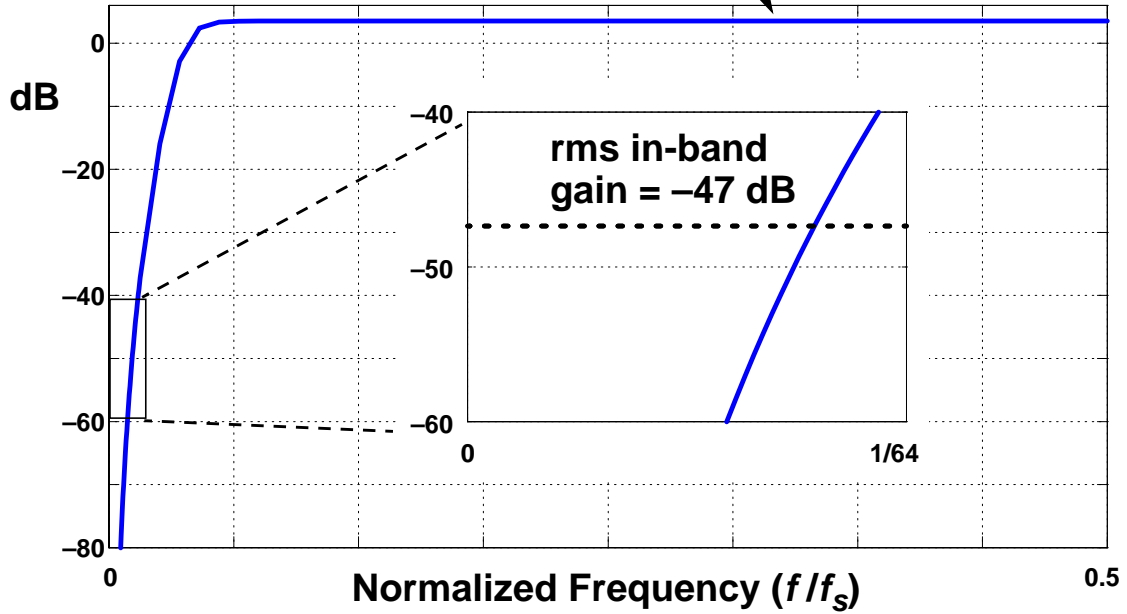
```
OSR = 32;  
H = synthesizeNTF(5);  
plotPZ(H);  
  
f = linspace(0,0.5);  
z = exp(2i*pi*f);  
H_z = evalTF(H,z);  
plot(f,dbv(H_z));  
g = rmsGain(H,0,0.5/OSR)
```

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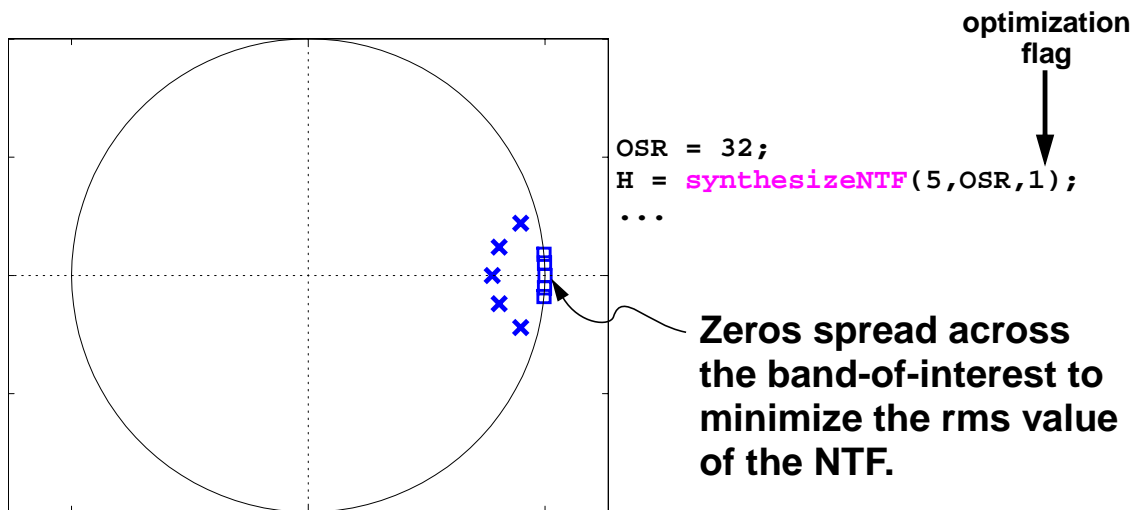
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Lowpass NTF

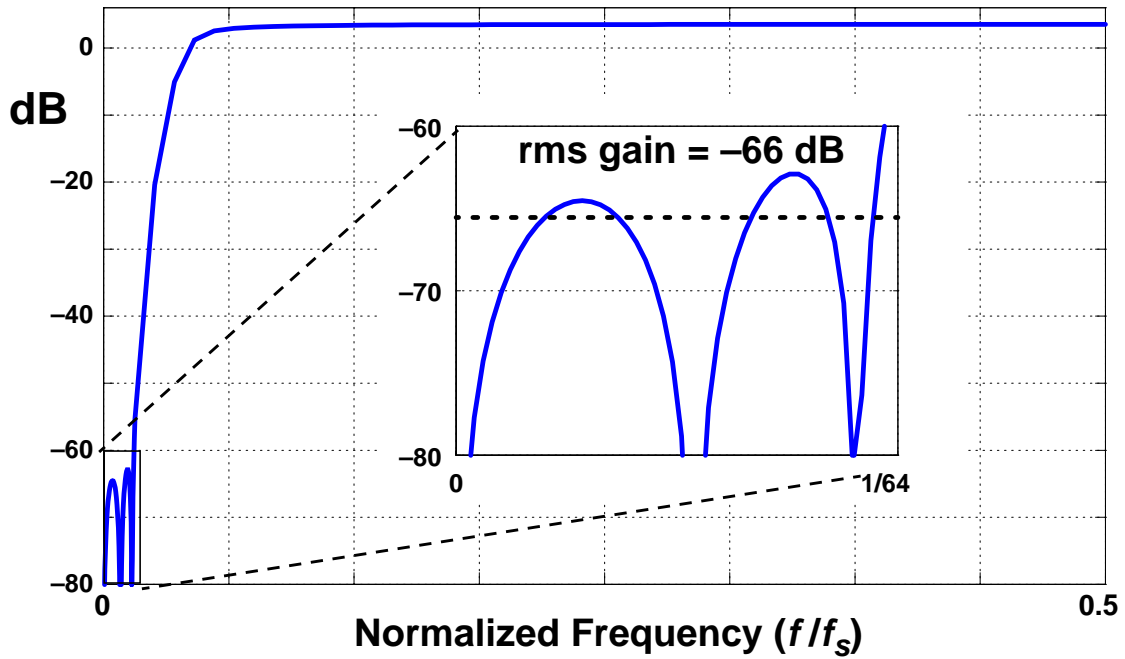
Out-of-band gain = 1.5



Improved 5th-Order Lowpass NTF Zeros optimized for OSR=32



Improved NTF



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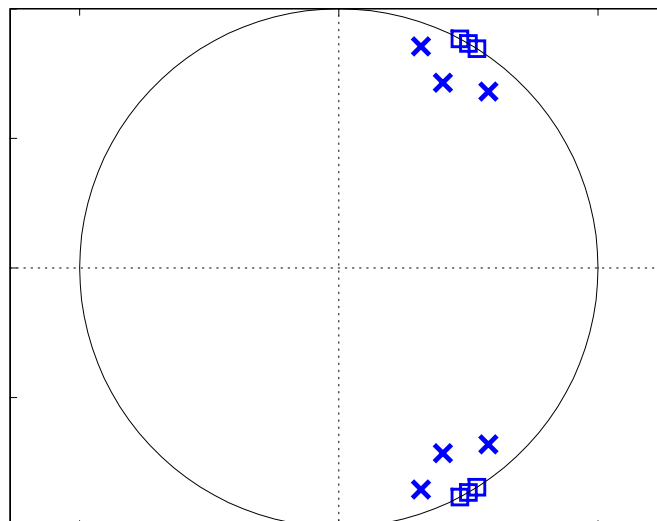
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Bandpass Example

```
OSR = 64;
f0 = 1/6;
H=synthesizeNTF(6,OSR,1,[],f0);...
```

center frequency

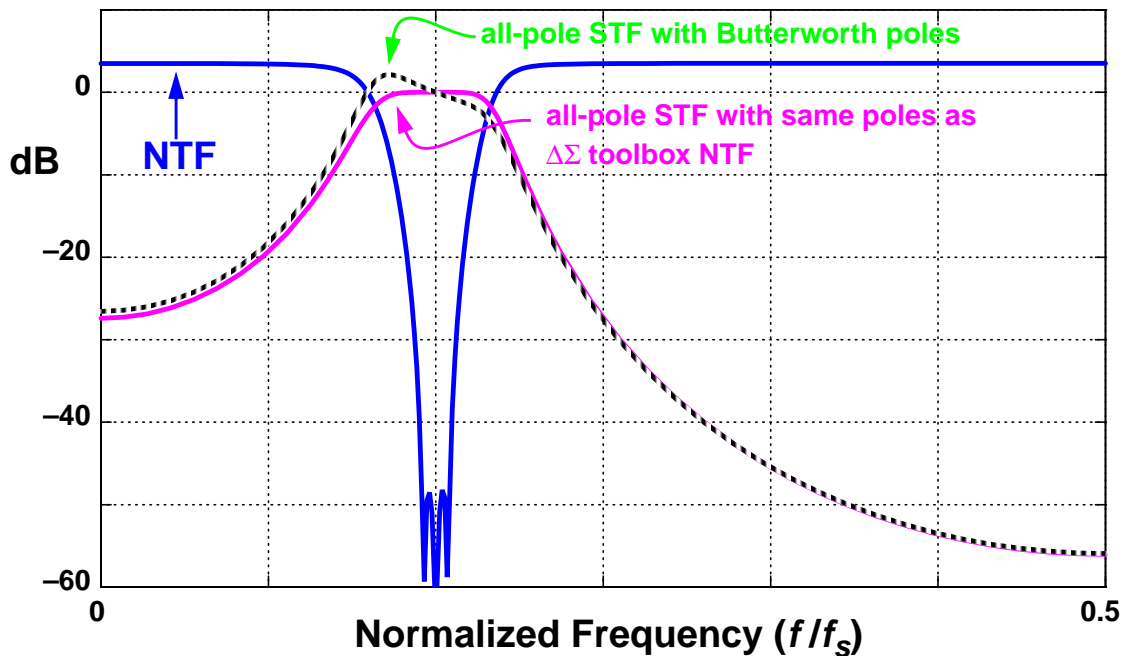
[] or NaN means use default value, i.e. Hinf = 1.5



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Bandpass NTF and STF



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Summary: NTF Selection

- If OSR is high, consider using a single-bit modulator
- To improve SQNR,
 - Optimize zeros,
 - Increase $\|H\|_{\infty}$, or
 - Increase order.
- If SQNR is insufficient, must use a multi-bit design
 - Can turn all the above knobs to enhance performance.
- Feedback DAC assumed to be ideal

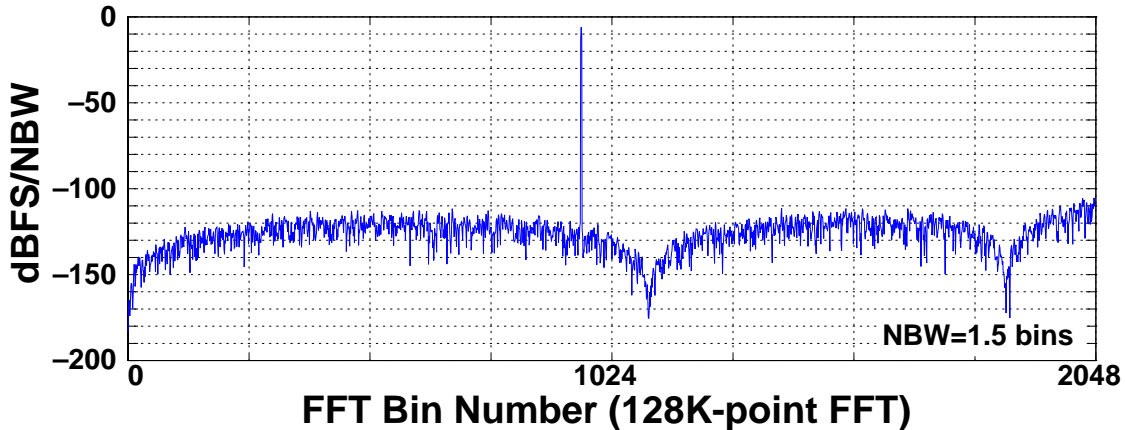
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NTF-Based Simulation [dsdemo2]

```
order=5; OSR=32;  
ntf = synthesizеNTF(order,OSR,1);  
N=2^17; fbin=959; A=0.5; % 128K points  
input = A*sin(2*pi*fbin/N*[0:N-1]);  
output = simulateDSM(input,ntf);  
spec = fft(output.*ds_hann(N)/(N/4));  
plot(dbv(spec(1:N/(2*OSR))));
```

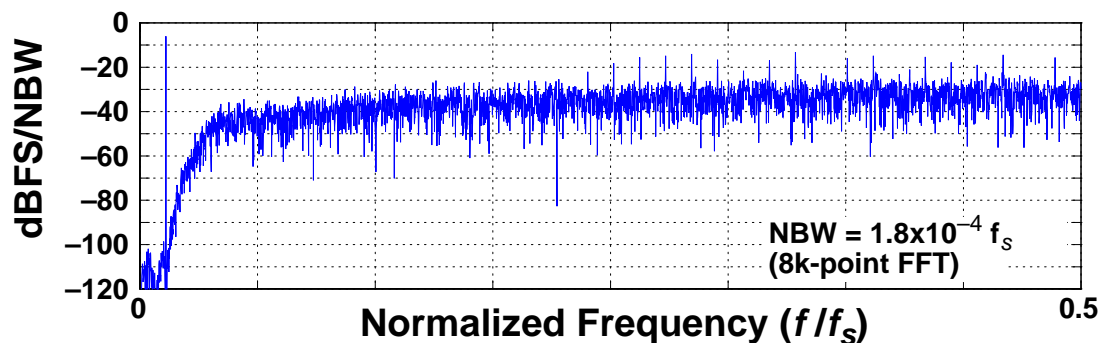
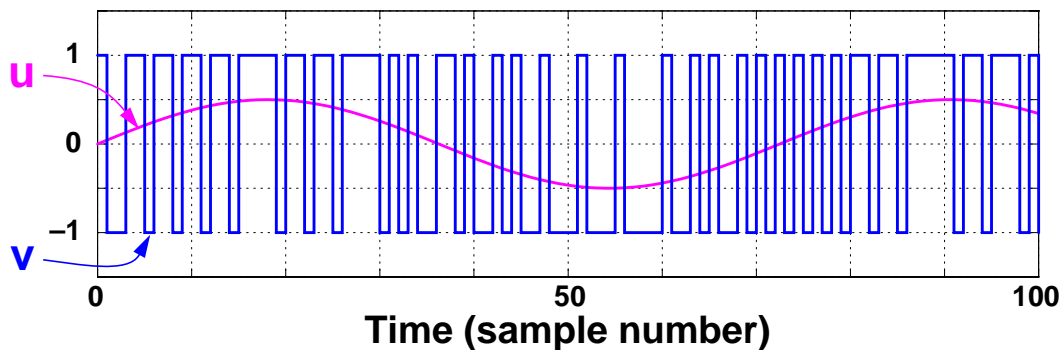
- In mex form; 128K points in < 0.1 sec



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Simulation Example Cont'd

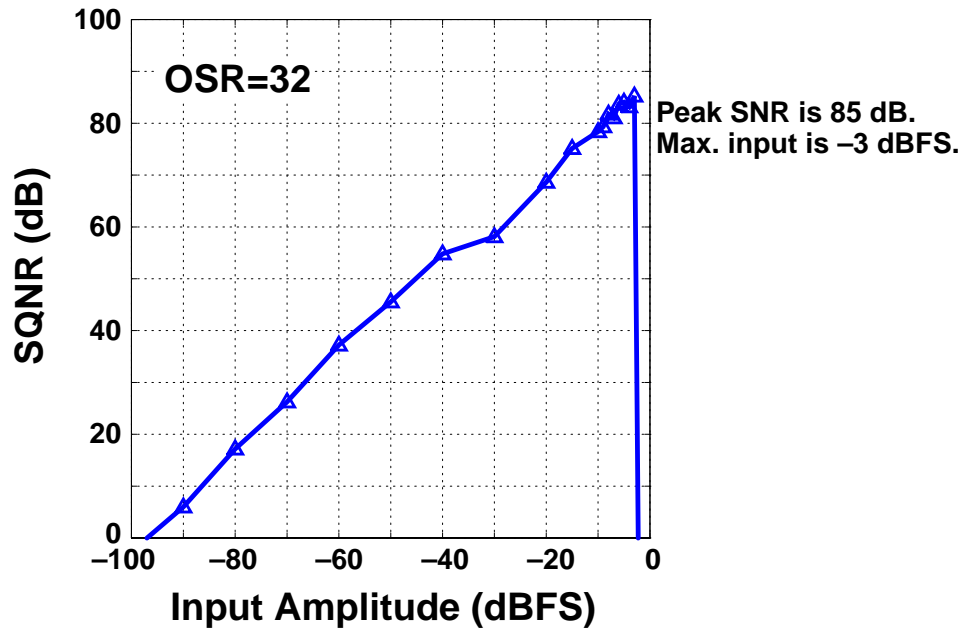


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SNR vs. Amplitude: simulateSNR

```
[snr amp] = simulateSNR(ntf,OSR);  
plot(amp,snr,'b-^');
```

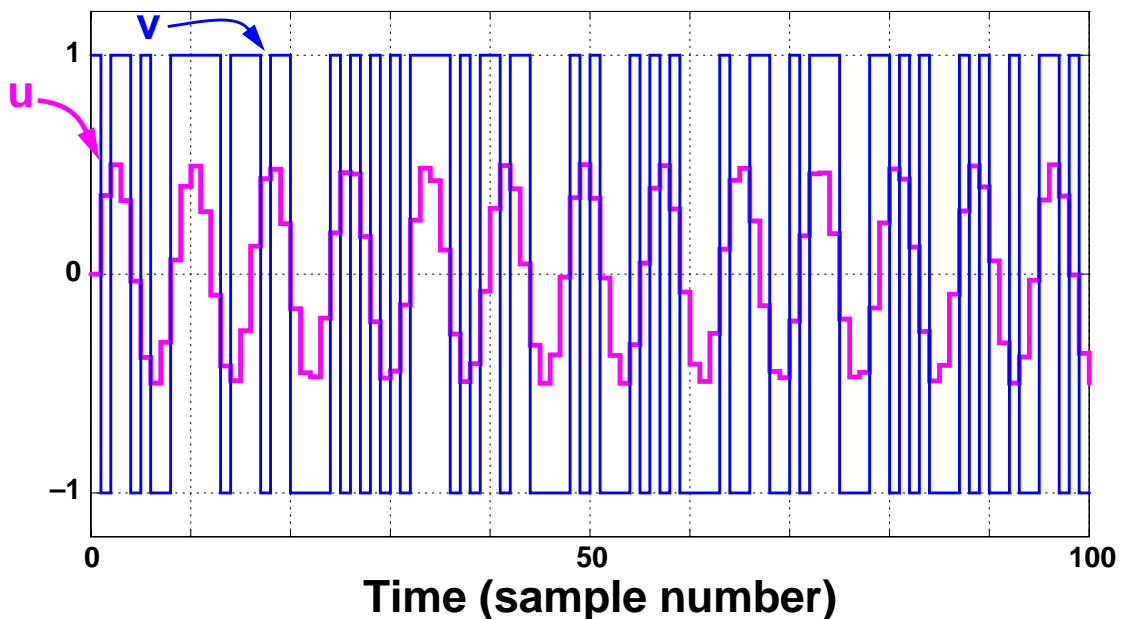


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Bandpass Simulation Example

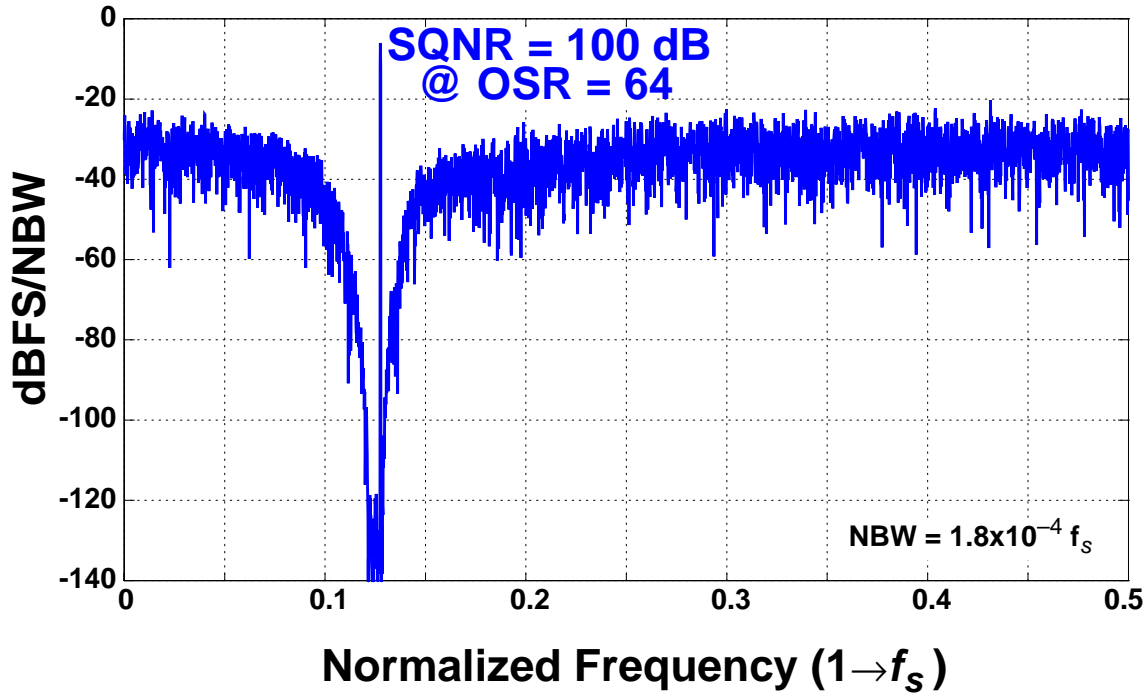
8th-order, $f_0 = f_s/8$



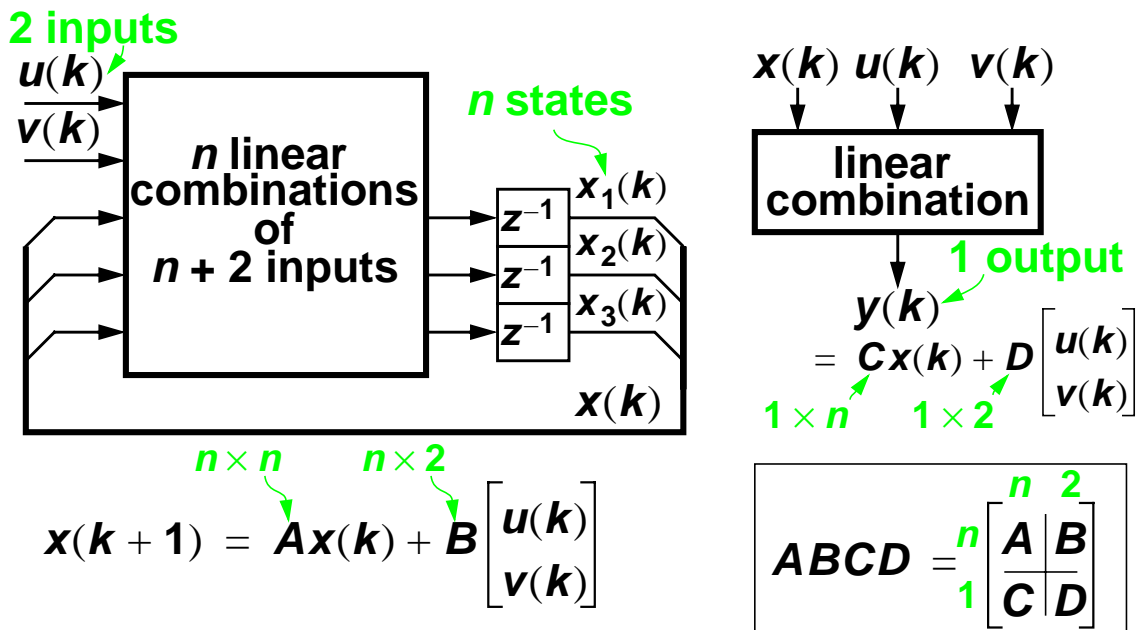
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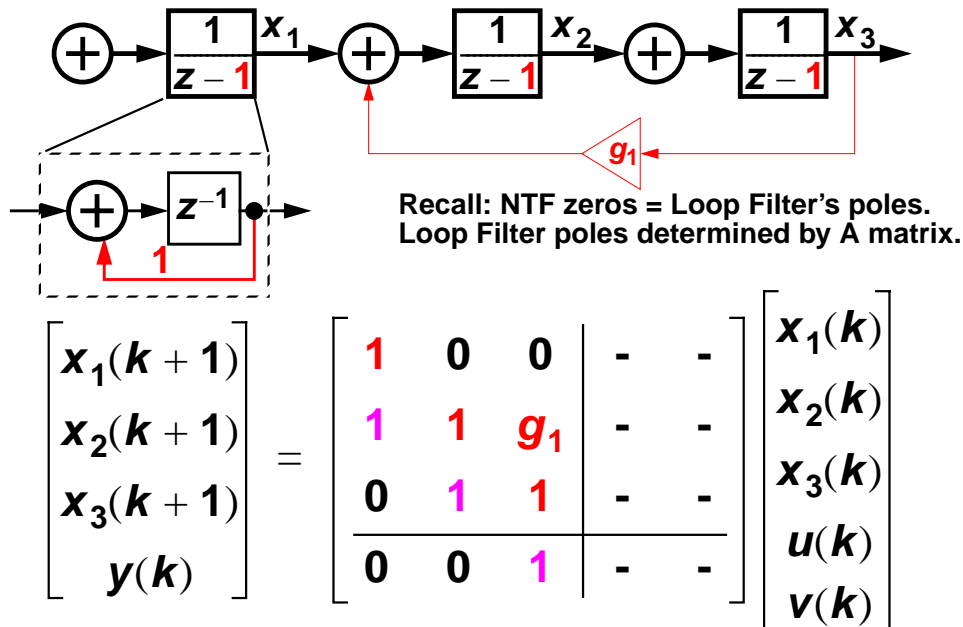
Example Bandpass Spectrum



ABCD: A *State-Space* Representation of the Loop Filter



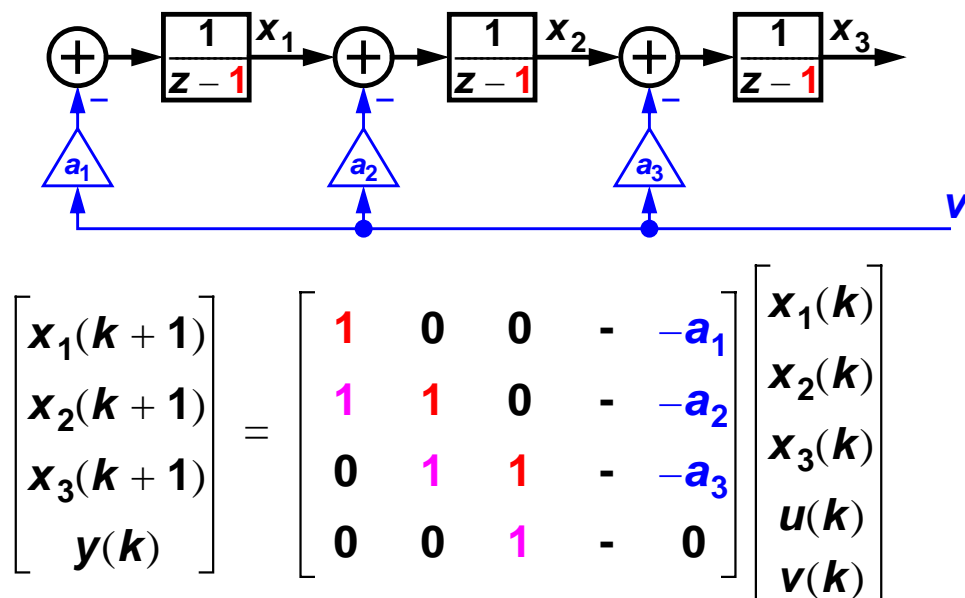
Ex.: Cascade of Integrators Feedback (CIFB) Topology



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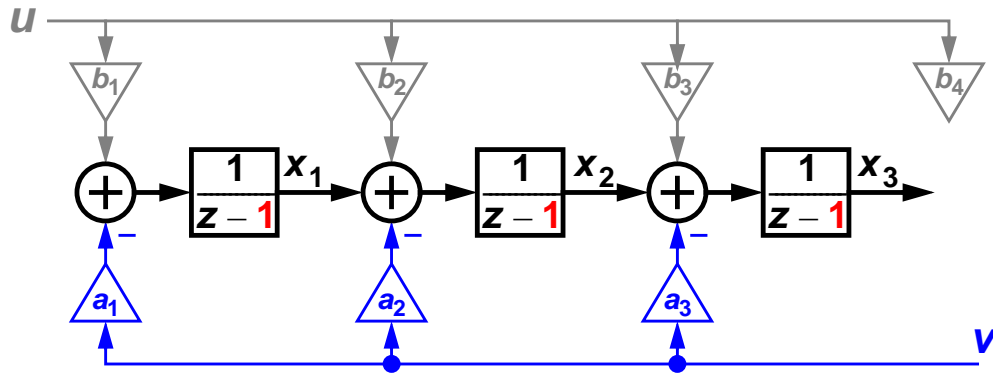
CIFB cont'd: a_i Control NTF & STF Poles



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b_i Control STF Zeros



$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & b_1 & -a_1 \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & b_2 & -a_2 \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & b_3 & -a_3 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & b_4 & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ u(k) \\ v(k) \end{bmatrix}$$

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ABCD and the Toolbox

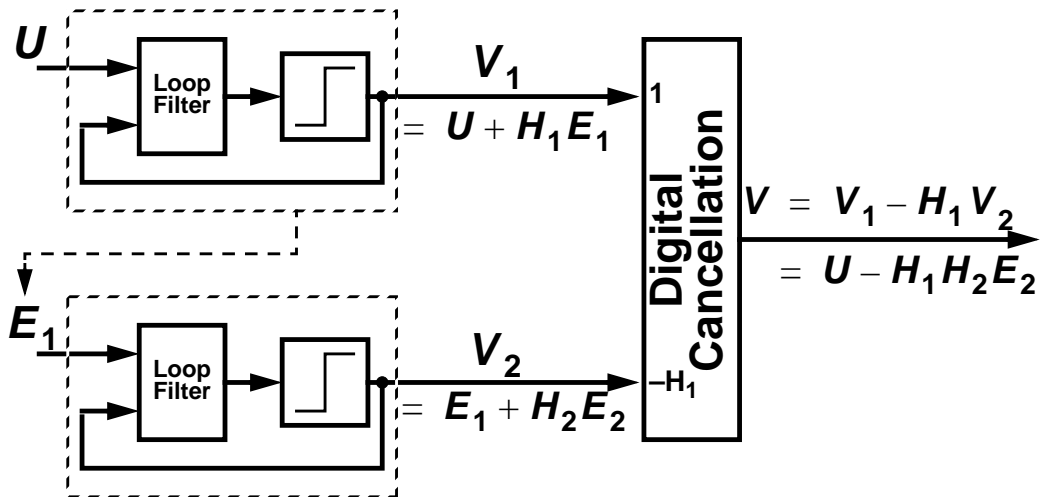
- `simulateDSM` simulates a modulator given an ABCD description of its loop filter
- `realizeNTF` gives (unscaled) coefficients for any of the supported topologies
- `stuffABCD` produces an ABCD matrix given the coefficients for one of the supported topologies
`mapABCD` performs the inverse operation.
- `scaleABCD` does dynamic range scaling on any ABCD matrix
- `calculateTF` calculates the NTF and STF from ABCD
 Useful for checking implementation of new topologies.

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Cascade (MASH) Modulators

- Put two (or more) modulators in “series”



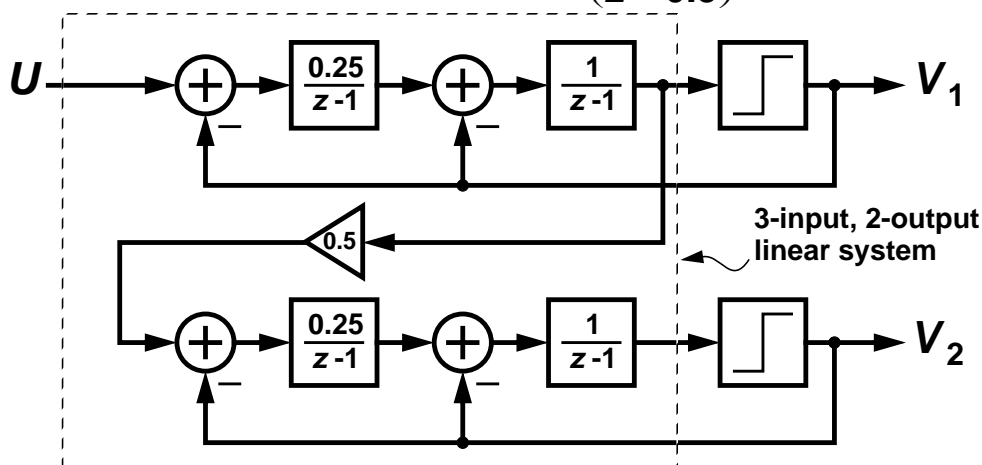
- The resulting NTF is the *product* of the individual NTFs

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Example: 2-2 Cascade

- Use Two MOD2b: $H(z) = \left(\frac{z-1}{z-0.5}\right)^2$

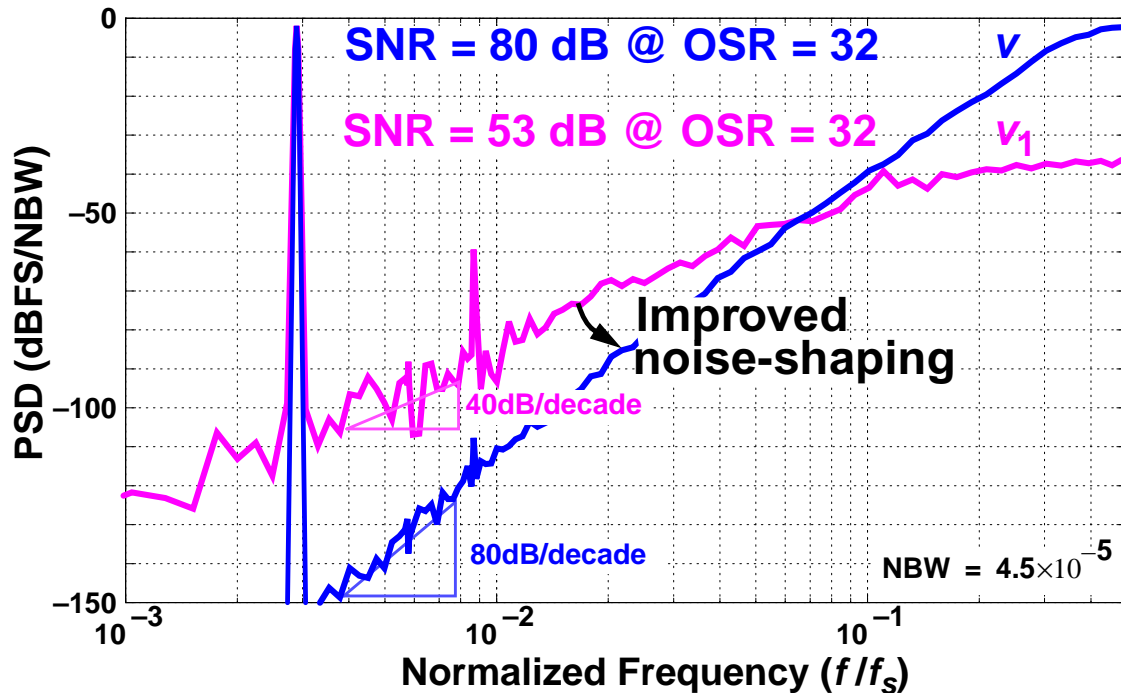


$$V = \frac{1}{z^3} V_1 + \frac{8(z-1)^2(z-0.5)^2}{z^3(z-0.75)} V_2 \Rightarrow \left(H(z) = \frac{8(z-1)^4}{(z-0.75)} \right)$$

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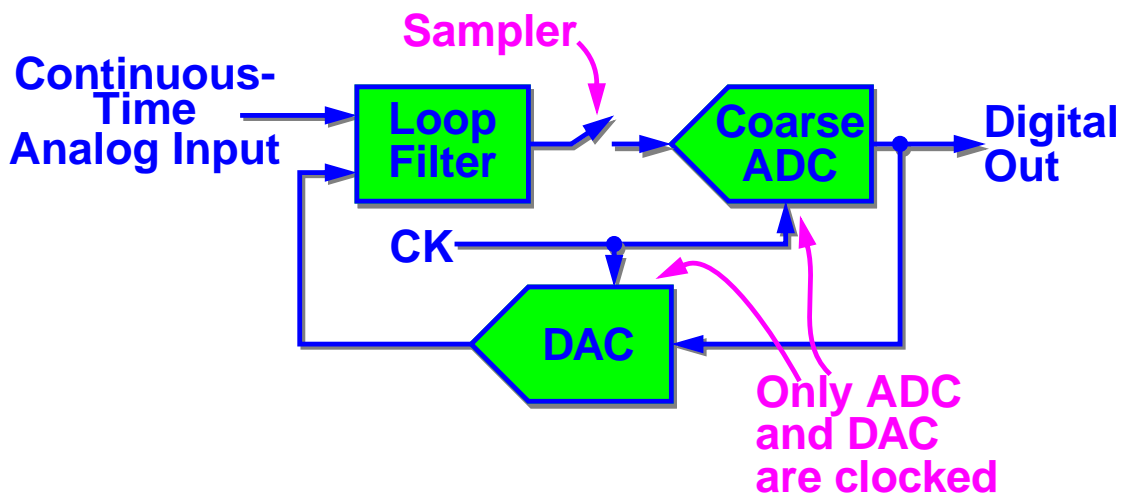
Example MASH Spectra



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4. A Continuous-Time $\Delta\Sigma$ ADC



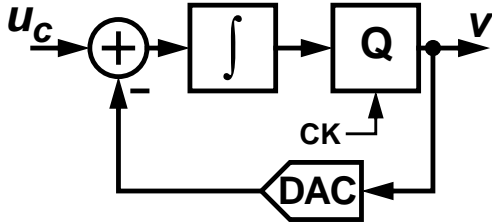
- Loop filter implemented with continuous-time circuitry
- Sampling occurs after the loop filter

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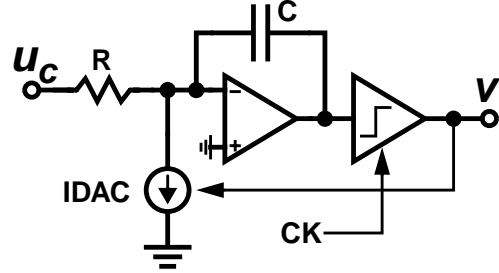
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Example: MOD1-CT

Block Diagram



Schematic



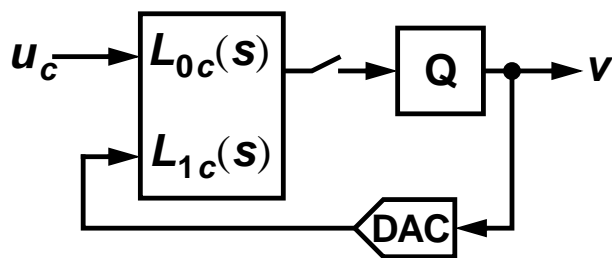
- **Note: Input is a simple resistor, not a switched capacitor**
CT ADCs are easier to drive than DT ADCs

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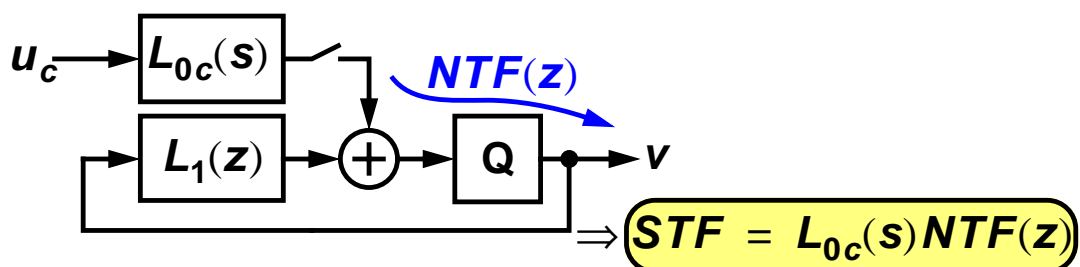
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Inherent Anti-Aliasing

- $\Delta\Sigma$ ADC with CT Loop Filter



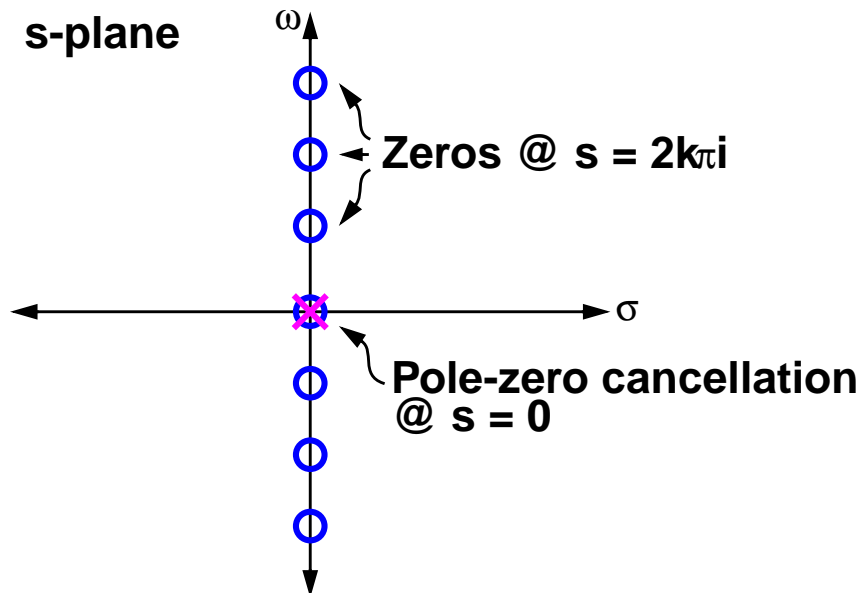
- Equivalent system with DT feedback path



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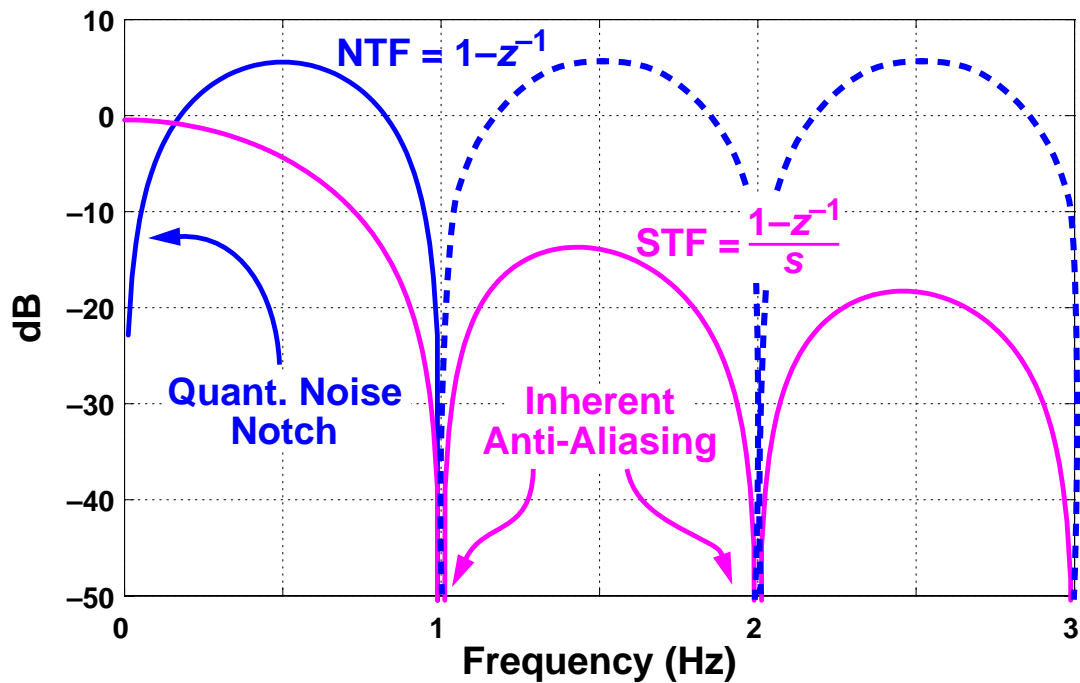
Example: MOD1-CT STF = $\frac{1 - z^{-1}}{s}$
 Recall $z = e^s$



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MOD1-CT Frequency Responses



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Inherent AAF Summary

$$STF = L_{0c}(s)NTF(z)$$

- STF contains the zeros of the NTF
- Any frequency which aliases to the passband is attenuated by at least as much as the quantization noise
 - Anti-alias performance tracks modulator order.
 - Also true for MASH systems.

- The effective anti-alias filter is

$$AAF(f) = \frac{STF(f)}{STF(f_{alias})} = \frac{L_{0c}(f)}{L_{0c}(f_{alias})}$$

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What You Learned Today And what the homework should solidify

1 High-Order $\Delta\Sigma$ Modulation

NTF, SQNR, instability, $\|H\|_{\infty}$, $\|h\|_1$

2 Feedback, Feedforward and Generic $\Delta\Sigma$ Topologies

NTF selection

3 $\Delta\Sigma$ Toolbox

Demos: NTF synthesis and simulation

Examples: Lowpass and Bandpass, MASH

ABCD: State-space representation

4 Continuous-Time $\Delta\Sigma$

Inherent anti-aliasing

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Homework #5

- 1 Verify that with ABCD set to $ABCD = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

`simulateDSM` is the same as your `simulateMOD2` from Homework #1.

One simulation is sufficient, just be sure to compare the internal states as well as the bit-stream output.

- 2 Obtain an NTF for MOD2b using `synthesizENTF` with $\|H\|_{\infty} = 2.5$, $OSR = 128$ and `opt=1`.

Plot the NTF's poles and zeros as well as its frequency response. Include those of MOD2.

Compare MOD2b's SQNR-vs.-input-amplitude curve with MOD2's using `simulateSNR`.

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- 3 Construct the block diagram of a 4th-order single-loop 9-level $\Delta\Sigma$ ADC employing the CRFB topology that achieves SQNR > 100 dB at OSR = 32 and verify it.

Choose $\|H\|_{\infty}$ such that the stable input range is maximized.

Plot the SQNR-versus-amplitude curve and note the value of u_{max} . Compare with the $\|h\|_1$ criterion.

Realize your NTF with a CRFB-style modulator having a single input feed-in. Ensure $STF(dc) = 1$.

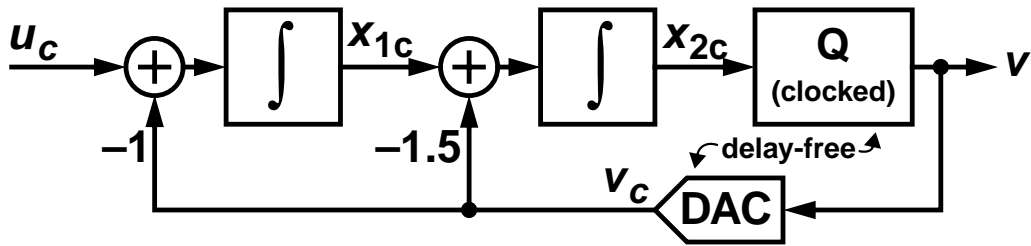
Do dynamic range scaling for $u_{max} = -1$ dBFS and state maxima less than unity. You may use the $\Delta\Sigma$ toolbox function `scaleABCD`. Plot the state maxima as functions of the input amplitude.

Implement your block diagram with C/Matlab code and verify that your coded modulator behaves the same as your MATLAB modulator.

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4 Verify that the following is a continuous-time implementation of MOD2



It is sufficient to verify that the sampled pulse response of the loop filter is the same as MOD2's.

Show that the STF is $\left(\frac{1 - z^{-1}}{s}\right)^2$

If this modulator is used at an oversampling ratio of 64, what is its minimum alias attenuation?