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Simultaneous search with heterogeneous firms and ex post competition ☆

Pieter A. Gautier [a,*,1], Ronald P. Wolthoff [b]

[a] Department of Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, CEPR, The Netherlands
[b] Department of Economics, University of Chicago, 1126 E. 59th Street, Chicago, IL 60637, United States

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ABSTRACT

In this paper we study the allocation of workers over high and low productivity firms in a labor market with coordination frictions. Specifically, we consider a search model where workers can apply to high and or low productivity firms. Firms that compete for the same candidate can increase their wage offers as often as they like. We show that if workers apply to two jobs, there is a unique symmetric equilibrium where workers mix between sending both applications to the high and sending both to the low productivity sector. But, efficiency requires that they apply to both sectors because a higher matching rate in the high-productivity sector can then be realized with fewer applications (and consequently fewer coordination frictions) if workers always accept the offer of the most productive firm. However, in the market the worker's payoff is determined by how much the firm with the second highest productivity is willing to bid. This is what prevents them from applying to both sectors. For many configurations, the equilibrium outcomes are the same under directed and random search so our results are not driven by random search. We discuss the effects of increasing the number of applications and show that our results can easily be generalized to N-firms.

1. Introduction

In an environment where the selection process of workers takes time, workers have strong incentives to simultaneously search for jobs in order to increase the expected number of offers. Firms on their turn have an incentive to increase their initial wage offers if their candidate has multiple offers. A firm that commits to its initial wage, irrespective of the number of other offers its candidate has, may loose its candidate to a firm that is willing to increase its initial bid. However, as Postel-Vinay and Robin (2002) and Albrecht, Gautier and Vroman (2006) show in different settings, allowing firms to make ex post counter offers can, in equilibrium, make the workers worse off because it gives the firms the opportunity to extract more rents from the workers ex ante. So ex post competition reduces ex ante competition.

In this paper we study the effects of allowing firms to increase their initial offers if their candidate has multiple offers on the application portfolio of workers. We show that under arguably small coordination frictions, portfolios are socially inefficient and the assignment of workers over sectors is suboptimal. We consider the following deviations from the competitive model: (i) workers do not know to which firms other workers apply to, (ii) firms do not know which candidates receive offers, (iii) applications are costly and firms can consider only a fraction of their candidates. While keeping our model as simple as possible we want to capture a number of factors that we feel are important in real world labor markets like heterogeneity, the possibility of simultaneous search and ex post competition for workers with multiple offers. At the same time we want to rigorously model the matching process, the strategic interactions between workers with each other and with the firms.

Specifically, we study a portfolio problem where identical unemployed workers must decide in which sector(s) to search; the high and or the low productivity sector. Within a sector, all firms are identical. Workers can send 0, 1 or 2 applications at a cost $k>0$ for each application. Each vacancy that receives one or more candidates randomly picks a candidate and offers the job to him. The other applications are rejected. In the simplest version of the model, workers know the productivity in each sector but only learn about the wage at a specific firm after applying there. We then show that our results still hold in the much more complicated case where search is fully directed: i.e. firms can ex ante post a wage which is observed by

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* Corresponding author.
E-mail addresses: pgautier@feweb.vu.nl (P.A. Gautier), wolthoff@uchicago.edu (R.P. Wolthoff).
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all workers before they decide where to send their applications. Firms that compete for the same candidate can increase their offers as often as they like, so we do not restrict the firm's strategy space in this dimension. We are interested in symmetric pure strategy equilibria (in terms of the number of applications) and their efficiency properties.

Interestingly, in the simplest version of our model it cannot be an equilibrium for workers to send just one application because then firms have no incentives to offer a positive wage. This is basically the Diamond (1971) paradox. Therefore, if k is sufficiently low, workers always send two applications, hoping to get a positive payoff by receiving two offers. But this in turn implies that workers will never apply to both sectors (HL) because this strategy is strictly dominated by sending both applications to the low productivity sector (LL). The intuition behind this result is that in any equilibrium where workers are willing to apply to the low productivity sector, the expected number of applications must be lower there. However, the expected payoffs of receiving an offer from a high and a low productivity firm is the same as receiving offers from two low productivity firms because a high productivity firm that (Bertrand) competes with a low productivity firm for the same candidate will win and pay the productivity level of the worker at the low productivity firm. So, the worker's payoffs conditional on getting two offers are the same for a worker who sends both applications to the low productivity sector (LL) and a worker who plays HL, but the probability of receiving two offers is higher for the first worker. We then show that there is a unique mixed strategy equilibrium where workers send both applications with probability q^*_{HH} to the high productivity sector and with probability $1-q^*_{\mathrm{HH}}$ to the low productivity sector where q^*_{HH} depends on the relative productivity and the relative supply of vacancies in each of the sectors. As in Albrecht et al. (2006) there are two coordination problems in the matching process: (1) workers do not know where other workers apply to and (2) firms do not know which candidate other firms consider.

By allowing workers to apply to different sectors, the degree of coordination frictions becomes partly endogenous, even for a given number of applications per worker. However, workers do not internalize the effects of their portfolio choice on the employment opportunities of other workers. They just want to maximize the productivity-weighted probability to receive multiple offers. We show that the resulting equilibrium is not efficient. An important reason for the inefficiency is that a social planner would like some or all workers to apply to both sectors in order to reduce the coordination problems in the matching process. More H matches can be realized by letting workers accept the job in the most productive sector in case of multiple offers. In the market, workers never play HL because the expected payoffs of this strategy are too low, since high productivity firms would either pay the monopsony wage or the productivity level of a low productivity firm in case the worker has two offers. Since the expected payoff of playing HL is independent of high productivity output, workers incentives are distorted. Another source of inefficiency is that because of the coordination frictions, the matching function is non-monotonic in the number of applications. When there are relatively few vacancies, the second coordination problem is severe and the matching rate is decreasing in the number of applications. The planner internalizes this while individual workers diversify too little and apply too often to the high productivity sector. A similar problem arises in the academic job market or the market for Ph.D. candidates where the top universities typically receive (too) many applicants.[2]

If the number of firms in the market or the difference in productivity between both sectors is not too large, the equilibrium outcomes under random search are the same as in the directed search equilibrium where firms can post a wage ex ante and workers observe all wages.[3] The reason for this is the same as the one in Albrecht et al. (2006) where posted wages are zero. They consider the case where all workers and firms are identical and show that the existence of ex post competition makes it still attractive for workers to apply to firms who offer the monopsony wage. Offering a higher wage than the monopsony wage only marginally increases the number of applicants in expectation, because workers mainly care about the probability to get multiple offers, while the expected firm payoffs in case of a match drop linearly. This implies that our results are not driven by the fact that search is random because for a fixed supply of vacancies and applications, the Albrecht et al. (2006) model is constraint efficient while the directed search version of our model is not.

There are a couple of other papers related to what we do. First, Shimer (2005) and Shi (2002) consider a directed search model with two-sided heterogeneity where workers can only apply to one job and ex post competition is irrelevant. They find that the decentralized market outcome is constrained efficient. We show that this result may break down if workers can simultaneously apply to multiple jobs and there is ex post competition for their services. In Gautier and Moraga-Gonzalez (2004) workers and firms are also identical and workers only learn about the wage after a firm is contacted. There, wages and the number of applications are determined in a simultaneous move game and the worker's portfolio problem is trivial: each application should go to a random vacancy. Chade and Smith (2006) and Galenianos and Kircher (in press) also consider portfolio problems of workers who can apply to multiple jobs. In the latter paper, all jobs have the same productivity but because firms must commit to their posted wages they respond to the worker's desire to diversify. This desire to diversify is driven by the fact that the expected payoff is equal to the maximum wage offer of a worker and not to the average one. This also creates non-trivial portfolio problems. Interestingly, because of the ex ante wage commitment of firms, workers diversify as much as possible over the different wages that are offered by the firms. Chade and Smith (2006) is not an equilibrium model but it considers a general class of portfolio problems in the absence of ex post competition. Finally, Davis (2001) analyzes a model in which workers and firms can decide to invest in human capital and job quality respectively. Because they cannot capture the full increase of the match surplus generated by these investments, both firms and workers tend to underinvest. In equilibrium there is excessive supply of inferior jobs and inferior workers.

The paper is organized as follows. Section 2 describes the model. We derive the equilibrium and determine whether it is efficient. In Section 3 we check whether our conclusions are sensitive to the simplifying assumptions we make. Finally, Section 4 concludes.

2. Model

2.1. Labor market

Consider a labor market with u risk neutral workers and v risk neutral single worker firms with a vacancy. All workers are identical, but the firms are divided into two different types. There are v_{H} high-productivity firms and v_{L} low-productivity firms, with $v=v_{\mathrm{H}}+v_{\mathrm{L}}$. We refer to those firms as *highs* and *lows*.

Workers can send zero, one, or two applications at costs $k>0$. Those applications can be directed to a specific type of vacancy/firm, but workers do not observe ex ante the wage that a particular firm offers. If a worker receives multiple job offers, there is Bertrand competition for his services. Basically, workers must decide whether they want to send both applications to high type vacancies, both applications to low type vacancies, or one application to a high type

[2] In small labor markets, more matches are realized if all workers play HL than if 50% plays LL and 50% plays HH. However, in larger labor markets there is no difference between these two cases.

[3] Usually, the equilibrium in directed search models is constrained efficient, e.g. Burdett et al. (2001), Moen (1997), Montgomery (1991), Peters (1991).

and one to a low type vacancy. In section 5 we show that if there are not too many firms in the market and if the productivity of the low type firms is not too small, our results carry over to a directed search setting, where workers observe ex ante the wage offered by each individual firm.

We make four important further assumptions. First, we assume that the labor market is large, i.e. $u \to \infty$ and $v \to \infty$, keeping $\theta_i \equiv v_i/u$ fixed $\forall i \in \{H,L\}$. Second, we assume that θ_H and θ_L are exogenously given.[4] Besides simplicity, this allows us to focus on the portfolio inefficiency which is absent in Albrecht et al. (2006) where firms are identical and workers always fully mix their applications. Third, we focus on symmetric equilibria, which means that identical agents must have identical strategies. Fourth, we assume that the labor market is anonymous: firms must treat identical workers identically and vice versa. So, a worker's strategy may only be conditioned on the type (H or L) of the firm. This excludes equilibria that require a lot of coordination amongst workers, something that seems hard to imagine in a large labor market.

2.2. Setting of the game

The model most closely related to ours is that of Albrecht et al. (2006). There are two important differences: (i) we allow for heterogeneity amongst firms and (ii) search is not fully directed (in Section 2.5 we discuss the directed search equilibrium). The setting of the game is as follows:

1. Each vacancy posts a wage.[5]
2. Workers observe all vacancy types, i.e. high or low, (but not the wage) and send $a \in \{0,1,2\}$ applications. In Section 2.5 we allow workers to also ex ante observe the wage.
3. Each vacancy that receives at least one application, randomly selects a candidate. Applications that are not selected are returned as rejections.
4. A vacancy with a processed application offers the applicant the job. If the applicant receives more than one offer, the firms in question can increase their bids as often as they like.
5. A worker that receives one job offer will accept that offer as long as the offered wage is non-negative. A worker with two offers will accept the one that gives him the highest wage, or will select a job randomly if the offered wages are equal.

If a type i firm matches with a worker, it produces y_i units of output. Without loss of generality we assume that $y_L < y_H = 1$. The payoff of a firm that matches with a worker equals $y_i - w$, where w denotes the wage that the firm pays. A worker hired at wage w receives a payoff that is equal to that wage. Workers and firms that fail to match receive payoffs of zero.

2.3. Decentralized market

First, note that in the decentralized market equilibrium no firm posts a positive wage. This is basically the Diamond (1971) paradox. Workers can direct their applications to a specific kind of vacancy, but not to a particular firm. So, posting a higher wage does not attract more applicants and does not affect the matching probability. This implies that there is no incentive for a firm to offer the worker more than zero.[6] A direct result of this is that workers never send only one application because then there will never be ex post competition for his services. Firms offer a wage equal to zero in that case, so the worker's payoff always equals $-k$. Hence, applying to one job is strictly dominated by not applying at all and therefore never part of an equilibrium strategy.

Whether a worker applies twice or not at all depends on the cost k of sending an application. For example if $k > 0.5$, each worker will decide not to apply, because applying twice costs more than the competitive wage ($2k > 1 = y_H$). On the other hand, all workers apply to two jobs if k is sufficiently small, because this gives a strictly positive expected payoff, while not applying results in a payoff of zero. In this paper we restrict ourselves to the situation in which k is small enough to guarantee that $a = 2$ with probability 1.[7] In this respect our model differs from Shimer (2005) and Shi (2002) where $a = 1$.

Three different strategies are possible: a worker can either apply to two high type vacancies, two low type vacancies, or one high type and one low type of vacancy. Denote the respective probabilities by q_{HH}, q_{LL}, and q_{HL}, where $q_{HH} + q_{LL} + q_{HL} = 1$.[8] Using the fact that each worker uses the same strategies, this implies that the total number of applications to firms of type i is equal to $(2q_{ii} + q_{HL})u$. The expected number of applications a specific vacancy receives, is therefore given by

$$\phi_i(q_{ii}, q_{HL}, \theta_i) = \frac{2q_{ii} + q_{HL}}{\theta_i}. \tag{1}$$

Since our labor market is large, the actual number of applications to a specific vacancy follows a Poisson distribution with mean ϕ_i.[9] Next, consider an individual who applies to a type i firm. The number of competitors for the job at that firm also follows a Poisson distribution with mean ϕ_i, because there is an infinite number of workers. In case of n other applicants, the probability that the individual in question will get the job equals $\frac{1}{n+1}$. Therefore, the probability that an application to a type i firm results in a job offer equals

$$\psi_i = \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{e^{-\phi_i}\phi_i^n}{n!} = \frac{1}{\phi_i}\left(1 - e^{-\phi_i}\right). \tag{2}$$

Note that this expression is not well defined for $\phi_i = 0$. For convenience we define $\psi_i(0) = \lim_{\phi_i \to 0} \psi_i(\phi_i) = 1$.

Whether a worker's second application results in an offer does not depend on whether the first application was successful or not. A worker who plays ij (i.e. applies to a type i firm and a type j firm) with $i,j \in \{H,L\}$ therefore has a probability $\psi_i\psi_j$ of getting two job offers and a probability $\psi_i(1-\psi_j) + \psi_j(1-\psi_i)$ of getting one job offer. The matching probability of such a worker equals one minus the probability that he does not get a job offer and is therefore equal to $1-(1-\psi_i)(1-\psi_j)$ (see Albrecht et al., 2006 for a proof in the case with homogenous firms). This matching probability is obviously strictly increasing in both ψ_i and ψ_j and depends on the worker's portfolio choice.

If a worker receives two high job offers, Bertrand competition between the two firms results in a wage equal to $y_H = 1$. In case of two low offers, the firms increase their bids until the worker's wage equals y_L. A combination of one high and one low offer also implies a wage of y_L, because at that wage level the low type firm is no longer willing to increase its bid. This is the standard result from Bertrand competition. As shown above, a worker who receives only one job offer gets a wage equal to zero.

Next, we prove that workers never send one application to a high and one to a low productivity firm:

Lemma 1. *Workers never play HL, since this strategy is strictly dominated.*

4 In the working paper version of this paper (see Gautier and Wolthoff, 2007) we show that the inefficiency result remains under free entry of vacancies.

5 Our results continue to hold if firms post wage mechanisms.

6 Note that this argument implies that posting a wage equal to zero does not only dominate posting a strictly positive wage, but also all other feasible wage mechanisms.

7 An explicit expression for the upperbound K on k in that case is derived below.

8 Note that the order of the two applications is irrelevant. The worker only cares about the application portfolio. Hence, the strategy space contains strategies like: the first application is made to a particular firm for sure and the second application is send with probability p to an H and with probability $(1-p)$ to an L firm.

9 For ease of exposition we omit the arguments of functions whenever this does not lead to confusion.

Proof. The expected payoff for a worker who plays HL is $\psi_H\psi_L y_L - 2k$, i.e. the probability that he receives two job offers times the productivity of the low type firm minus the application cost. Likewise, the expected payoffs of playing HH and LL are $\psi_H^2 y_H - 2k$ and $\psi_L^2 y_L - 2k$ respectively. Suppose that $\psi_H \geq \psi_L$. In that case all workers play HH, since that strategy gives a strictly higher payoff than HL and LL. This however implies that $\phi_L = 0$ and thus that $\psi_L = 1$, which contradicts $\psi_H \geq \psi_L$. Hence, in equilibrium it must be the case that $\psi_L > \psi_H$. Then, playing LL gives a strictly higher payoff than HL. So, HL is strictly dominated. □

In the following proposition we show that the model has a unique equilibrium for all parameter values.

Proposition 1. *A unique equilibrium exists for any $\theta_H > 0$, $\theta_L > 0$, and $y_L \in (0,1)$. The equilibrium is a pure strategy equilibrium if and only if the following condition holds:*

$$\frac{\theta_H^2}{4}\left(1 - \exp\left(-\frac{2}{\theta_H}\right)\right)^2 \geq y_L. \tag{3}$$

Otherwise, the equilibrium is a mixed strategy equilibrium, which can be characterized by the value q_{HH}^* that solves the equality $\psi_H^2 = \psi_L^2 y_L$.

Proof. We can rule out the possibility that workers play HL because of Lemma 1. First, note that an equilibrium in which $q_{LL} = 1$ does not exist, since a deviant that applies twice to a high firm gets a higher payoff (y_H) than the equilibrium payoff $\psi_L^2 y_L < y_L$.[10]

On the other hand, $q_{HH} = 1$ can be an equilibrium if y_L is low enough. The equilibrium payoff in this case equals $\psi_H^2 = \frac{\theta_H^2}{4}\left(1 - \exp\left(-\frac{2}{\theta_H}\right)\right)^2$. Deviating to LL gives a wage y_L for sure. So, $q_{HH}^* = 1$ is an equilibrium if condition (3) holds.

If condition (3) does not hold, only a mixed strategy equilibrium can exist, in which the workers are indifferent between playing HH and LL, i.e. where $\psi_H^2 = \psi_L^2 y_L$. If we substitute $q_{LL} = 1 - q_{HH}$, the only free parameter in this condition is q_{HH}. To see that a unique equilibrium value q_{HH}^* exists, note that the left hand side of the condition is continuous and strictly decreasing in q_{HH}, while the right hand side is continuous and strictly increasing in q_{HH} (see Fig. 1). Furthermore, we have

$$\lim_{q_{HH}\to 0} \psi_H^2 = 1 > \frac{\theta_L^2}{4}\left(1 - \exp\left(-\frac{2}{\theta_L}\right)\right)^2 y_L = \lim_{q_{HH}\to 0} \psi_L^2 y_L$$

and

$$\lim_{q_{HH}\to 1} \psi_H^2 = \frac{\theta_H^2}{4}\left(1 - \exp\left(-\frac{2}{\theta_H}\right)\right)^2 < y_L = \lim_{q_{HH}\to 1} \psi_L^2 y_L.$$

Applying the Intermediate Value Theorem now shows that there exists a unique value $0 < q_{HH}^* < 1$ such that $\psi_H^2 = \psi_L^2 y_L$ holds. □

Hence, we have a pure strategy equilibrium in which all firms post a wage equal to zero and all workers apply twice to high type vacancies if condition (3) holds. This condition imposes very low upper bounds on y_L for any reasonable value of θ_H (e.g. $\theta_H = 0.5$ implies $y_L < 0.06$). The case in which the condition does not hold is therefore more interesting. Unfortunately, we are not able to derive an explicit expression for q_{HH}^*. Fig. 1 shows the equilibrium as the intersection point of the ψ_H^2-curve and the $\psi_L^2 y_L$-curve for $\theta_H = \theta_L = 0.5$ and $y_L = 0.5$. For those values 63% of the workers plays HH, while 37% plays LL.

In equilibrium the expected payoff for a worker equals $\psi_H^2 - 2k = \psi_L^2 y_L - 2k$. The requirement that this value should be larger than the payoff of not applying at all, i.e. zero, implies that k should be smaller than $\frac{1}{2}\psi_H^2 = \frac{1}{2}\psi_L^2 y_L$. This assumption seems reasonable. It is hard to imagine that the cost of a particular application exceeds half the expected wage of a job.

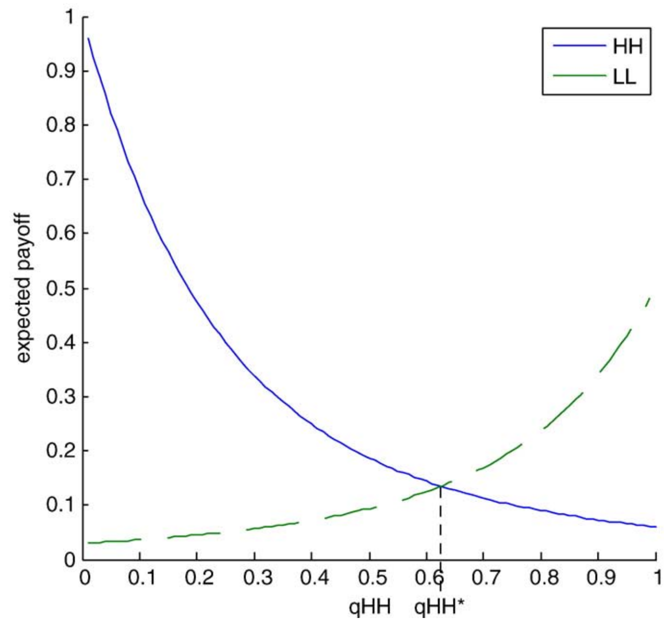

Fig. 1. Expected payoff of playing HH and LL for $\theta_H = \theta_L$ and $y_L = 0.5$.

2.4. Efficiency

In the mixed strategy equilibrium that we derived in the previous subsection, a fraction q_{HH}^* of the workers matches with probability $1-(1-\psi_H^*)^2$ to a high firm and produce output $y_H = 1$. The remaining workers match with probability $1-(1-\psi_L^*)^2$ to a low firm and produce output y_L. The total output Y^* per worker in this equilibrium is therefore given by

$$Y^* = q_{HH}^*\left(1 - \left(1 - \psi_H^*\right)^2\right) + \left(1 - q_{HH}^*\right)\left(1 - \left(1 - \psi_L^*\right)^2\right) y_L.$$

The main question of this paper is whether the equilibrium value q_{HH}^* is constrained efficient. In order to answer this question we consider a social planner who maximizes total output in the economy. The planner cannot eliminate the coordination frictions, but he can decide to which firms the workers apply. In other words, he can control q_{HH}, q_{LL}, and q_{HL}. In order to focus on the (in) efficiency of application portfolios we also restrict the planner to let workers send exactly two applications. We assume that the social planner can decide which job a worker will take if he receives both a high and a low job offer. Suppose that he sends a fraction α of those workers to the high type firm and a fraction $1-\alpha$ to the low type firm. Then we can derive χ_{ij}^k, $i,j,k \in \{H,L\}$, which represents the probability that playing ij results in a match with a type k firm. These probabilities are functions of α, ψ_H, and ψ_L:

$$\chi_{HH}^H = 1 - (1 - \psi_H)^2 \tag{4a}$$

$$\chi_{HL}^H = \alpha\psi_H\psi_L + \psi_H(1 - \psi_L) \tag{4b}$$

$$\chi_{LL}^L = 1 - (1 - \psi_L)^2 \tag{4c}$$

$$\chi_{HL}^L = (1 - \alpha)\psi_H\psi_L + \psi_L(1 - \psi_H). \tag{4d}$$

[10] Since we only consider strategies in which workers apply twice, we can safely ignore the application cost k in the proof. This parameter only plays a role in comparing the playoffs of strategies that differ in the number of applications sent.

The remaining probabilities, like χ^{L}_{HH} are equal to zero. Using this notation, we can write the per-worker output created by the high and the low type firms as respectively:

$$Y_H = q_{HH}\chi^{H}_{HH} + q_{HL}\chi^{H}_{HL} \tag{5}$$

and

$$Y_L = \left(q_{LL}\chi^{L}_{LL} + q_{HL}\chi^{L}_{HL}\right)y_L. \tag{6}$$

This implies that the social planner's problem is:

$$\max_{q_{HH},q_{LL},q_{HL},\alpha} Y = \max_{q_{HH},q_{LL},q_{HL},\alpha} q_{HH}\chi^{H}_{HH} + q_{HL}\chi^{H}_{HL} + \left(q_{LL}\chi^{L}_{LL} + q_{HL}\chi^{L}_{HL}\right)y_L, \tag{7}$$

subject to $q_{HH}+q_{LL}+q_{HL}=1$.

Solving this maximization problem gives us the optimal values q^{**}_{ij} and α^{**}, which can be used to calculate Y^{**}, the level of output. First note that $\alpha^{**}=1$, i.e. when a worker gets a job offer from both a high type and a low type firm, the planner wants him to take the high type job. The intuition for this result is clear. If a worker receives a job offer from both a high and a low type firm, he must always take the job at the high type firm because his marginal productivity is higher there. Next, we can formally prove that the mixed strategy market equilibrium is inefficient: the social planner creates a higher output.

Proposition 2. *The equilibrium described in* Proposition 1 *is not constrained efficient if* $y_L > \exp\left(-\frac{2}{\theta_H}\right)$.

Proof. Note that $\exp\left(-\frac{2}{\theta_H}\right) < \frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2 \forall\theta_H$. First, consider the pure strategy equilibrium in which $q^{*}_{HH}=1$. Let the planner instead impose $\alpha=1$, $q_{HH}=q^{*}_{HH}-q_{HL}$, q_{HL}, and $q_{LL}=0$. This generates output equal to

$$Y = (1-q_{HL})\left(1-(1-\psi_H)^2\right) + q_{HL}(\psi_H + \psi_L(1-\psi_H)y_L)$$

Taking the derivative with respect to q_{HL} and evaluating the result in $q_{HL}=0$ gives

$$\frac{\partial Y}{\partial q_{HL}}\Big|_{q_{HL}=0} = (1-\psi_H)\left(y_L - \exp\left(-\frac{2}{\theta_H}\right)\right),$$

which is positive if $y_L > \exp\left(-\frac{2}{\theta_H}\right)$. Hence, the pure strategy equilibrium is not constraint efficient if this condition holds.

Second, consider the case in which $y_L > \frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2$. This implies a mixed strategy market equilibrium in which a strictly positive fraction of the workers sends two applications to the high sector and another strictly positive fraction sends two applications to low type firms. Next, consider a social planner who faces this equilibrium. One way in which he can increase output is by selecting a worker that plays HH and a worker that plays LL and by letting them both diversify their applications amongst the sectors. By matching HH-workers and LL-workers in this way, the total number of vacancies in each sector remains constant, implying that the matching probabilities ψ^{*}_{H} and ψ^{*}_{L} do not change. So, let the planner impose $\alpha=1$, $q_{HH} = q^{*}_{HH} - \frac{1}{2}q_{HL}, q_{HL},$ and $q_{LL} = q^{*}_{LL} - \frac{1}{2}q_{HL} = 1 - q^{*}_{HH} - \frac{1}{2}q_{HL}$, where the market equilibrium corresponds to $q_{HL}=0$. The output Y in that case equals

$$Y = \left(q^{*}_{HH} - \frac{1}{2}q_{HL}\right)\left(1-\left(1-\psi^{*}_{H}\right)^2\right) + q_{HL}\left(\psi^{*}_{H} + \psi^{*}_{L}\left(1-\psi^{*}_{H}\right)y_L\right) + \left(q^{*}_{LL} - \frac{1}{2}q_{HL}\right)\left(1-\left(1-\psi^{*}_{L}\right)^2\right)y_L.$$

Taking the derivative with respect to q_{HL} gives

$$\frac{\partial Y}{\partial q_{HL}} = \frac{1}{2}\left(\psi^{*2}_{H} - 2\psi^{*}_{H}\psi^{*}_{L}y_L + \psi^{*2}_{L}y_L\right) > \frac{1}{2}\left(\psi^{*}_{H} - \psi^{*}_{L}\sqrt{y_L}\right)^2 \geq 0.$$

This expression is strictly positive for all q_{HL}. Hence, the mixed strategy market equilibrium is not constrained efficient. □

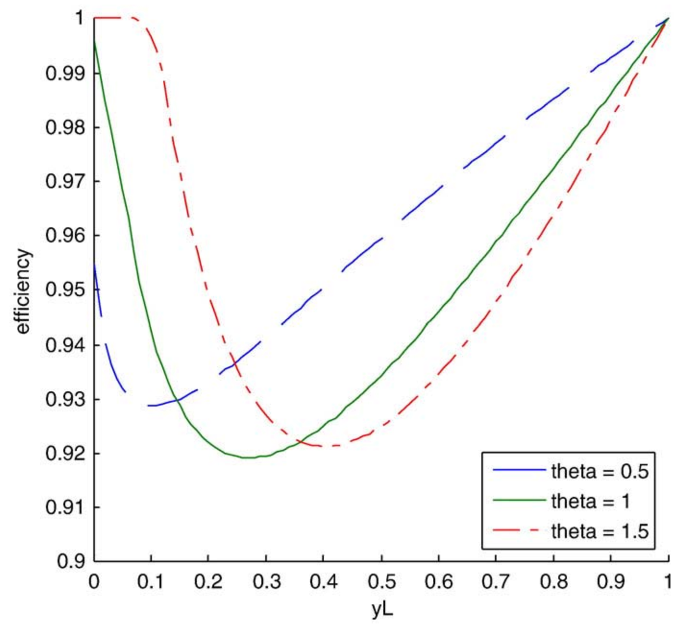

Fig. 2. Efficiency of the decentralized equilibrium (Y^{*}/Y^{**}) as a function of y_L for several values of $\theta_H=\theta_L=1/2\theta$.

From this proof it is immediately clear that q_{HH} and q_{LL} cannot both be strictly larger than zero in the planner's solution. The planner can match HH-workers and LL-workers and thereby increase output until one of both groups is completely exhausted. Note that although the resulting situation generates a higher social welfare than the market equilibrium, there is no reason to believe that it is the optimum. Other strategies might increase welfare even more. Unfortunately, an explicit expression for the planner's solution cannot be derived, because of the noninvertibility of ψ_i and χ^{i}_{ij}. Therefore, we maximize Eq. (10) numerically.[11]

We find that for many values of $\{\theta_H,\theta_L,y_L\}$ the planner lets all workers play HL. This is for example the case for $\theta_H=\theta_L\leq 0.5$ and $y_L \in \left(\frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2, 1\right)$. As mentioned above, this contrasts with the decentralized market where nobody plays HL. Workers do not play HL because they are only interested in getting two job offers in the same sector. However, from the planner's point of view two job offers to the same worker is always inefficient, because in that case one firm remains unmatched, while it could have matched with a worker without any job offers. Hence, all workers ideally receive only one job offer. The planner can however not coordinate the job offers, so the only way in which he can reduce the coordination problem is by spreading the applications as much as possible, i.e. by playing HL. The planner only considers HH or LL if (i) the productivity of the *L*-types firms is very low, (ii) the number of firms in the market is very large, or (iii) there is a large difference between the number of high type firms and the number of low type firms.

Next, we consider the ratio $\frac{Y^{*}}{Y^{**}}$, i.e. the ratio between the total output in the decentralized equilibrium and the output level created by the social planner. This ratio is displayed in Fig. 2. This figure confirms that the decentralized equilibrium is in general not efficient. The output in the mixed strategy equilibrium is only equal to the optimal level for $y_L=1$ because then there is essentially no difference between high and low firms. For $y_L=0$, the market equilibrium is not efficient for $\theta=\theta_H+\theta_L=\frac{1}{2}$ or 1 because the optimal number of applications per worker to the *H*-sector is smaller than 2 for those values of θ. The planner can use the *L*-sector as "garbage can" to reduce the number of applications to the *H*-sector which reduces the

[11] The numerical results in this paper are obtained using Ox version 3.40.

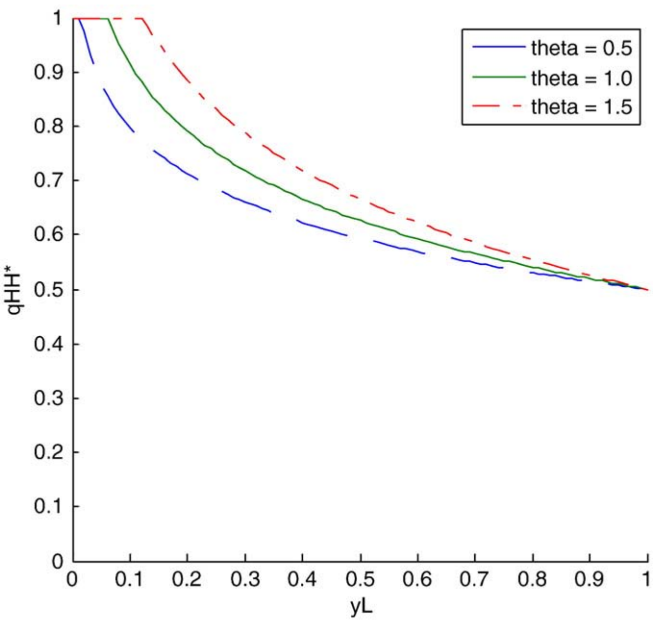

Fig. 3. q^*_{HH} as a function of y_{L} for several values of $\theta_{\mathrm{H}}=\theta_{\mathrm{L}}=1/2\theta$.

probability that two firms consider the same candidate. For $\theta=\frac{3}{2}$, the optimal number of applications is equal to 2 and the market equilibrium is constrained efficient. We also see that for low values of y_{L}, the equilibria with high θ perform relatively well as compared to the planner's choice, while for high values of y_{L}, the equilibria with low θ are closer to the constrained optimum. In the first case, almost all workers play HH, which makes the second coordination friction large (many H-firms loose their candidate to a rival firm). When θ is large, this second coordination friction is less severe. For larger values of y_{L}, it is less desirable to play HH because L-firm matches become more valuable but for high θ $\frac{\partial_{qHH}}{\partial_{yL}}$ is smaller (see Fig. 3), so q^*_{HH} adjusts too slow and therefore the low-θ equilibria are closer to the planner's solution.[12]

The model has two important characteristics that could both potentially cause the inefficiency: (i) the fact that workers in the decentralized market never play HL, while the social planner does and (ii) the fact that workers cannot direct their applications to specific firms. Below we prove that our results are not driven by (ii).

2.5. Directed search equilibrium

In this subsection, we investigate to what extent the inefficiency in our model depends on the assumption of random search. In other words, we check whether efficiency would be restored if we allow workers to direct their applications to specific wages. We find that this is not the case. General expressions for an equilibrium in a directed search framework are hard to derive, but the equilibrium outcomes of our model coincide with the equilibrium outcomes of a directed search model for many values of θ_{H}, θ_{L}, and y_{L}.

Compared to the setup described in the previous section there is one important difference: workers can observe the wages posted by the firm before they send out their applications. This allows the worker to choose not only the sectors but also the wages to which he wants to apply.[13] Let $\psi_i(w)$ denote the probability that an application send to a firm in sector i offering wage w results in a match. Then, the worker's problem is to choose sectors i and j and wages w_1 and w_2 that maximize his expected payoff:

$$\psi_i(w_1)\Big(1-\psi_j(w_2)\Big)w_1+\psi_j(w_2)(1-\psi_i(w_1))w_2+\psi_i(w_1)\psi_j(w_2)\min\Big\{y_i,y_j\Big\}.$$

Firms take this into account when they decide which wages to post. This provides firms with an incentive to consider positive wages because a higher wage leads to a larger arrival rate of applicants. Nevertheless, for many parameter configurations, all firms post wages equal to zero, as we state in the following proposition.

Proposition 3. *Assume that k is small enough to guarantee that all workers send two applications.*[14] *Then, for θ_H and θ_L sufficiently small or for y_L sufficiently large, the equilibrium outcomes described in section 3 are the same as in the directed search version of our model where workers observe all wages before they apply.*

Proof. See Appendix A. □

Fig. 4 shows for which values of $\theta_H=\theta_L=\frac{1}{2}\theta$ and y_{L} the random search equilibrium values are the same as the directed search equilibrium values. The intuition is the same as in Albrecht et al. (2006). First, posted wages are lower if workers apply to multiple jobs than if they apply to one job because Bertrand competition makes it very valuable to have multiple offers. So workers place a relatively larger weight on short expected queue length than on posted wages. The reason that wages go down all the way to zero is that the benefits of a downward deviation are constant but the cost of a downward deviation (in terms of less applications) are decreasing in the wage. As we prove in the appendix, only for the low type sector there exist configurations for which there is a profitable deviation from the candidate equilibrium where all firms post $w_{\mathrm{L}}=0$. For example, if there are many firms relative to workers or if the low type firms have a low productivity, which makes it unattractive for the workers to apply there, $w_{\mathrm{L}}>0$ and the standard positive relation between posted wages and productivity can break down. In Postel-Vinay and Robin (2002) this happens for similar reasons. In their model, workers agree to accept a lower initial wage at high productivity firms because of future possibilities of wage increases through Bertrand competition with rival firms. In the directed search version of our model, high productivity firms always get away with posting the reservation wage while low productivity firms do not because the payoff of receiving multiple offers from high productivity firms is more attractive than from low productivity firms.

The fact that the equilibrium values under random search and directed search can coincide implies that the inefficiency of the decentralized equilibrium cannot be eliminated by making search fully directed. This result is contradictory to for example Burdett et al. (2001), Kircher (2008) and Moen (1997), who found that the equilibrium in their directed search models was constrained efficient. In Burdett et al. (2001) buyers could only send one application so there can never be inefficient portfolios at the individual level. Kircher (2008) does allow for multiple applications and he also allows for complete recall (firms can go to the next applicant if they fail to hire the first one) but in his model, firms are identical and he assumes that firms commit to their initial wage in all bidding subgames. Camera and Selcuk (in press) do consider limited commitment but they again allow buyers to only contact one seller at a time so there are no portfolio problems in their setting.

To sum up, for a fixed supply of vacancies the market equilibrium is inefficient predominantly owing to workers never playing HL. Playing HL has the advantage that more H-matches can be realized by setting $\alpha=1$ (in case of two offers, always take the H-offer). Therefore, the

[12] Note that we do not say the low θ equilibria are more desirable. Decreasing θ lowers output, but the planner's output decreases as well.

[13] Note that given the anonimity assumption, a worker randomizes over all firms in a specific sector that are offering the same wage.

[14] Under directed search we can have an equilibrium with $a=1$ for some values of k. Since this is a special case of the model described in Shimer (2005), we focus on sufficiently low values of k such that $a=2$.

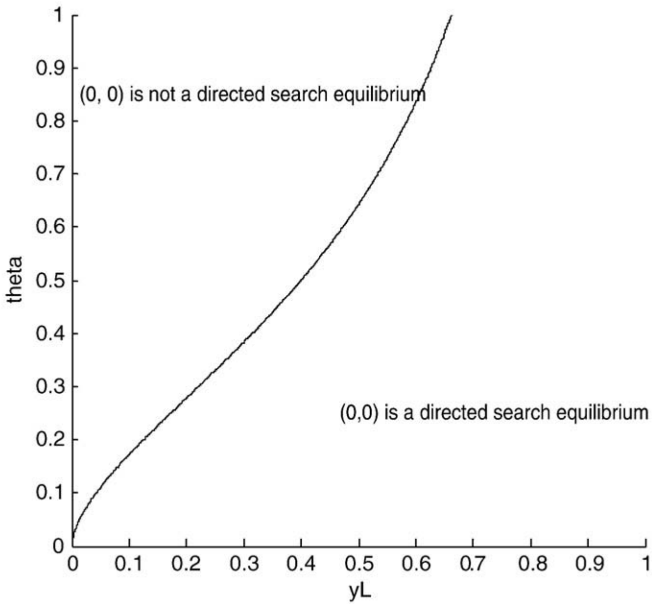

Fig. 4. Combinations of y_L and $\theta_H = \theta_L = 1/2\theta$ for which $\{w_H = 0, w_L = 0\}$ is a directed search equilibrium.

coordination frictions are larger than necessary. Interestingly, Galenianos and Kircher (in press) also find that worker's market portfolios of applications are socially inefficient. They only have ex ante competition for workers and show that even if workers and firms are homogeneous, workers have a desire to diversify and firms respond to this desire by offering different wages. In their model, workers choose to apply both to the high and the low wage firms but with a higher probability to the high wage firms whereas it would be socially efficient if workers apply to each firm with equal probability. Finally, note that in Albrecht et al. (2006) the portfolio inefficiency is absent because they consider both identical workers plus jobs and allow for ex post competition.

3. Robustness

In this section we discuss to what extent our results are sensitive to the following four simplifying assumptions we made: (i) there are only two firm types, (ii) a worker cannot send more than two applications, (iii) if a firm fails to hire its candidate it cannot make an offer to the next candidate, and (iv) firms that compete for the same worker engage in Bertrand competition.

3.1. More than two firm types

Suppose there are N rankable firm types where $y_{n+1} > y_n$. Then it is straightforward to show that workers never diversify because the application-portfolio strategy, $(n+i,n)$, is dominated by (n,n). The only way for workers to receive a positive payoff is by getting two job offers. For both portfolios, Bertrand competition leads to a wage of y_n but because the expected queue length is shorter in the least productive sector, the probability of receiving two offers is larger for the (n,n) than for the $(n+i,n)$ portfolio. One can easily generalize Proposition 2 to show that also in this case the market outcome is inefficient. Therefore, considering only two firm types is not restrictive.

3.2. More than two applications

The second simplifying assumption is that a worker cannot send more than two applications. Allowing workers to apply to more than two jobs makes the analysis more difficult but does not change the nature of the portfolio problem. Still workers are only interested in the productivity-weighted probability to get more than one job offer, while the social planner wants to spread applications in order to reduce the coordination frictions. So, the fact that we restrict the workers to at most two applications is not driving our main result. If we allow workers to send three applications, (HHL) can be a symmetric equilibrium portfolio for very large θ_L and θ_H and y_L. The L-application is used to increase the probability of two offers. θ_L must be sufficiently large to make this effect large enough, y_L must be sufficiently large to make the payoffs of HL-offers close to the payoffs of HH-offers and θ_H should be sufficiently large that it is not profitable to play (HHH). If workers apply to four jobs there exist more equilibria with diversification. Suppose $\theta_L \to \infty$, then for y_L sufficiently high, workers will send two applications to the L-sector which will result in two offers with a probability close to one. The marginal contribution of sending the remaining two applications to the L-sector are close to zero so they can best be sent to the H-sector. For five and more applications we cannot rule out regions where workers send three applications to the L-sector and the rest to the H-sector. This only happens for θ_L sufficiently large but smaller than one. The L-applications are used to secure a job while the H-applications are used to get a large payoff. We do know for sure that workers never send just one application to the H-sector $\forall a$ because the resulting wage in case of HL-offers equals the wage in case of LL-offers but the probability of occurrence is higher for the LL-portfolio.

The desire to diversify in our model is less than in Chade and Smith (2006) or Galenianos and Kircher (in press) who only have ex ante competition but no ex post competition for workers. This is caused by the fact that in our model the wage is not determined by the productivity at the most productive firm but by the productivity of the second-highest-productivity firm that makes an offer. In the portfolio problems that they consider, the firms commit ex ante to a wage. Under ex post competition, workers have incentives to generate similar offers. Allowing workers to send more than two applications will not restore efficiency because the planner will reduce coordination frictions by letting workers diversify as much as possible between sectors while workers have strong incentives to send applications to the same sector.

Finally, note that in our setting the marginal improvement algorithm (MIA) of Chade and Smith (2006) does not work. This algorithm first picks the application with the highest expected payoff, the next application is sent to the location with the highest marginal improvement and so on and so forth. If the marginal contribution of an application is negative then the previous one is the final application. In our setting, the first application has a negative marginal payoff. Moreover, if an agent has played LL, an additional H-application always has a smaller marginal contribution to the portfolio than a single L-application but as we argued before, for some configurations, the LLHH-portfolio dominates the LLLL-portfolio. This makes it computanionally hard to find the optimal portfolio for the case with many firm types and many applications.[15]

4. Final remarks

We presented a simple model where workers can apply to multiple, heterogeneous jobs and where firms can increase their initial bids when their candidate has multiple offers. Workers do not apply to firms with the highest expected payoffs for an individual application but rather maximize the value of their portfolio. The resulting equilibrium is not efficient because workers want to maximize the productivity-weighted probability to get two job offers,

[15] There may exist algorithms where the marginal contribution of pairs or triples of applications can be used rather than comparing complete portfolios with each other but we have not been able to prove this.

while the planner aims to maximize the productivity-weighted number of matches. This conflict of interest results in too little matches and excessive unemployment. We showed that this result is not driven by the fact that search is random in our model. For a large share of parameter values the posted wages are also zero in the directed search version of our model as in Albrecht et al. (2006). The workers' portfolio distortions cannot easily be corrected. Governments may have instruments to make one of the sectors more attractive, but this will only increase the fraction of workers who send both applications to this sector without increasing the fraction of workers that mixes between sectors.

Appendix A. Proof of Proposition 3

Proof. Suppose that all firms posting a wage equal to zero is not a directed search equilibrium. Then a profitable deviation must exist for either the high type firms or the low types firms. Consider a deviation by a high type firm first. Instead of 0 it posts a strictly positive wage: $w'_H>0$. Workers now have two additional application strategies: they can send (i) one application to the deviant and the other one to a high firm or (ii) one application to the deviant and the other one to a low firm. Denote the former strategy by H′H and the latter by H′L. The payoff of playing H′H equals

$$\psi'_H\psi_H + \psi'_H(1-\psi_H)w'_H \tag{8}$$

and the payoff of H′L equals

$$\psi'_H\psi_L y_L + \psi'_H(1-\psi_L)w'_H, \tag{9}$$

where ψ'_H is defined in the usual way and denotes the probability that an application to the deviant results in a job offer.

Since we consider a large labor market, a specific worker applies with probability zero to the deviant. So, the presence of a deviant does not affect the average number of applications received by the other non-deviant high or low firms. Therefore, the indifference condition $\psi_H^2=\psi_L^2 y_L$ must still hold. By substituting $\psi_H=\psi_L\sqrt{y_L}$ in Eq. (8) and using the fact that $1>\sqrt{y_L}>y_L$, one can easily see that H′L is dominated by H′H.

In response to the deviation by one of the high firms, workers will adjust their application strategies such that they are indifferent between HH, LL and H′H. The new equilibrium is therefore defined by the following two equations:

$$\psi_H^2 = \psi_L^2 y_L$$
$$\psi_H^2 = \psi'_H\psi_H + \psi'_H(1-\psi_H)w'_H$$

Let ϕ'_H denote the expected number of applications that the deviant receives. Then, by substituting $\psi'_H=\frac{1}{\phi'_H}\left(1-e^{-\phi'_H}\right)$ in the second condition and rearranging the result, we can derive the following relation between the posted wage w'_H and ϕ'_H:

$$w'_H = \frac{1}{1-\psi_H}\left(\frac{\phi'_H\psi_H^2}{1-e^{-\phi'_H}} - \psi_H\right). \tag{10}$$

The first derivative of this function with respect to ϕ'_H equals

$$\frac{\partial w'_H}{\partial\phi'_H} = \frac{\psi_H^2}{\psi_H-1}\frac{e^{-\phi'_H}+\phi'_H e^{-\phi'_H}-1}{e^{-2\phi'_H}-2e^{-\phi'_H}+1} > 0\ \forall\phi'_H > 0.$$

Hence, w'_H is a monotonic function of ϕ'_H: the higher the wage set by the deviant, the higher the expected number of applications it receives. The fact that w'_H is monotonically increasing in ϕ'_H also implies that rather than deriving the optimal wage for a deviant, we can derive the optimal queue length. The one implies the other.

After substituting Eq. (10), the profit function for a high type deviant equals

$$\pi'_H = \left(1-e^{-\phi'_H}\right)(1-\psi_H)\left(1-w'_H\right)$$
$$= \left(1-e^{-\phi'_H}\right)(1-\psi_H)\left(1-\frac{1}{1-\psi_H}\left(\frac{\phi'_H\psi_H^2}{1-e^{-\phi'_H}}-\psi_H\right)\right).$$

Differentiating this profit function with respect to ϕ'_H yields the following expression:

$$\frac{\partial\pi'_H}{\partial\phi'_H} = e^{-\phi'_H} - \psi_H^2,$$

which is a strictly decreasing function of ϕ'_H that equals zero for $\phi'_H=-2\log(\psi_H)$. Therefore, the profit function has a global maximum in this point. The corresponding value of w'_H follows from evaluating Eq. (10) in this maximum:

$$w'_H = \frac{\psi_H\left(\psi_H^2-2\psi_H\log(\psi_H)-1\right)}{(1-\psi_H)^2(1+\psi_H)}. \tag{11}$$

This expression has the same sign as $\psi_H^2-2\psi_H\log(\psi_H)-1$. The first derivative of this equation is equal to $2(\psi_H-\log\psi_H-1)$, which easily can be shown to be positive for all ψ_H in the interval (0,1). Together with the fact that $\lim_{\psi_H\to 1}\psi_H^2-2\psi_H\log(\psi_H)-1=0$, this implies that the right hand side of Eq. (11) is negative $\forall\psi_H\in(0,1)$. Since we do not allow for negative wages, this optimal value of w'_H is not feasible. Given that the profit is strictly decreasing in $\phi'_H>-2\log(\psi_H)$ and that w'_H is strictly increasing in ϕ'_H, the profit function maximization problem therefore has a boundary solution: the deviant maximizes its profit by posting $w'_H=0$. This implies that the best response for a potential deviant is to also post w_H.

Now we perform the same analysis for a low type deviant. Suppose that it posts a wage $w'_L>0$. In that case the payoff of playing LL′ equals

$$\psi_L\psi'_L y_L + \psi'_L(1-\psi_L)w'_L = \psi'_L w'_L + \psi'_L\psi_L\left(y_L - w'_L\right)$$

and the payoff of HL′ equals

$$\psi_H\psi'_L y_L + \psi'_L(1-\psi_H)w'_L = \psi'_L w'_L + \psi'_L\psi_H\left(y_L - w'_L\right),$$

where ψ'_L denotes the probability that an application to the deviant results in a job offer.

In a similar way as we described above, one can show that the strategy HL′ is dominated by LL′. The new equilibrium is therefore defined by the following two indifference conditions:

$$\psi_H^2 = \psi_L^2 y_L$$
$$\psi_L^2 y_L = \psi_L\psi'_L y_L + \psi'_L(1-\psi_L)w'_L$$

Let ϕ'_L denote the expected number of applications that the deviant receives. Then, by substituting $\psi'_L=\frac{1}{\phi'_L}\left(1-e^{-\phi'_L}\right)$ in the second condition and rearranging the result, we can derive the following relation between the posted wage w'_L and ϕ'_L:

$$w'_L = \frac{1}{1-\psi_L}\left(\frac{\phi'_L\psi_L^2 y_L}{1-e^{-\phi'_L}} - \psi_L y_L\right). \tag{12}$$

The first derivative of this function with respect to ϕ'_L equals

$$\frac{\partial w'_L}{\partial\phi'_L} = \frac{\psi_L^2 y_L}{\psi_L-1}\frac{e^{-\phi'_L}+\phi'_L e^{-\phi'_L}-1}{e^{-2\phi'_L}-2e^{-\phi'_L}+1} > 0\ \forall\phi'_L > 0.$$

Hence w'_L is a monotonic function of ϕ'_L: the higher the wage set by the deviant, the higher the expected number of applications it receives.

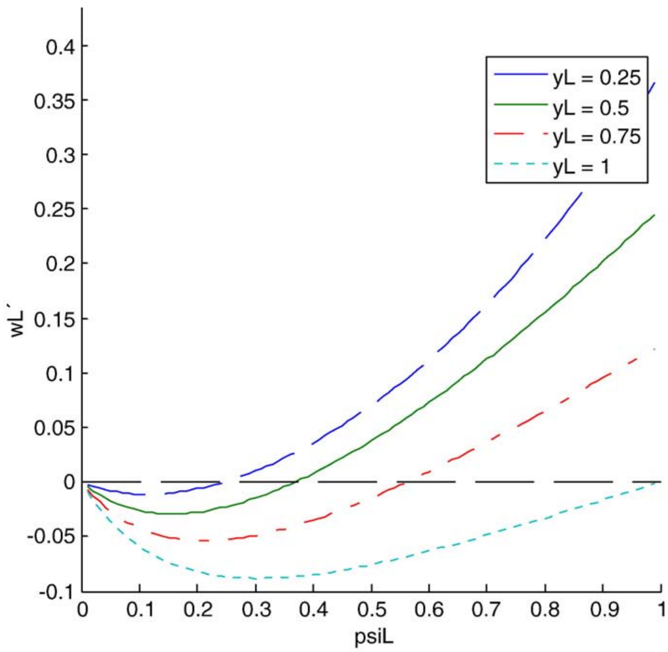

Fig. 5. w'_L as a function of ψ_L for several values of y_L. Positive values of w'_L imply that a profitable deviation exists for a low type firm.

The profit function for the deviant equals

$$\pi'_L = \left(1 - e^{-\phi'_L}\right)(1 - \psi_L)\left(1 - w'_L\right)$$
$$= \left(1 - e^{-\phi'_L}\right)(1 - \psi_L)\left(1 - \frac{1}{1 - \psi_L}\left(\frac{\phi'_L \psi_L^2 y_L}{1 - e^{-\phi'_L}} - \psi_L y_L\right)\right).$$

Differentiating this profit function with respect to ϕ'_L yields the following expression:

$$\frac{\partial \pi'_L}{\partial \phi'_L} = e^{-\phi'_L}(1 - (1 - y_L)\psi_L) - \psi_L^2 y_L,$$

which is a strictly decreasing function of ϕ'_L that equals zero for $\phi'_L = -\log \kappa$, where $\kappa \equiv \frac{\psi_L^2 y_L}{1 - (1 - y_L)\psi_L}$. Therefore the profit function has a global maximum in this point. The corresponding value of w'_L follows from evaluating Eq. (12) in this maximum:

$$w'_L = \frac{-\psi_L y_L}{1 - \psi_L}\left(\frac{\psi_L \log \kappa}{1 - \kappa + 1}\right).$$

One can check that $\lim_{\psi L \to 0} w'_L = 0$, $\lim_{\psi_L \to 0} \frac{\partial w'_L}{\partial \psi_L} = -y_L < 0$ and, by applying l'Hospital's Rule twice, $\lim_{\psi_L \to 1} w'_L = \frac{1 - y_L}{2} > 0$ (see Fig. 5). Therefore, it depends on the equilibrium value ψ_L^* whether a profitable deviation exists. For ψ_L^* close to 0 the optimal value for w'_L is negative. Given the fact that $\frac{\partial \pi'_L}{\partial \phi'_L}$ for $\phi'_L > \log \kappa$ and that $\frac{\partial \pi'_L}{\partial \phi'_L} > 0\ \forall \phi'_L > 0$, this implies that low type firms have no incentive to post a wage that is different from 0. On the other hand, for ψ_L^* close to 1, it is profitable for a low firm to deviate by posting a wage that is strictly positive. It is straightforward to show that both cases can occur. For example, $\psi_L^* \to 0$ if $\theta_H \to 0$, $\theta_L \to 0$ and $y_L \to 1$, while $\psi_L^* \to 1$ if $\theta_H \to \hat{\theta}_H$ where $\hat{\theta}_H$ is such that $\frac{\hat{\theta}_H^2}{4}\left(1 - \exp\left(-\frac{2}{\hat{\theta}_H}\right)\right)^2 = y_L$. □

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