FFT-Based Option Pricing under Mean-Reverting Lévy Processes

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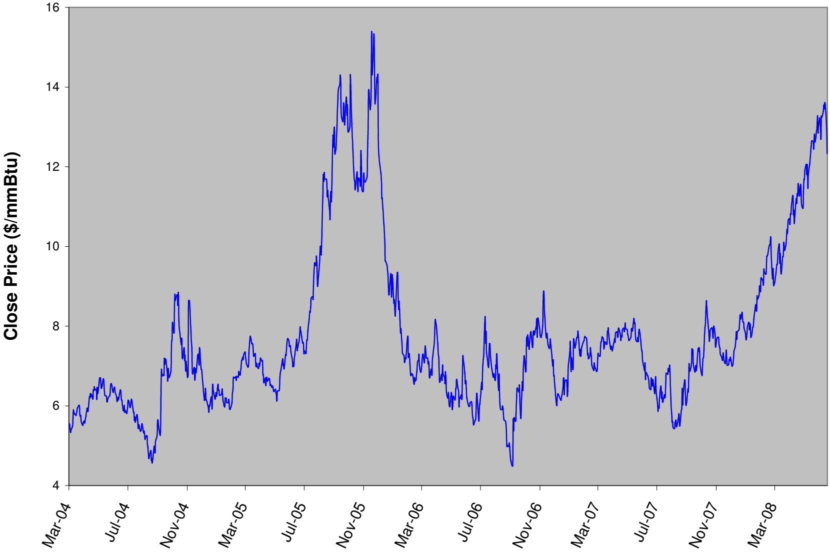
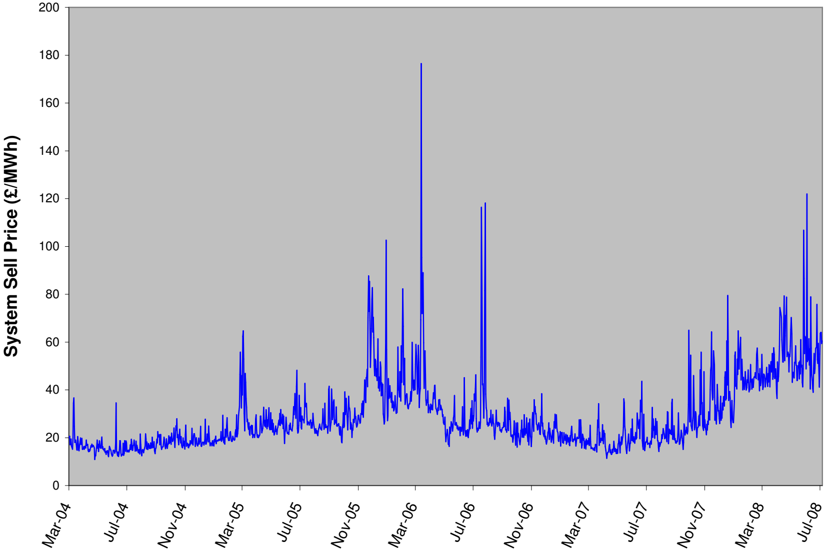
Commodities - Oil, Gas, Electricity,...

- Exhibit high volatilities and spikes in prices
- Tend to revert to long run equilibrium prices
- Many complex commodity contingent claims exist in the markets, such as swing and interruptible options

Henry Hub Natural Gas Prices



UK National Grid Electricity Prices



Existing Methods for Option Pricing

- Monte Carlo and tree methods
  - Slow convergence and expensive in computing the Greeks
- Finite difference methods
  - Integral term computationally expensive to handle; difficult to extend to multi-dimensional setting
  - References: *Andersen and Andreasen (2000), Cont and Tankov (2004), Briani, Natalini, and Russo (2004), d'Halluin, Forsyth, and Vetzal (2005)*
- Early (fast) Fourier transform methods
  - Limited to European options; require Fourier transform of the payoff function
  - References: *Carr and Madan (1999), Dempster and Hong (2000), Raible (2000), Lewis (2001)*
- Current fast Fourier transform (FFT) methods
  - Can only be applied to Lévy processes with independent increments
  - References: *Lord, Fang, Bervoets and Oosterlee (2007), Jackson, Jaimungal and Surkov (2007)*

1. Motivation

2. Mean-Reverting Fourier Space Time-stepping method
   - One-dimensional mean reversion with jump-diffusion
   - Multi-dimensional general framework

3. Numerical Results and Applications
   - Single-asset European, American and barrier options
   - Multi-asset spread options
   - Swing options

4. Conclusions

The Option Pricing Problem

- Option payoff is given by $\varphi(S)$
- The commodity spot price $S_t = e^{X_t}$ is driven by mean-reverting jump-diffusion process

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t + dJ_t, \quad X_0 = \ln S_0$$

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Option Value PIDE

$$\begin{cases} \partial_t v + \mathcal{L}v & = 0 \\ v(T,x) & = \varphi(e^x) \end{cases}$$

where $\mathcal{L}$ is the infinitesimal generator:

$$\mathcal{L}f(x) = \kappa(\theta - x)\partial_x f(x) + \frac{1}{2}\sigma^2\partial_{xx}f(x) + \int (f(x+y) - f(x))\,\nu(dy)$$

Solving for the Characteristic Function

- Let $h_{t,x}(\omega, T) \triangleq \mathbb{E}^{\mathbb{Q}}[e^{i\omega X_T}|\mathcal{F}_t]$ be the characteristic function of the log-stock price density under the risk-neutral measure

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- Assume ansatz form:

$$h_{t,x}(\omega, T) = e^{\Psi_t^T(\omega)+\Phi_t^T(\omega)x}$$

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- Assume ansatz form:

$$h_{t,x}(\omega, T) = e^{\Psi_t^T(\omega) + \Phi_t^T(\omega) x}$$

- Since $h_{t,x}(\omega)$ is a martingale, it satisfies the PIDE for all x:

$$(\partial_t + \mathcal{L}) h_{t,x} = \Big(\dot{\Psi}_t^T + \dot{\Phi}_t^T x + \kappa(\theta - x)\Phi_t^T + \frac{1}{2}\sigma^2 (\Phi_t^T)^2 + \int \big(e^{\Phi_t^T y} - 1 \big) \nu(dy) \Big) h_{t,x} = 0$$

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- $\Psi_t^T(\omega)$ and $\Phi_t^T(\omega)$ satisfy a system of Riccati ODEs:

$$\begin{cases} \dot{\Psi}_t^T + \kappa\theta\Phi_t^T + \frac{1}{2}\sigma^2(\Phi_t^T)^2 + \int(e^{\Phi_t^T y} - 1)\nu(dy) & = 0 \\ \dot{\Phi}_t^T - \kappa\Phi_t^T & = 0, \end{cases}$$

subject to $\Psi_T^T(\omega) = 0$ and $\Phi_T^T(\omega) = i\omega$

Solving for the Characteristic Function (cont.)

- We can solve for $\Phi_t^T(\omega)$ analytically:

$$\Phi_t^T(\omega) = i\omega e^{-\kappa(T-t)}$$

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$$\Phi_t^T(\omega) = i\omega e^{-\kappa(T-t)}$$

- $\Psi_t^T(\omega)$ can then be solved:

$$\Psi_t^T(\omega) = \kappa\theta \int_t^T \Phi_s^T ds + \frac{1}{2}\sigma^2 \int_t^T (\Phi_s^T)^2 ds + \int_t^T \int (e^{\Phi_s^T y} - 1)\nu(dy) ds$$
$$= i\omega\theta(1 - e^{-\kappa(T-t)}) - \frac{\omega^2\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)}) + \int_t^T \tilde{\psi}(\omega e^{-k(T-s)}) ds$$

where $\tilde{\psi}$ is the characteristic function of the jump distribution.

Solving for the Characteristic Function (cont.)

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- $\Psi_t^T(\omega)$ can then be solved:

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  where $\tilde{\psi}$ is the characteristic function of the jump distribution.

- $\int \tilde{\psi}(\omega e^{-ku})du$ can be computed explicitly in terms of an exponential integral for double-exponential distribution (Kou model) and numerically using quadrature for Gaussian distribution (Merton model)

Solving the PIDE

- Expand the payoff (assume paid at $t + \Delta t$) in a Fourier basis:

$$\varphi(x) = v_{t+\Delta t}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} \, \mathcal{F}[v_{t+\Delta t}](\omega) \, d\omega$$

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- Assuming no decisions (such as barrier breach or optimal exercise) are made during the interval $(t, t + \Delta t]$:

$$v_t(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h_{t,x}(\omega, t + \Delta t) \, \mathcal{F}[v_{t+\Delta t}](\omega) \, d\omega$$

The above satisfies the PIDE and the boundary condition

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  The above satisfies the PIDE and the boundary condition

- Apply Fourier transform to $v_t(x)$:

$$\mathcal{F}[v_t](\omega) = \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\Psi_t^{t+\Delta t}(\omega') + \Phi_t^{t+\Delta t}(\omega') x} \mathcal{F}[v_{t+\Delta t}](\omega') d\omega' \right] e^{-i\omega x} dx$$

Mean-Reverting FST Method

PIDE Solution in Fourier space

$$\mathcal{F}[v_t](\omega) = e^{\Psi_t^{t+\Delta t}(\omega\, e^{\kappa \Delta t}) + \kappa \Delta t} \mathcal{F}[v_{t+\Delta t}](\omega\, e^{\kappa \Delta t})$$

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- Since $\kappa > 0$, $e^{\kappa \Delta t} > 1$, extrapolation in frequency space of $\mathcal{F}[v_{t+\Delta t}]$ is required

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- Since $\kappa > 0$, $e^{\kappa\Delta t} > 1$, extrapolation in frequency space of $\mathcal{F}[v_{t+\Delta t}]$ is required
- Using the scaling property of the Fourier transform, this can be obtained by interpolating in real space $v_{t+\Delta t}$

Mean-Reverting FST Method

$$v^{m-1}(x) = \text{FFT}^{-1}\left[e^{\Psi(\omega e^{\kappa\,\Delta t})}\cdot \text{FFT}[v^m(x\, e^{-\kappa\Delta t})]\right]$$

- Without mean reversion, mrFST reduces to the standard FST method of Jackson, Jaimungal and Surkov (2007) since $\Phi_t^T(\omega) \to i\omega$

Spot Price Model

$$d\mathbf{Y}_t = \boldsymbol{\kappa}\left(\boldsymbol{\theta} - \mathbf{Y}_{t-}\right)dt + d\mathbf{J}_t$$
$$\mathbf{X}_t = \mathbf{B}\mathbf{Y}_t$$
$$S_t^i = \exp\{X_t^i\} \qquad i = 1, \dots, n$$

- n log spot-prices $\mathbf{X}_t$ are modeled as a linear transformation of a set of $d-$fundamental market factors $\mathbf{Y}_t$
- $\boldsymbol{\theta}$ a d-dimensional vector of long-run means
- $\boldsymbol{\kappa}$ a $d \times d$ matrix with positive eigenvalues representing the mixing of the market factors
- $\mathbf{B}$ a $d \times n$ matrix representing the linear transformation of the market factors into the observed log-prices
- $\mathbf{J}_t$ a d-dimensional Lévy process with Lévy triple $(\gamma, \mathbf{C}, \boldsymbol{\nu})$

Flexible Framework

- One factor mean-reverting model with jumps (*Clewlow and Strickland 2000*):
  $\boldsymbol{\theta} = \theta$, $\boldsymbol{\kappa} = \kappa$, $\mathbf{C} = \sigma^2$, $\mathbf{B} = 1$, and $\nu(d\mathbf{Z}) = \lambda F(dz)$

$$dX_t = \kappa(\theta - X_t)\,dt + \sigma\,dW_t \; + dJ_t$$

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$$dX_t = \kappa(\theta - X_t)\,dt + \sigma\,dW_t \;+ dJ_t$$

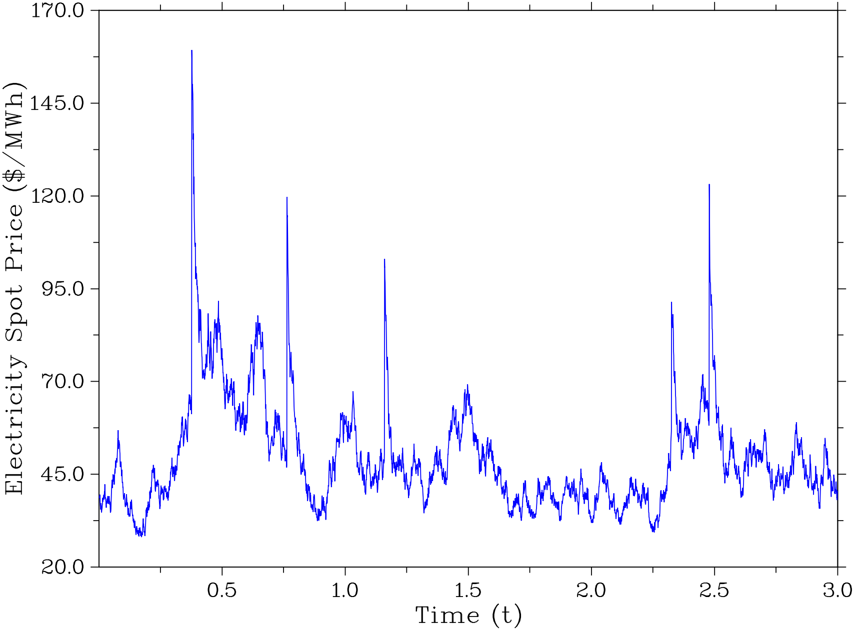
- Mean-reverting jump-diffusion model (*Hikspoors and Jaimungal 2007*) with different decay rates for the jumps and diffusion.

$$\boldsymbol{\theta} = \begin{pmatrix}\theta \\ 0\end{pmatrix} \boldsymbol{\kappa} = \begin{pmatrix}\alpha & 0 \\ 0 & \beta\end{pmatrix} \mathbf{C} = \begin{pmatrix}\sigma^2 & 0 \\ 0 & 0\end{pmatrix} \mathbf{B} = \begin{pmatrix}1 & 1\end{pmatrix}$$

and $\nu(dZ_1 \times dZ_2) = \lambda\,\delta_{Z_1}\,dF(Z_2)$.

$$\begin{aligned} dY_t^1 &= \alpha(\theta - Y_t^1)\,dt + \sigma\,dW_t \\ dY_t^2 &= -\beta Y_t^2\,dt + dJ_t \\ X_t &= Y_t^1 + Y_t^2 \end{aligned}$$

Simulated Electricity Spot Prices



Flexible Framework (cont.)

- Two factor mean-reverting model (*Barlow, Gusev, and Lai 2004*) with log-prices mean-revert to a stochastic long-run mean, which itself mean-reverts to a fixed level:

$$\boldsymbol{\theta} = \begin{pmatrix} \theta \\ \theta \end{pmatrix} \boldsymbol{\kappa} = \begin{pmatrix} \alpha & -\alpha \\ 0 & \beta \end{pmatrix} \mathbf{C} = \begin{pmatrix} \sigma^2 & \rho\sigma\eta \\ \rho\sigma\eta & \eta^2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 & 0 \end{pmatrix} \nu(d\mathbf{Z}) = 0$$

$$dX_t = \alpha(Y_t - X_t)dt + \sigma\, dW_t^X$$
$$dY_t = \beta(\theta - Y_t)dt + \eta\, dW_t^Y$$

Flexible Framework (cont.)

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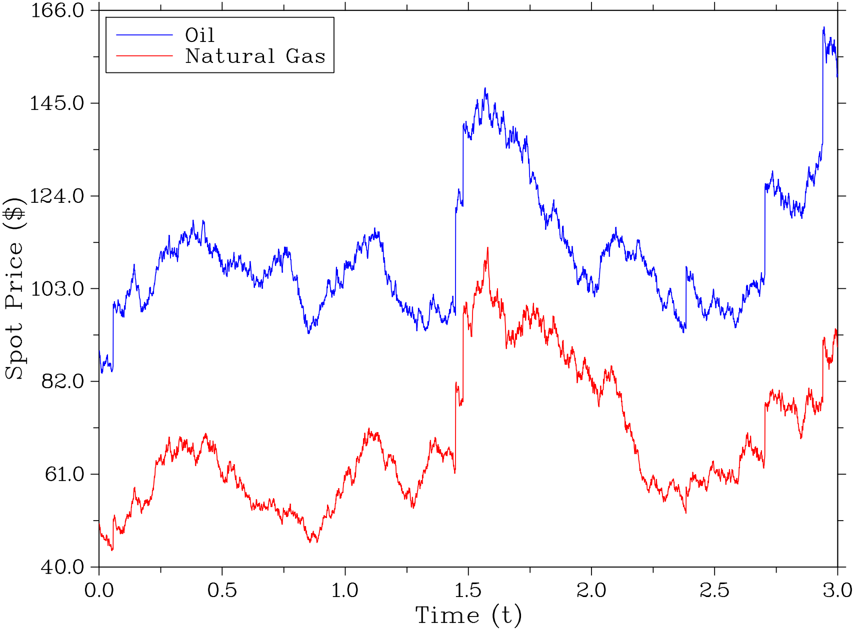
$$dX_t = \alpha(Y_t - X_t)dt + \sigma\, dW_t^X$$
$$dY_t = \beta(\theta - Y_t)dt + \eta\, dW_t^Y$$

- Jump diffusion model where the diffusions are correlated, and jumps may have codependent pieces. Noise driven by a copula to introduce co-dependence in the innovations:

$$\boldsymbol{\theta} = \begin{pmatrix} \theta \\ \phi \end{pmatrix} \boldsymbol{\kappa} = \begin{pmatrix} \alpha & \gamma \\ \delta & \beta \end{pmatrix} \mathbf{C} = \begin{pmatrix} \sigma^2 & \rho\sigma\eta \\ \rho\sigma\eta & \eta^2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and $\nu(dZ_1 \times dZ_2) = dC(F_1(Z_1), F_2(Z_2))$ with $C(u, v)$ a copula and $F_i(z)$ two marginal distribution functions.

Simulated Energy Spot Prices



Multi-Dimensional Mean-Reverting FST Method

PIDE Solution in Fourier Space

The discounted price $v_t(\mathbf{Y}_t)$ of a European option written on the vector of price processes $\{S_t^1 = e^{X_t^1}, \ldots, S_t^n = e^{X_t^n}\}$ where $\mathbf{X}_t = \mathbf{B}\,\mathbf{Y}_t$ with payoff function $\varphi(\mathbf{X}_T) = \varphi(\mathbf{B}\,\mathbf{Y}_T) = \phi(\mathbf{Y}_T)$ is

$$\mathcal{F}[v_t(\mathbf{Y}_t)] = e^{\Psi_t(\boldsymbol{\omega},T)+(T-t)\mathrm{Tr}\,\boldsymbol{\kappa}}\;\mathcal{F}[\phi(\mathbf{Y}_t)](e^{\boldsymbol{\kappa}'(T-t)}\,\boldsymbol{\omega})$$

where,

$$\Psi_t(\boldsymbol{\omega},T) = \int_t^T \psi(e^{\boldsymbol{\kappa}'(u-t)}\,\boldsymbol{\omega})\,du$$

$$\psi(\boldsymbol{\omega}) = i\boldsymbol{\omega}'\boldsymbol{\kappa}\boldsymbol{\theta} - \tfrac{1}{2}\boldsymbol{\omega}'\mathbf{C}\boldsymbol{\omega} + \int \left(e^{i\boldsymbol{\omega}'\mathbf{y}} - 1 - i\,\mathbb{I}_{\{|\mathbf{y}|<1\}}\,\boldsymbol{\omega}'\mathbf{y}\right)\nu(d\mathbf{y})$$

Multi-Dimensional Mean-Reverting FST Method

By discretizing space and frequency, prices for a full spectrum of spot values are computed using two FFT evaluations

$$V_{n-1}(\mathbf{X}) = \text{FFT}^{-1}\left[e^{\Psi_0(\boldsymbol{\omega},\Delta t)+\Delta t \operatorname{Tr}\boldsymbol{\kappa}} \cdot \text{FFT}[V_n](e^{\boldsymbol{\kappa}'\Delta t}\,\boldsymbol{\omega})\right](\mathbf{X})$$

The transform of the price at time-step n is required at scaled frequencies. Using the scaling property of Fourier transforms

$$\mathcal{F}\,[g]\,(e^{\boldsymbol{\kappa}'\Delta t}\,\omega) = \mathcal{F}\,[\widetilde{g}]\,(\omega)\,e^{-\Delta t\,\operatorname{Tr}\boldsymbol{\kappa}}\,, \quad \widetilde{g}(\mathbf{X}) \triangleq g(\mathbf{X}\,e^{-\kappa'\Delta t})$$

these are conveniently computed from interpolated prices at time-step n:

Multi-Dimensional Mean-Reverting FST Method

$$V_{n-1}(\mathbf{X}) = \text{FFT}^{-1}\left[e^{\Psi_0(\boldsymbol{\omega},\Delta t)} \cdot \text{FFT}[\widetilde{V}_n](\boldsymbol{\omega})\right](\mathbf{X})$$

European Option Results

N	Value	Change	Convergence order	Time (msec.)
4096	10.47317181			5.546
8192	10.47299262	0.0001792		9.358
16384	10.47295334	0.0000393	2.1895	18.719
32768	10.47294517	0.0000082	2.2656	37.984

- *Option:* European put $S = 100, K = 105, T = 1$
- *Model:* Merton jump-diffusion with mean reversion $\sigma = 0.2, \lambda = 0.25, \tilde{\mu} = 0.3, \tilde{\sigma} = 0.5, \theta = 90, \kappa = 0.75, r = 0.05$
- *Monte Carlo:* 95% CI $(10.47243025, 10.47356975)$ @ 114 sec.

American Option Results

N	M	Value	Change	Convergence order	Time (sec.)
4096	256	12.23654432			0.107
8192	512	12.23592062	0.0006237		0.432
16384	1024	12.23559075	0.0003299	0.9190	2.214
32768	2048	12.23542014	0.0001706	0.9511	9.546

- *Option:* American put $S = 100, K = 105, T = 1$
- *Model:* Merton jump-diffusion with mean reversion $\sigma = 0.2, \lambda = 0.25, \tilde{\mu} = 0.3, \tilde{\sigma} = 0.5, \theta = 90, \kappa = 0.75, r = 0.05$
- *Monte Carlo:* 95% CI $(12.2330185, 12.2361035)$ @ 53 min.

Discrete Barrier Option Results

N	Value	Change	Convergence order	Time (sec.)
4096	3.05084502			0.842
8192	3.05259256	0.0017475		1.712
16384	3.05298286	0.0003903	2.1626	3.777
32768	3.05308576	0.0001029	1.9234	12.369

- *Option:* Down-and-out barrier put $S = 100, K = 105, T = 1, B = 85, R = 2.5$ with hourly monitoring
- *Model:* Merton jump-diffusion with mean reversion $\sigma = 0.2, \lambda = 0.25, \tilde{\mu} = 0.3, \tilde{\sigma} = 0.5, \theta = 90, \kappa = 0.75, r = 0.05$
- *Monte Carlo:* 95% CI $(3.04777486, 3.05662484)$ @ 19.1 min.

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Spread Option Results

N	Value	Change	Convergence order	Time (sec.)
512	20.40021368			0.717
1024	20.39217503	0.0080386		3.945
2048	20.39000196	0.0021731	1.8872	20.362
4096	20.38935935	0.0006426	1.7577	80.414

- *Option:* European spread call $S_1 = 100, S_2 = 100, K = 3.5, T = 1$
- *Model:* 2D Kou jump-diffusion with mean reversion and copula
  $\sigma_1 = 0.2, \lambda_1 = 0.75, p_1 = 0.45, \eta_{1+} = 0.25, \eta_{1-} = 0.125, \theta_1 = 92, \kappa_1 = 0.5,$
  $\sigma_2 = 0.3, \lambda_2 = 0.5, p_2 = 0.55, \eta_{2+} = 0.3, \eta_{2-} = 0.2, \theta_2 = 110, \kappa_2 = 0.75$
  $\rho = 0.7, r = 0.05$
  $\lambda_c = 0.5, \widehat{\mu}_{c1} = -0.1, \widehat{\sigma}_{c1} = 0.2, \widehat{\mu}_{c2} = 0.075, \widehat{\sigma}_{c2} = 0.15, \rho_c = 0.7$
- *Monte Carlo:* 95% CI $(20.378096, 20.431361)$ @ 35 minutes

American Spread Option Results

N	M	Value	Change	Convergence order	Time (sec.)
512	64	23.55544302			4.1
1024	128	23.50622121	0.0492218		40.7
2048	256	23.48240921	0.0238120	1.0476	353.4

- *Option:* American spread call $S_1 = 100, S_2 = 100, K = 3.5, T = 1$
- *Model:* 2D Kou jump-diffusion with mean reversion and copula
  $\sigma_1 = 0.2, \lambda_1 = 0.75, p_1 = 0.45, \eta_{1+} = 0.25, \eta_{1-} = 0.125, \theta_1 = 92, \kappa_1 = 0.5,$
  $\sigma_2 = 0.3, \lambda_2 = 0.5, p_2 = 0.55, \eta_{2+} = 0.3, \eta_{2-} = 0.2, \theta_2 = 110, \kappa_2 = 0.75$
  $\rho = 0.7, r = 0.05$
  $\lambda_c = 0.5, \widehat{\mu}_{c1} = -0.1, \widehat{\sigma}_{c1} = 0.2, \widehat{\mu}_{c2} = 0.075, \widehat{\sigma}_{c2} = 0.15, \rho_c = 0.7$

Overview

- Common in electricity and natural gas markets
- Provides constrained flexibility with respect to volume and timing of energy delivery
- Two components: a pure forward agreement and a swing option

Example: Simple Swing Option

The holder agrees to buy 100MWh at a cost of \$45/MWh over a period of 1 month. At the start of each hour, the holder has the right to increase power consumption to 110MW for that hour (swing up) or decrease to 90MW (swing down) at the same cost. The total number of swings is limited to 50.
The swing component is the right to change consumption at holder's choosing.

For overview of swing options and their valuation see Ware (2007)

Pricing

At each swing opportunity, a choice to exercise q swing "options" for immediate cashflow Υ must be made:

Dynamic Programming Equation

$$v_{t_m}(x, Q) = \max_q \left\{ \Upsilon_{t_m}(x, q) + e^{-r\Delta t}\mathbb{E}[v_{t_{m+1}}(x, Q + q)] \right\}$$

where the expectation is readily computed using the mrFST method

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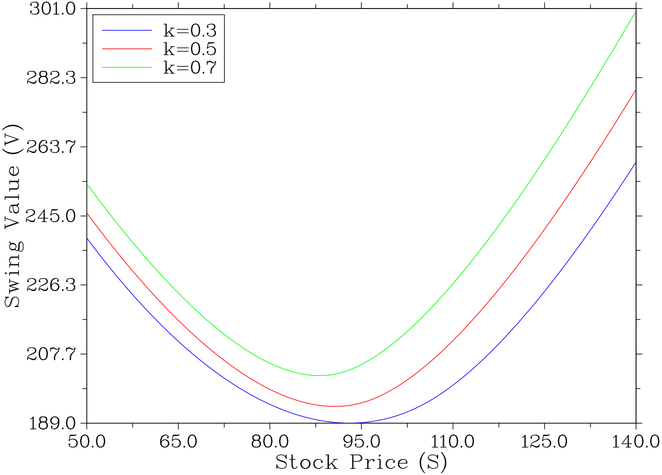
where the expectation is readily computed using the mrFST method

- The available choices are to do nothing $q = 0$, swing up $q > 0$ or swing down $q < 0$
- The amount of swings may be bounded $|Q_{t_m}| \leq \overline{Q}$ where $Q_{t_m} = \sum_{j=1}^{m} q_{t_j}$ or $Q_{t_m} = \sum |q_{t_j}|$
- The cashflow function $\Upsilon_t(x, q)$ may include a penalty term to enforce additional limits on Q or may be as simple as $\Upsilon_t(x, q) = q(e^x - K)$

Results



- *Option:* Swing $S = 100, K = 100, T = 2, -3 \leq Q \leq 5$
- *Model:* Kou jump-diffusion with mean reversion
  $\sigma = 0.3, \lambda = 0.7, p = 0.45, \eta_+ = 0.25, \eta_- = 0.2, \theta = 90, r = 0.05$

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   - Single-asset European, American and barrier options
   - Multi-asset spread options
   - Swing options

4. Conclusions

mrFST Method

- Naturally applied to
  - Path-dependent options with discontinuous/irregular payoffs
  - Multi-dimensional problems
  - Exotic options such as swing
- Computationally efficient
  - 2 FFTs required to obtain option values computed for a range of spots
  - European options priced in a single time-step
  - Bermudan style options do not require time-stepping between monitoring dates

Thank You

Questions?