Search Frictions, Competing Mechanisms and Optimal Market Segmentation

Online Appendix

Xiaoming Cai* Pieter Gautier† Ronald Wolthoff‡

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B Omitted Proofs

The results in lemma 1 and proposition 3 follow from Cai et al. (2016). For completeness, we provide short proofs here, referring to their paper for additional detail.

B.1 Proof of Lemma 1

The maximum valuation at a seller who meets $n \in \mathbb{N}_1$ buyers is an order statistic, distributed according to $G^n(x)$. Taking the expectation over x and n, followed by integration by parts and using the Dominated Convergence Theorem to interchange summation and integration, yields

$$S(\lambda, G) = \sum_{n=1}^{\infty} P_n(\lambda) \int_0^1 x \, dG^n(x) = \int_0^1 \left(1 - \sum_{n=0}^{\infty} P_n(\lambda) G^n(x)\right) dx.$$

The result then follows because the rightmost integrand equals $\phi(\lambda(1 - G(x)), \lambda)$. □

B.2 Proof of Proposition 3

The proof consists of two parts. First, we consider a seller who can choose the length and composition of his queue directly in a competitive market ("relaxed maximization problem"). By the first welfare theorem, the equilibrium in this market is Pareto optimal, which necessarily

*Tongji University. E-mail: xiaoming@tongji.edu.cn.
†Vrije Universiteit Amsterdam and Tinbergen Institute. E-mail: p.a.gautier@vu.nl.
‡University of Toronto. E-mail: ronald.p.wolthoff@gmail.com.

implies that it maximizes social net output as there is only one consumption good. Subsequently, we establish that a seller who posts the proposed equilibrium mechanism to attract an endogenously determined queue of buyers ("constrained maximization problem") implements the same solution.

Part 1 (relaxed maximization problem). For a given market utility function $\overline{U}(x)$, a seller chooses the queue (λ, G) that maximizes his expected payoff, which equals the difference between surplus $S(\lambda, G)$ and the expected payoff that the seller has to offer to each of the buyers. That is,

$$\int_0^1 \phi\left(\lambda\left(1 - G\left(z\right)\right), \lambda\right)\, dz - \int_0^1 \overline{U}\left(z\right)\, d\lambda G(z).$$

Because the seller takes the market utility function as given, he is a residual claimant on any extra surplus that he creates. Hence, the seller will compare the marginal cost $\overline{U}(x)$ of attracting a buyer with valuation x to this buyer's marginal contribution to surplus $T(x)$. To calculate $T(x)$, increase the measure of buyers with values around x, formally $[x, x+\Delta x]$, by ε and denote the new queue length and buyer value distribution as λ' and G' respectively. That is, $\lambda' = \lambda + \varepsilon$, while $\lambda'(1 - G'(z)) = \lambda(1 - G(z))$ for $z > x$ and $\lambda'(1 - G'(z)) = \lambda(1 - G(z)) + \varepsilon$ for $z < x$. By lemma 1, the average contribution to surplus by buyers with values around x is

$$\begin{aligned}\frac{S(\lambda', G') - S(\lambda, G)}{\varepsilon} &= \frac{1}{\varepsilon}\left(\int_0^x \phi(\lambda(1 - G(x)) + \varepsilon, \lambda + \varepsilon) - \phi(\lambda(1 - G(x)), \lambda)\right) \\ &+ \frac{1}{\varepsilon}\left(\int_x^1 \phi(\lambda(1 - G(x)), \lambda + \varepsilon) - \phi(\lambda(1 - G(x)), \lambda)\right)\end{aligned}$$

Let $\varepsilon \to 0$, then the above equation converges to

$$T(x) = \int_0^1 \phi_\lambda\left(\lambda\left(1 - G\left(z\right)\right), \lambda\right)\, dz + \int_0^x \phi_\mu\left(\lambda\left(1 - G\left(z\right)\right), \lambda\right)\, dz. \tag{20}$$

The solution to the relaxed maximization problem must therefore satisfy

$$\overline{U}(x) \geq T(x) \text{ for all } x, \text{ with equality for all } x \in \operatorname{supp} G \tag{21}$$

Part 2 (constrained maximization problem). Consider now a seller who posts a second-price auction and a meeting fee τ, attracting a queue (λ, G). A buyer with valuation x in the support of G meets the seller together with $n - 1$ other buyers with probability $\frac{nP_n(\lambda)}{\lambda}$.[1] Hence, he pays the meeting fee τ with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n\left(\lambda\right) = \phi_\mu\left(0, \lambda\right)$ and trades with

[1]See Eeckhout and Kircher (2010) or Lester et al. (2015).

probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda)\, G(x)^{n-1} = \phi_\mu(\lambda(1-G(x)),\lambda)$. As a result, his expected payoff is

$$U(x,\tau,\lambda,G) = -\phi_\mu(0,\lambda)\,\tau + \int_0^x \phi_\mu(\lambda(1-G(y)),\lambda)\; dy, \tag{22}$$

where the second term is the payoff from the auction, which—by standard results in auction theory—equals the integral over the trading probabilities (see e.g. Peters, 2013). A queue (λ, G) is therefore compatible with an auction with fee τ if and only if

$$\overline{U}(x) \geq U(x,\tau,\lambda,G) \text{ for all } x, \text{ with equality for all } x \in \operatorname{supp} G \tag{23}$$

Clearly, if a queue (λ, G) satisfies (21), then by setting the entry fee τ in equation (22) equal to

$$\tau = -\frac{\int_0^1 \phi_\lambda(\lambda(1-G(x)),\lambda)\; dx}{\phi_\mu(0,\lambda)},$$

it also satisfies (23). Therefore, any queue chosen by an unconstrained seller who can buy queues directly at a price $\overline{U}(x)$ is also compatible with an auction with an entry fee. □

C Joint Concavity Using $P_n(\lambda)$

In the main text, we define joint concavity in terms of ϕ, but an equivalent condition in terms of P_n, the actual primitive of the model, can be derived.[2] Starting from the definition of ϕ, taking partial derivatives yields

$$\begin{aligned}
\phi_{\mu\mu} &= -\sum_{n=0}^{\infty}(n+2)(n+1)\frac{P_{n+2}}{\lambda^2}\left(1-\frac{\mu}{\lambda}\right)^n,\\
\phi_{\mu\lambda} &= \sum_{n=0}^{\infty}\left[(n+1)\frac{\lambda P'_{n+1}-P_{n+1}}{\lambda^2} + (n+2)(n+1)P_{n+2}\frac{\mu}{\lambda^3}\right]\left(1-\frac{\mu}{\lambda}\right)^n,\\
\phi_{\lambda\lambda} &= -\sum_{n=0}^{\infty}\left[P''_n + 2\mu(n+1)\frac{\lambda P'_{n+1}-P_{n+1}}{\lambda^3} + \frac{(n+2)(n+1)P_{n+2}\mu^2}{\lambda^4}\right]\left(1-\frac{\mu}{\lambda}\right)^n.
\end{aligned}$$

Using the fact that $\sum_{n=0}^{\infty} a_n y^n \sum_{n=0}^{\infty} b_n y^n - \left(\sum_{n=0}^{\infty} c_n y^n\right)^2 = \sum_{n=0}^{\infty}\sum_{i=0}^{n}(a_i b_{n-i} - c_i c_{n-i})\, y^n$, the condition for joint concavity can then be written as

$$\phi_{11}\phi_{22} - \phi_{12}^2 = \sum_{n=0}^{\infty} Z_n \left(1-\frac{\mu}{\lambda}\right)^n \geq 0,$$

[2] To save on notation, we suppress the argument of P_n throughout this derivation.

where, after some simplification, Z_n equals

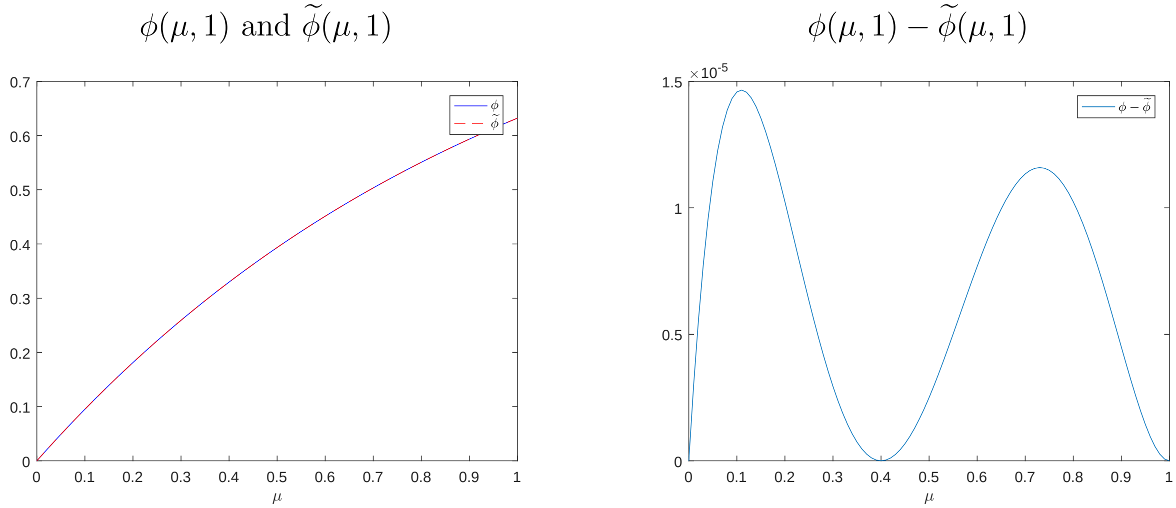
$$Z_n = \sum_{i=0}^{n} \left[\frac{(i+2)(i+1)P_{i+2}P''_{n-i}}{\lambda^2} - (i+1)(n-i+1)\frac{\lambda P'_{i+1} - P_{i+1}}{\lambda^2}\frac{\lambda P'_{n-i+1} - P_{n-i+1}}{\lambda^2} \right].$$

D Additional Details on Proposition 3

To illustrate the proof of proposition 3, consider a market with a measure $\Lambda = 1$ of buyers whose valuations are either low (with probability 0.6) or high (with probability 0.4), such that $N = 2$, $\zeta_1 = 1$ and $\zeta_2 = 0.4$. This implies $a_0 = a_1 = 0$, $a_2 = 0.36$, $a_3 = -1.56$, $a_4 = 2.2$ and $a_5 = -1$. Since urn-ball implies $P_5(1) = 0.0031$, we set $\varepsilon(1) = 0.002$ to make sure that $\widetilde{P}_n(1) \geq 0$ for all n.

The left panel of figure 1 then plots $\phi(\mu, 1)$ and $\widetilde{\phi}(\mu, 1)$ as functions of μ. The two functions are nearly indistinguishable, so we plot the difference $\phi(\mu, 1) - \widetilde{\phi}(\mu, 1)$ in the right panel. This reveals that the difference is strictly positive, except when $\frac{\mu}{\lambda} \in \{0, \zeta_1, \zeta_2\}$, just as we expected from the proof of proposition 3.

Figure 1: Perturbation of the Meeting Technology



Unfortunately, this method does not work when considering a distribution for which the support contains an interval. To see this, consider for example a distribution where $x = 0$ with probability 1/2 and x has a density function f on the interval $[\underline{x}, 1]$ for $\underline{x} > 0$. Constructing $\widetilde{P}$ then requires that

$$\widetilde{\phi}(\mu, \lambda) \leq \phi(\mu, \lambda) \quad \text{with equality if and only if} \quad \frac{\mu}{\lambda} \in \left[\frac{1}{2}, 1\right]$$

However, for a given λ, both $\widetilde{\phi}(\mu, \lambda)$ and $\phi(\mu, \lambda)$ are analytic functions of μ. If two analytic functions agree on an interval, then they agree everywhere (see p127 of Ahlfors, 1979 or Theorem 10.18 of Rudin, 1987). Therefore, there doesn't exist a $\widetilde{P}_n$ such that the above equation holds.

E Mechanisms and Equilibrium Conditions

Our description of the mechanism space and equilibrium conditions is similar to Eeckhout and Kircher (2010) and Auster and Gottardi (2016). See those papers for additional details.

Mechanism Space. A direct mechanism m is an extensive form game, which determines buyers' trading probabilities and payoffs. To be precise, for a buyer with valuation x, meeting the seller joint with $n-1$ other buyers whose valuations are $(x_1, \ldots, x_{n-1})$, the mechanism specifies

1. a trading probability $\theta_n(x; x_1, \ldots, x_{n-1}; m)$, symmetric in $x_1, \ldots, x_{n-1}$.

2. a transfer $t_n(x; x_1, \ldots, x_{n-1}; m)$, symmetric in $x_1, \ldots, x_{n-1}$.

Feasibility requires that the sum of the trading probabilities across all buyers meeting a seller does not exceed 1, i.e.

$$\sum_{i=1}^{n} \theta_n(x_i; x_1, \ldots, x_{i-1}, x_{i+1}, \ldots x_n; m) \leq 1 \; \forall (x_1, \ldots, x_n) \in [0,1]^n .$$

Incentive Compatibility. Given a queue (λ, G), the expected probability of trade for a buyer with valuation x equals

$$\theta(x; m, \lambda, G) = \sum_{n=1}^{\infty} \frac{nP_n(\lambda)}{\lambda} \int \ldots \int \theta_n(x; x_1, \ldots, x_{n-1}; m) \; dG(x_1) \ldots dG(x_{n-1}),$$

while the expected transfer equals

$$t(x; m, \lambda, G) = \sum_{n=1}^{\infty} \frac{nP_n(\lambda)}{\lambda} \int \ldots \int t_n(x; x_1, \ldots, x_{n-1}; m) \; dG(x_1) \ldots dG(x_{n-1}).$$

Incentive compatibility then specifies that buyers maximize their payoff by by truthfully reporting their type. That is,

$$\theta(x; m, \lambda, G) x_i - t(x; m, \lambda, G)$$

is maximized in $x = x_i$ for all x_i.

Payoffs. If a mechanism m is incentive compatible and attracts a queue (λ, G) , then the expected payoff of a buyer with valuation x is

$$U(x; m, \lambda, G) = \theta(x; m, \lambda, G)\, x - t(x; m, \lambda, G),$$

while the expected payoff of the seller is

$$R(m, \lambda, G) = \lambda \int t(x; m, \lambda, G)\; dG(x).$$

Beliefs. Given a mechanism $m(j)$ and a queue $(\lambda(j), G(j, \cdot))$ for each seller $j \in [0, 1]$, define the market utility $\overline{U}(x)$ as the highest payoff that a buyer with valuation x can obtain, i.e.

$$\overline{U}(x) = \max_{j \in [0,1]} U(x; m(j), \lambda(j), G(j, \cdot)).$$

A queue (λ, G) solves the market utility condition for a mechanism m if

$$U(x; m, \lambda, G) \leq \overline{U}(x) \text{ with equality for each } x \in \operatorname{supp} G. \tag{24}$$

Denote the set of queues (λ, G) that solve (24) for a mechanism m by $Q(m)$. A seller who posts a mechanism m then expects to obtain a payoff

$$\max_{\lambda, G} \{R(m, \lambda, G) \mid (\lambda, G) \in Q(m)\},$$

where the max operator represents sellers' optimism in case $Q(m)$ contains more than one element. By convention, sellers expect a non-positive payoff if $Q(m)$ is empty.[3]

Aggregate Consistency. Consistency requires that aggregating queues across sellers does not exceed the total measure of buyers of each type. That is,

$$\int_0^1 \lambda(j)\nu(j, B)\, dj \leq \Lambda \nu_F(B)$$

for any Borel-measurable set B, where ν_F is the measure associated with F and $\nu(j, \cdot)$ is the measure associated with $G(j, \cdot)$.

[3] The motivation for this assumption is the following. A seller who offers all buyer types a payoff less than their market utility, irrespective of the queue, expects to attract no buyers, which yields a zero payoff. A seller who offers certain buyer types a payoff higher than their market utility, irrespective of the queue, expects to attract an infinite queue, which yields the seller a payoff that approaches minus infinity.

F Invariance, Non-Rivalry and Pooling

Invariance. Proposition 2 and 4 jointly imply that invariance is a sufficient condition for a pooling equilibrium, but not a necessary condition. The intuition for sufficiency is straightforward. Invariance implies that the presence of low-type buyers in a submarket has no effect on the meetings between high-type buyers and sellers. Surplus is therefore maximized by spreading high-type buyers evenly across all sellers, as opposed to concentrating them at a subset, to maximize the number of high-type buyers that will trade. A single market results.

To see why invariance is not necessary, consider again the pairwise urn-ball technology. As discussed in the main text, $\phi_\lambda > 0$ for this technology. That is, adding low-type buyers to a submarket increases the probability that a seller will meet a high-type buyer. This feature violates invariance, but not joint concavity: the fact that the addition of low-type buyers to the submarket helps to spread the high-type buyers better across sellers strengthens the incentive to send all buyers to the same market.[4]

Non-Rivalry. Proposition 2 and 5 jointly imply that non-rivalry is a necessary condition for a pooling equilibrium, but not a sufficient condition. To understand why non-rivalry is necessary, consider a submarket with a single high-type buyer with valuation $x_2 > 0$ and a number of low-type buyers with valuation $x_1 \to 0$, such that surplus only depends on the trading probability of the high-type buyer. Violation of non-rivalry would imply that this probability could be increased by sending either some low-type buyers (if $\phi_{\mu\lambda}(0, \lambda) < 0$) or some sellers (if $\phi_{\mu\lambda}(0, \lambda) > 0$) to a different submarket, contradicting the optimality of the single market associated with joint concavity.

To see why non-rivalry is not sufficient, consider again the multi-platform technology. As discussed in the main text, this technology is non-rival. However, the presence of low-type buyers in the submarket increases the chances for high-type buyers to be crowded out at one of the αs sellers in the first round, concentrating them at the $(1-\alpha)s$ second-round sellers in higher numbers than optimal. It is therefore better to send at least some low types to a separate submarket.

G Spatial Meeting Technology

Consider the following technology.

> *Spatial.* This technology features two distinct locations within every submarket. We call these locations *near* and *remote*, respectively. Each location always attracts exactly half of

[4] This may raise the question how $\phi_\lambda \geq 0$ relates to joint concavity. We prove in section H of this online appendix that it is a necessary but not a sufficient condition.

all sellers in the submarket, allocated in a random way. In contrast, the number of buyers in each location depends on the queue length λ. In particular, buyers disproportionally visit the sellers in the near location when λ is small, but spread more evenly across locations when λ increases. When the queue length reaches (or exceeds) a critical level $\overline{\lambda} > 0$, half of all buyers visit either location. To formalize this, let $\gamma(\lambda)$ be the fraction of buyers in the submarket visiting the remote location. We then assume that i) $\gamma(0) < \frac{1}{2}$, ii) $\gamma(\lambda) = \frac{1}{2}$ for all $\lambda \geq \overline{\lambda}$, and iii) $\gamma(\lambda) = \gamma(0) + \frac{\lambda}{\overline{\lambda}}\left(\frac{1}{2} - \gamma(0)\right)$ for all $0 \leq \lambda \leq \overline{\lambda}$. Within each location, buyers and sellers are allocated according to an urn-ball technology.[5] Hence,

$$P_n(\lambda) = \begin{cases} \frac{1}{2} e^{-2\lambda\gamma(\lambda)} \frac{[2\lambda\gamma(\lambda)]^n}{n!} + \frac{1}{2} e^{-2\lambda(1-\gamma(\lambda))} \frac{[2\lambda(1-\gamma(\lambda))]^n}{n!} & \text{for } \lambda < \overline{\lambda} \\ e^{-\lambda} \frac{\lambda^n}{n!} & \text{for } \lambda \geq \overline{\lambda}. \end{cases}$$

Verification of the conditions in definition 1 and lemma 2 then yields the following result.

Proposition 7. *The spatial technology is jointly concave but not invariant, if $\gamma(0)$ is below but sufficiently close to $\frac{1}{2}$.*

Proof. It immediately follows from earlier results that the technology satisfies the conditions for invariance and joint concavity for $\lambda \geq \overline{\lambda}$. We therefore focus on $\lambda < \overline{\lambda}$ in the remainder of the proof. For these values of λ,

$$\phi(\mu, \lambda) = \frac{1}{2}\left(1 - e^{-2\gamma(\lambda)\mu}\right) + \frac{1}{2}\left(1 - e^{-2(1-\gamma(\lambda))\mu}\right).$$

Taking the derivative with respect to λ yields

$$\phi_\lambda = \frac{\frac{1}{2} - \gamma(0)}{\overline{\lambda}} \mu \left[e^{-2\gamma(\lambda)\mu} - e^{-2(1-\gamma(\lambda))\mu}\right] > 0,$$

which reveals that the technology is not invariant. Similarly, taking second-order derivatives reveals that $\phi_{\mu\mu}\phi_{\lambda\lambda} - \phi_{\mu\lambda}^2$ equals

$$\phi_{\mu\mu}\phi_{\lambda\lambda} - \phi_{\mu\lambda}^2 = \left(\frac{\partial \gamma}{\partial \lambda}\right)^2 e^{-2\mu}\left[4\mu^2 - 4\mu + 2 + e^{2\mu(1-2\gamma)}(4\mu\gamma - 1) + e^{-2\mu(1-2\gamma)}(4\mu(1-\gamma) - 1)\right],$$

where we omit the argument of $\gamma(\lambda)$ to simplify notation. Note that if γ is sufficiently close to $\frac{1}{2}$, then we can use the Taylor expansion $e^x = 1 + x + \frac{x^2}{2}$ to approximate $e^{2\mu(1-2\gamma)}$ and $e^{-2\mu(1-2\gamma)}$.

[5] Strictly speaking, the resulting technology is not (twice-continuously) differentiable in $\overline{\lambda}$, which violates one of our assumptions. However, at the expense of additional notation, it can be approximated arbitrarily closely by a technology that satisfies this condition. We omit this step to keep the exposition as simple as possible.

After simplification, this yields

$$\phi_{\mu\mu}\phi_{\lambda\lambda} - \phi_{\mu\lambda}^2 \approx \left(\frac{\partial\gamma}{\partial\lambda}\right)^2 e^{-2\mu} 4\mu^2 \left[1 + (1-2\gamma)^2 (2\mu - 3)\right],$$

which is strictly positive for such γ. Hence, by choosing $\gamma(0)$ below but sufficiently close to $\frac{1}{2}$, we obtain a meeting technology that is jointly concave but not invariant. □

H Necessity and Insufficiency of $\phi_\lambda \geq 0$

The fact that the pairwise urn-ball technology satisfies $\phi_\lambda \geq 0$ as well as joint concavity may raise the question how these two properties are related. The following proposition establishes that $\phi_\lambda(\mu, \lambda) \geq 0$ for all $0 \leq \mu \leq \lambda < \infty$ is a necessary but not a sufficient condition for joint concavity.

Proposition 8. *Joint concavity implies $\phi_\lambda \geq 0$, but $\phi_\lambda \geq 0$ does not imply joint concavity.*

Proof. We prove this result in two steps.

Part 1 (joint concavity implies $\phi_\lambda \geq 0$). To derive a contradiction, suppose that there exists a meeting technology for which $\phi(\mu, \lambda)$ is concave in (μ, λ), but $\phi_\lambda(\mu_0, \lambda_0) < 0$ in some point (μ_0, λ_0). Note that $\phi_{\mu\mu} < 0$ for all technologies that exhibit joint concavity, hence $\phi(\mu, \lambda)$ must also be concave in λ alone, i.e. $\phi_{\lambda\lambda} \leq 0$. In other words, $\phi_\lambda(\mu, \lambda)$ is a non-increasing function of λ, such that $\phi_\lambda(\mu_0, \lambda) \leq \phi_\lambda(\mu_0, \lambda_0) < 0$ for all $\lambda > \lambda_0$. This implies that $\phi(\mu_0, \lambda) \leq \phi(\mu_0, \lambda_0) + \phi_\lambda(\mu_0, \lambda_0)(\lambda - \lambda_0)$ for all $\lambda > \lambda_0$. Let $\lambda \to \infty$ and thus $\phi_\lambda(\mu_0, \lambda_0)(\lambda - \lambda_0) \to -\infty$, such that $\phi(\mu_0, \lambda) \to -\infty$. Since ϕ is a probability, this leads to the required contradiction. Hence, concavity of $\phi(\mu, \lambda)$, i.e. joint concavity, implies $\phi_\lambda \geq 0$.

Part 2 ($\phi_\lambda \geq 0$ does not imply joint concavity). Consider the following technology:

> *Minimum Demand.* This technology consists of two rounds. In the first round, the b buyers in the submarket are allocated to the s sellers according to the urn-ball technology. In the second round, each seller draws a minimum demand requirement and operates only if the number of buyers that came to him weakly exceeds this minimum.[6] We assume that the minimum demand requirements follow a geometric distribution, such that the minimum is weakly less than $n \in \mathbb{N}_1$ with probability $1 - (1-\psi)^n$ for $0 < \psi < 1$. Hence, $P_n(\lambda) = e^{-\lambda} \frac{\lambda^n}{n!} (1 - (1-\psi)^n)$ for $n \in \mathbb{N}_1$ and $P_0(\lambda) = 1 - \sum_{n=1}^{\infty} P_n(\lambda) = e^{-\psi\lambda}$.

[6] Geromichalos (2012) analyzes minimum demand requirements in a different context. Minimum class size requirements are also common in the matching between students and schools.

This technology gives $\phi(\mu,\lambda) = 1 - e^{-\mu} - e^{-\psi\lambda} + e^{-\lambda\psi - \mu(1-\psi)}$. Hence, $\phi_\lambda = \psi e^{-\psi\lambda}\left(1 - e^{-\mu(1-\psi)}\right)$, which is strictly positive. However, the determinant of the Hessian of ϕ, evaluated in $\mu = 0$, equals $-\psi^2(1-\psi)^2 e^{-2\lambda\psi} < 0$, which means that ϕ is not concave. Hence, $\phi_\lambda \geq 0$ does not imply joint concavity. □

I Urn-Ball with Capacity Constraints

A particularly intuitive class of meeting technologies is the following.[7]

> *Urn-Ball with Capacity Constraints.* Meetings between buyers and sellers are governed by an urn-ball technology, except for the fact that each seller can meet at most $L \in \mathbb{N}_1$ buyers. That is,

$$P_n(\lambda) = \begin{cases} e^{-\lambda}\frac{\lambda^n}{n!} & \text{for } n \in 0,\ldots,L \\ \sum_{i=L}^{\infty} e^{-\lambda}\frac{\lambda^i}{i!} & \text{for } n = L \\ 0 & \text{for } n \in \{L+1, L+2, \ldots\}. \end{cases} \tag{25}$$

Clearly, if $L \to \infty$, then this technology reduces to standard urn-ball, which satisfies joint concavity, such that perfect pooling is optimal for any F and Λ. However, for any $L < \infty$, joint concavity is violated, as the following lemma establishes.

Lemma 4. *For any $L < \infty$, the meeting technology* (25) *violates joint concavity.*

Proof. Note that

$$\begin{aligned} \phi(\mu,\lambda) &= 1 - \sum_{n=0}^{L} P_n(\lambda)\left(1 - \frac{\mu}{\lambda}\right)^n \\ &= 1 - \frac{e^{-\mu}\Gamma(L,\lambda-\mu) + \left(1-\frac{\mu}{\lambda}\right)^L (\Gamma(L) - \Gamma(L,\lambda))}{\Gamma(L)}, \end{aligned}$$

where $\Gamma(L)$ and $\Gamma(L,\lambda)$ denote the Gamma function $\int_0^\infty z^{L-1}e^{-z}\,dz$ and the upper incomplete Gamma function $\int_\lambda^\infty z^{L-1}e^{-z}\,dz$, respectively. Taking the derivative with respect to λ yields

$$\phi_\lambda(\mu,\lambda) = \frac{e^{-\lambda}\mu\left(1-\frac{\mu}{\lambda}\right)^L\left[\lambda^L - e^{\lambda}L(\Gamma(L) - \Gamma(L,\lambda))\right]}{\lambda(\lambda-\mu)\Gamma(L)} < 0,$$

[7] We thank Philipp Kircher for suggesting an analysis of this meeting technology.

for any $0 < \mu < \lambda$, where the inequality follows from the fact that

$$\lambda^L - e^{\lambda} L \left(\Gamma(L) - \Gamma(L, \lambda)\right) = \lambda^L - e^{\lambda} L \int_0^{\lambda} e^{-z} z^{L-1} \, dz = \lambda^L - \int_0^{\lambda} e^{\lambda - z} \, dz^L$$
$$= \int_0^{\lambda} \left(1 - e^{\lambda - z}\right) dz^L < 0.$$

By proposition 8, $\phi_\lambda < 0$ implies that joint concavity is violated. □

Given the violation of joint concavity, proposition 2 then directly implies that, for any $L < \infty$, there exists an F and Λ such that perfect pooling is not optimal. However, it turns out that one can prove an even stronger result: for any $L < \infty$, there exist F and Λ such that perfect separation is optimal. The following proposition formalizes this by providing an example.[8]

Proposition 9. *If meetings are governed by* (25) *with* $L < \infty$*, there exist* F *and* Λ *for which the planner creates a separate submarket for each type of buyer.*

Proof. Consider a two-point distribution with support $\{x_1 = 0, x_2 > 0\}$. By lemma 1, surplus in a submarket with a queue μ of high-type buyers and a queue $\lambda - \mu$ of low-type buyers equals $x_2 \phi(\mu, \lambda)$. As established in the proof of lemma 4, the meeting technology satisfies $\phi_\lambda < 0$. Hence, as long as $\lambda - \mu > 0$, a planner can increase surplus by removing low-type buyers from the submarket and allocating them to their own submarket (which will not feature any sellers as $x_1 = 0$). In other words, perfect separation is optimal. □

While we use a specific distribution to prove proposition 9, for a given $L < \infty$, other distributions may exist for which perfect separation is optimal. A complete characterization is not feasible, but one can show that the lower L is, the more prevalent perfect separation becomes, in the following sense:

Proposition 10. *For given* F *and* Λ*, if perfect separation is optimal for* L'*, then perfect separation is optimal for all* $L < L'$*.*

Proof. Denote by A an allocation of buyers to sellers and let $\mathcal{S}(A, L)$ be the surplus that this allocation creates when the capacity constraint is L. Further, let A_S denote the optimal (perfectly separating) allocation when the capacity is L'. We then prove the desired result by establishing that

$$\mathcal{S}(A_S, L) = \mathcal{S}(A_S, L') \geq \mathcal{S}(A, L') \geq \mathcal{S}(A, L) \tag{26}$$

[8] As the proof of the proposition reveals, this result does not only hold for (25), but for any meeting technology violating joint concavity while satisfying $\phi_\lambda < 0$.

for any A. The first equality holds because the value of the capacity constraint is irrelevant under perfect separation: surplus per seller is $(1 - P_0(\lambda))\,x$, which is independent of L. The second inequality follows from the assumption that perfect separation is optimal for meeting technology L'. The final inequality holds because $\phi(\mu, \lambda)$ is weakly increasing in L. Hence, by lemma 1, the meeting technology L' generates weakly more surplus than L for any allocation (and strictly more if there is buyer heterogeneity in some submarket). Comparing the first and last term in (26) yields $\mathcal{S}(A_S, L) \geq \mathcal{S}(A, L)$ for any A, i.e., perfect separation is optimal for the meeting technology L. □

References

Ahlfors, L. (1979). *Complex Analysis*. McGraw-Hill.

Auster, S. and Gottardi, P. (2016). Competing mechanisms in markets for lemons. mimeo.

Cai, X., Gautier, P. A., and Wolthoff, R. P. (2016). Meetings and mechanisms. mimeo.

Eeckhout, J. and Kircher, P. (2010). Sorting vs screening - search frictions and competing mechanisms. *Journal of Economic Theory*, 145:1354–1385.

Geromichalos, A. (2012). Directed search and optimal production. *Journal of Economic Theory*, 147:2303–2331.

Lester, B., Visschers, L., and Wolthoff, R. (2015). Meeting technologies and optimal trading mechanisms in competitive search markets. *Journal of Economic Theory*, 155:1–15.

Peters, M. (2013). *The Handbook of Market Design*, chapter Competing Mechanisms. Oxford University Press.

Rudin, W. (1987). *Real and complex analysis*. McGraw-Hill.