Spatial Search
—Online Appendix—

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Appendix B Additional Results

B.1 Derivations for Section 2.5

As in the case of homogeneous sellers, we can analyze the effect of making goods more niche. To do so, we again increase z and simultaneously reduce $x(z)$, keeping the distribution $G(q)$ of expected buyer value $q = zx(z)$ fixed. Because we fix the distribution of q, the correspondence between sellers and locations does not depend on γ and can thus be denoted by $q^*(s)$, which is given by a variant of equation (8), i.e., $1 - G(q^*(s)) = 1 - L(s)$.

Let $\gamma^* = 0$ for the first example, and $\gamma^* = \alpha/\beta$ for the second example; then the first-order approximation of equation (9) is

$$Y(\lambda, \gamma^* + \Delta\gamma) \approx Y(\lambda, \gamma^*) + \int_s q^*(s) \frac{1 - e^{-\lambda s x^*(s)} - \lambda s x^*(s) e^{-\lambda s x^*(s)}}{(x^*(s))^2} \Delta x^*(s)\, dL(s), \tag{B.1}$$

where $\Delta x^*(s) = x(q^*(s), \gamma^* + \Delta\gamma) - x(q^*(s), \gamma^*)$. Given $x(q, \gamma) = (q/z_0)^{-\gamma/(1-\gamma)}$, we have

$$\Delta x^*(s) = -\frac{1}{(1-\gamma^*)^2} \left(\frac{z_0}{q}\right)^{\frac{\gamma^*}{1-\gamma^*}} \log\left(\frac{q}{z_0}\right) \Delta\gamma. \tag{B.2}$$

First-order approximation for Example 1. We now consider a first-order approximation around $\gamma = 0$ for Example 1 above. To simplify the analysis we set $s_0 = 0$, so $L(s) = 1 - e^{-s}$. Furthermore, we fix the distribution of expected buyer value ($q = zx(z)$): $G(q) = 1 - \left(\frac{z_0}{q}\right)^\alpha$ with $\alpha > 1$ and $q \geq z_0$. By equation (11), the assignment between sellers

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and locations is then $q^*(s) = z_0 e^{s/\alpha}$, which, by equation (B.2), implies that $\Delta x^*(s) \approx s/\alpha \cdot \Delta\gamma$. The first-order approximation in (B.1) then becomes

$$Y(\lambda, \Delta\gamma) = \overline{z}\frac{\alpha\lambda}{\alpha\lambda + \alpha - 1} + \overline{z}\frac{\alpha^2\lambda^2(\alpha\lambda + 3(\alpha - 1))}{(\alpha - 1)(\alpha\lambda + \alpha - 1)^3}\Delta\gamma,$$

where $\overline{z} = z_0\alpha/(\alpha - 1)$, and the first term on the right-hand side corresponds to Y in equation (12) with $s_0 = 0$. Note that we have included $\Delta\gamma$ as an argument of Y to emphasize its dependence on γ. By the above equation, the percentage increase in Y is then given by

$$\frac{\partial \log Y(\lambda, \gamma)}{\partial \gamma}\Big|_{\gamma=0} = \frac{\alpha\lambda(\alpha\lambda + 3(\alpha - 1))}{(\alpha - 1)(\alpha\lambda + \alpha - 1)^2}, \tag{B.3}$$

which is strictly positive since $\alpha > 1$. Note that the above equation is strictly decreasing in α.[1] Recall that a higher γ leads to a higher percentage change of $z(q)$ for a higher q. Therefore, the effect of γ on total surplus is stronger when α is smaller (when the quality distribution $G(q)$ is more dispersed). When the quality distribution is concentrated at z_0 ($\alpha \to \infty$), equation (B.3) converges to zero, since $x(z_0)$ is always 1 for any γ.

First-order approximation for Example 2. At $\gamma^* = \alpha/\beta$, the distribution of the expected buyer value $q = zx(z)$ is given by $1 - G(q) = \mathbb{P}(zx(z) \geq q) = \mathbb{P}(z/z_0 \geq (q/z_0)^{1/(1-\gamma^*)}$, which implies that $G(q) = 1 - (z_0/q)^{\alpha/(1-\gamma^*)}$. Furthermore, by equation (14), we have $q^*(s) = z_0(\frac{s}{s_0})^{\frac{1-\gamma}{\gamma}}$ and $x^*(s) = (q^*(s)/z_0)^{-\gamma/(1-\gamma)} = s_0/s$.

We now fix $G(q)$ and increase γ from γ^* to $\gamma^* + \Delta\gamma$. Since $q^*(s) = z_0(\frac{s}{s_0})^{\frac{1-\gamma}{\gamma}}$, by equation (B.2) we have

$$\Delta x^*(s) = -\frac{s_0 \log(\frac{s}{s_0})}{s\gamma^*(1 - \gamma^*)}\Delta\gamma$$

which implies that the percentage reduction $\Delta x^*(s)/x^*(s) = -\Delta\gamma \cdot \log(\frac{s}{s_0})/\gamma^*(1-\gamma^*)$, which is higher for sellers with higher quality.

Plugging the above expression of $\Delta x^*(s)$ into (B.1) yields

$$Y(\lambda, \gamma^* + \Delta\gamma) \approx \left(1 - e^{-\lambda s_0}\right)\overline{z} + \frac{\overline{z}\left(1 - e^{-\lambda s_0} - \lambda s_0 e^{-\lambda s_0}\right)}{(1 - \gamma^*)(\alpha - 1)}\Delta\gamma$$

Again, increasing γ while holding the distribution of q constant increases the expected total

[1] To see this, note that the derivative of (B.3) with respect to α is given by

$$-\frac{\Delta\gamma\lambda^3}{(\alpha-1)^2(\alpha\lambda+\alpha-1)^3} - \frac{\Delta\gamma\lambda\left((\alpha-1)^2(\lambda+1)(\lambda+3) + (\alpha-1)(3\lambda^2+7\lambda+6) + 3\lambda(\lambda+1)\right)}{(\alpha-1)(\alpha\lambda+\alpha-1)^3}$$

which is strictly negative since $\alpha > 1$.

surplus. The effect is smaller when α is higher.

B.2 Price Posting

Below we assume that $G(\tilde{z}, z)$ is a continuous distribution function, the support of $G(\tilde{z}, z)$ is $[a(z), b(z)]$, and the density of $G(\tilde{z}, z)$ is $g(\tilde{z}, z)$, i.e., $g(\tilde{z}, z) = \partial G(\tilde{z}, z)/\partial \tilde{z}$.

B.2.1 The Planner's Problem

The expected surplus for a seller type z in location s is given by

$$
\begin{aligned}
S(s,z) &= \sum_{n=1}^{\infty} e^{-\lambda s} \frac{(\lambda s)^n}{n!} \int_{a(z)}^{b(z)} \tilde{z} dG\left(\tilde{z}, z\right)^n = \int_{a(z)}^{b(z)} \tilde{z} d\left(\sum_{n=1}^{\infty} e^{-\lambda s} \frac{(\lambda s)^n}{n!} G\left(\tilde{z}, z\right)^n\right) \\
&= \int_{a(z)}^{b(z)} \tilde{z} d\left(e^{-\lambda s(1-G(\tilde{z},z))}\left(1 - e^{-\lambda s G(\tilde{z},z)}\right)\right) = \int_{0}^{b(z)} 1 - e^{-\lambda s(1-G(\tilde{z},z))} d\tilde{z} \qquad \text{(B.4)}
\end{aligned}
$$

where for the last equality we used integration by parts. The above equation generalizes equation (2) where $G(\cdot, z)$ is a two-point distribution.

The partial derivative of $S(s, z)$ with respect to s is then given by

$$
\frac{\partial S(s,z)}{\partial s} = \frac{1}{s} \int_{0}^{b(z)} \lambda s \left(1 - G(\tilde{z}, z)\right) e^{-\lambda s(1-G(\tilde{z},z))} d\tilde{z}, \qquad \text{(B.5)}
$$

where the integrand above is the probability that a seller meets exactly one buyer with a value above $\tilde{z}$.

Denote the inverse function of $G(\tilde{z}, z)$ with respect to $\tilde{z}$ by $G^{-1}(t, z)$. That is, if $G(\tilde{z}, z) = t$, then $\tilde{z} = G^{-1}(t, z)$. With the above notation, we can then rewrite equation (B.5) as follows.

$$
\begin{aligned}
\frac{\partial S(s,z)}{\partial s} &= \frac{1}{s}\left(\lambda s e^{-\lambda s} a(z) + \int_{a(z)}^{b(z)} \frac{1 - G(\tilde{z}, z)}{g(\tilde{z}, z)} d\left(1 - e^{-\lambda s(1-G(\tilde{z},z))}\right)\right) \\
&= \frac{1}{s}\left(\lambda s e^{-\lambda s} a(z) + \int_{0}^{1} \frac{1-t}{g(G^{-1}(t,z), z)} d\left(1 - e^{-\lambda s(1-t)}\right)\right) \qquad \text{(B.6)}
\end{aligned}
$$

where for the second equality we changed the variable of integration.

Therefore, if i) $a(z)$ is weakly increasing, and ii) $g(G^{-1}(t, z), z)$ is weakly decreasing in z for any $t \in [0, 1]$, then the two terms in the parenthesis of equation (B.6) are increasing in z, and hence $S(s, z)$ is supermodular. If either condition is strict, then $S(s, z)$ is strictly supermodular.

B.2.2 The Decentralized Equilibrium

The problem of an (s, z) seller is to choose p to maximize expected profit:

$$\pi(s,z) \equiv \max_p p \sum_{n=1}^{\infty} e^{-\lambda s} \frac{(\lambda s)^n}{n!} \left(1 - G(p,z)^n\right) = pQ(p,s,z)$$

where

$$Q(p,s,z) = 1 - e^{-\lambda s(1-G(p,z))}. \tag{B.7}$$

The seller's problem above is similar to a standard profit maximization problem of a monopolist where the demand function is given by $Q(p, s, z)$. Denote the seller's optimal price by $p^*(z)$ (we ignore its dependence on s to simplify notation). By the envelope theorem,

$$\frac{\partial \pi(s,z)}{\partial s} = p^*(z) Q_s\left(p^*(z), s, z\right) = p^*(z)\lambda(1 - G(p^*(z), z))e^{-\lambda s(1-G(p^*(z),z))} > 0 \tag{B.8}$$

where Q_s is the partial derivative with respect to s.

If for some seller type z, the optimal price $p^*(z) = a(z)$, the corner solution, then equation (B.8) becomes $\pi_s(s, z) = a(z)\lambda e^{-\lambda s}$, which implies that $\pi_{sz}(s, z) > 0$ if and only if $a'(z) > 0$.

If the optimal price $p^*(z)$ is interior, then it must satisfy the following first-order condition,

$$0 = 1 - e^{-\lambda s(1-G(p,z))} - pg(p,z)\lambda s e^{-\lambda s(1-G(p,z))}. \tag{B.9}$$

From the above first-order condition we can solve for the derivative $dp^*(z)/dz$.

$$\frac{dp^*(z)}{dz} = -\frac{G_z(p^*,z)\left(\lambda s p^* g(p^*,z) + 1\right) + p^* g_z(p^*,z)}{g(p^*,z)\left(\lambda s p^* g(p^*,z) + 2\right) + p^* g_p(p^*,z)} \tag{B.10}$$

where the subscripts refer to partial derivatives, e.g., $G_z(p^*, z) = \partial G(p^*, z)/\partial z$. To determine the supermodularity of $\pi(s, z)$, differentiating equation (B.8) with respect to z yields

$$\frac{\partial^2 \pi}{\partial s \partial z} = \lambda e^{-\lambda s(1-G)} \left(\frac{dp^*}{dz}(1-G) - p^* \left(\frac{dp^*}{dz} g + G_z \right) (1 - \lambda s(1-G)) \right), \tag{B.11}$$

where dp^*/dz is given by equation (B.10), and we have suppressed the arguments of $G(p^*, z)$. In general, it is difficult to sign (B.11). Below we discuss three special cases where we can analyze it.

Finally, we show that if $G(p, z)$ is a uniform distribution, then the first-order condition for $p^*(z)$ is both necessary and sufficient. Furthermore, we derive the condition under which

$p^*(z)$ is interior. Assume that $G(p,z)$ is $U[a(z),b(z)]$, the uniform distribution between $a(z)$ and $b(z)$. Then

$$\frac{\partial}{\partial p}\left(\frac{Q(p,s,z)}{-Q_p(p,s,z)}\right) = e^{\lambda s \frac{b(z)-p}{b(z)-a(z)}}$$

where $Q(p,s,z)$ is the demand function defined by equation (B.7), and Q_p is its derivative with respect to p (density function). Since the above equation is strictly decreasing in p, the sellers' first-order condition is both necessary and sufficient for the optimal price $p^*(z)$.

The optimal price $p^*(z)$ is interior if and only if at $p = a(z)$, the right-hand side of equation (B.9) is strictly positive, which is equivalent to

$$\frac{a(z)}{b(z)-a(z)} < \frac{e^{\lambda s}-1}{\lambda s}. \tag{B.12}$$

Hence, given λs, $p^*(z)$ is interior if and only if $a(z)$ is sufficiently small; similarly, given $a(z)$ and $b(z)$, the same conclusion holds if and only λs is sufficiently large.

B.2.3 Efficiency: Three Examples

We now discuss three special cases where $S(s,z)$ is always strictly supermodular, while $\pi(s,z)$ is strictly supermodular for the first two cases, and neither supermodular nor submodular for the third case.

Case 1: z acts as a multiplicative shifter in $G(\tilde{z},z)$. In this case, we have $G(\tilde{z},z) = H(\tilde{z}/z)$, $a(z) = a_0 z$, and $b(z) = b_0 z$, where $a_0 \geq 0$, b_0 can be ∞, and $H(\cdot)$ is a univariate cdf.

Note that $\tilde{z} = G^{-1}(t,z)$ if and only if $\tilde{z}/z = H^{-1}(t)$. Furthermore, $g(G^{-1}(t,z),z) = g(\tilde{z},z) = \frac{1}{z}H'(\tilde{z}/z) = \frac{1}{z}H'(H^{-1}(t))$, which is inversely proportional to z, and hence strictly decreasing. Hence, in the planner's problem $S(s,z)$ is always strictly supermodular.

In the decentralized market, $p^*(z)$ and hence $\pi(s,z)$ must be proportional to z, since z scales up buyers' values proportionally. Therefore, $\pi(s,z)$ must be strictly supermodular. Hence, the planner's solution and the decentralized equilibrium coincide.

Case 2: z acts as an additive shifter in $G(\tilde{z},z)$. In this case, we have $G(\tilde{z},z) = H(\tilde{z}-z)$, $a(z) = a_0 + z$, and $b(z) = b_0 + z$, where $a_0 \geq 0$, b_0 can be ∞, and $H(\cdot)$ is a univariate cdf.

Note that $\tilde{z} = G^{-1}(t,z)$ if and only if $\tilde{z} - z = H^{-1}(t)$. Furthermore, $g(G^{-1}(t,z),z) = g(\tilde{z},z) = H'(\tilde{z}-z) = H'(H^{-1}(t))$, which is independent of z. Hence, the first term in between parenthesis of equation (B.6) is strictly increasing in z, and the second term is independent of z, which implies that in the planner's problem $S(s,z)$ is always strictly supermodular.

The analysis of the decentralized equilibrium is more complicated, since in general we can not derive an explicit expression for $p^*(z)$. Suppose that $H(\cdot)$ is uniform, $U[a_0,b_0]$. When both a_0 and z are small, then $p^*(z)$ is interior, i.e., $p^*(z) > a_0 + z$; otherwise we have a corner solution $p^*(z) = a_0 + z$. When $p^*(z)$ is interior, then by equation (B.10) and (B.11) we have

$$\frac{\partial^2 \pi}{\partial s \partial z} = \frac{\lambda(b_0 + z)}{\lambda s p^*(z) + 2b_0 - 2a_0} e^{-\frac{\lambda s(b_0 + z - p^*(z))}{b_0 - a_0}} > 0$$

When $p^*(z)$ is a corner solution, as we argued before, $\pi_{sz}(s, z) > 0$ if and only if the lower bound $a(z)$ is strictly decreasing, which holds trivially here. Thus $\pi(s, z)$ is always strictly supermodular.

Case 3: z acts as mean-preserving spread in $G(\tilde{z}, z)$. Assume that $G(\tilde{z}, z)$ is uniform, $U[1 - z, 1 + z]$ with $0 \leq z < 1$; hence $a(z) = 1 - z$ and $b(z) = 1 + z$. The mean of $\tilde{z}$ is always 1 but as z increases, the distribution of $\tilde{z}$ becomes more dispersed, i.e., the goods become more niche.

We first consider the planner's problem. Since $a(z) = 1 - z$, which is strictly decreasing, we can not apply the conditions that are sufficient for $S(s, z)$ to be supermodular. Instead, we calculate $S(s, z)$ directly. By equation (B.4), we have

$$S(s, z) = 2z \frac{e^{-\lambda s} + \lambda s - 1}{\lambda s}$$

Since $S(s, z)$ is linear in z, it is then strictly supermodular (recall that $S(s, z)$ is always increasing in s).

Next, consider the decentralized equilibrium. By equation (B.12), $p^*(z)$ is interior if and only if

$$z > \frac{\lambda s}{\lambda s + 2e^{\lambda s} - 2}. \tag{B.13}$$

Note that the right-hand side above is strictly decreasing in λs, and its maximum is $1/3$. Thus when $z \geq 1/3$, the optimal $p^*(z)$ is always interior.

When z is close to 0 (so that the distribution of buyers is highly concentrated), we have a corner solution $p^*(z) = 1 - z$. Since $a(z)$ is strictly decreasing, $\pi_{sz}(s, z) < 0$.

Now suppose that the optimal price $p^*(z)$ is interior. By equation (B.10) and (B.11) we have

$$\frac{\partial^2 \pi}{\partial s \partial z} = \lambda e^{-\frac{\lambda s(1 + z - p^*(z))}{2z}} \frac{(1 + z - p^*(z))p^*(z)(\lambda s p^*(z) + 4z) - 2z(z + 1)}{2z^2(\lambda s p^*(z) + 4z)}$$

which is strictly positive if and only if

$$\lambda s > \frac{2z(1+z-2p^*(z)(1+z-p^*(z)))}{(1+z-p^*(z))p^*(z)^2}$$

Note that the optimal price $p^*(z)$ also depends on λs. In Figure 1 we plot $\pi_{sz}(s,z)$ where $s \in [0,2]$, $z \in [0.5,1]$, and we normalize $\lambda = 1$. In the figure we set $z \geq 0.5$ to ensure that the optimal $p^*(z)$ is interior so that equation (B.11) is valid, and to increase the figure's visibility. In general, when s is small, $\pi_{sz}(s,z) < 0$, $\pi_{sz}(s,z)$ is not supermodular and spatial sorting is inefficient. This finding echoes the result of our benchmark model, where supermodularity of $\pi(x,z)$ failed when $zx(z)$ is constant and λs is small.

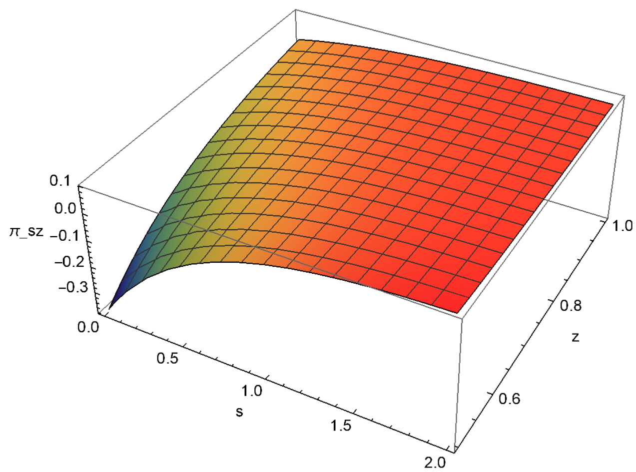

Figure 1: Inefficient spatial sorting under price posting

B.3 Relation with Invariance

In our model, the probability that a seller meets n buyers is given explicitly by equation (16). In earlier literature (see, e.g., Eeckhout and Kircher, 2010), it has been common to start with $P_n(\lambda)$ unspecified. An invariant meeting technology is then defined as one for which equation (20) holds for any λ and x, with (set $x = 1$) $m(\lambda) = 1 - P_0(\lambda)$ (Lester et al., 2015; Cai et al., 2017). Furthermore, the n^{th} derivative of equation (20) with respect to x, evaluated at $x = 1$, equals $P_n(\lambda) = (-1)^{n+1}\frac{\lambda^n}{n!}m^{(n)}(\lambda)$.

Therefore, for any invariant meeting technology, its associated function $m(\lambda)$ has the following properties: i) It is non-negative, and ii) it is infinitely differentiable and $(-1)^{n+1}\frac{d^n}{d\lambda^n}m(\lambda) \geq 0$ for $n \geq 1$. That is, $m(\lambda)$ is a *Bernstein function*. In addition, $m(\lambda)$ is bounded between 0 and 1, $m(0) = 0$, and $m'(0) \leq 1$. By Bernstein's theorem, $m(\lambda)$ has the following representation.[2]

[2] See page 21 of Schilling et al. (2012) for the definition of Bernstein functions and Bernstein's Theorem.

Theorem 1. *A function $m(\lambda)$ generates an invariant meeting technology if and only if there exists a probability measure $\widetilde{L}(s)$ on $[0,\infty)$ (the positive real half-line) with $\int_{[0,\infty)} s\, d\widetilde{L}(s) \leq 1$ such that*

$$m(\lambda) = \int_{[0,\infty)} \left(1 - e^{-\lambda s}\right) d\widetilde{L}(s). \tag{B.14}$$

Proof. **Invariance implies (B.14).** For the first part of the proof, consider an invariant meeting technology defined by the condition that $\{P_0(\lambda), P_0(\lambda), \dots\}$ satisfies equation (20). As we argued before Theorem 1, $m(\lambda)$ is a Bernstein function. By Bernstein's theorem, the function $m(\lambda)$ has the following Lévy-Khintchine representation:

$$m(\lambda) = a_1 + a_2\lambda + \int_{(0,\infty)} \left(1 - e^{-\lambda s}\right) dL(s),$$

where $a_1, a_2 \geq 0$ and L is a measure on $(0,\infty)$ satisfying $\int_{(0,\infty)} \min\{1,t\}\, dL(s) < \infty$ (see Theorem 3.2 of Schilling et al., 2012).

Since $m(0) = 0$, it follows that $a_1 = 0$. Moreover, since $m(\lambda)$ is bounded from above by 1, a_2 must equal 0 as well. Further, if $\lambda \to \infty$, we have $1 - e^{-\lambda s} \nearrow 1$ for any $t > 0$, and therefore $m(\lambda) \to \int_{(0,\infty)} 1\, dL(s)$ by the monotone convergence theorem. Since $m(\lambda)$ cannot exceed 1, the total measure of $L(\cdot)$ must be less or equal to 1: $\int_{(0,\infty)} 1\, dL(s) \leq 1$. If the total measure is strictly less than 1, without loss of generality we can assign measure $1 - \int_{(0,\infty)} 1\, dL(s)$ on point $s = 0$. Therefore, it is without loss of generality to assume that $L(\cdot)$ is a probability measure on $[0,\infty)$.

Next, the probability that a worker meets a firm is $m(\lambda)/\lambda = \int_0^\infty \left(1 - e^{-\lambda s}\right)/\lambda\, dL(s)$, which cannot exceed 1 for any $\lambda \geq 0$. One can easily verify that when $\lambda \searrow 0$, we have $\left(1 - e^{-\lambda s}\right)/\lambda \nearrow s$. Therefore, $\lim_{\lambda \to 0} m(\lambda)/\lambda = \int_0^\infty s\, dL(s)$ by the monotone convergence theorem. Hence, $L(\cdot)$ must satisfy $\int_0^\infty s\, dL(s) \leq 1$.

(B.14) implies invariance. For the second part of the proof, assume that $m(\lambda)$ is given by equation (B.14) where L is a probability measure on $[0,\infty)$ satisfying $\int_{[0,\infty)} s dL(s) \leq 1$. Since this corresponds to our meeting process on the circle, the resulting meeting technology is invariant (see equation (20)) and $P_n(\lambda)$ is given by equation (16). □

If $\int_{[0,\infty)} s\, d\widetilde{L}(s) = 1$, then the class of invariant technologies corresponds exactly to the above search process on the circle with $\widetilde{L}(s) = L(s)$.[3] We can also use our model to understand the general case. If $\int_{[0,\infty)} s\, d\widetilde{L}(s) < 1$, then with probability $1 - \int_{[0,\infty)} s\, d\widetilde{L}(s)$ buyers do not arrive on the circle and are passive. This probability is independent of λ. Given the set of buyers who arrive on the circle, the matching process specified by an invariant technology

[3] We use the notation $\int_{[0,\infty)}$ to emphasize that there can be a mass point at 0.

is again equivalent to our search process on the circle. Thus it is without loss of generality to assume that $\int_{[0,\infty)} s\, d\widetilde{L}(s) = 1$. Given the correspondence in Theorem 1, $P_n(\lambda)$ can also be calculated by equation (16) (a mixture of the corresponding Poisson probabilities).

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