Knowledge Spillovers in Cities: An Auction Approach

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Abstract: I propose a new micro-foundation for knowledge spillovers. I specify a model of a city in which uncompensated knowledge transfers to entrepreneurs are bids by experts in auctions for jobs. The model derives from the key ideas about how knowledge differs from other inputs of production, namely that knowledge must be possessed for its value to be assessed, and that knowledge is freely reproducible (Arrow 1962). The model identifies conditions under which knowledge spills through non-market interactions, as opposed to being transacted in markets. Endogenous agglomeration economies result from growth in the number of meetings between experts and entrepreneurs and from heightened competition for jobs among experts.

Key words: agglomeration, auctions, knowledge transfers.

JEL classification: D44, D83, R23, R39

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1. Introduction

Knowledge externalities, or spillovers, feature prominently in theoretical models able to explain the growth or the location of production in modern economies. The main objective of this paper is to explain why, how and when knowledge 'spills', as opposed to being bought and sold in markets. In the model, an uncompensated knowledge transfer is a bid by an expert to an entrepreneur who auctions a job. The model features increasing returns in aggregate production to the number of experts and entrepreneurs interacting in a city, and demonstrates how the key properties of knowledge as an input of production lead naturally to a micro-foundation for agglomeration economies.

Empirical studies highlight the very localized nature of production externalities, a finding consistent with the idea that densely populated urban areas intensify knowledge diffusion by facilitating face-to-face interactions.[1] But empirical evidence is still lacking on the exact mechanism through which knowledge spills, adding to the interest in theoretical modeling of knowledge transfers in cities.

I develop a model in which individuals meet each other in a city, and then start to work. During a meeting, an expert chooses how much knowledge to freely transfer to an entrepreneur. After learning from all experts, an entrepreneur hires one of them and produces a good with a constant returns to scale technology, using both the knowledge obtained during meetings and that acquired from the hired expert. The model's defining assumptions - that an expert's knowledge is unobservable and can be communicated at no cost - derive from the key properties of knowledge as an input of production, namely the impossibility to assess the value of knowledge without possessing it (Arrow, 1962), and free reproducibility. Arrow's property suggests the motivation behind uncompensated knowledge transfers. For an expert, a free transfer can be thought of as a bid for a job contract; as a mean to prove how knowledgeable he is. For an entrepreneur, who auctions a job to experts, the free knowledge transfers are an opportunity to accumulate knowledge, and to find the expert who is best for the job. In equilibrium, because of free reproducibility and the greater number of possible meetings between experts and entrepreneurs in a larger city, these transfers generate agglomeration economies: an explanation for the existence of cities. The

[1] See for instance Rosenthal and Strange (2008), Jaffe, Trajtenberg, and Henderson (1993) on the geographical location of patents' citations, and especially Arzaghi and Henderson (2008) on networking by advertising agencies in Manhattan.

use of auction theory uncovers another source of increasing returns in aggregate production from agglomeration, stemming from competitive behavior. As the number of experts competing for jobs in a city increases, their willingness to freely transfer knowledge - the size of their knowledge bid - also increases. The model delineates the incentives of experts to communicate some, but not all of their valuable knowledge without direct payment, and identifies the conditions under which knowledge spills in non-market interactions, as opposed to market transactions which take place once the best expert for a job is revealed and starts to work for an entrepreneur. Auction theory appeals to intuitive notion of people's behavior in competitive environment, and captures the strategic incentive to give valuable knowledge in a framework with many agents. As such, the model evokes so-called 'industrial clusters', in which networking is essential, job changes are frequent, and, as shown in Freedman (2008), workers initially accept lower wages while they develop their reputation.

In the first extension of the model, I analyze the equilibrium size and composition of an open city for the case in which experts, who differ in the value of their knowledge, can choose to migrate to a city before learning their type. The results for city composition confirm the presumption that increasing the proportion of experts - who provide the externality - can improve a city's efficiency. The results on city size, however, upsets the standard outcome, in that there can be cities with larger population than the equilibrium one, and with larger aggregate welfare. The simplest remedy would be a tax redistributing income from entrepreneurs to experts, but a micro-founded model allows the design of even better tax policies that target the incentives of experts to transfer knowledge. In the second extension of the model, experts can choose to migrate after learning their type, which leads to sorting of the best experts into the city, an outcome consistent with empirical results (Combes, Duranton, and Gobillon, 2008). In this extension, there is a trade-off between the quality and the quantity of experts moving into a city, when either the cost of urban living or competition for a place in the city varies. The cost of urban living also has a direct impact on aggregate production, because it affects the knowledge bid of experts, just like an entry cost would in a standard auction.

The theory presented here draws from, and in some cases contributes to, many strands of literature in urban economics, innovation, knowledge markets and mechanism design (auctions). A vast empirical literature in urban economics establishes that firms and individuals in larger agglomerations are more productive; see Melo, Graham, and Noland (2009) for a meta-analysis.

The classic explanations for this productivity advantage are so-called 'agglomeration economies'; processes through which interactions between firms or individuals are facilitated in cities, in a way that makes these units more productive. However, identification of knowledge spillovers and of their contribution to agglomeration economies stays largely elusive. Jaffe *et al.* (1993) are able to measure a locational component of knowledge transfers from the tendency of patents to cite other patents whose origin is geographically close. Perhaps the key piece of empirical evidence in support of knowledge spillovers as a determinant of the productivity advantage of cities is the finding that production externalities are very localized. Both Rosenthal and Strange (2008) and Arzaghi and Henderson (2008) find that the productivity gains from agents' proximity to one another decays rapidly with distance. This decay suggest that face-to-face interactions - whose cost is especially sensitive to distance - are at the heart of the complex mix of factors rendering larger cities more productive. Regarding the incentives of agents to engage in knowledge transfers, the innovation literature emphasizes the role of reciprocity. In a seminal paper, von Hippel (1987), based on survey evidence, finds that engineers in the steel industry who share knowledge with their peers are motivated by the expectation of reciprocal transfers.

With one notable exception, reciprocity is also the motivation for knowledge transfers in theoretical models of knowledge diffusion and creation.[2] Jovanovic and Rob (1989) present a model in which agents not only can develop ideas privately, but can also share their ideas with other agents, such interactions leading both to imitation (diffusion) and invention (creation). Rob and Jovanovic's insight that new and better ideas originate from the contact of different ideas is refined in a series of papers by Berliant and co-authors (2006, 2008 and 2009), which provide an analysis of the costs and benefits of urban diversity for knowledge creation. Helsley and Strange's (2004) model is the first in which the choice of how much knowledge to transfer is endogenous. Agents living in a city can barter knowledge and develop a reputation for cooperation. Their model is directly based on Von Hippel's finding on the importance of reciprocity. The model that I propose purposely excludes the possibility of sharing or reciprocity, to provide an alternative explanation for uncompensated knowledge transfers, based on the incentives of agents to communicate how knowledgeable they are. Interestingly, reciprocity is found to be less important in high-tech or other rapidly evolving industries (Appleyard, 1996), precisely the kind of industries in which

[2]In Glaeser's (2001) model of learning in cities, knowledge is transfered by imitation of the old by the young - with the old getting a share from the young's returns to a successful skill transfer. In this model, an agent only decide whether or not to live in a city.

agglomeration economies, that reciprocity-based models are meant to explain, are found to be largest (Moretti (2004), Fu (2007), and others). Ignoring reciprocity reconciles theory with these empirical findings.

The model presented here also relates to the literature on knowledge markets, which contains two influential ideas about how knowledge differs from other inputs of production. These ideas provide the intellectual foundation for the model, and they map into its key assumptions. The first idea, from Arrow (1962), is that one often has to possess knowledge in order to assess its value, or to reveal an idea in order to demonstrate its usefulness. The notion that knowledge transfered for free cannot be given back led Anton and Yao (2002) to designate knowledge as 'expropriable'.[3] Anton and Yao (2002) and others - an early example is Bhattacharya and Ritter (1983) - draw from Spence (1973, 1974) work on market signaling to show that Arrow's property can lead, as it does in my model, to a 'voluntary disclosure of knowledge' (another expression for 'free knowledge transfer'). My model differs from existing models of voluntary knowledge disclosure, in that it defines a free knowledge transfers not as a 'signal', but rather as a 'bid', a simpler concept that is better suited to competitive urban environments. The second idea is that knowledge is freely reproducible, or has a very low marginal cost of production once a blue-print exists. The discovery of a link between the free reproducibility of knowledge and increasing returns can be attributed to Romer (1990).

Finally, the paper draws from the literature on auction theory. The solution to the basic version of the model only involves the simplest case of a first-price auction. To study sorting by skills in an open city, I use the reasoning in Samuelson (1985), who solves an auction model with an entry cost and a finite number of bidders who know their valuation before entering the auction. However, knowledge auctions in an urban environment differs in two ways from existing auction models. First, a knowledge auction is all-pay from the auctioneer perspective, but not from that of the bidder. Second, entry costs in urban environments are endogenous, because congestion in a city depends on the number of inhabitants.

[3]Note how the problem is not the non-contractibility of knowledge, but rather the unobservability of knowledge and the impossibility to give it back once transfered.

2. A model of knowledge transfers

I consider a closed city with N inhabitants, a number X of which are experts who hold some knowledge, and a number E of which are entrepreneurs who can use that knowledge in production, so that $X + E = N$.

The game played by experts and entrepreneurs has two stages: first a meeting stage, then a production stage. At the meeting stage, experts choose how much knowledge to freely transfer to entrepreneurs that they meet in the city. Entrepreneurs learn from the knowledge transfered by experts. At the production stage, each entrepreneur hires one expert and pays him for the knowledge that he has not already given at the meeting stage. Entrepreneurs then produce a consumption good, using the knowledge acquired both at the meeting stage and at the production stage of the game.

The X experts are indexed by i and the E entrepreneurs are indexed by j. The value of expert i's knowledge to entrepreneur j is denoted by k_{ij}, so expert i's type is a E-vector of knowledge $k_i = (k_{i1}...,k_{ij},...k_{iE})$. k_{ij} is independently and identically distributed over the [0,1] uniform distribution, for all $i \in \{1,...,X\}$ and $j \in \{1,...,E\}$. The value of an expert's knowledge to an entrepreneur is independent of its value to another entrepreneur, to reflect the idea that experts are knowledgeable about different issues, and that each entrepreneur puts a different value on any expert's knowledge.[4] Knowledge has two key properties: there is no cost of communicating it (free reproducibility), and the value of an expert's knowledge is unobservable to an entrepreneur or to other experts (Arrow's property). The meeting technology is such that the number of meetings is $E * X$, so that every expert meets every entrepreneur in the first stage of the game. I define b_{ij}, a function of k_{ij} and X, as the knowledge transfered for free by expert i to an entrepreneur j that he meets in the first stage of the game. Each entrepreneur uses the same constant returns to scale, additive production function to produce a single good y, with knowledge as the only input. The amount of the good produced by entrepreneur j (denoted by y_j), is equal to the knowledge that he received for free at the meeting stage of the game, plus that he knowledge paid for at the production stage (denoted by K_j), so that:

$$y_j = \sum_{i=1}^{X} [b(k_{ij},X)] + K_j. \tag{1}$$

[4]This independence does not affect any of the results in this, or the next section. It matters in section 4, in which experts can choose to migrate into an open city after learning their type.

Both experts and entrepreneurs have utility function $u(y) = y$, where y is an agent's consumption of the good.

An expert hired by an entrepreneur earns a wage equal to his productivity at work.[5] Knowledge transfered for free cannot be given back (Arrow's property) and as such is never paid for. Note that the wage contract depends on the actual surplus created by the expert, not on the belief of an entrepreneur about his type. It follows from the CRS, additive production function that expert i hired by entrepreneur j earns a wage equal to $k_i - b_i(k_{ij},X)$; the value of the knowledge that he has not already given during the meeting stage of the game. The wage is paid in unit of the good y, so that $k_i - b_i(k_i,X) = K_j$, which, because of the linear utility function, is also the utility of an expert from obtaining a job. An expert can work for any number of entrepreneurs.

Entrepreneurs are the mechanism designers. The unobservability of experts' knowledge presents an opportunity to ask for free knowledge, with a job offered as a reward. Instead of solving for an optimal job allocation mechanism, I propose the following mechanism: it is common knowledge that an entrepreneur gives the job to the expert who transmits the most knowledge during the meeting stage.[6] This job allocation mechanism - a first-price auction - is motivated both by its simplicity and by its properties in equilibrium. A simple rule, allowing entrepreneur both to obtain free knowledge and to find and hire the best expert (in equilibrium), is perhaps a more realistic representation of entrepreneurs' behavior in informal or semi-formal meetings than the complex incentives scheme that optimal mechanisms could suggest. Also, the first-price auction is arbitrarily close to an optimal mechanism - both in the sense that it maximizes an entrepreneur's payoff and that it makes a city efficient - as X becomes large.

Interpretation: auction framework

Auction theory is only relevant to knowledge transfers in cities if its main concepts have a plausible interpretation in the context of urban working and networking environments. Most important, an auction implies that a free transfer is a bid, rather than a signal, or a reciprocal

[5]Equivalently, the wage is settled through Nash bargaining and λ, the bargaining power of an entrepreneur, is equal to 0. A different value of λ would not affect any of the main result of this section.

[6]Note that commitment to hire only one expert is not needed for a Bayesian equilibrium because entrepreneurs are indifferent between hiring zero, one or many experts (entrepreneurs do not get a share λ of experts' surplus). In fact, hiring just one expert - auctioning one job as opposed to zero or many - maximizes an entrepreneur's equilibrium payoff when λ is small enough, because it maximizes the bids of experts. For instance, experts do not transfer any free knowledge in an equilibrium in which entrepreneurs hire all experts. The result is intuitive, but a formal proof requires solving auctions with many goods, in which winners pay different prices (so-called 'discriminatory auctions').

transfer, or something else. Whenever an environment is competitive, so that experts care about being recognized as more knowledgeable than others they compete with for jobs or fame, a bid probably captures the motivation behind knowledge transfers better than a signal or a purely reciprocal exchange. Also, an auction implies that the value of an expert's knowledge corresponds to his valuation for a job. Whenever better performance earns better rewards, which seems to be the norm for industries in which agglomeration economies are observed, more knowledgeable experts, who perform better at work, must value a job more. Finally, an auction implies that the number of experts competing for a job affects the size of their knowledge bid. Whenever the winner of a job cannot be paid for the knowledge already transfered for free (expropriability), experts optimally transfer just enough knowledge to outbid other experts, so that the number of competing experts matters. Theoretically, expropriability is a corollary of the Arrow property, and it has been invoked in another model by Anton and Yao (2002). Empirically, expropriability is hard to verify, and in many industries, ethical concerns or social norms might supercede entrepreneur's pecuniary interest in expropriating experts.[7]

Results

I first solve for an expert's equilibrium strategy at the meeting stage. Using auction terminology, k_{ij} becomes expert i's valuation for a job with entrepreneur j, and $b_{ij}(k_{ij},X)$ becomes his bid for a job. The strategy of an expected utility maximizing expert i, when meeting entrepreneur's j, is to transfer the amount of free knowledge that maximizes the probability that he gets the job times the utility (wage) that he obtains if he is hired. So for each $k_{ij} \in [0,1]$, the expert submits a bid $b_{ij} \geq 0$ that solves:

$$\max_{b_{ij}} prob[b_{ij} > \max_{l \in \{1,\ldots,X\} \setminus \{i\}} b_{lj}(k_{lj},X)] * (k_{ij} - b_{ij}) \tag{2}$$

From this expression, it appears that a meeting for an expert is equivalent to participation in a first-price auction. The highest bidder wins and 'pays' his bid, because an expert cannot be paid at the production stage for the knowledge already transfered at the meeting stage. An expert optimally works for every entrepreneur who offers him a job, so his E meetings are strategically

[7]There is certainly much anecdotal evidence, for instance, that designers providing free designs in the hope of winning a contest are paid below market rates. For a discussion see the article by Michelle Goodman *When to work for nothing* in the New York Times 'Shifting Careers' blog, November 9, 2008 [http://shiftingcareers.blogs.nytimes.com/2008/11/09/when-to-work-for-free/].

identical and independent. The following lemma contains the solution to an expert's problem of how much knowledge to transfer in a meeting:

Lemma 1 *There exists a unique symmetric equilibrium bidding function that is strictly increasing in k and with $b(0,X) = 0$, given by $b(k,X) = \frac{X-1}{X}k$.*

Proof The proof of Lemma 1 follows directly from that of Lemma 3 (in-text), which covers a more general case, with non-negative entry costs[8]. ■

An important property of the bidding function is that $\frac{\partial b(k,X)}{\partial X} > 0$, for all $k \in [0,1]$ and all $X > 0$, so that for expert of each type, the value of the free knowledge transfered increases with the number of other experts in the city. In other words, the more intense the competition for jobs, the better the disposition of experts towards free knowledge transfer. Before stating and proving the main proposition of this section, I derive a few intermediate results for later use.

To compute W, the expected wage paid by an entrepreneur, recall that the largest knowledge bidder wins the job auction and that the bidding function is strictly increasing in k, so that in equilibrium an expert is hired if he is the best among X experts for a given entrepreneur. Denote the type of this expert by $k_{(X)}$, the value of the X^{th} order statistics from X draws from the $i.i.d.$ $U[0,1]$ distribution, with probability distribution function $f(k_{(X)}) = Xk_{(X)}^{X-1}$. The expected wage W - which is equal to $\mathbb{E}[K_j]$, the expectation of K_j - can then be computed as:

$$\begin{aligned} W(X) &= \int_0^1 (k_{(X)} - b(k_{(X)},X))f(k_{(X)})dk_{(X)} \\ &= \int_0^1 ((k_{(X)} - \frac{X-1}{X}k_{(X)})Xk_{(X)}^{X-1})dk_{(X)} \\ &= \frac{1}{X+1}. \end{aligned} \tag{3}$$

To study the migration of experts into an open city in section 3 and 4, it is necessary to distinguish between the expected utility of experts before and after they learn their type. The derivation of expected utilities only uses the fact $\mathbb{E}$ is a linear operator, that $k_{ij} \sim i.i.d.\ U[0,1]$, and that the utility function is linear. In the equation below, $\mathbb{I}\{\cdot\}$ is an indicator function equal to 1 if expert i wins a job from entrepreneur j. The expected utility of an expert i of type k after he

[8] $b(k,X) = \frac{X-1}{X}k$ is the unique symmetric equilibrium of a first-price auction in the independent private value model with quasi-linear utility and a $iidU[0,1]$ distribution of valuation, see for instance Jehle and Reny (2000). That the result holds for all quasi-linear utility functions implies that λ, the bargaining power of an entrepreneur, does not affect the bidding function experts, which stays the same for wages equal to $(1-\lambda)*(k - b(k))$ and $0 \leq \lambda < 1$.

learns his type is equal to:

$$\begin{aligned} A_X(k_{i1},...,k_{iE},X,E) &= \mathbb{E}[\sum_{j=1}^{E}(k_{ij} - b(k_{ij},X)\mathbb{I}\{k_{ij} > k_{hj}, \forall h \in \{1,...,X\}\backslash\{i\}\})] \\ &= \sum_{j=1}^{E} k_{ij}^{X-1}(k_{ij} - b(k_{ij},X)) \\ &= \sum_{j=1}^{E} \frac{k_{ij}^{X}}{X}. \end{aligned} \tag{4}$$

Note that k_{ij}^{X-1} in the second line of equation (4) is the probability that expert i wins the auction of entrepreneur j. A_X increases with the number of entrepreneurs and with an expert's value of knowledge, and it decreases with the number of other experts. The expected utility of an expert before he learns his type is equal to the expected wage from a job given by equation (3), times the expected number of jobs each expert wins ($\frac{E}{X}$):

$$B_X(X,E) = \frac{1}{1+X} * \frac{E}{X}. \tag{5}$$

Recall that entrepreneurs do not get a share of the surplus produced by working experts. So the expected utility of an entrepreneur is equal to the expected value of the knowledge transfered during meetings:

$$\begin{aligned} B_E(X) &= \mathbb{E}[\sum_{i=1}^{X} b(k_{ij})] = X\int_0^1 b(z)dz \\ &= X\int_0^1 \frac{X-1}{X} z dz = \frac{X-1}{2}. \end{aligned} \tag{6}$$

$B_E(X)$ displays increasing returns to the number of experts. The result stems from the competitive behavior of experts who bid more as the number of other expert increases, and from the special nature of knowledge; *the knowledge auction is 'all-pay' from the perspective of an entrepreneur.* B_E does not depend on E because of the absence of competition between entrepreneurs in the model.

Finally, an expression for expected total production in the city results from simple arithmetics.

Proposition 1 *Expected total production in the city is equal to* $Y = E(\frac{X-1}{2} + \frac{1}{X+1})$.

Proof *The term inside the bracket is expected production per entrepreneur, which is equal to the expected sum of the knowledge transfered to each entrepreneur during the meeting stage (*$\frac{X-1}{2}$*, from equation (6)), plus the expected knowledge used by each entrepreneur during the production stage (*$\frac{1}{X+1}$*, equivalent to* *the expected wage paid by an entrepreneur, from equation (3)). As all entrepreneurs are identical, expected total production is equal to E times per entrepreneur production.* ■

The defining feature of total expected production is the presence of external returns to scale. That is, doubling the number of experts and entrepreneurs in the city more than doubles total expected production, despite the fact the production function itself has constant returns to scale. There are two sources of increasing returns to the number of city inhabitants: an increase in the number of meetings (extensive margin) and an increase in the the productivity of each meeting (intensive margin).

First, increasing returns at the extensive margin come from the matching function. Indeed, the expression for Y, which is equal to the number of entrepreneurs times the sum of the knowledge transfered in both stages by all experts, closely resembles the expression for the number of meetings ($E * X$).

Second, increasing returns at the intensive margin come from the decision of experts to reveal more knowledge for free as competition for jobs heats up. That is, each meeting becomes more productive as the number of other experts increases, which is an implication of the standard auction result that bids increase as the number of participants in an auction increases. At the intensive margin, there are only increasing returns to X, the number of city experts, not necessarily to the number of city inhabitants. The term $\frac{X-1}{2}$ in the expression for Y represents the expected knowledge freely transfered by all experts to each entrepreneur and it displays increasing returns for the reasons mentioned above. However, the relevant term from the perspective of the city is the expected per entrepreneur productivity of each expert (including knowledge that is paid for): $\frac{1}{X}(\frac{X-1}{2}+\frac{1}{X+1})$. As $\frac{d}{dX}[\frac{1}{X}\frac{X-1}{2}+\frac{1}{X}\frac{1}{X+1}] = \frac{X^2+4-4X}{2X^2(X-2)^2} - \frac{1+2X}{X^2(X+1)^2} = \frac{1}{2X^2(X+1)^2}\left(X^2-2X-1\right) > 0$ for all integers $X > 2$, there is in fact increasing returns to X[9]. Increasing returns at the intensive margin vanish as X becomes large and experts bid all their knowledge at the meeting stage. That is, $\lim_{X\to\infty} b(k) = \lim_{X\to\infty} \frac{X-1}{X}k = k$, which also implies that each entrepreneur extracts all the knowledge of every expert as X tends to infinity, in which case the mechanism becomes arbitrarily close to being optimal for entrepreneurs. As every entrepreneur uses the total amount of knowledge in the economy (equal to $\frac{X}{2}$, because the average quantity of knowledge of an expert is $\frac{1}{2}$) in his production, the city becomes efficient as Y tends to its maximum level of $\frac{E*X}{2}$.

[9]There is no increasing returns from competition when starting from a single expert because if there is only one expert in the city, he bids nothing but he is hired by everyone, which is efficient compared to the case in which there are two experts who do bid against each other, but with only one of them working.

Increased competition in larger cities presupposes some form of increasing returns from the matching function. In other words, increasing returns at the extensive margin is a necessary condition for increasing returns at the intensive margin. Intuitively, the possibility of increasing returns to city size comes from the potential for more interactions between experts and entrepreneurs in larger cities. In the model, this intuition is captured by a matching function such that everyone can meet once, at no cost. The realization of increasing returns however, comes from combining strategic behavior - auction theory - with the key properties of knowledge as an input of production. The incentive of an expert to transfer knowledge relates to Arrow's idea that valuable knowledge must be given in order to demonstrate how knowledgeable one really is, and increasing returns obtains because knowledge is freely reproducible, so that an experts can give it to many entrepreneurs without loosing it. In fact, one could define conditions under which Arrow's property enhances the efficiency of cities. The unobservability of experts' knowledge enables entrepreneurs to ask for free transfers, and if experts' knowledge were observable, meetings in which free knowledge is transfered would not happen.

Industry examples

The auction framework offers answers to the why, when and how of uncompensated knowledge transfers. For an expert, a meeting is an opportunity to provide information on the extent of his knowledge by transferring some of that knowledge for free, in a bid to impress an entrepreneur enough to get a job. For an entrepreneur, a meeting is an opportunity to ask for free knowledge - with a job offered as a reward - and to find the best expert for the job. Market interactions, in the form of compensated knowledge transfers at the production stage of the game, occur only once the type of an expert is revealed.[10] While the model represents informal or semi-formal interactions between holders and users of knowledge, the division between an uncompensated knowledge transfers stage and an employment stage has formal equivalent in many 'creative' industries in which pitches are frequent (advertising, architecture, graphic design, investments, etc) as well as in many job interview situations. Studying how these industries cope with free

[10]In equilibrium, an entrepreneur does infer the type of an expert from his bid, and he knows that he is hiring the best expert, even if the wage does not depend on that inference. The incentive of an entrepreneur to hire the best expert would be clearer if the wage was $(1-\lambda) * (k_i - b_i(k_i))$, with $\lambda > 0$ (it is easy to show that in equilibrium, the surplus produced by the best expert is larger than that of the second best). In the model's simple setting however, 'hiring the best expert' is a good strategy for an entrepreneur only because it provides incentives for experts to transfer knowledge.

knowledge transfers allows to determine whether the strategic incentives of the agents in my model relate to those of real-world individuals. In the absence of hard data, the most supportive evidence for the theory paradoxically comes from industries in which free knowledge transfers are controversial. In these industries, as in the model, the unobservability of experts knowledge combines with competitive pressure to lead to outcomes that are considered unfair, especially in the presence of entry or communication cost, which are shown in section 4 to drive some experts completely out of the market/city.

Graphic design is such an industry, in which free transfers - called 'spec(ulative) work' or 'working on spec' - are deemed so problematic that some professional associations explicitly prohibit their members from providing free designs.[11] While a design or a logo does not have all the properties of knowledge defined in section 4.3 - designs are not necessarily freely reproducible, and there is a design cost - the bans highlight the incentives of experts to give knowledge for free in a competition for jobs.

In advertising and marketing, free knowledge transfers are less controversial, perhaps because of the low cost of creating a marketing strategy or a slogan for a product. Unlike a design, a marketing advice can often be used productively even when the advisor does not win the contract, so free transfers can potentially make a city more efficient. Of course, it can be precisely when a free advice is used profitably, without leading to a job offer, that the situation is considered unfair. That being said, there are nevertheless vocal opponents to free knowledge transfers within the advertising community. For instance, Win Without Pitching is a consulting firm heading an online movement whose grievances are very explicit. An extract from their mission statement reads as follow:

"The forces of the creative industry are aligned against the artist. These forces pressure him to give his work away for free as a means of proving his worthiness of the assignment. Clients demand it. Industry associations deride it but offer alternatives that are just as costly and commoditizing. Agencies resign themselves to it [...]".[12]

Competitive pressures to give free knowledge in a bid to get a contract can indeed impose

[11] In Canada, the Society of Graphic Designers of Canada and the Association of Registered Graphic Designers of Ontario strictly prohibit their members from providing free designs, or from entering into a design contest in which only the winner is remunerated. In the US, the largest such professional organization, AIGA, only goes so far as stating, somewhat mysteriously, that: "Instead of working speculatively, AIGA strongly encourages designers to enter into projects with full engagement to continue to show the value of their creative endeavor".

[12] www.winwithoutpitching.com

a significant burden on small players. From an efficiency perspective, the concern is that such competition can discourage agents from acquiring knowledge, from actively developing creative ideas, or from incurring the cost of moving to a city. The issue deserves more discussion than what appears in this paper, but I do show, in section 4, that there are always benefits from competition - increasing returns to the number of experts - when entry cost in the city are low enough.

Finally, I did not uncover any evidence that uncompensated knowledge transfers were contentious in most consulting industries, and among professionals such as lawyers, accountants and financial advisors. Free knowledge transfers seem to be a part of standard networking practices. For instance, lawyers are legally prevented from pitching free ideas to individuals or firms (to prevent 'ambulance chasing'), but most large firms have an in-house lawyer who can be legally contacted, and it is common enough for lawyers to offer free legal advice in the hope of landing a contract (Asher, 2004). Financial consulting is an industry that appears to embrace free knowledge transfers. An example is the mergers and acquisitions' (M&A) pitch, through which a firm specializing in M&A services provides a candidate firm with free information about the benefits of merging with, or acquiring another firm. Another example is 'stock pitching', an art that, for expert practitioners at least, seems closer to a recreational exercise than to a burdening assignment: there are M&A pitching and stock pitching contest organized for MBA students. Generally, consulting industries tend to share the model's main features: knowledge is (almost) freely reproducible and it is impossible to take back knowledge transfered for free. Also, there are often obvious benefits from hiring one consultant, ideally the best, while getting knowledge from many. For instance, at least one lawyer is needed to sign legal papers, one broker is needed to buy stocks, but the potential for legal actions to take or stocks to buy is almost unlimited.

By and large, the entrepreneurs versus experts set-up is representative of the general motivation behind urban networking. Casual meetings and professional encounters are often opportunities to let others know what and how much one knows, and to learn what and how much others know. These are the essential ingredients of successful networking; of a process that can lead to a job offer directly as in the model or indirectly through name-dropping by others, or even to recognition as a valuable partner in a reciprocal relationship. The experts in the model strive to develop a reputation for being knowledgeable because of competition for entry into a contractual relationship with an entrepreneur, but it is likely that competition for entry into reciprocal relationships fosters the same incentives for knowledge transfers. The behavior of agents in the model is perhaps closest to that of their real-world counterparts in environments in which agglomeration economies are found to be strongest, in high-tech industries and clusters like Silicon Valley. In these milieus, a high premium is placed on having a reputation for being 'smart', it is crucial to know who knows what and how good they are, and, as shown in Freedman (2008), job changes are frequent and workers are willing to accept initially lower wages while hoping for a lucrative contract once their reputation is made.

3. Migration without sorting by skills

I now consider a country in which experts and entrepreneurs can choose to migrate to an open city. Experts make their migration decision before learning their type, which implies that there is no sorting of experts by skills into the city. The main result is that if the urban cost of living does not increase too fast with population, there exist cities with aggregate welfare larger than that in equilibrium, that have a larger proportion of experts and a larger population. The auction framework permits a tax policy that renders the city more efficient both by attracting experts into the city and by enhancing their incentives to transfer knowledge for free.

There is a large number of potential experts and entrepreneurs in the countryside, whose utility is normalized to 0. As before, $N = E + X$ is the city's population, and let $z = \frac{E}{X}$ denote the city's composition. The number of entrepreneurs and experts in the city can be expressed as a function of its population and composition, as $E = \frac{Nz}{z+1}$ and $X = \frac{N}{z+1}$. The mechanism for knowledge transfers and the production technology, and the benefit of living in the city for experts and entrepreneurs are as defined in section 2. I introduce a cost of urban living $C(N)$, which can be thought of as a congestion cost, or a transportation cost arising from the need to drive to the city center to meet and work. C is a continuous, convex and increasing function of a city's population, with $C(0) = 0$, $C'(N) > 0$, $C''(N) > 0$ and $\lim_{N \to 0} C'(N)$ finite. $C(N)$ is the same for experts and entrepreneurs and it covers the cost of all meetings, work and production.[13] Using equation (5), the expected utility of an expert who migrates to a city with population N

[13]This last assumption is consistent with the definition of the matching function, which realizes all meetings. Note that even if the urban cost was divided into a up-front part incurred when migrating and an extra cost per meeting, in equilibrium every agent who incurs the up-front cost would find it profitable to attend every meeting (all meetings have the same expected utility).

and composition z is:

$$U_X(z,N) = B_X(X,E) - C(N) = \frac{E}{X(X+1)} - C(N) = \frac{z(z+1)}{N+(z+1)} - C(N), \tag{7}$$

and using equation (6), the expected utility of an entrepreneur is:

$$U_E(z,N) = B_E(X) - C(N) = \frac{(X-1)}{2} - C(N) = \frac{N-(z+1)}{2(z+1)} - C(N). \tag{8}$$

Experts and entrepreneurs migrate if they obtain a positive expected payoff in the city. The equilibrium condition is therefore:

$$\begin{aligned} U_E(z,N) &= 0 \\ U_X(z,N) &= 0. \end{aligned} \tag{9}$$

The easiest way to demonstrate the existence of a unique stable equilibrium is to represent the indifference curves $U_E(z,N) = 0$ and $U_X(z,N) = 0$ in the (z,N) space.[14] From (7), it is immediate that $\frac{\partial U_X}{\partial N} < 0$ and $\frac{\partial U_X}{\partial z} > 0$, for all $z > 0$ and $N > 0$, so that the indifference curves of an expert slopes upward from the origin (remind that $C(0) = 0$, so that $U_X = 0$ at $(z,N) = (0,0)$). To determine the shape of an entrepreneur's indifference curve, I fix his utility level at $U_E = 0$, and using equation (8), I isolate z from $\frac{N-(z+1)}{2(z+1)} - C(N) = 0$ to find:

$$z = \frac{N}{2C(N)+1} - 1 \tag{10}$$

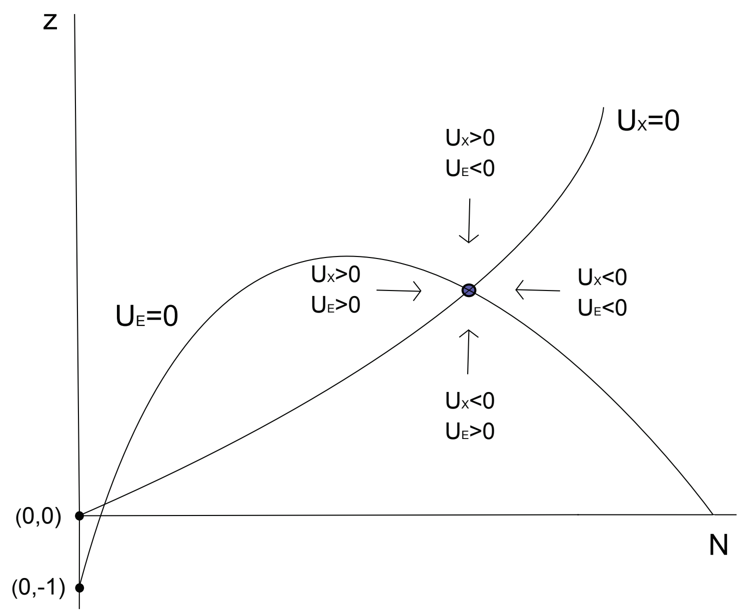
$$\frac{\partial z}{\partial N} = \frac{2C(N)+1-2NC'(N)}{4(C(N)+0.5)^2}. \tag{11}$$

The denominator of (11) is always positive. The limit of the numerator as $N \to 0$ is also positive, because $C(N) = 0$ and $\lim_{N\to 0} C'(N)$ is finite. As $C(N)$ is continuous, the indifference curve must slope up for small N. For N large enough, $\frac{\partial z}{\partial N}$ becomes negative, because $C'(N) > 0$ and $C''(N) > 0$ imply that the negative term $-2NC'(N)$ grows faster than the term $C(N)$.[15] Finally, note that $(-1,0)$ satisfies equation (10), and suppose that there are points such that $z(N) > 0$ that also satisfy it (which is necessary for an equilibrium to exist). I conclude that the indifference curve $U_E = 0$ in the (z,N) space slopes up, then down, in the first quadrant. Both indifference curves are represented in Figure 1.

[14] I now drop the dependence of U_X and U_E on z and N to economize on notation.

[15] The derivative of $NC'(N)$ is $NC''(N) + C'(N)$, and the derivative of $C(N)$ is $C'(N)$, so that $C'(N) > 0$ and $C''(N) > 0$ imply that $NC''(N) + C'(N) > C'(N)$, for all $N > 0$.

Figure 1: Unique stable equilibrium in open city



From Figure 1, a stable equilibrium exists if the peak (at $\frac{\partial z}{\partial N} = 0$) of the indifference curve of an entrepreneur lies above the indifference curve of an expert.[16] For instance, if $C(N) = \tau N^2$, as it would in a linear city of size N with transport costs $\tau > 0$, then it is not too hard to show that an equilibrium exists if $\tau < 0.02$ i.e. if transport costs are low enough.

The equilibrium is locally stable because agents move in or out of the city when their net payoff from such a move is positive. The four arrows of motion in Figure 1 illustrate that through such migration, the city returns to equilibrium after any small perturbation away from it. When both $U_X > 0$ and $U_E > 0$, N must increase, because both experts and entrepreneurs benefit from moving into the city. When $U_X < 0$ and $U_E < 0$, N must decrease, because both experts and entrepreneurs benefit from moving out of the city. When $U_X < 0$ and $U_E > 0$, z must increase, because entrepreneurs benefit from moving into the city while experts benefit from moving out of the city. By a similar reasoning, z decreases when $U_X > 0$ and $U_E < 0$.

If E and X are required to be integers, then the equilibrium population would not be that identified in Figure 1. Instead, equilibrium population would be equal to the largest integer that is the sum of two other integers (number of experts and entrepreneurs) whose ratio z is in the part of the graph with $U_X > 0$ and $U_E > 0$ - or it would equal 0.

[16] There may also be another - unstable - equilibrium with smaller N and lower z, represented in Figure 1.

City size

There are cities, displayed in Figure 1, with $U_X > 0$ and $U_E > 0$, in which both experts and entrepreneurs are better-off than in equilibrium, at which $U_X = U_E = 0$. All these cities have a population smaller than that of the equilibrium city. Excess migration is a standard result for an open city in a self-organizing equilibrium, and it occurs because agents do not internalize the negative congestion externality that they impose on others, and migrate until the benefits from city living is exactly offset by its cost.

But if wealth can be redistributed among entrepreneurs and experts, then the optimal city must only maximize aggregate welfare - total expected production minus total urban costs - and in this case the equilibrium city can be too small. The model is not tractable enough to allow the analytical derivation of optimal or equilibrium size and composition, but the following argument demonstrates the existence of cities with aggregate welfare larger than that in equilibrium. For N large enough, the growth of Y with respect to N for a city whose composition (z) stays in equilibrium is slower than the growth of Y with respect to N for any city with fixed - not even optimal - z. This implies that if the cost function does not increase too fast, there always exist cities for which $Y - N * C(N)$ is positive, instead of 0 in an equilibrium with a smaller population. To see why the above is true, plug $E = \frac{Nz}{z+1}$ and $X = \frac{N}{z+1}$ in the equation for Y given in Proposition 1, to obtain Y as a function of N and z:

$$Y = \frac{N^3 z + N^2 z + 2Nz + N}{2(z+1)^2(N+z+1)}. \tag{12}$$

The largest exponent on N in the expression for Y represents the strenght of the returns to city size (increasing returns if this exponent is larger than 1). In equation (12), the largest exponent on N is 2, so that total expected production with *fixed* z grows like N^2. In any equilibrium however, it must be, from equation (9), that $z^{eq} = \{z : U_E = U_X\}$. Using equation (7) and (8) to equate U_X and U_E, z^{eq} can be defined implicitly as $N = \sqrt{(z^{eq}+1)^2(2z^{eq}+1)}$. From this expression, z^{eq} grows like $N^{2/3}$, so the equilibrium ratio of entrepreneurs to experts becomes very large as city size increases.[17] Through tougher competition, the payoff of experts decreases fast as the number of other experts in the city increases, so there must be an even larger increase in the number of entrepreneurs for experts to brake even. The key to the argument is that after plugging z^{eq} into equation (12), it appears that Y must grow like $N^{4/3}$ in equilibrium, as opposed to N^2 when

[17]For instance, cities with a lower $C(N)$ growing slower are larger in equilibrium, and therefore would contain a larger proportion of entrepreneurs.

z if fixed. That is, increasing returns are weaker under self-organized migration, because of an imbalance in the ratio of entrepreneurs to experts that worsens as population increases.[18]

City composition and taxation policy

In the cities with positive aggregate welfare described in the argument above, N is larger than its value in equilibrium, so that from Figure 1, either experts or entrepreneurs must earn negative payoffs, and some redistribution is necessary. Moreover, the argument implies that in those cities, z is smaller than its value in equilibrium, so that from Figure 1 again, it is experts who earn negative payoffs.[19]

The simplest policy recommendation is therefore to reduce the cost of urban living for experts, for instance by taking lump-sum payments from entrepreneurs and giving the proceeds to experts.[20]

However, a micro-founded model allows the design of better, targeted subsidies acting directly on the incentives of experts to transfer knowledge for free. One such policy is to subsidize only the work of an expert; to offer a wage top-up σ. In equilibrium, only the best experts, when they win a knowledge auction, are subsidized. This policy is more efficient than just a lump-sum transfer because the wage subsidy, by increasing the valuation of experts for a job, increases the knowledge transfered by every expert, of any type. In a first price auction, the bid of an expert increases, with respect to the case with no subsidy, by exactly σ, the value of the subsidy (i.e. $b(k,X) = \frac{X}{X+1}k + \sigma$, unless σ is so large that experts transfer all their knowledge and $b(k,X) = k$). Unsurprisingly, in a regular first price auction an auctioneer cannot increase his payoff by subsidizing the winner, because the bid of the winner increases by exactly the value of the subsidy. But a knowledge auction is all-pay for an entrepreneur, so such a subsidy is worth implementing as it motivates every experts to transfer more knowledge, even those who fail to get the job. The result relates to Lazear and Rosen (1981) 'tournament' theory, which successfully explained how the oversized wages observed at the top end of a firm's hierarchy motivate workers in the lower ranks. The auction framework is simplified, but it conveys the idea that providing the best expert with a wage

[18]Intuitively the optimal z stays close to 1, because of the symmetry in the matching function.

[19]To understand this last point, recall that the growth of z with N in equilibrium is what limits increasing returns under self-organized migration relative to increasing returns in a city with fixed z.

[20]As an example, suppose that the cost of urban living grows like N^{α} , where $3/4 < \alpha < 2$, so that there exist an equilibrium city (in which production grows like $N^{\frac{3}{4}}$) in which experts and entrepreneurs earn 0 payoff. If a policy transfers income to experts in a way that maintain z fixed, the city (in which production grows like N^2) would grow to an infinite size, as would aggregate welfare.

above his productivity is a sensible policy inasmuch as it increases the productivity of everyone else. Such a subsidy both improves the composition of a city by increasing the expected payoff of experts, and enhances the incentives of expert to transfer knowledge for free, rendering the city more efficient on both accounts. In fact, one can show that entrepreneurs themselves have an incentive to implement this policy, although they optimally choose a subsidy too small to make the city's knowledge market efficient. That is, an entrepreneur would choose a subsidy smaller than that at which every type of expert transfers all its knowledge.

Interpretation: experts vs entrepreneurs framework

The model's dichotomy of the city into holders and users of knowledge - into experts and entrepreneurs - is just an expository device, but it captures some features of the fabric of a city. Individuals are not equally endowed with entrepreneurial skills. At least since the work of Jacobs (1968), cities are recognized as hotbeds of entrepreneurial activities, in part because densely populated areas provide entrepreneurs with more opportunities to learn. In the model, the possibility to learn from experts is exactly what motivates an entrepreneur's move to a city.

The decision of experts to locate into cities has become a current public policy issue since the work of Florida (2002) on those he refers to as the 'creative class' and whom, he argues, cities should strive to attract. Florida considers tolerance, talent and technology as factors drawing creatives to a city, while in the model above, the possibility to meet entrepreneurs and to get a job drives experts to migrate. Florida recommends raising the value of urban living for the creative class by subsidizing amenities that they are thought to like more than normal people do. Such a policy is reasonable if creatives, like experts in the model, provide benefits for which they are not compensated.[21] But with an empirical understanding of the exact mechanism through which such benefits arise, better targeted subsidies could be designed. For instance, the theoretical model above suggests that subsidizing the best experts enhances the incentives for uncompensated knowledge transfers.[22]

[21]Even in a system of cities, experts should be subsidized lest they stay in the countryside. In the real world, or in a different model in which all experts already live in cities, place-based policies designed to attract experts to a particular city can turn into beggar-thy-neighbor policies, whereas subsidies designed to enhance incentives for knowledge transfers would still produce efficiency gains.

[22]However, I show in the next section that if experts move after learning their type, then the cost of living has a direct effect on the knowledge bid of experts, and in this case the simple policy of reducing that cost has more to recommend it.

4. Migration with sorting by skills

This section covers the more complicated case of a country in which experts can decide to migrate to an open city *after* learning the value of their knowledge. In this setting, the knowledge transfer mechanism becomes a first-price auction with endogenous entry cost, and only the best experts self-select into the city. Section 4.1 and 4.2 focuses on the impact of the model's two parameters - competition and cost of urban living - on the number and on the quality of experts living in the city (which impacts production indirectly), and on the knowledge bid of experts (which impacts production directly). The direct impact of the cost of urban living on production occurs for the same reason an entry cost affects bids in a standard auction. Section 4.3 confirms that one of the key result of the paper, increasing returns from competition between experts, is robust the addition of a small enough entry - or communication - cost.

There is a number X_p of 'potential' experts living in the country, an expected number X_c of which decide to migrate to an open city.[23] I use a subscript c to denote city level variables. The number of entrepreneurs in the city is fixed at E_c, which simplifies the problem and focuses the discussion on the behavior of experts, who are heterogeneous.

The utility of experts outside the city is normalized to 0. The cost of living in the city, $C(N_c)$, is a positive, increasing function of its number of inhabitants, $N_c = X_c + E_c$. Let $c = \frac{C(N_c)}{E_c}$ denote, for an expert, the per meeting cost of living in the city. Note that $\frac{dc}{dX_c} < 0$ for all $X_c > 0$ and all $E_c > 0$.[24]

The key assumption of this section is that experts move to a city after learning their type. An implication is that the correlation between the values of an expert's knowledge for different entrepreneurs affects the model's results. To simplify the presentation, I only consider the case in which $corr\,(k_{ij},k_{ih}) = 1$, for all $j,h \in \{1,...,E\}$ so that in equilibrium the same expert is best for every entrepreneur, wins every auction and works for every entrepreneur. This expert, being the best owner of a freely reproducible good, is what Rosen (1981) called a 'Superstar'. Besides, the assumption that an expert moves to a city after learning the value of his knowledge is more plausible when this value is the same for every entrepreneur. Otherwise the mechanism for

[23] Note that unlike in section 3, the number of potential experts matters, and is not assumed to be unlimited. Also, I often drop the 'expected' when I refer to the number of experts.

[24] If E_c was not fixed, $\frac{\partial c}{\partial E_c}$ could be positive or negative, as more entrepreneurs leads to a larger cost of living, but not necessarily to a larger cost per meeting, because of the matching function.

knowledge transfers and the production technology are exactly as in Section 4, except for the presence of the entry cost.

Number and quality of experts

The problem of an expert is to decide whether or not to migrate, and then if he migrates, to decide how much knowledge to transfer in each meeting. It is easier to solve by considering the migration decision separately. I analyze how the number and types of experts moving to a city depend on competition (X_p) and on the per meeting cost of urban living (c). I find a quantity/quality trade-off as either X_p or c varies.

Each meeting corresponds to participation in a knowledge auction, so that c has the interpretation of an entry cost, sometimes called 'bidding' or 'communication' costs. In contrast with standard auction theory, entry costs in urban environment are endogenous, because congestion depends on the number of agents moving to the city.

Define $k^* \in (0,1)$ as a type cut-off: a value such that all experts with a quantity of knowledge below k^* earn a negative expected payoff in the city and stay in the countryside, all experts with types above k^* earn a positive expected payoff in the city and migrate, and and an expert with a type k^* is indifferent as he earns an expected payoff of 0 in the city. All entrepreneurs have the same valuation for an expert's knowledge, so an expert's meetings must all have the same expected utility. Therefore, for an expert of type k^*, the expected utility from any meeting must also be 0. I prove the existence of k^* in the next section, but for now I assume that it exists and I find an expression for it as a function of c and X_p. The following reasoning is adapted from Samuelson (1985). With symmetric bidding functions, a type k^* expert only wins the auction if every other potential expert has a valuation below k^*. This implies that the probability that a type k^* expert gets the job and wins the auction is k^{*X_p-1}. Because a type k^* expert only wins if he is the sole participant, the bidding function (which now depends on c) must be such that $b(k^*,X_p,c) = 0$, so that the wage obtained by this expert should he win is equal to his full valuation k^*. The expected utility from a meeting for an expert of type k^* must equal $k^{*X_p-1}(k^* - 0) - c$, the probability that he wins times the wage minus the per meeting cost. By definition of k^*, this expected utility must equal 0 in equilibrium. Isolating k^* from $k^{*X_p-1}(k^* - 0) - c = 0$ leads to:

$$k^* = c^{\frac{1}{X_p}}. \tag{13}$$

The share of potential experts moving to the city is $(1-k^*)$, so (13) establishes the equilibrium relationship between X_c, the expected number of experts in the city, and X_p, the number of potential experts in the country, as:

$$X_c = (1-k^*)X_p = (1-c^{\frac{1}{X_p}})X_p. \tag{14}$$

The best experts self-select into the (single) city. This sorting by skills is consistent with the evidence in Combes et al. (2008) that more productive workers self-select in larger cities. From (13), an exogenous change in the entry cost increases the quality of experts in the city ($\frac{\partial k^*}{\partial c} > 0$) and, from (14), decreases their number ($\frac{\partial X_c}{\partial c} < 0$).

From (14), the impact of competition (X_p) on the number of city expert (X_c) is:

$$\frac{dX_c}{dX_p} = \frac{1 - c^{1/X_p} + c^{1/X_p}\frac{\ln c}{X_p}}{1 + c^{\frac{1+X_p}{X_p}}\frac{dc}{dX_c}}.[25] \tag{15}$$

The first term in the numerator, $1 - c^{1/X_p} = 1 - k^*$, is the direct effect of an increase in X_p; it increases X_c according to the proportion of experts who move. The second term in the numerator, $c^{1/X_p}\frac{\ln c}{X_p} = -X_p\frac{\partial k^*}{\partial X_p}$, is negative and represent the effect of X_p on k^*; competition increases the average quality but reduces the proportion of experts who move to the city. The term $c^{\frac{1+X_p}{X_p}}\frac{dc}{dX_c}$ in the denominator is positive; c is endogenous and an increase in X_c increases the urban cost of living (and therefore c) which reduces the number of experts moving to the city. The next lemma is on the sign and size of $\frac{dX_c}{dX_p}$ and $\frac{\partial k^*}{\partial X_p}$.

Lemma 2 *If the number of city entrepreneurs is fixed, the number of city experts always increases with the number of potential experts,* ($0 < \frac{dX_c}{dX_p} < 1$, $\forall c \in (0,1)$, $\forall X > 1$), *albeit less than proportionally* ($\frac{dk^*}{dX_p} > 0$, $\forall c \in (0,1)$, $\forall X > 1$).

Proof In Appendix A. ■

The results of this section reveal a trade-off, in an equilibrium in which the number of entrepreneurs is fixed, between the quality and the quantity of experts migrating to the city. To summarize:

- An exogenous increase in the entry cost leads to an increase in the average quality of experts in the city ($\frac{\partial k^*}{\partial c} > 0$) and to a decrease in their number ($\frac{\partial X_c}{\partial c} < 0$).

[25] The exact derivation of $\frac{\partial X_c}{\partial X_p}$ is included in the proof of the next lemma.

- An exogenous increase in the number of potential experts (competition) leads to an increase in the average quality of experts in the city, a decrease in the proportion of experts who move, and an increase their absolute number (Lemma 2).

This comparative analysis suggests that city characteristics like low communication cost (which decrease the quality of experts in the city) or high competition for entry (which decrease the proportion of expert who migrate) are not *a priori* desirable.

Bidding function

Competition (X_p) and the per meeting cost of urban living (c), in addition to their effect on the number and quality of experts in the city, can also have a direct impact on aggregate production, because they affect the knowledge bid of experts moving to the city. The equilibrium strategies of experts constitute a Bayesian equilibrium of a one-stage game in which experts simultaneously decide whether to move or not, and how much to bid if they do move. An expert does not observe the exact number of other experts who move to the city, so he chooses the value of his bid based on his - correct - assessment of the expected value of c. Finding the equilibrium involves solving a first-price auction, but this time with an entry cost.[26] All auctions/meetings are exactly the same for an expert, because the value of his knowledge is the same for every entrepreneur. So expert i enters the city if his expected utility from any meeting is positive. For each $k_i \in [0,1]$, he can stay in the countryside and obtain a payoff of 0, or enter the city and submit a bid $b_i \geq 0$ that maximizes the probability that he wins a job times his wage, minus the entry cost. So expert i solves:

$$\max_{stay,enter}\{0,\ \max_{b_i} prob[b_i > \max_{l \in X_c \setminus \{i\}} b_l(k_l,X_p,c)] * (k_i - b_i) - c\}. \tag{16}$$

Lemma 3 *There exists a unique symmetric equilibrium with $k^* = c^{1/X_p}$, a bidding function that is strictly increasing in k, and $b(k,X_p,c) = 0$ for $k \in [0,k^*]$, in which experts of type $k < k^*$ do not move to the city, experts of type $k = k^*$ are indifferent between not moving and moving with a bid equal to $b(k^*,X_p,c) = 0$, and experts of type $k > k^*$ move to the city and bid $b(k,X_p,c) = \frac{X_p-1}{X_p}(k - \frac{c}{k^{X_p-1}})$.*

[26]The endogeneity of c does not matter when solving for the bidding function of experts. It needs to be taken into account when analyzing, for instance, how exogenous changes in X_p affect the bid.

Proof (To prove Lemma 1, set c equals to 0.) The expected utility of an expert with knowledge k who moves to the city but bids like an expert with knowledge s, when all other experts follow the strategy outlined in the proposition, is $u(s,k) = s^{X_p-1}(k - b(s,X_p,c)) - c$. The probability of winning is s^{X_p-1} because the best expert wins and bidding functions are symmetric and strictly increasing. If $b(s,X_p,c)$ is an equilibrium bidding function, then $u(s,k)$ must be maximized at $s = k$. Taking the derivative of $u(s,k)$ with respect to s and setting it to 0 leads to:

$$(X_p - 1)s^{X_p-2}(k - b(s,X_p,c)) - s^{X_p-1}\frac{\partial b(s,X_p,c)}{\partial s} = 0.$$

Setting $s = k$ leads to

$$(X_p - 1)k^{X_p-2}b(k,X_p,c) + k^{X_p-1}\frac{\partial b(s,X_p,c)}{\partial s} = (X_p - 1)k^{X_p-1},$$

or equivalently,

$$\frac{\partial(k^{X_p-1}b(k,X_p,c))}{\partial k} = (X_p - 1)k^{X_p-1}.$$

Integrating on both sides (types below $k^* = c^{\frac{1}{X_p}}$ bid 0, which sets the lower bound of integration) leads to:

$$b(k,X_p,c)k^{X_p-1} = (X_p - 1)\int_{k^*=c^{\frac{1}{X_p}}}^{k} z^{X_p-1}dz.$$

Solving this integral and isolating $b(k,X_p,c)$ leads to the equilibrium bidding function $b(k,X_p,c) = \frac{X_p-1}{X_p}(k - \frac{c}{k^{X_p-1}})$. I now prove that the strategy outlined in the lemma is the unique subgame perfect Nash equilibrium. The objective function of an expert who moves is $u(s,k) = s^{X_p-1}(k - b(s,X_p,c)) - c = s^{X_p-1}(k - \frac{X_p-1}{X_p}(s - \frac{c}{s^{X_p-1}})) - c$. The first derivative of this function with respect to s is $s^{X-2}\,(X_p - 1)\,(k - s)$, which is positive for $s < k$, negative for $s > k$ and equal to 0 at $s = k$, which must maximizes the function, so that $b(k,X_p,c)$ is the only Nash equilibrium strategy for experts who move. Replacing s by k in $u(s,k)$, I obtain type k's maximum attainable utility in the city: $k^{X_p-1}(k - \frac{X_p-1}{X_p}(k - \frac{c}{k^{X_p-1}})) - c = \frac{k^{X_p}-c}{X_p}$. I conclude that an expert is indifferent between moving or not at $k = k^* = c^{\frac{1}{X_p}}$ (because $\frac{k^{X_p}-c}{X_p} = 0$), prefers to move at $k > k^*$ (because $\frac{k^{X_p}-c}{X_p} > 0$) and not to move at $k < k^*$ (because $\frac{k^{X_p}-c}{X_p} < 0$). ■

An important property of the bidding function is that it decreases linearly with c. Therefore, aggregate production depends directly on the cost of urban living, because this cost affects the willingness of experts to transfer knowledge for free. The indirect effect of c on production, through the number and quality of experts, is a feature of any open city model with sorting by skills, but the direct effect just exposed is special.

The effect of an increase in competition (increase in X_p) is not to unambiguously increase the bid of experts, as in the model without an entry cost of section 2. Instead, $\frac{\partial b(k,X_p,c)}{\partial X_p} = -\frac{k^{1-X_p}}{X_p^2}\left(c - k^{X_p} + cX_p \ln k - cX_p^2 \ln k\right)$ is positive for low X_p and negative for high X_p.[27] This result does not necessarily imply the presence of increasing returns to X_p at low levels of competition, because unlike in section 2, X_p also affects the number and on the quality of experts in the city.

Concerning public policies designed to attract experts into the city, the simple policy of reducing the cost of entry for all experts gains an edge when there is sorting by skill, because reducing c also increases knowledge bids.

Increasing returns

A key feature of the knowledge transfer mechanism presented in section 2 is that competition led to increasing returns to the number of experts. I now show that the result is robust to the addition of a small enough entry cost. It facilitates the exposition to define P_p as the average per entrepreneur productivity of a potential expert, and to write expected total production as:

$$Y = P_p E_c X_p. \tag{17}$$

In the closed-city model of section 2, $Y = E(\frac{X-2}{1} - \frac{1}{X+1})$ so that $P = \frac{1}{X}(\frac{X-1}{2} - \frac{1}{X+1})$. Note that $\frac{dP_p}{dX_p} > 0$ corresponds to increasing returns in aggregate production to the number of experts. The next proposition confirms that under a mild requirement on the shape of $C(N_c)$, there are increasing returns to X_p whenever c is small enough.[28]

Proposition 2 *If the number of city entrepreneurs is fixed and if* $0 < \frac{C'(N_c)}{C(N_c)} < 1$ *(a sufficient but not a necessary condition) then* $\frac{dP_p}{dX_p} > 0$*, for all* $c \in [0,c^*)$ *and for all integers* $X_p > 2$.

Proof In Appendix B. ■

The notation in the proposition can be confusing, because $c = \frac{C(N_c)}{E_c}$ is an endogenous variable that also appears as a fixed real number in the conclusion of the proposition that $\frac{dP_p}{dX_p} > 0$ for c

[27] One can verify that for all $k \in [0,1]$, and all $c \in (0,1)$, the expression for $\frac{\partial b(k,c)}{\partial X_p}$ is positive for $X_p = 1$, and that $\frac{\partial^2 b(k,c)}{\partial X_p^2}$ is always negative.

[28] There is an alternative interpretation of the result, as it also applies to a closed-city model with a per meeting communication cost. In this case, X_p becomes the number of experts already in the city, and X_c becomes the number of experts with enough knowledge to benefit from meeting entrepreneurs.

small enough. To place ideas, consider a case in which the cost of urban living is $C(N_c) = \tau N_c^2$, as it would be in a linear city of size N with transport cost τ. Then $\frac{C'(N_c)}{C(N_c)} = \frac{1}{N_c} < 1$, so that the assumption in the proposition is satisfied. Also, $c = \frac{\tau N_c}{E_c}$, and the proposition can be stated as: $\frac{dP_p}{dX_p} > 0$ for all $\tau \in [0,\tau^*)$ and for all $X_p > 2$, and there is no confusion possible. The proof of the proposition, perhaps surprisingly, is not trivial. One cannot conclude, from the presence of increasing returns to the number of experts in the model with $c = 0$, that the same is true for c small enough. The reason is the endogeneity of c. An increase in X_p leads to an increase in X_c (Lemma 3), which in turns increases c and decreases the proportion of experts who migrate to the city and become productive. To obtain increasing returns to X_p, c must not increase too fast with X_c, for instance $\frac{C'(N_c)}{C(N_c)} < 1$ is a sufficient condition.

Because the best experts sort into the city, and because of the implicit assumption that experts staying in the countryside are not productive, it would also be possible to show that very low or decreasing returns to the number of potential experts in a country - which corresponds to empirical observation of small or non-existent (dis)agglomeration economies at the country level - are magnified into large increasing returns at the city level.[29]

Interpretation: effect of entry cost on knowledge transfers

Urban living is expensive, especially in large cities, and face-to-face interactions are often costlier than other modes of communication. The impact of entry costs on the quality of experts attending urban meetings relates to Storper and Venables (2004) quip that for those participating in costly face-to-face interactions, "the medium *is* the message". While the message itself matters in the model, there are potential benefits from the presence of high entry or communication costs, as it encourages stricter self-selection of the best experts. In terms of mechanisms for screening valuable communication partners, the leading alternative to voluntary self-selection through increased entry costs is the design of reputation systems. For instance, mechanisms that allow for reputation formation on the Internet are key prerequisite for productive use of that very low cost medium. Of course, self-selection and reputation formation are not mutually exclusive and can be part of the same process. In the model, experts are willing to incur an entry cost *because* it eventually allows them to form a valuable reputation for being knowledgeable.

[29]This general phenomenon is well-known in the literature. For instance, in a dynamic setting with many cities, Rossi-Hansberg and Wright (2007) show that a balanced growth path at the country level can coexist with increasing returns at the city level.

Lower entry or communication cost also carry benefits. While the model is not about knowledge creation and economic growth, the direct impact of entry or communication costs on expert's readiness to transfer knowledge relates to the idea that for technological progress to take place, incentives to disseminate knowledge must be in place. For knowledge to diffuse, communication cost must be low enough to allow experts and entrepreneurs, creatives and innovators, to meet and to learn about others' endeavors, and to freely pitch their ideas, knowing that the expected rewards are larger than the costs.

5. Conclusion

Starting from the key properties of knowledge as an input in production, I model knowledge transfers as bids in first-price auctions for jobs, to illustrate why, how and when knowledge 'spills' in non-market interactions as opposed to being bought and sold in markets. This knowledge transfer mechanism leads to endogenous agglomeration economies, from the greater number of possible meetings in urban areas, and from heightened competition between experts. The model is too stylized to lead to clear-cut policy recommendations, but it can inform discussions on the incentives of individuals to move to a city and to transfer free knowledge, and on how these incentives are affected by public policy, by the cost of urban living, and by the number of other experts and entrepreneurs.

The opportunities that urban environments offer to learn and network, as featured in this paper, take center stage in recent thinking about the purpose and the future of cities. The model, however, overlooks a potentially substantial component of the productivity gains from agglomeration, because it ignores the complementarities between the skills of different knowledge workers. Through such complementarities, an expert could derive direct benefits from being located around other experts.

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Appendix A. Proof of Lemma 2

Starting from $X_c = (1-k^*)X_p$ and differentiating with respect to X_p, I obtain $\frac{dX_c}{dX_p} = (1-k^*) - X_p\frac{dk^*}{dX_p}$. To find $\frac{dk^*}{dX_p}$, I use $k^* = c^{\frac{1}{X_p}}$ (note that c itself is a function of X_c) and I differentiate with respect to X_p to obtain $\frac{dk^*}{dX_p} = c^{\frac{1}{X_p}}(\frac{\frac{dc}{dX_c}\frac{dX_c}{dX_p}}{Xc} - \frac{\ln c}{X^2})$. Given that $\frac{dc}{dX_c} > 0$, $c \in (0,1)$ and $X_p > 1$, showing that $0 < \frac{dX_c}{dX_p}$, as I do next, will imply the result that $\frac{dk^*}{dX_p} > 0$. Plugging $\frac{dk^*}{dX_p}$ into the expression for $\frac{dX_c}{dXp}$ above, I obtain $\frac{dX_c}{dX_p} = \frac{1-c^{1/X_p}+c^{1/X_p}\frac{\ln c}{X_p}}{1+c^{\frac{1+X_p}{X_p}}\frac{dc}{dX_c}}$. It is immediate that $\frac{dX_c}{dX_p} < 1$. To show that $0 < \frac{dX_c}{dX_p}$, I need to prove that $\frac{-\ln c}{X_p} < c^{-1/X_p} - 1$. Both sides of this inequality are positive, so it holds if and only if $e^{\frac{-\ln c}{X_p}} < e^{c^{-1/X_p}-1}$, where e is Euler's number. I rewrite this inequality as $ec^{-1/X_p} < e^{c^{-1/X_p}}$. Note that if $c \in (0,1)$ and $X_p > 1$, then $c^{-1/X_p} \in (1,\infty)$. Both sides of the inequality are just equal if $c^{-1/X_p} = 1$, because $e * 1 = e^1$. To complete the proof, I define a variable $x = c^{-1/X_p}$ and I show that the right-hand side of the inequality increases faster than the left-hand side i.e. that $\frac{\partial(e^x - ex)}{\partial x} = e^x - e > 0$ for $x > 1$, so that the inequality holds for $c^{-1/X_p} > 1$.

Appendix B. Proof of Proposition 2

I make the dependence of P_p and c on X_p explicit and I decompose P_p into two components such that $P_p(X_p,c(X_c(X_p))) = A_p(X_p,c(X_c(X_p))) + R_p(X_p,c(X_c(X_p)))$, with A_p as the per entrepreneur average knowledge transfered at the meeting stage by a potential expert in X_p, and R_p as the per entrepreneur average knowledge transfered at the production stage by a potential expert in X_p. I write c as $c(X_c(X_p))$ to emphasize that it is a function of X_c, which itself is a function of X_p (and of c itself). However, for any given X_p, c takes a value that is a real number, so that statements such as: "$\frac{dP_p(X_p,c(X_c(X_p)))}{dX_p} > 0$ for c small enough" are well-defined, albeit a bit awkward. It is useful to consider the example given in the text, with $c = \frac{\tau(X_c+E_c)}{E_c}$, and to think of the proposition as: "$\frac{dP_p(X_p,c(X_c(X_p)))}{dX_p} > 0$ for τ small enough". I sometimes denote the partial derivative of a function f with respect to it's i^{th} argument as f'_i.

The proposition is that if $0 < \frac{C'(N_c)}{C(N_c)} < 1$, then for all $X_p > 2$ there is a number c^* such that

$$\begin{aligned} & \frac{dP_p(X_p,c(X_c(X_p)))}{dX_p} \\ = \quad & A'_{p1}(X_p,c(X_c(X_p))) + A'_{p2}(X_p,c(X_c(X_p)))\frac{dc(X_c(X_p))}{dX_c}\frac{dX_c}{dX_p} + \\ & R'_{p1}(X_p,c(X_c(X_p))) + R'_{p2}(X_p,c(X_c(X_p)))\frac{dc(X_c(X_p))}{dX_c}\frac{dX_c}{dX_p} \end{aligned} \tag{B1}$$

is larger than 0 for all $c \in [0,c^*)$. To do so, I show that $\lim_{c \to 0^+} \frac{dP'_p(X_p,c(X_c(X_p)))}{dX_p} > 0$ for all $X_p > 2$, and I conclude that $\frac{dP'_p(X_p,c(X_c(X_p)))}{dX_p} > 0$ holds for c small enough from the fact all 4 element that sums up to $\frac{dP'_p(X_p,c(X_c(X_p)))}{dX_p}$ in equation (18) are continuous with respect to c.

First I compute

$$\begin{aligned} & A_p(X_p,c(X_c(X_p))) \\ = \quad & \int_{c^{\frac{1}{X_p}}}^{1} b(k,X_p,c)dk \\ = \quad & \int_{c^{\frac{1}{X_p}}}^{1} (\frac{X_p - 1}{X_p}(k - \frac{c}{k^{X_p - 1}}))dk \\ = \quad & (\frac{X_p - 1}{X_p})(\frac{1}{2} - \frac{c}{2 - X_p} - \frac{c^{\frac{2}{X_p}}}{2} + \frac{c^{\frac{1}{X_p}(2 - X_p)+1}}{2 - X_p}) \end{aligned} \tag{B2}$$

and

$$\begin{aligned} & R_p(X_p,c(X_c(X_p))) \\ = & \int_{c^{\frac{1}{X_p}}}^{1} (k_{(X)} - b(k_{(X)},X_p,c)) f(k_{(X)}) dk_{(X)} \\ = & \int_{c^{1/X_p}}^{1} ((k_{(X)} - \frac{X-1}{X}(k_{(X)} - \frac{c}{k_{(X)}^{X-1}})) X k_{(X)}^{X-1}) dk_{(X)} \\ = & \frac{1}{X_p}((c + c^{\frac{1+X_p}{X_p}})(X_p - 1) + (\frac{c^{\frac{1+X_p}{X_p}} + 1}{X_p + 1})). \end{aligned} \tag{B3}$$

Next, I show that the the limit as $c \to 0^+$ of the first term in equation (18) is $\lim_{c\to 0^+} A'_{p1}(X_p,c(X_c(X_p)) = \frac{X_p^2+4-4X_p}{2X_p^2(X_p-2)^2}$ and that the limit of the third term in equation (18) is $\lim_{c\to 0^+} R'_{p1}(X_p,c(X_c(X_p)) = -\frac{1+2X_p}{X_p^2(X_p+1)^2}$ so that the sum of both limits, $\frac{X_p^2+4-4X_p}{2X_p^2(X_p-2)^2} - \frac{1+2X_p}{X_p^2(X_p+1)^2} = \frac{1}{2X^2(X+1)^2}\left(X^2 - 2X - 1\right)$ is positive for any integers $X_p > 2$ (note that the values of the limits are exactly those found in section 4 for the case in which $c = 0$).

$$\begin{aligned} & \lim_{c\to 0^+} A'_{p1}(X_p,c(X_c(X_p))) \\ = & \lim_{c\to 0^+} \frac{1}{2X^2(X-2)^2} \begin{pmatrix} 4c^{\frac{2}{X}} \ln c - 4c - 4X - 2X^2 c + X^2 c^{\frac{2}{X}} + \\ 4Xc + X^2 - 6Xc^{\frac{2}{X}} \ln c + 2X^2 c^{\frac{2}{X}} \ln c + 4 \end{pmatrix} \\ = & \frac{X_p^2 + 4 - 4X_p}{2X_p^2(X_p-2)^2} \end{aligned} \tag{B4}$$

$$\begin{aligned} & \lim_{c\to 0^+} R'_{p1}(X_p,c(X_c(X_p))) \\ = & \lim_{c\to 0^+} \frac{1}{X_p^2(X_p+1)^2} \begin{pmatrix} c - 2X_p + X_p^2 c + 2X_p c + X_p^2 c c^{\frac{1}{X_p}} - \\ X_p^2 c c^{\frac{1}{X_p}} \ln c - X_p c c^{\frac{1}{X_p}} \ln c - 1 \end{pmatrix} \\ = & -\frac{1 + 2X_p}{X_p^2(X_p+1)^2} \end{aligned} \tag{B5}$$

Note that the second line in both equation (21) and (22) is a continuous function of c. It remains to show that the limit as $c \to 0^+$ of the second and fourth terms in equation (18) is 0 to conclude that the sum of the limits of all four terms is positive for $X_p > 2$ and c small enough. The second term, $A'_{p2}(X_p,c(X_c(X_p)))\frac{dc(X_c(X_p))}{dX_c}\frac{dX_c}{dX_p}$, is the trickiest. First note that

$$\lim_{c\to 0^+} A'_{p2}(X_p,c(X_c(X_p))) = \lim_{c\to 0^+} \frac{1}{Xc}\left(c - c^{\frac{2}{X}}\right)\frac{X-1}{X-2} = -\infty. \tag{B6}$$

The assumption that $0 < \frac{C'(N_c)}{C(N_c)} < 1$, when the number of entrepreneur is fixed, can be rewritten as $0 < \frac{\frac{dc(X_c(X_p))}{dX_c}}{c(X_c(X_p))} < 1$ so that the term $A'_{p2}(X_p,c(X_c(X_p)))\frac{dc(X_c(X_p))}{dX_c}$ tends to 0, i.e.

$$\lim_{c \to 0^+} \frac{\frac{dc}{dX_c}}{Xc} \left(c - c^{\frac{2}{X}}\right) \frac{X-1}{X-2} = 0, \tag{B7}$$

where I dropped the dependence of c on X_c from the notation.

To see why equation (24) holds, note that $\lim_{c \to 0^+} \frac{\frac{dc}{dX_c}}{c} = 0$ if $0 < \frac{dc}{dX_c} < c$. Finally, from Lemma 3, we have that:

$$\lim_{c \to 0^+} \frac{dX_c}{dX_p} = \lim_{c \to 0^+} \frac{1 - c^{1/X_p} + c^{1/X_p} \frac{\ln c}{X_p}}{1 + c^{\frac{1+X_p}{X_p}} \frac{dc}{dX_c}} = 1, \tag{B8}$$

so that, as claimed,

$$\lim_{c \to 0^+} A'_{p2}(X_p,c(X_c(X_p))) \frac{dc(X_c(X_p))}{dX_c} \frac{dX_c}{dX_p} = 0 * 1 = 0. \tag{B9}$$

Note that the product of the three terms in equation (26) is a continuous functions of c. To show that the limit of the fourth term, $R'_{p2}(X_p,c(X_c(X_p)))\frac{dc(X_c(X_p))}{dX_c}\frac{dX_c}{dX_p}$, is 0 as $c \to 0^+$, it suffices to show that

$$\lim_{c \to 0^+} R'_{p2}(X_p,c(X_c(X_p))) = \lim_{c \to 0^+} \frac{1}{X_p} \left(X_p + X_p c^{\frac{1}{X}} - 1\right) = \frac{X_p - 1}{X_p}, \tag{B10}$$

which is finite. As already shown, $\lim_{c \to 0^+} \frac{dX_c}{dX_p} = 1$, and $\lim_{c \to 0^+} \frac{dc}{dX_c} = 0$ because $0 < \frac{dc}{dX_c} < c$. I conclude that:

$$\lim_{c \to 0^+} R'_{p2}(X_p,c(X_c(X_p))) \frac{dc(X_c(X_p))}{dX_c} \frac{dX_c}{dX_p} = (\frac{X_p - 1}{X_p}) * 0 * 1 = 0, \tag{B11}$$

and again, the product of the three terms in equation (28) is a continuous function of c.