It's About Time: Implications of the Period Length in an Equilibrium Job Search Model*

Ronald P. Wolthoff†

University of Toronto

September 19, 2011

Abstract

This paper analyzes the role of the period length in a search model of the labor market and argues that it has profound implications for the market equilibrium. In the model, job offers and job destruction shocks arrive according to a Poisson process in continuous time, but institutional factors and/or informational frictions may delay workers' transitions into or out of a job. This effectively creates discrete time periods of arbitrary length, with continuous time being the limit case when the period length goes to zero. Longer periods introduce the possibility of simultaneity or recall of job offers, affecting the labor share, the amount of wage dispersion, as well as the allocation of workers over jobs with different productivity levels. Misspecification of the period length may therefore lead to inconsistent estimates of structural parameters and wrong conclusions on optimal policy.

*I thank Carlos Carillo-Tudela and Pieter Gautier for their feedback on an earlier draft of this paper, which circulated under a different title. I am also grateful for helpful comments from Jim Albrecht, Ken Burdett, Philipp Kircher, Susan Vroman, Randy Wright, and the participants of the La Pietra-Mondragone Workshop in Florence, the Penn Search & Matching Workshop, and a seminar at the Tinbergen Institute. A first version of this paper was written during a visit to the Economics Department of the University of Pennsylvania, which I thank for its hospitality.

†E-mail: ronald.p.wolthoff@gmail.com

1 Introduction

1.1 Motivation and Summary

The frequency with which firms with vacancies and and workers searching for a job meet each other differs vastly across occupations. An extremely low frequency is found in e.g. the market for academic economists, in which hiring decisions are typically made only once a year. In most other occupations, meetings take place much more often: a hairstylist who has been unsuccessful in his/her applications does not need to wait a full year to have another chance of finding a job. In fact, it is perfectly possible that he/she sends a new application the next day and is employed by the next week. Of course, the jobs of economists and hairstylists differ in many ways and it is not hard to think of explanations why their labor markets have such different characteristics. Differences in the screening process, institutional factors, the monthly variation in the need for their services, the size of the market, and the amount of coordination are all likely to play a role. Much less clear however is what - everything else being equal - the consequences of the differences in interaction frequency are: if hairstylists started organizing annual job markets or economists started hiring during the academic year, how would that change the equilibrium in their respective labor markets?

The literature on this topic is limited. Some authors have specified continuous time models in which one meeting can take place today and the next meeting tomorrow. Other authors have used discrete time models in which a worker meeting a firm today has to wait a certain amount of time (e.g. a week or a month) before he can meet another firm. However, there exist no models which nest these approaches by introducing a parameter for the period length.[1]

In this paper, I present such a model and I argue that even in the simplest setting, abstracting from exacerbating issues like screening, the period length has profound implications for the characteristics of the decentralized market equilibrium. For example, it determines the labor share (as measured by the average wage), the amount of frictional wage dispersion, and whether the allocation of workers over firms is efficient. An economist making the wrong assumption about the nature of time may obtain incorrect estimates of productivity distributions or draw wrong conclusions on the optimal level of unemployment benefits or the minimum wage.

The main intuition for these findings is that discrete time creates a possibility of simultaneity and recall of job offers which is typically ruled out by assumption in continuous time models. When time is discrete, workers may receive multiple job offers in a given period which allows them to compare these offers to each other. In the standard models of continuous time, workers make at most one comparison, i.e. between their (single) offer and their outside option. This difference changes the structure of the equilibrium.[2]

[1] See the next subsection for an overview of the literature.

[2] Of course, other assumptions can be made. For example, one can construct continuous time models in which job offers arrive in bunches. The equilibrium in such a model would be similar to the one in the discrete time version of my model, implying a possible indeterminacy about the best way of modeling the simultaneity that arises. In this paper, I focus on the most standard interpretation.

Consequently, it is crucial to specify the period length correctly. Given the lack of empirical evidence for most labor markets, I present a simple model that allows for time periods of arbitrary length. Workers receive wage offers according to a continuous time Poisson process but they cannot immediately accept these positions. Transitions are only possible at certain equidistant points in time, which effectively create time periods. If the period length is strictly positive, the model behaves like discrete time model in which workers may collect multiple job offers before deciding which one to accept. On the other hand, the model converges to a continuous time model if the length of a period goes to zero.[3]

I characterize the equilibrium and I show how its properties depend on the period length. First, I consider how the surplus created by a match is divided between the worker and the firm. I find that the wage distribution is degenerate if periods are infinitesimally short or infinitely long. With intermediate period lengths however, there is wage dispersion. The wage density is non-monotonic and the wage distribution resembles a wage ladder in the sense that not all wages can be reached directly after unemployment. The results on wage dispersion are important since in empirical applications the distribution of worker and/or firm productivities is often estimated from wage data after controlling for the frictional wage dispersion implied by a structural model (see e.g. Bontemps et al., 1999, 2000; Postel-Vinay and Robin, 2002). The assumed length of a time period will affect the estimates that are obtained in such a way.

Second, I study the allocative efficiency of the model. In particular, I analyze whether job-to-job transitions are efficient in the sense that workers always flow from less productive to more productive firms. I find that the answer depends on the values of the parameters and that the period length again plays a crucial role. For some parameter values, the job-to-job transitions are efficient for short period lengths, but not for long period lengths. For other parameter values, the exact opposite result applies. These results imply that the effectiveness of unemployment benefits or a minimum wage, which can be instrumental in restoring efficiency in my model, depends on the nature of time as well.

1.2 Related Literature

This paper adds to the existing literature in multiple ways. First, as described in the introduction, the strong focus on the role of time is new. Several authors have discussed how simultaneity or recall of job offers can change equilibrium outcomes Burdett and Judd (1983); Burdett and Mortensen (1998); Galenianos and Kircher (2009); Carrillo-Tudela et al. (2011), but to the best of my knowledge, the model presented here is the first to nest continuous and discrete time in a labor market setting. A second novelty of the model is that it allows workers to receive job offers simultaneously (i.e. multiple in one period) as well as sequentially (i.e. in subsequent periods in the form of on-the-job search). Past literature has considered at most one of both types along both dimensions: typically either time is

[3] It is important to emphasize that the job offer arrival rate per real unit of time does not depend on the length of a period. Some papers in the literature make a comparison between e.g. one job offer per week and one job offer per two weeks. That is not what I do in this paper. Instead, I compare one job offer per week to *two* job offers per two weeks.

discrete and multiple job offers may arrive simultaneously (see e.g. Kircher, 2009; Wolthoff, 2010), or time is continuous and job offers arrive according to a Poisson process and workers search on-the-job (e.g. Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002). Table 1 presents a classification for a large number of search models.

The random search model that I present in the next section can be described as a combination of the simultaneous search aspects of Burdett and Judd (1983) and the on-the-job search component of Burdett and Mortensen (1998). A difference with Burdett and Mortensen (1998) is that - under my assumption about what constitutes a period - an employer is always fully informed about the current employment contract of a worker that he meets. In that sense, my model is related to the work by Carrillo-Tudela (2009a,b), who allows firms to observe the employment status and/or experience of workers (but not the contract). The observability of the employment contracts also implies that the equilibrium in my model shares some features with the directed search literature, as I will discuss in more detail towards the end of this paper. Finally, this paper is related to Eeckhout and Kircher (2010) who study how a seller's optimal strategy depends on whether he can meet multiple buyers simultaneously or not. Since continuous time naturally leads to bilateral meetings and discrete time to multilateral meetings, there is a potential role for time in their model.

2 Model

2.1 Overview

Starting point for my analysis is a standard continuous time model. The main novelty is that I allow for discretization of time by introducing information frictions. Suppose that workers and firms are not immediately informed about relevant events that take place in the decentralized market, in particular about job offers and job destruction shocks. Instead they only learn about them at the earliest of a number of predefined points in time. This yields a meaningful definition of a period. At any moment, workers and firms are perfectly informed about the job offers and job destruction shocks that took place *before* the current period, but they are still unaware of the ones that arose *during* the current period.

Discretization can also be motivated in various other ways without changing the fundamental structure of the model. For example, screening of applicants typically takes time; decisions in the hiring process may be delayed by legal or corporate requirements regarding the recruitment procedure; workers may not be able to immediately quit their current job upon receiving a better offer because they are required to give an advance notice to their employer; or institutional factors may cause a profession to have an annual job market. The relevant element in each of these motivations is the simultaneity which is ruled out by assumption in continuous time models: at some moment in time, workers may learn that they have received multiple job offers and that they can choose which one they want to accept.

	Time		Job offers	
	Cont.	Disc.	Simult.	Seq.
Burdett and Judd (1983)		•	•	
Albrecht and Axell (1984)		•		
Mortensen and Pissarides (1994)	•			
Pissarides (1994)	•			•
Moen (1997)	•			
Burdett and Mortensen (1998)	•			•
Acemoglu and Shimer (1999)	•			
Julien et al. (2000, 2006)		•	•	
Burdett et al. (2001)		•		
Postel-Vinay and Robin (2002)	•			•
Gautier and Moraga-Gonzalez (2005)		•	•	
Albrecht et al. (2006)		•	•	
Delacroix and Shi (2006)		•		•
Carrillo-Tudela (2009a,b)	•			•
Galenianos and Kircher (2009)		•	•	
Gautier et al. (2009)		•	•	
Gautier and Wolthoff (2009)		•	•	
Kircher (2009)		•	•	
Shi (2009)	•			•
Menzio and Shi (2010a,b,c)		•		•
Kaas (2010)		•	•	
Wolthoff (2010)		•	•	
Carrillo-Tudela et al. (2011)	•		•	
This paper	•	•	•	•

Relationship between this paper and the existing literature. Cont. = continuous time model; Disc. = discrete time model / static model; Simult. = model allows for simultaneous arrival of job offers; Seq. = model allows for on-the-job search. Burdett and Judd (1983) deals with consumer markets but can be rewritten in labor market terms.

Table 1: Related literature

In the model that I present here, I abstract from various issues that might be important in reality. For example, the period length can have implications for the quality of the screening of applicants or for how well firms are able to deal with fluctuations in the demand for output. However, these aspects only exacerbate the difference between continuous time and discrete time. I therefore focus on the question how the nature of time affects the market equilibrium in the absence of these factors.

2.2 Setting

Consider the steady state in a labor market with a measure 1 of identical workers and a measure v of firms. Initially I will assume that all firms are identical. I analyze the implications of firm heterogeneity in section 5. All agents are infinitely-lived and risk-neutral. Firms employ workers in order to produce output according to a production technology that exhibits constant returns to scale. Each worker can supply one indivisible unit of labor and is therefore at a given moment in time either employed at one of the firms or unemployed. I denote the measures of employed and unemployed workers by $1-u$ and u respectively.

Time is denoted by t and is continuous. Both firms and workers discount future payoffs at rate $\widehat{\rho}$. A worker who is employed by a firm produces a flow output $\widehat{y}$. From this output, the firms pays the worker a wage $\widehat{w}$ at each instant for as long as the match lasts. Unemployed workers have a flow payoff equal to $\widehat{h}$, consisting of unemployment benefits, home production, and the value of leisure. In order to guarantee the existence of a market, assume that $\widehat{h}$ is strictly smaller than $\widehat{y}$.

Workers and firms meet each other according to a Poisson process. When a meeting takes place, the firm makes a take-it-or-leave-it job offer.[4] As common in the literature, I allow the job offer arrival rate to differ across employment state. Unemployed workers receive job offers at rate λ_U, while employed workers meet new firms at rate λ_E. To keep notation as simple as possible, assume $\lambda_E \leq \lambda_U$. Each job offer consists of a wage that the firm promises to pay to the worker for as long as the match lasts.[5] Jobs are subject to job destruction shocks which arrive at rate δ.

As discussed above, the main difference between my model and Burdett and Mortensen (1998) is that due to either informational frictions or institutional factors, transitions between employment states are only possible at certain predetermined time points. I assume that these moments occur every $\tau \geq 0$ units of time, i.e. at $t \in \{\tau, 2\tau, 3\tau, ...\}$. This essentially creates discrete time periods of length τ. If τ goes to zero, we get convergence to a continuous time model and if τ becomes really large we get a static (one-shot) model.

In the remainder of this paper, I will simplify notation by using the discrete-time equivalents of the model's parameters when possible. I denote the periodical discount factor by $\rho(\tau) = e^{-\widehat{\rho}\tau}$. The

[4]To keep the exposition as simple as possible, I maintain the assumption that each meeting results in a job offer. This assumption can be motivated with a CRS production technology. Direct competition between workers through e.g. an urn-ball matching technology would merely exacerbate the frictions.

[5]Burdett and Coles (2003) study a model in which the firm can condition the worker's wage on tenure within the firm.

discounted value of periodical output equals $y(\tau) = \beta(\tau)\widehat{y}$, where

$$\beta(\tau) = \int_0^\tau e^{-\widehat{\rho}t} dt = \frac{1}{\widehat{\rho}}\left(1 - e^{-\widehat{\rho}\tau}\right).$$

Likewise, I define $w(\tau) = \beta(\tau)\widehat{w}$ and $h(\tau) = \beta(\tau)\widehat{h}$. Time aggregation also needs to be taken into account for the transition rates. Workers may receive multiple job offers per period. To be precise, the number of firms that a worker with employment state $s \in \{U, E\}$ (U = unemployment, E = employment) meets in any given period follows a Poisson distribution with mean $\lambda_s\tau$. I denote the probability that the worker obtains j job offers by $p_s(j|\tau) = e^{-\lambda_s\tau}\frac{(\lambda_s\tau)^j}{j!}$. Likewise, the number of job destruction shocks in a period follows a Poisson distribution with mean $\delta\tau$.

The assumed information structure implies that a firm making a job offer does not know what other job offers the worker gets in the current period. However, the firm is always fully informed about the worker's current employment status and wage, since all information becomes public at the end of each period. So, a wage offer x can be conditioned on the worker's current wage w (or on the fact that he is unemployed), but not on the number of other firms that the worker meets in the current period. Let $F(x|w)$ be the equilibrium distribution of wage offers x to workers earning w and $F(x|h)$ the distribution of wage offers to the unemployed.[6] The corresponding supports are $\mathcal{F}(w)$ and $\mathcal{F}(h)$ respectively. In equilibrium these distribution will imply a steady state earnings distribution $G(w)$, representing the fraction of employed workers earning a wage lower than w.[7] Denote the corresponding density by $g(w)$ and its support by $\mathcal{G}$. Sometimes I will use expressions that apply to both employed and unemployed workers. For this purpose, define $\mathcal{G}' = \mathcal{G} \cup h$.

Towards the end of the period, each worker learns about the wage offers that he has received and accepts the best one, as long as it gives him a higher value than staying in his current job (or staying unemployed). Let $r(w)$ be the reservation wage of a worker currently earning w. I denote the workers' matching probability by $\mu_s(\tau)$. An expression for $\mu_s(\tau)$ will be derived in the next section. In the very last phase of the period, job destruction takes places. A job is destroyed if it was hit by at least one job destruction shock, which happens with probability $\phi(\tau) = 1 - e^{-\delta\tau}$. Workers hit by the job destruction shock flow back into unemployment, after which a new period starts.[8]

[6]Although unemployed workers technically do not earn a salary, I will in the remainder of this paper often simplify notation by treating them as workers earning h.

[7]I characterize the earnings distribution in section 4.

[8]The timing that I impose here implies that workers may lose their job immediately after accepting it. This assumption does not affect any of the conclusions in this paper and can easily be adjusted. The main advantage of this specification is that it simplifies some of the expressions by maintaining symmetry between unemployed and employed workers.

Note further that periodical output does not depend on the exact moment at which the job destruction shocks arrive within a period, implying that the shocks cannot easily be interpreted as shocks to the workers' productivity. A preferred interpretation is therefore that they represent shocks to (the price of) the firm's capital which destroy the surplus of the match, but only at the next transition moment because contracts regarding capital are only renewed then.

2.3 Value Functions

To facilitate the equilibrium derivation, I specify the workers' and firms' Bellman equations. Consider the workers first. Note that since no transitions are possible within a period, attention can be restricted to the time points τ, 2τ, 3τ, etc. Hence, let V_U denote a the value of unemployment and $V_E(w)$ the value of being employed at a wage $w \in \mathcal{G}$ at the beginning of a period. During the period, each worker receives a flow payoff, either from his job or from household production. The discounted values of these payoffs are w and h respectively. An unemployed worker gets $j \in \mathbb{N}_0$ job offers from the distribution $F(\cdot|h)$ with probability $p_U(j)$. He accepts the best wage offer as long as the associated payoff is higher than the payoff of remaining unemployed. After matching, job destruction may occur in which case the worker flows back to unemployment. So, the expected value of an offer x equals[9]

$$\widetilde{V}_E(x) = (1-\phi)V_E(x) + \phi V_U.$$

The value of unemployment is then equal to

$$V_U = h + \rho \sum_{j=1}^{\infty} p_U(j) \int_{x \in \mathcal{F}(h)} \max\left\{\widetilde{V}_E(x), V_U\right\} dF^j(x|h) + \rho p_U(0) V_U. \tag{1}$$

A similar expression holds for the value of employment. The worker gets $j \in \mathbb{N}_0$ job offers with probability $p_E(j)$. Again, the worker accepts the best offer, but only if it gives a higher payoff than rejecting it. Hence, $V_E(w)$ equals

$$V_E(w) = w + \rho \sum_{j=1}^{\infty} p_E(j) \int_{x \in \mathcal{F}(w)} \max\left\{\widetilde{V}_E(x), \widetilde{V}_E(w)\right\} dF^j(x|w) + \rho p_E(0) \widetilde{V}_E(w). \tag{2}$$

Formally, the strategy of workers can be described by acceptance sets, consisting of the wage offers that they are willing to accept given their current wage. A worker rejects all wage offers that are not part of his acceptance set. Given monotonicity of the value functions, each acceptance set can be characterized by a reservation wage. A worker currently earning $w \in \mathcal{G}'$ accepts his best wage offer x if it is higher than his reservation wage $r(w)$ and rejects it otherwise.[10]

Next, consider the firms' Bellman equations. Firms may employ multiple workers, but the payoff of hiring a specific worker is independent of the number or employment conditions of other workers in the firm, since the production function exhibits constant returns to scale. I therefore specify the value functions per individual job. Specifically, denote the firm's value of giving employment to a worker at wage w by $V_F(w)$. The firm gets an instantaneous payoff of $y - w$. The match continues in the next

[9]To simplify notation, I will suppress the dependence of parameters on the period length τ when no confusion is possible.

[10]Workers that get an offer equal to their reservation wage are indifferent between accepting and rejecting the offer. As a tie-breaking rule I assume that workers always reject in this event of measure zero. This assumption does not affect any conclusions, but simplifies notation because under the opposite assumption both unemployed and employed workers could have a periodical income equal to h.

period if the worker does not make a job-to-job transition and no job destruction shock takes place. In equilibrium, any new wage offer will always be higher than the current wage of a worker, since firms can condition their wage offers on the current wage. Hence, the worker only does not make a job-to-job transition if he does not meet any new firm. The worker's value function $V_F(w)$ therefore equals

$$V_F(w) = y - w + \rho p_E(0)(1-\phi) V_F(w). \tag{3}$$

A firm that meets a worker currently earning $w \in \mathcal{G}'$ and makes a job offer x obtains a positive payoff if it manages to hire the worker. Let $m(x|w)$ denote the firm's hiring probability. The firm faces a trade-off. Offering a higher wage increases the matching probability $m(w|x)$, since workers can compare offers. However, simultaneously it lowers the value $V_F(w)$ of the future match. Firms will offer wages such that they maximize

$$V_V(x|w) = \rho m(x|w)(1-\phi) V_F(x) \tag{4}$$

This determines the wage offer distribution F_w.

I now define an equilibrium as follows.

Definition 1. A steady state market equilibrium ('equilibrium') is a tuple $\left\{u, \{F(\cdot|w), r(w)\}_{w \in [h,y]}\right\}$ such that

1. Profit maximization: $V_V(x|w) = \max_{x'} V_V(x'|w)$ for all $x \in \mathcal{F}(w)$, and for all $w \in \mathcal{G}'$.
2. Optimal reservation wage: $\begin{cases} V_E(x) = V_U & \text{iff } x = r(h) \\ V_E(x) = V_E(w) & \text{iff } x = r(w), \text{ for all } w \in \mathcal{G}. \end{cases}$
3. Steady state: u consistent with labor market flows.

3 Equilibrium

3.1 Firms' Wage Setting

Consider a firm which meets a worker currently earning w and which has to decide what wage to offer. The worker will reject wage offers below $r(w)$, so $m(x|w) = 0$ for $x \leq r(w)$. However, wage offers in the interval $(r(w), y]$ may be acceptable to the worker. The firm faces a trade-off in this interval. A higher wage offer lowers the future per-period profit, but is more likely to be accepted by the candidate. To be precise, the worker will accept the wage offer if it is higher than all $j-1$ other wage offers. If a firm offers the job to a worker, the conditional probability that he has $j-1$ other job offers equals $\frac{jp_s(j)}{\sum_{j=1}^{\infty} jp_s(j)} = \frac{jp_s(j)}{\lambda_s \tau} = e^{-\lambda_s \tau} \frac{(\lambda_s \tau)^{n-1}}{(n-1)!}$. For $x > r(w)$, $m(x|w)$ therefore equals

$$
\begin{aligned}
m(x|w) &= \sum_{j=1}^{\infty} e^{-\lambda_s \tau} \frac{(\lambda_s \tau)^{j-1}}{(j-1)!} F^{j-1}(x|w) \\
&= e^{-\lambda_s \tau (1-F(x|w))}. \qquad (5)
\end{aligned}
$$

Hence, the number of other, better offers follows a Poisson distribution with mean equal to $\lambda_s \tau (1 - F(x|w))$. Substituting (5) and the solution to (3) into (4), gives the following expression for the expected discounted payoff for a firm offering $x > r(w)$ to a worker earning w:

$$
V_V(x|w) = e^{-\lambda_s \tau (1-F(x|w))} \frac{\rho e^{-\delta}(y-x)}{1 - \rho p_E(0) e^{-\delta}}.
$$

The firm maximizes this expression with respect to x, which determines the wage offer distribution $F(x|w)$. First I show that there is wage dispersion and that $F(x|w)$ is continuous and has connected support.

Lemma 1. *Given $r(h) < y$, in any market equilibrium, $\mathcal{F}(w)$ is not a singleton but a convex set of positive measure. Further, $F(x|w)$ is continuous.*

Proof. The proof is similar to the one of lemma 1 in Burdett and Judd (1983).[11] The intuition is as follows. The matching technology implies that a worker gets at least two job offers with probability $1 - p_s(0) - p_s(1) > 0$. In this case, the worker will compare the offers and accept the best one. This feature implies that if all firms offer the same wage $r(w) < x < y$, a deviant can do better by offering a marginally higher wage, which allows it to attract workers that compare multiple job offers with probability 1. Offering $x = y$ however, leading to a match payoff of zero, is dominated by posting $x = r(w)$ since there is a strictly positive probability $p_s(1)$ that the candidate does not compare wages. Hence, firms post wages according to a mixed strategy. Similar arguments rule out mass points and gaps in the support. □

Using this lemma, one can derive the equilibrium wage offer distribution.

Lemma 2. *Given $v > 0$ and $r(h) < y$, in any market equilibrium, firms post prices according to*

$$
F(x|w) = \begin{cases} 0 & \text{if } x \le r(w) \\ \frac{1}{\lambda_s \tau} \log\left(\frac{y-r(w)}{y-x}\right) & \text{if } x \in \mathcal{F}(w) = (r(w), \overline{x}(w)], \text{ for all } w \in [h, y] \\ 1 & \text{if } x > \overline{x}(w), \end{cases} \qquad (6)
$$

where $\overline{x}(w)$ equals

$$
\overline{x}(w) = e^{-\lambda_s \tau} r(w) + \left(1 - e^{-\lambda_s \tau}\right) y. \qquad (7)
$$

[11] See also lemma 1 in Gautier and Moraga-Gonzalez (2005).

Proof. First, note that the infimum of the support of $F(x|w)$ must equal $r(w)$. Offers below $r(w)$ are rejected and give a payoff of zero. On the other hand, if the infimum of the support would be strictly larger than $r(w)$, the firm posting the lowest wage in the market could decrease its offer and make a higher profit. Second, let $\bar{x}(w)$ denote the upper bound of the support of $F(x|w)$, hence $\mathcal{F}(w) = (r(w), \bar{x}(w)]$. The equilibrium definition now implies that the payoff for the firm must be the same for each $x \in \mathcal{F}(w)$. Hence, an expression for the wage offer distribution $F(x|w)$ for $x \in \mathcal{F}(w)$ follows from the equal profit condition$V_V(r(w)|w) = V_V(x|w)$, which is equivalent to

$$\frac{e^{-\lambda_s \tau}\rho(1-\phi)(y-r(w))}{1-\rho p_E(0)(1-\phi)} = \frac{e^{-\lambda_s \tau(1-F(x|w))}\rho(1-\phi)(y-x)}{1-\rho p_E(0)(1-\phi)}.$$

Solving for $F(x|w)$ yields

$$F(x|w) = \frac{1}{\lambda_s \tau}\log\left(\frac{y-r(w)}{y-x}\right).$$

The presented upper bound $\bar{x}(w)$ of the support of $F(x|w)$ follows from solving $F(\bar{x}(w)|w) = 1$. □

3.2 Workers' Reservation Wage

The strategy of workers consists of the decision whether or not to accept the best wage offer x given their current wage or value of home production. It is straightforward that the workers' value function of employment $V_E(w)$ is strictly increasing in w. This implies that workers follow a reservation wage strategy. The reservation wage $r(w)$ is defined by the reservation wage property

$$\begin{cases} V_U = V_E(r(h)) \\ V_E(w) = V_E(r(w)) \quad \text{for all } w \in \mathcal{G}. \end{cases} \tag{8}$$

It is immediate that $r(w) = w$ for all $w \in \mathcal{G}$, meaning that employed workers accept all wage offers higher than their current wage. Unemployed workers on the other hand take into account that if they accept a job, their job offer arrival rate might change. The following lemma shows that their reservation wage is a weighted average of h and y.

Lemma 3. *In any market equilibrium, for all $w \in \mathcal{G}'$, the workers' reservation wage function is given by*

$$r(w) = \begin{cases} \frac{h+\rho\gamma(1-\phi)(\Phi_U-\Phi_E)y}{1+\rho\gamma(1-\phi)(\Phi_U-\Phi_E)} & \text{for } w = h \\ w & \text{for all } w \in \mathcal{G}, \end{cases} \tag{9}$$

where

$$\gamma = \frac{1}{1-\rho(1-\phi)(1-\Phi_E)}. \tag{10}$$

and

$$\Phi_s = 1 - e^{-\lambda_s \tau} - \lambda_s \tau e^{-\lambda_s \tau}.$$

Proof. See appendix A.1. □

It is straightforward to check that $r(h) > h$ if $\lambda_U > \lambda_E$. In other words, workers are choosy because they realize that their job offer arrival rate will fall after accepting a job. If $\lambda_U = \lambda_E$, this motivation disappears and $r(h) = h$. This result is analogous to Burdett and Mortensen (1998).

3.3 Existence of an Equilibrium

After characterizing the firms' wage offer distribution and the workers' reservation wage function, one can now assess the existence of the steady state equilibrium. This is done in the following proposition.

Proposition 1. *A unique steady state equilibrium exists.*

Proof. Lemmas 2 and 3 describe the unique equilibrium candidates for the wage offer distribution and the reservation wage function. In order to establish existence, we still need to consider the unemployment rate u. In steady state, it can be calculated by equating inflow and outflow. Workers who are unemployed at the beginning of a period are no longer unemployed one period later if they get at least one job offer and are not immediately hit by a job destruction shock. Hence, their matching probability is $\mu_U(\tau) = \left(1 - e^{-\lambda_U \tau}\right)(1 - \phi(\tau))$, implying that outflow is equal to $\mu_U u$. On the other hand, a fraction ϕ of the employed workers loses its job such that inflow equals $\phi(1-u)$. Solving for u yields

$$u = \frac{\phi}{\phi + \mu_U}. \tag{11}$$

Hence, an equilibrium exists and it is unique. □

4 Division of Surplus

In this section, I consider how the surplus generated by a match is divided between the worker and the firm, and how this depends on the period length. I derive the earnings distribution $G(w)$, i.e. the cross-sectional wage distribution that arises in equilibrium. Before deriving this distribution it is helpful to consider two important differences between my model and standard search models.

4.1 Full Support

The support of the earnings distribution $G(w)$ ranges from the reservation wage of an unemployed worker up to the productivity level, i.e. $\mathcal{G} = (r(h), y]$. This contrasts most other search models with homogeneous agents and wage posting. In those models, the upper bound of the support of the wage distribution is typically strictly smaller than the productivity level.[12] This result is the immediate

[12] A notable exception is Delacroix and Shi (2006) when the application cost in their model goes to zero. In models that allow for some type of Bertrand competition, like Julien et al. (2000) and Postel-Vinay and Robin (2002), the wage can also equal the productivity level. However, the support of the earnings distribution typically consists of two mass points only in that case, one at $r(h)$ and one at y.

consequence of an equal profit condition. All equilibrium wage offers must give the same expected payoff to the firms, otherwise some firms would have a profitable deviation. Since wages sufficiently close to the productivity level y give a payoff of zero with probability 1, they cannot be part of the support of the wage distribution.

However, the assumption that firms are indifferent between all equilibrium wage offers may be unrealistically strong. After all, in a model with frictional wage dispersion agents who are identical ex ante can be different ex post, just because agents are luckier in the matching process than others. The different wage offers may therefore reflect the differences in outside options across workers and do not necessarily need to yield the same expected payoff. In other words, if a firm meets a worker with a high current wage (a good outside option), then it is irrelevant that matching with this worker may lead to a lower payoff than what some other firms are getting. The only relevant criterion is whether there is a positive surplus from matching and this is the case as long as the worker's current wage is below y.

4.2 Wage Ladder

A second difference between my model and models like Burdett and Mortensen (1998) is that not all wage levels can be reached directly from unemployment. In the first job after unemployment, a worker can never earn more than $\overline{x}(h)$, which is strictly lower than y. A worker who earns that upper bound and receives a job offer in the next period, can never earn more than $\overline{x}(\overline{x}(h))$ in that new job. By induction, one can show that the maximum wage $\hat{w}_{U,n}$ a worker can earn in his n^{th} job after unemployment is equal to

$$\hat{w}_{U,n} = \overline{x}^n(h) = y - e^{-\lambda_U \tau - \lambda_E \tau (n-1)} (y - r(h)) \text{ for } n \in \mathbb{N}_1.$$

Hence, the wage distribution can be seen as a wage ladder. Workers who earn a wage w have to climb one rung (i.e. find a job in $(w, \overline{x}(w)]$) before they can climb the next (a job in $(\overline{x}(w), \overline{x}(\overline{x}(w))]$).[13]

It is well established in the literature that workers coming out of unemployment earn lower wages than the population of workers as a whole. For example, Jolivet et al. (2006) documents that the wage distribution of all workers first-order stochastically dominates the distribution of entry wages in eleven different countries.[14] This fact in itself is consistent with the model of Burdett and Mortensen (1998) as well as my model. Buchinsky and Hunt (1999) however provide evidence for the existence of a wage ladder by estimating the transition probabilities between the quintiles of the wage distribution. They find that most wage mobility takes place either within a quintile or from one quintile to an adjacent quintile.

[13] Note that my definition of a wage ladder differs from the one used by authors who call the Burdett and Mortensen (1998) equilibrium a wage ladder.

[14] Arulampalam (2001), Farber (1999), Gregory and Jukes (2001) and Kletzer (1998) also report lower wages after unemployment.

Nevertheless, equilibrium search models featuring a wage ladder are rare in the search literature. In (a homogeneous-agent version of) the sequential auction model of Postel-Vinay and Robin (2002) the ladder has two rungs: workers start an employment spell at $r(h)$, but jump to y if they get an outside offer. In Carrillo-Tudela (2009a), firms observe the employment status (but not the wage) of each worker they meet and offer a low wage to unemployed workers and higher wages to employed workers. Only directed search models with on-the-job search yield a larger number of rungs. For example, Delacroix and Shi (2006) find an equilibrium wage distribution with a support consisting of a potentially large but finite number of mass points. Workers start an employment spell at a low wage, but choose to only apply to firms that offer a wage that is one rung higher than their current wage level when searching on-the-job. Consequently, the speed with which workers climb the ladder is fixed (i.e. the distribution of the current wage conditional on the previous wage is degenerate). My model allows for variation in this speed. Some workers can experience larger wage increases between two jobs than others, which is in line with empirical evidence.[15]

4.3 Earnings Distribution

I now derive the earnings distribution by considering the set $\mathcal{Z}(w)$ of employed workers earning a wage lower than w. By definition, the probability that a worker is included in this set is given by $G(w)$. I exploit the fact that in steady state inflow into and outflow from $\mathcal{Z}(w)$ should be equal. Note that workers never move to jobs paying a lower wage, implying that inflow only occurs from unemployment. Further, note that unemployed workers flow into $\mathcal{Z}(w)$ if the best wage offer they get is lower than w and if they are not immediately hit by the job destruction shock. Hence, inflow into the set of workers earning less than w, denoted by $I(w)$, equals

$$I(w) = u(1-\phi)\sum_{j=1}^{\infty} p_U(j)\,F^j(w|h). \tag{12}$$

On the other hand, outflow from $\mathcal{Z}(w)$ can occur for two reasons. First, all employed workers, irrespective of whether they make a job-to-job transition, are subject to a job destruction shock with probability ϕ in which case they flow back to unemployment. Second, workers may move to a better paying job if they get job offer paying more than w and are not hit by the shock. So, outflow $O(w)$ equals

$$O(w) = (1-u)\int_{r(h)}^{w}\left(\phi + (1-\phi)\sum_{j=1}^{\infty} p_E(j)\left(1-F^j(w|z)\right)\right)g(z)\,dz. \tag{13}$$

[15] See Shi (2009) for a model in which wage-tenure contracts create a similar mechanism.

Equating inflow and outflow and rewriting the result gives the following integral equation:

$$\frac{u}{1-u}(1-\phi)\sum_{j=1}^{\infty}p_U(j)F^j(w|h)=\phi G(w)+(1-\phi)\int_{r(h)}^{w}\sum_{j=1}^{\infty}p_E(j)\left(1-F^j(w|z)\right)g(z)\,dz. \tag{14}$$

Solving this equation gives the expression for the earnings distribution $G(w)$ and taking the first derivative of this expression yields the density $g(w)$. Note that the support of each $F(w|z)$ does not correspond to $\mathcal{G}$. Therefore, the functional form of $G(w)$ varies across subintervals on its support.

In order to derive these subintervals, I define two sets of cutoff points. The first set consists of the maximum wages $\hat{w}_{U,n}$, $n\in\mathbb{N}_1$ a worker can earn in his n^{th} job after unemployment , as derived in section 4.2. The second set follows from considering a worker that is employed at the lowest wage in the economy, i.e. $w=r^+\equiv\lim_{\varepsilon\downarrow 0}r(h)+\varepsilon$. He can never earn more than $\bar{x}(r^+)$ in his next job and never more than $\bar{x}(\bar{x}(r^+))$ in the job after that. Let $\hat{w}_{E,n}$ denote the maximum wage this worker can earn in his n^{th} job. Hence,

$$\hat{w}_{E,n}\equiv\bar{x}^n\left(r^+\right)=y-e^{-\lambda_E\tau n}\left(y-r^+\right)\text{ for }n\in\mathbb{N}_1.$$

Next, combine both sets of cutoff points and let $\hat{w}_n$ be the n^{th} order statistic of the new set, i.e. the n^{th} smallest value. Note that $\hat{w}_{U,1}>\hat{w}_{E,1}$, implying that $\hat{w}_1=\hat{w}_{E,1}$. Let $\hat{w}_{E,\hat{n}}$ be the largest $\hat{w}_{E,n}$ smaller than $\hat{w}_{U,1}$. Then it is straightforward to show that the cutoff points $\hat{w}_{U,n}$ and $\hat{w}_{E,n}$ alternate from $\hat{w}_{U,1}$ onwards. Hence

$$\begin{aligned}&\{\hat{w}_0,\hat{w}_1,\hat{w}_2,\ldots,\hat{w}_{\hat{n}},\hat{w}_{\hat{n}+1},\hat{w}_{\hat{n}+2},\hat{w}_{\hat{n}+3},\ldots\}\\&=\{r(h),\hat{w}_{E,1},\hat{w}_{E,2},\ldots,\hat{w}_{E,\hat{n}},\hat{w}_{U,1},\hat{w}_{E,\hat{n}+1},\hat{w}_{U,2},\ldots\}.\end{aligned}$$

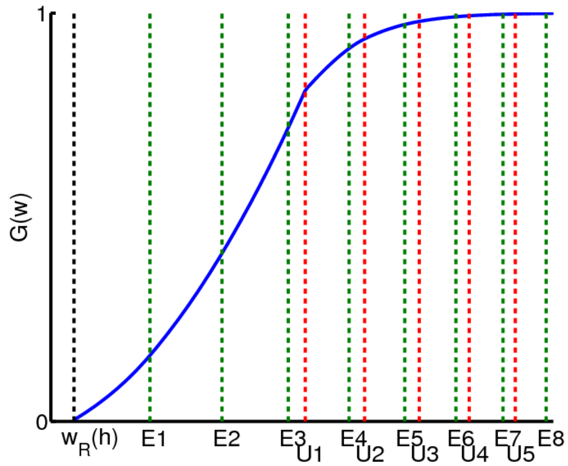
Figure 1 illustrates this by showing the cutoff points for arbitrary parameter choices.

In appendix A.2, I show that the functional form of $G(w)$ is different on each interval $(\hat{w}_{n-1},\hat{w}_n]$. Since $\lim_{n\to\infty}\hat{w}_n=y$, there exist infinitely many of such intervals. Hence, I partition the earnings distribution as follows

$$G(w)=G_n(w)\text{ for }w\in(\hat{w}_{n-1},\hat{w}_n],\,n\in\mathbb{N}_1.$$

First, I obtain a closed-form expression for $G_1(w)$. Then, I show that the elements of the wage distribution satisfy a recursive structure: knowledge of $G_{n-1}(w)$ or $G_{n-2}(w)$ is sufficient to derive $G_n(w)$. Taking derivatives yields expressions for $g_1(w)$, $g_2(w)$, $g_3(w)$, et cetera. Hence, the entire earnings density can be characterized by the initial element $G_1(w)$ and a recursive equation. This is summarized in the following lemma.

Lemma 4. *In market equilibrium, the earnings distribution is characterized by the following recursive*



Cutoff points $\{\hat{w}_0, \hat{w}_1, \hat{w}_2, \ldots, \hat{w}_{\hat{n}}, \hat{w}_{\hat{n}+1}, \hat{w}_{\hat{n}+2}, \hat{w}_{\hat{n}+3}, \ldots\} = \{r(h), \hat{w}_{E,1}, \hat{w}_{E,2}, \ldots, \hat{w}_{E,\hat{n}}, \hat{w}_{U,1}, \hat{w}_{E,\hat{n}+1}, \hat{w}_{U,2}, \ldots\}$ of the earnings distribution. Note that $\hat{w}_{E,n} < \hat{w}_{U,1}$ for $n \leq \hat{n}$ (here $\hat{n} = 3$). After $\hat{w}_{E,\hat{n}}$, the cutoff points formed by $\hat{w}_{U,i}$ and $\hat{w}_{E,\hat{n}+i}$ alternate.

Figure 1: Illustration of cutoff points.

system

$$\begin{cases} G_1(w) &= \frac{\phi}{\mu_U}(1-\phi-\mu_U)\left(\left(\frac{y-r(h)}{y-w}\right)^{\Delta_E} - 1\right) \\ G_n(w) &= C_n - \frac{\phi}{\mu_U}(1-\phi-\mu_U) + (1-\phi)\left(G_{n-1}(\underline{w}(w)) - (y-w)^{-\Delta_E}\int (y-w)^{\Delta_E} dG_{n-1}(\underline{w}(w))\right) \\ & \quad \text{if } n \in \{2,\ldots,\hat{n}+1\} \\ G_n(w) &= C_n + \phi + (1-\phi)\left(G_{n-2}(\underline{w}(w)) - (y-w)^{-\Delta_E}\int (y-w)^{\Delta_E} dG_{n-2}(\underline{w}(w))\right) \\ & \quad \text{if } n \in \{\hat{n}+2,\ldots\}, \end{cases}$$

where $\Delta_E = \frac{1}{\phi+\mu_E}$ *denotes the expected duration of a match,* $\underline{w}(w) = y - e^{\lambda_E \tau}(y-w)$*, and* C_n *is determined by* $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$*.*

Proof. See appendix A.2. □

$G(w)$ is continuous and differentiable except at $\hat{w}_{U,1}$. The complexity of $G_n(w)$ increases rapidly in n, which impedes derivation of analytical expressions. However, one can easily show that together they create a non-monotonic wage density $g(w)$, as formalized in the following lemma.

Lemma 5. *For all* $0 < \tau < \infty$*, the wage density* $g(w)$ *is 1) increasing for* w *sufficiently close to* $r(h)$*, and 2) decreasing for* w *sufficiently close to* y*.*

Proof. Note that the second derivative of $G_1(w)$ equals

$$g_1'(w) \quad = \quad \frac{\phi}{\mu_U}(1-\phi-\mu_U)\Delta_E(\Delta_E+1)(y-r(h))^{\Delta_E}(y-w)^{-\Delta_E-2}.$$

This is positive for all $w \in (r(h), \hat{w}_1]$, implying that the first statement holds.

Further, earning a wage $w > \hat{w}_{U,n}$ requires at least n consecutive job-to-job transitions without experiencing a job destruction shock. The conditional probability that a match ends with a job-to-job transition is $\frac{\mu_E}{\phi+\mu_E}$. Hence, the probability of experiencing n job-to-job transitions in a row is $\left(\frac{\mu_E}{\phi+\mu_E}\right)^n$ which tends to 0 if $n \to \infty$. Hence, $g(w) \downarrow 0$ if $w \to y$. □

Even though non-monotonic wage densities are in line with empirical evidence, very few equilibrium search models generate them without allowing for ex-ante heterogeneity. Most models with wage posting give wage densities that are strictly increasing (e.g. Burdett and Judd, 1983; Burdett and Mortensen, 1998; Kircher, 2009) or strictly decreasing (e.g. Delacroix and Shi, 2006; Galenianos and Kircher, 2009). Mortensen (2000) shows that adding match-specific capital to a Burdett-Mortensen type of framework may help. The intuition for the non-monotonicity in my model is as follows. In order to be employed at a low wage, a worker must have gotten a low wage offer after an unemployment spell and have remained there since that moment. On the other hand, in order to earn a really high salary, the worker must have experienced many consecutive job-to-job transitions without a job destruction shock in between. The probability of both events is relatively small and therefore the equilibrium fractions of workers earning these wages are small. Intermediate wage levels are much more common, also because there are several ways in which one can obtain such a salary. Some workers get this wage directly after unemployment, while others experience a couple of job-to-job transitions before finding a job paying this wage.

4.4 Effect of the Period Length

I now consider the effect of the period length τ on the wage distribution. It is important to note that the intuition for the non-monotonic shape of the wage density given above only holds for positive but finite values of τ. I therefore start by considering the limit cases $\tau \to 0$ and $\tau \to \infty$.

If $\tau \to 0$, the model basically converges to a standard continuous-time model without recall. In each infinitesimally short period, a worker receives either zero or one job offer, so there are never multiple firms competing to hire the same worker. Moreover, firms know the workers' outside options at each moment in time. These facts together imply that each wage offer is equal to the reservation wage of the worker in question. Hence, we are back in the Diamond equilibrium and get an earnings distribution that is degenerate at the level of household production h. Workers flow from unemployment to employment at rate λ_U and flow back at rate δ. Hence, the unemployment rate becomes $\frac{\delta}{\delta+\lambda_U}$. These results are summarized in the following claim.

Claim 1. If $\tau \to 0$, the reservation wage function, the offer distribution and the earnings distribution converge to

$$r(w) = w, \text{ for all } w \in \mathcal{G}'.$$

$$F(x|w) = \begin{cases} 0 & \text{if } x < w \\ 1 & \text{if } x \geq w, \text{ for all } w \in \mathcal{G}'. \end{cases}$$

$$G(w) = \begin{cases} 0 & \text{if } w < h \\ 1 & \text{if } w \geq h \end{cases}$$

At the beginning of each period, the equilibrium unemployment rate equals

$$u = \frac{\delta}{\delta + \lambda_U}.$$

Next consider what happens if the period length becomes infinitely long. In that case, workers match with probability 1 ($\lambda_U \tau, \lambda_E \tau \to \infty$). Nevertheless, unmatched agents assign zero weight to the payoffs of a potential match for two reasons. First, the payoffs of a match are only realized in the next period and the discount factor $\rho = e^{-\widehat{\rho}\tau}$ tends to zero. Second, the probability to be hit by a job destruction shock tends to one, which given the assumptions about the timing (with production taking place after job destruction) means that no output will ever be produced in the first place. As a result, all possible strategies give zero payoff to the firms and h to the workers. However, as long as the agents assign some weight to what happens in the next period, the equilibrium is unique. Unemployed workers are willing to accept any wage higher than h, since they no longer face a lower job offer arrival rate than employed workers, hence $r(h) = h$. The severe competition between firms however guarantees that all workers are offered the competitive wage, which equals the productivity level y since there are more potential job openings than workers.

Claim 2. If $\tau \to \infty$, the reservation wage function, the offer distribution and the earnings distribution converge to

$$r(w) = w, \text{ for all } w \in \mathcal{G}'.$$

$$F(x|w) = \begin{cases} 0 & \text{if } x < y \\ 1 & \text{if } x = y, \text{ for all } w \in \mathcal{G}'. \end{cases}$$

$$G(w) = \begin{cases} 0 & \text{if } w < y \\ 1 & \text{if } w = y \end{cases}$$

At the beginning of each period, the equilibrium unemployment rate equals

$$u = 1.$$

Hence, the amount of frictional wage dispersion crucially depends on the length of a period. When periods are infinitesimally short or infinitely long, the wage distribution is degenerate. For intermediate period lengths, wage dispersion is a fundamental characteristic of the equilibrium.

I illustrate the effect of time by calibrating the model. For the baseline calibration, let the period length equal one month, $\tau = 1$. This corresponds with the frequency of the employment data provided by the US Bureau of Labor Statistics, such that there are no complications arising from time aggregation. Following a similar approach as Shimer (2005, 2007), I then find an average monthly job finding probability $\mu_U(\tau)$ of 0.443 and a monthly job destruction probability $\phi(\tau)$ of 0.024 between January 1994 and December 2004.[16] These values imply $\lambda_U = 0.605$ and $\delta = 0.024$. In line with the estimates by Moscarini and Vella (2008), I set the monthly job-to-job transition probability $\mu_E(\tau)$ equal to 0.032, which corresponds to $\lambda_E = 0.033$. I further set $\widehat{h} = 0.04$, $\widehat{y} = 1$ and $\widehat{\rho} = 0.05/12$.

Taking the transition rates as given, one can then calculate the equilibrium outcomes for other values of τ. The results of this exercise are presented in table 2 for τ between 0 (continuous time) and 12 (annual periods). The table confirms that the workers' reservation wage, the average wage and the amount of wage dispersion all vary with τ. The last column reports the mean-min ratio, a measure of wage dispersion introduced by Hornstein et al. (2011). In the class of models that they consider, this statistic is independent of the period length. This no longer holds in the model presented here: an increase in the period length increases the distance between the average and the lowest wage in the market.[17]

5 Efficiency of Job-to-Job Transitions

I now extend the basic model to allow for heterogeneity in firm productivity. This makes it possible to study the allocative efficiency of the market equilibrium. In particular, I assess whether job-to-job transitions always take place from less productive to more productive firms, or also in the opposite direction.

5.1 Model with Heterogeneity

In order to keep the analysis as simple as possible, I introduce firm heterogeneity with a two-point distribution: a fraction α_i of the firms creates a periodical output equal to y_i, where $y_2 > y_1 > h$ and

[16] Note that Shimer (2005, 2007) needed to control for time aggregation since he assumed a continuous time model, i.e. $\tau = 0$. Obviously if $0 < \tau < 1$, a different correction for time aggregation is required.

[17] Note that like the models considered by Hornstein et al. (2011), my model generates a rather small value for the Mm-ratio.

τ	$\mu_0(\tau)$	$\mu_1(\tau)$	$\phi(\tau)$	Min	Mean	Std. dev.	Mm-ratio
0.00	0.000	0.000	0.000	0.400	0.401	0.000	1.001
0.25	0.140	0.008	0.006	0.754	0.773	0.010	1.025
0.50	0.258	0.016	0.012	0.832	0.858	0.013	1.030
1.00	0.443	0.032	0.024	0.884	0.918	0.015	1.038
2.00	0.668	0.061	0.047	0.909	0.955	0.016	1.051
4.00	0.827	0.113	0.093	0.905	0.979	0.016	1.082
6.00	0.841	0.157	0.136	0.885	0.990	0.014	1.118
12.00	0.747	0.246	0.253	0.799	0.999	0.005	1.250

Equilibrium outcomes for various values of the period length τ. Reported are the periodical transition probabilities ($\mu_0(\tau)$, $\mu_1(\tau)$, and $\phi(\tau)$) as well as some statistics regarding the earnings distribution (all relative to y to ease the comparison). The Mm-ratio refers to the mean-min ratio introduced by Hornstein et al. (2011).

Table 2: Calibration results

$\alpha_1+\alpha_2=1$. However it is important to stress that all intuition behind the results that I derive below holds for more general heterogeneity distributions as well. I further simplify notation by assuming that the job offer arrival rate is the same for all workers, i.e. $\lambda=\lambda_U=\lambda_E$, which implies that $r(h)=h$. Workers meet low type firms at a rate $\lambda_1=\alpha_1\lambda$ and high type firms at a rate $\lambda_2=\alpha_2\lambda$. The equilibrium distribution of wage offers by firms of type $i\in\{1,2\}$ meeting a worker earning w is denoted by $F_i(x|w)$ with support $\mathcal{F}_i(w)$. I further define $\underline{x}_i(w)=\inf\{x\in\mathcal{F}_i(w)\}$ and $\overline{x}_i(w)=\sup\{x\in\mathcal{F}_i(w)\}$, respectively representing the infimum and the supremum of the support.

Many equilibrium characteristics carry over from the homogeneous model, but a few things change. For example, consider a worker currently earning $w<y_1<y_2$. Compared to the homogeneous case, high type firms may now have an additional reason to post a high wage: an offer $x\geq y_1$ eliminates the competition from the low types firms, not only in this period but also in all future periods. In other words, the high type firms' Bellman equation of giving employment to a worker at a wage x contains a positive discrete jump at $x=y_1$ since the arrival rate of better job offers drops at that wage:

$$V_{F,2}(x)=\begin{cases}\frac{y_2-x}{1-\rho(1-\phi)e^{-\lambda\tau}} & \text{if } x<y_1\\ \frac{y_2-x}{1-\rho(1-\phi)e^{-\lambda_2\tau}} & \text{if } x\geq y_1\end{cases}.$$

Whether $x\geq y_1$ is optimal behavior for the high type firms depends on y_1 and y_2 as well as the other parameters of the model and needs to be verified. As a first result, I show that any wage offered by a high type firm is weakly larger than any wage offered by a low type firm. Hence, $x_2\geq x_1$ for all $x_2\in\mathcal{F}_2(w)$ and $x_1\in\mathcal{F}_1(w)$, or equivalently $\underline{x}_2(w)\geq\overline{x}_1(w)$.

Lemma 6. *In any equilibrium, $x_2\geq x_1$ for all $x_1\in\mathcal{F}_1(w)$ and $x_2\in\mathcal{F}_2(w)$.*

Proof. Note that $x_1 \in \mathcal{F}_1(w)$ implies $x_1 \leq y_1$, so the result is immediate if $x_2 \geq y_1$. The proof for $x_2 < y_1$ resembles equation (34) of Burdett and Mortensen (1998). The Bellman equation of a firm of type i employing a worker at a wage $x < y_1$ equals

$$V_{F,i}(x) = \frac{y_i - x}{1 - \rho(1-\phi)e^{-\lambda\tau}}.$$

If x_i is in the support of wage offers by firm of type i, it must maximize $V_{V,i}(x_i|w) = \rho m(x_i|w)(1-\phi)V_{F,i}(x_i)$ which is proportional to $m(x_i|w)(y_i - x_i)$. Hence,

$$\begin{aligned} m(x_2|w)(y_2 - x_2) &\geq m(x_1|w)(y_2 - x_1) \\ &> m(x_1|w)(y_1 - x_1) \\ &\geq m(x_2|w)(y_1 - x_2). \end{aligned}$$

Comparing the difference between the first and the last term of this equation with the difference between the middle two yields

$$m(x_2|w)(y_2 - y_1) \geq m(x_1|w)(y_2 - y_1).$$

Since $m(x|w)$ is strictly increasing in x, this implies $x_2 \geq x_1$. □

Hence, the highest wage offer that a worker receives always comes from the most productive firm that he meets in a given period. A worker's new job is therefore always welfare maximizing conditional on the worker making a transition. However, it is not obvious that the job-to-job transition itself will be efficient. Suppose for example that a worker is employed at a high type firm earning a wage $w < y_1$. With positive probability, the worker will only meet low type firms in the next period. They will offer wages $w < x < y_1$, which makes a job-to-job transition beneficial from the worker's point of view, but inefficient from a welfare perspective. Efficiency of all job-to-job transitions therefore requires a stronger condition than the one derived in lemma 6. It requires that in equilibrium high type firms eliminate the competition from low type firms by offering above y_1 for any worker that they meet, i.e. $\underline{x}_2(w) \geq y_1$ for all $w \in \mathcal{G}'$.

To analyze under what conditions this holds, first consider the optimal strategy for a low type firm meeting a worker earning a wage $w < y_1$. By lemma 6, low type firms will not be able to hire the worker if that worker also meets high type firms in the same period. Hence, a low type firm offering x matches with probability

$$m(x|w) = e^{-\lambda_2\tau - \lambda_1\tau(1-F_1(x|w))}.$$

Lemma 1 continues to hold, so solving for the distribution of wage offers in an analogous way to lemma 2 yields

$$F_1(x|w) = \frac{1}{\lambda_1\tau}\log\left(\frac{y-w}{y-x}\right)$$

with upper bound

$$\bar{x}_1(w) = e^{-\lambda_1 \tau} w + \left(1 - e^{-\lambda_1 \tau}\right) y_1.$$

Next consider a high type firm meeting a worker earning a wage $w < y_1$. The expected payoff of offering x equals

$$V_{V,2}(x|w) = \begin{cases} \rho(1-\phi) e^{-\lambda_2(1-F_2(x|w))\tau} \frac{y_2 - x}{1-\rho e^{-\lambda\tau}(1-\phi)} & \text{if } x < y_1 \\ \rho(1-\phi) e^{-\lambda_2(1-F_2(x|w))\tau} \frac{y_2 - x}{1-\rho e^{-\lambda_2\tau}(1-\phi)} & \text{if } x \geq y_1. \end{cases}$$

Because of the discontinuity in $V_{V,2}(x|w)$ at y_1, wage offers just below y_1 can never be part of the equilibrium strategy of a high type firm. Let $\widetilde{x}(w)$ denote the highest possible wage below y_1 that high type firms are willing to offer. Hence, $\widetilde{x}(w)$ follows from $V_{V,2}(\widetilde{x}(w)|w) = V_{V,2}(y_1|w)$ and $F_2(\widetilde{x}(w)|w) = F_2(y_1|w)$, which yields

$$\widetilde{x}(w) = y_2 - \frac{1-\rho(1-\phi)e^{-\lambda\tau}}{1-\rho(1-\phi)e^{-\lambda_2\tau}}(y_2 - y_1) < y_1.$$

Depending on the values of $\bar{x}_1(w)$ and $\widetilde{x}(w)$, one can distinguish between three different scenarios. The following lemma present the support $\mathcal{F}_2(w)$ of the equilibrium wage offer distribution for each of the three cases.

Lemma 7. *In equilibrium, the support $\mathcal{F}_2(w)$ of the wage offer distribution of the high type firms equals*

$$\mathcal{F}_2(w) = \begin{cases} \left(\bar{x}_1(w), e^{-\lambda_2\tau}\bar{x}_1(w) + \left(1-e^{-\lambda_2\tau}\right)y_2\right] & \text{for } w \in [h, \widetilde{w}_1] \\ \left(\bar{x}_1(w), \widetilde{x}(w)\right] \cup \left(y_1, e^{-\lambda_2\tau(1-F_2(y_1|w))}y_1 + \left(1-e^{-\lambda_2\tau(1-F_2(y_1|w))}\right)y_2\right] & \text{for } w \in (\widetilde{w}_1, \widetilde{w}_2] \\ \left(y_1, e^{-\lambda_2\tau}y_1 + \left(1-e^{-\lambda_2\tau}\right)y_2\right] & \text{for } w \in (\widetilde{w}_2, y_2] \end{cases}$$

where

$$\widetilde{w}_1 = y_1 - \left(\frac{1-\rho(1-\phi)e^{-\lambda\tau}}{1-\rho(1-\phi)e^{-\lambda_2\tau}}\frac{1}{e^{-\lambda_2\tau}} - 1\right)\frac{y_2 - y_1}{e^{-\lambda_1\tau}} < y_1$$

and

$$\widetilde{w}_2 = y_1 - \left(\frac{1-\rho(1-\phi)e^{-\lambda\tau}}{1-\rho(1-\phi)e^{-\lambda_2\tau}} - 1\right)\frac{y_2 - y_1}{e^{-\lambda_1\tau}} < y_1.$$

Proof. See appendix A.3. □

So, when the current wage w of the worker is sufficiently low, the high type firm is not affected by the discontinuity in the value function. The advantage of eliminating competition from low type firms is dominated by the large flow payoff that can be obtained by offering lower wages. Hence, the entire support will be below the low productivity level. When the current wage of the worker increases, the discontinuity starts to be become relevant. The discontinuity will split the support into two disjoint intervals, one below y_1 and one above y_1. If the current wage is sufficiently close to y_1, eliminating future interaction with low type firms becomes the most profitable strategy and the entire support will be above y_1.

This lemma now allows us to assess the efficiency of the job-to-job transitions. As argued above, efficiency of all transitions requires that $\underline{x}_2(w) \geq y_1$ for all $w \in \mathcal{G}'$. The lemma implies that $\underline{x}_2(w) \geq y_1$ for all $w > \widetilde{w}_2$. In order to have efficient job-to-job transitions, we therefore need that the payoff of an unemployed worker is sufficiently high, i.e.

Corollary 1. *Efficiency of the market equilibrium requires*

$$\begin{aligned} h &\geq \widetilde{w}_2 \\ &= y_1 - \left(\frac{1 - e^{-\widehat{\rho}\tau} e^{-\delta\tau} e^{-\lambda\tau}}{1 - e^{-\widehat{\rho}\tau} e^{-\delta\tau} e^{-\lambda_2\tau}} - 1 \right) \frac{y_2 - y_1}{e^{-\lambda_1\tau}}. \end{aligned} \tag{15}$$

Condition (15) only contains exogenous variables and can therefore easily be verified. Suppose for example that a firm's productivity equals $\widehat{y}_2 = 1$ (normalization) or $\widehat{y}_1 = 0.8$. Set the unit of observation and the length of a period both equal to one month, i.e. $\tau = 1$. Assume a discount rate of $\widehat{\rho} = 0.0042$ (i.e. 0.05 annually), a job destruction rate of $\delta = 0.025$, and job offer arrival rates $\lambda_1 = \lambda_2 = 0.25$, such that an unemployed worker's monthly matching probability equals 0.384 and the steady state unemployment rate is equal to 0.060. Then the threshold value for $\widehat{h}$ above which job-to-job transitions are efficient equals 0.624. This value is decreasing in $\widehat{y}_2$ and λ_1, since larger values for those parameters make it more attractive for high type firms to extend match durations by eliminating offers from low type firms. On the other hand, high values of $\widehat{y}_1$, λ_2, δ and $\widehat{\rho}$ make it less attractive to do so and increase therefore the cutoff value. The effect of τ on the threshold value is the focus of the next subsection.

5.2 Effect of the Period Length

I now consider the influence of the period length on the efficiency of job-to-job transitions by analyzing how the cutoff value $\widetilde{w}_2$ varies with τ. The following lemma gives a complete description of the results.

Lemma 8. *$\widetilde{w}_2$ equals $y_1 - \frac{\lambda_1}{\widehat{\rho}+\delta+\lambda_2}(y_2 - y_1)$ for τ sufficiently close to 0. Further,*

1. *if $\lambda_1 = \lambda_2 + \widehat{\rho} + \delta$, then $\frac{\partial \widetilde{w}_2}{\partial \tau} = 0$ for all τ, and $\widetilde{w}_2 = 2y_1 - y_2$ for all τ;*
2. *if $\lambda_1 > \lambda_2 + \widehat{\rho} + \delta$, then $\frac{\partial \widetilde{w}_2}{\partial \tau} < 0$ for all τ, and $\lim_{\tau \to \infty} \widetilde{w}_2 = -\infty$;*
3. *if $\lambda_1 < \lambda_2 + \widehat{\rho} + \delta$, then $\frac{\partial \widetilde{w}_2}{\partial \tau} > 0$ for all τ, and $\lim_{\tau \to \infty} \widetilde{w}_2 = y_1$.*

Proof. See appendix A.4 □

Hence, the effect of a change in the period length depends on the frequency with which a worker meets a low type firm relative to (the sum of) the arrival rate of job offers from high type firms, the discount rate and the job destruction rate. If meetings with a low type firm are relatively common, an increase in the period length increases the set of values of h for which efficiency is obtained. The opposite is true if the worker meets low type firms relatively infrequently.

The intuition for the this result is as follows. A high type firm making a job offer cares about i) the flow payoff from the match as well as ii) the expected duration of the match. It realizes that offering a wage above y_1 lowers the flow payoff but increases the expected duration compared to a wage offer below y_1 since it eliminates the competition from low type firms in the future. The loss in flow payoff is smaller when λ_1 is larger, since in that case $\bar{x}_1(w)$ is closer to y_1. At the same time, the firm assigns less weight to the future when there is more discounting (ρ is high), more job destruction (δ is high) or more workers quiting to other high type firms (λ_2 is high) . An increase in the period length intensifies both effects.

Now consider a situation in which $\lambda_1 < \lambda_2 + \widehat{\rho} + \delta$ and τ is large. High type firms assign little weight to future time periods, so the best strategy consists of maximizing the payoff in the current time period, which is achieved by offering wages below y_1 for most values of h. On the other hand, when each period is short, it is relatively more attractive to eliminate competition from low types firm in order to extend the expected duration of a match. The opposite argument applies when $\lambda_1 > \lambda_2 + \widehat{\rho} + \delta$.

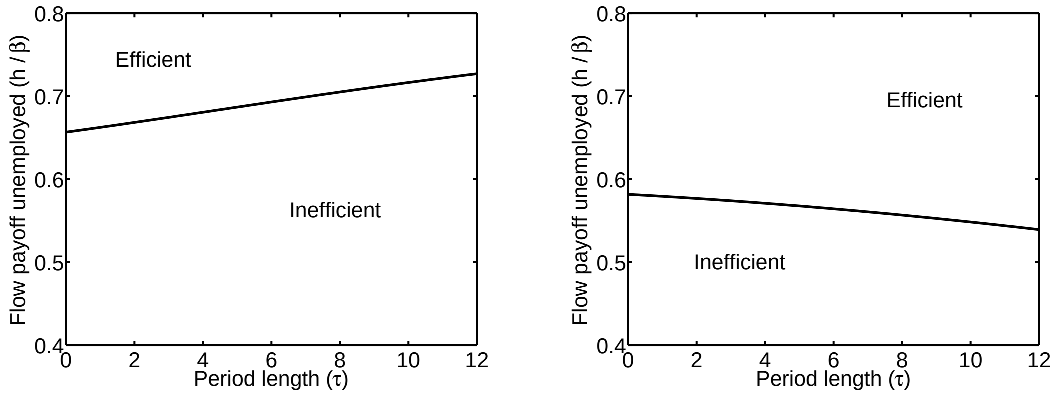
Figure 2 displays the relationship between the period length τ and the efficiency of the equilibrium for two sets of parameter values. In both panels, I keep the productivity levels, the discount rate and the job destruction rate at the same level as in the previous subsection, i.e. $\widehat{y}_2 = 1$, $\widehat{y}_1 = 0.8$, $\widehat{\rho} = 0.0042$ and $\delta = 0.025$. In the left panel, $\lambda_1 = 0.2$ and $\lambda_2 = 0.25$, such that $\lambda_1 < \lambda_2 + \widehat{\rho} + \delta$ and the cutoff value is increasing in τ. In the right panel, I assume $\lambda_1 = 0.25$ and $\lambda_2 = 0.2$, implying $\lambda_1 > \lambda_2 + \widehat{\rho} + \delta$ and therefore a negative relationship between the period length and the cutoff value.

6 Discussion

After deriving the decentralized equilibrium and analyzing the effect of a change in the period length, I now briefly discuss the relevance and the scope of the results. I argue that misspecifying the period length or the possibility of recall may affect estimates of structural parameters and may influence conclusions on optimal policy.

6.1 Estimation of Structural Parameters

The finding in section 4 that the wage distribution varies with the length of a period has potentially important implications for the estimation of structural parameters regarding the labor market. A prominent example is the estimation of productivity distributions. A popular way to estimate such distributions is from wage data, a technique explored by e.g. Bontemps et al. (1999, 2000) and Postel-Vinay



Efficiency of the equilibrium as a function of the period length τ and the payoff of an unemployed worker $\widehat{h}$. In both panels, $\widehat{y}_2 = 1$, $\widehat{y}_1 = 0.8$, $\widehat{\rho} = 0.0042$ and $\delta = 0.025$. In the left panel $\lambda_1 = 0.2$ and $\lambda_2 = 0.25$, while in the right panel the opposite holds.

Figure 2: Efficiency

and Robin (2002). The authors specify an equilibrium search model that allows for heterogeneity in productivity and estimate the parameters of the frictional wage distribution from transition rates. Subsequently, the productivity distribution can be identified from the observed wage distribution, controlling for the frictional wage dispersion. It follows from table 2 that if the data is generated by the model that I present here, misspecification of the period length will yield inconsistent estimates of the productivity distribution when they are obtained in this way.

A similar argument applies to the reservation wage and/or the flow payoff of an unemployed worker. These values play an important role in several papers analyzing the empirical content of job search models (see e.g. Shimer, 2005; Hornstein et al., 2011). However, table 2 again reveals that the link between the two strongly depends on the period length. Estimating or calibrating them is therefore not trivial.

6.2 Welfare Effects of Policy

The results derived in section 5 suggest a new potential role for policy instruments such as unemployment benefits or a binding minimum wage. Condition (15) shows that inefficiency may arise in the market equilibrium if the payoff of an unemployed worker h is too low. Higher unemployment benefits will increase h and will therefore cause low type firms to offer higher wages. This in turn makes it more attractive for high type firms to offer wages above y_1, which reduces the number of socially wasteful job-to-job transitions. A similar argument applies to a binding minimum wage.

Since the current model does not capture the negative effects of these policy instruments, such as a decrease in firm entry, I leave the determination of the optimal level for future research.[18] Lemma 8 however implies that such a study must take the nature of time into account , since the set of parameter values for which an increase in the equilibrium wage offers by low types firms is welfare enhancing crucially depends on the length of a period.

6.3 Scope of the Results

In section 5, I consider a particular distribution of firm productivity with only two points of support. However, it is easy to see that the mechanisms driving the results are valid more generally. Adding extra points of support does not change anything in the analysis. In fact, having smaller productivity differences between firms makes it harder to achieve efficiency, as can been seen from $\lim_{y_1 \to y_2} \widetilde{w}_2 = y_2$. In the limit, with a continuous distribution between h and the highest level of productivity $\bar{y}$ present in the market, efficiency is never obtained except when $\tau \to \infty$ and each firm offers the entire output of the match to the worker, i.e. $x = y$ for all $w \in \mathcal{G}'$. Nevertheless, policy will still be beneficial in reducing the level of the inefficiencies and the extent to which it can do this depends on τ.

The random search assumption is also less restrictive than it may seem at first sight. In fact, the model shares some important features with the equilibrium in directed search models because of the perfect observability of the employment contracts. In directed search models, this feature arises endogenously through the sorting of workers: in equilibrium, a firm always knows what the outside option of an applicant is. Consequently, wage offers are always acceptable and different for each worker type.[19]

7 Conclusion

This paper studies the role of the period length on the decentralized market equilibrium in a job search model. Unlike past literature, I do not impose ex ante that the interaction between workers and firms has to take place in either continuous or discrete time. Instead, I present a model in which job offers and job destruction shocks arrive according to a Poisson process, but in which information frictions may delay the workers' transition into or out of a job. This effectively results in discrete periods of arbitrary length. When the period length goes to zero, the information frictions disappear and the

[18] See also Galenianos et al. (2011) for an analysis of the effect of unemployment benefits and a minimum wage on the allocative efficiency in a directed search model.

[19] A directed search model allowing for both simultaneous and on the job search would be a combination of Delacroix and Shi (2006) and Kircher (2009). If all workers send one application in each period, the equilibrium wage distribution consists of a number of mass points, as shown by Delacroix and Shi (2006). When some or all workers send multiple applications, the number of mass points increases rapidly. An unemployed worker will apply to e.g. three different wage levels, say $w_1 < w_2 < w_3$. A worker who currently earns w_1 will apply to three higher wages, $w_{1,1} < w_{1,2} < w_{1,3}$. It follows from the analysis in Kircher (2009) however that $w_{1,1}$ does not coincide with w_2. Instead it must be the case that $w_{1,1} > w_2$ since the outside option of the employed worker while sending his lowest application is his current wage w_1, which is better than the outside option of an unemployed worker while sending his middle application (i.e. having a chance of getting w_1). Likewise $w_{1,2}$ does not coincide with w_3. Hence, a longer time period changes - and complicates - the structure of the equilibrium wage distribution significantly.

model converges to a standard continuous time model in which workers cannot recall past job offers. On the other hand, if the period length is strictly positive, a standard discrete time model with (recall of) simultaneous offers is obtained.

This setup makes it possible to analyze how the equilibrium properties vary with the length of a period. I find that the effects are profound. The length of a period determines the average wage, the amount of frictional wage dispersion and whether the allocation of workers over jobs with different productivities is efficient. As a consequence, I argue that misspecifying the period length while analyzing a labor market may lead to inconsistent estimates of productivity distributions or the flow payoff of unemployed workers. Further, wrong conclusions may be drawn on the optimal level of unemployment benefits or a minimum wage.

An empirical analysis of the period length is therefore an important avenue for further research. As discussed in the introduction, it seems that various factors may cause the period length - and therefore the importance of simultaneous search - to vary significantly across occupations. Empirical evidence supporting or contradicting this intuition is however extremely scarce. The collection of high quality data on the search and recruitment decisions of workers and firms is imperative for progress on this topic.

A Proofs

A.1 Proof of Lemma 3

In order to derive the reservation wage, consider the Bellman equations (1) and (2). Both satisfy Blackwell's (1965) sufficient conditions for a contraction mapping, implying that a unique solution for V_U and $V_E(w)$ exists. The reservation wage property implies that $V_E(r(h)) = V_U$. Conjecture that the solution for $V_E(w)$ is a linear function of w, i.e. $V_E(w) = V_U + \gamma(w - r(h))$ for some unknown constant γ. Substituting this into (2) yields

$$V_E(w) = (1 + \rho\gamma(1-\phi)\, p_E(0))\, w + \rho V_U - \rho\gamma(1-\phi)\, r(h) + \rho\gamma(1-\phi) \sum_{j=1}^{\infty} p_E(j) \int_{w}^{\bar{x}(w)} x dF^j(x|w) \tag{16}$$

Note that for all $r(w) < x \leq \bar{x}(w)$

$$\begin{aligned} \sum_{j=1}^{\infty} p_s(j)\, F^j(x|w) &= e^{-\lambda_s \tau(1-F(x|w))} - e^{-\lambda_s \tau}, \\ &= e^{-\lambda_s \tau} \frac{y - r(w)}{y - x} - e^{-\lambda_s \tau}. \end{aligned} \tag{17}$$

where the first equality follows from the fact that $p_s(j)$ is Poisson and the second from the equilibrium expression for $F(x|w)$ as given in (6). Consequently,

$$\begin{aligned} \sum_{j=1}^{\infty} p_E(j) \int_{r(w)}^{\bar{x}(w)} x dF^j(x|w) &= \int_{r(w)}^{\bar{x}(w)} x d \sum_{j=1}^{\infty} p_E(j)\, F^j(x|w) \\ &= e^{-\lambda_E \tau} \int_{r(w)}^{\bar{x}(w)} x d \frac{y - r(w)}{y - x}. \end{aligned}$$

Partial integration gives

$$\begin{aligned} \int_{r(w)}^{\bar{x}(w)} x d \frac{y - r(w)}{y - x} &= \left[x \frac{y - r(w)}{y - x} \right]_{r(w)}^{\bar{x}(w)} - \int_{r(w)}^{\bar{x}(w)} \frac{y - r(w)}{y - x} dx \\ &= \bar{x}(w) \frac{y - r(w)}{y - \bar{x}(w)} - r(w) + (y - r(w)) \left[\log(y - x) \right]_{r(w)}^{\bar{x}(w)} \\ &= \left(e^{\lambda_E \tau} - 1 \right) y - \lambda_E \tau \left(y - r(w) \right). \end{aligned}$$

Hence

$$\sum_{j=1}^{\infty} p_E(j) \int_{r(w)}^{\bar{x}(w)} x dF^j(x|w) = \lambda_E \tau e^{-\lambda_E \tau} r(w) + \left(1 - e^{-\lambda_E \tau} - \lambda_E \tau e^{-\lambda_E \tau} \right) y. \tag{18}$$

This result can be used to rewrite (16) as follows

$$\begin{aligned} V_E(w) &= w + \rho V_U + \rho\gamma(1-\phi)\left[(1-\Phi_E)(w - r(h)) + \Phi_E(y - r(h))\right] \qquad (19) \\ &= \left[r(h) + \rho V_U + \rho\gamma(1-\phi)\Phi_E(y - r(h))\right] + \left[1 + \rho\gamma(1-\phi)(1-\Phi_E)\right](w - r(h)). \end{aligned}$$

where

$$\Phi_s = 1 - e^{-\lambda_s \tau} - \lambda_s \tau e^{-\lambda_s \tau}.$$

This confirms the linear structure of $V_E(w)$. In a similar way, one can obtain the following expression for V_U

$$\begin{aligned} V_U &= h + \rho V_U - \rho\gamma(1-\phi)(1 - p_U(0))\, r(h) + \rho\gamma(1-\phi)\sum_{j=1}^{\infty} p_U(j)\int_{r(h)}^{\bar{x}(h)} x dF^j(x|h) \\ &= h + \rho V_U + \rho\gamma(1-\phi)\Phi_U(y - r(h)). \end{aligned}$$

The expressions for γ and the reservation wage $r(h)$ can be found by solving the system

$$\begin{aligned} (1-\rho)V_U &= r(h) + \rho\gamma(1-\phi)\Phi_E(y - r(h)) \\ \gamma &= 1 + \rho\gamma(1-\phi)(1-\Phi_E) \\ (1-\rho)V_U &= h + \rho\gamma(1-\phi)\Phi_U(y - r(h)). \end{aligned}$$

This yields

$$\gamma = \frac{1}{1 - \rho(1-\phi)(1-\Phi_E)}$$

and

$$r(h) = \frac{h + \rho\gamma(1-\phi)(\Phi_U - \Phi_E)\, y}{1 + \rho\gamma(1-\phi)(\Phi_U - \Phi_E)}.$$

A.2 Proof of Lemma 4

Recall that I define the following cutoff points:

$$\begin{aligned} &\{\hat{w}_0, \hat{w}_1, \hat{w}_2, \ldots, \hat{w}_{\hat{n}}, \hat{w}_{\hat{n}+1}, \hat{w}_{\hat{n}+2}, \hat{w}_{\hat{n}+3}, \ldots\} \\ &= \{r(h), \hat{w}_{E,1}, \hat{w}_{E,2}, \ldots, \hat{w}_{E,\hat{n}}, \hat{w}_{U,1}, \hat{w}_{E,\hat{n}+1}, \hat{w}_{U,2}, \ldots\}. \end{aligned}$$

The functional form of $G(w)$ and $g(w)$ is different on each interval $(\hat{w}_{n-1}, \hat{w}_n]$ and therefore I partition the earnings distribution and density as follows

$$G(w) = G_n(w) \text{ for } w \in (\hat{w}_{n-1}, \hat{w}_n], \ n \in \mathbb{N} \setminus \{0\}$$

and

$$g(w) = g_n(w) \text{ for } w \in (\hat{w}_{n-1}, \hat{w}_n], n \in \mathbb{N} \setminus \{0\}$$

I derive the earnings distribution in a recursive way. First, I obtain a closed-form expression for $G_1(w)$. After that, I derive a recursive relationship between $G_n(w)$ and $G_{n-1}(w)$ and/or $G_{n-2}(w)$. Throughout the proof I simplify notation by letting Δ_E denote the average duration of a match, i.e. $\Delta_E = \frac{1}{\phi+\mu_E}$.

A.2.1 Derivation of $G_1(w)$

I derive an expression for $G_1(w)$ by considering $\mathcal{Z}(w)$, the set of workers earning less than w, for values of w in the interval $(r(h), \hat{w}_1]$. First, recall from equation (17) that

$$\sum_{j=1}^{\infty} p_s(j) F^j(w|z) = \begin{cases} 0 & w \leq r(z) \\ e^{-\lambda_s \tau}\left(\frac{y-r(z)}{y-w} - 1\right) & w \in (r(z), \bar{x}(z)] \\ 1 - e^{-\lambda_s \tau} & w > \bar{x}(z). \end{cases} \tag{20}$$

Substituting this into the steady state equation (14) and multiplying both the left and the right hand side with $y-w$ gives

$$\frac{u}{1-u}(1-\phi-\mu_U)(w-r(h)) = (\phi+\mu_E)(y-w)G_1(w) - (1-\phi-\mu_E)\int_{r(h)}^{w}(w-z)g_1(z)\,dz.$$

Taking the first derivative with respect to w turns this expression into a differential equation. Simplifying the result by substituting $\frac{u}{1-u} = \frac{\phi}{\mu_U}$ yields

$$g_1(w) - \frac{\Delta_E}{y-w}G_1(w) = \frac{\phi}{\mu_U}(1-\phi-\mu_U)\frac{\Delta_E}{y-w}.$$

This is a first-order non-homogeneous linear differential equation. Hence, the solution for $G_1(w)$ is given by

$$G_1(w) = \exp\left(\Delta_E \int \frac{1}{y-w}dw\right)\left(\int \exp\left(-\Delta_E \int \frac{1}{y-w}dw\right)\frac{\phi}{\mu_U}(1-\phi-\mu_U)\frac{\Delta_E}{y-w}dw + C_1\right),$$

where C_1 is a constant. Solving the integrals and simplifying the resulting expression gives

$$G_1(w) = C_1(y-w)^{-\Delta_E} - \frac{\phi}{\mu_U}(1-\phi-\mu_U),$$

The value of the constant C_1 follows from the condition $G_1(r(h)) = 0$. This implies

$$C_1 = \frac{\phi}{\mu_U}(1-\phi-\mu_U)(y-r(h))^{\Delta_E}.$$

Hence,

$$G_1(w) = \frac{\phi}{\mu_U}(1-\phi-\mu_U)\left(\left(\frac{y-r(h)}{y-w}\right)^{\Delta_E}-1\right). \tag{21}$$

A.2.2 Derivation of $G_2(w),\ldots,G_{\hat{n}+1}(w)$

Next, consider $\mathcal{Z}(w)$ for $w \in (\hat{w}_{n-1}, \hat{w}_n]$, $n \in \{2,3,\ldots\hat{n}+1\}$. In these intervals the inflow still depends on w, as in the derivation of $g_1(w)$. However not all workers can leave $\mathcal{Z}(w)$ anymore by a job-to-job transition. The workers earning low wages will stay in $\mathcal{Z}(w)$ even if they experience the largest possible wage increase when moving to a new job. Let $\underline{w}(w)$ denote the minimum wage that a worker must earn in order to be able to earn $w > \hat{w}_1$ in his next job. By inverting equation (7), I obtain

$$\underline{w}(w) = e^{\lambda_E\tau}w + \left(1-e^{\lambda_E\tau}\right)y. \tag{22}$$

Note that $\underline{w}(w)$ is strictly increasing in w and that $\underline{w}(\hat{w}_{n-1}) = \hat{w}_{n-2}$, which confirms that workers earning less than $\hat{w}_{n-2}$ cannot leave $\mathcal{Z}(w)$ via a job-to-job transition. By using equation (22), I can let the integral in the right hand side of (14) start at $\underline{w}(w)$. Substituting the relevant cases of equation (20) then gives

$$\frac{u}{1-u}(1-\phi-\mu_U)\left(\frac{w-r(h)}{y-w}\right) = G(w)-(1-\phi)G(\underline{w}(w))-(1-\phi-\mu_E)\int_{\underline{w}(w)}^{w}\frac{y-z}{y-w}g(z)\,dz. \tag{23}$$

Note that $G(w) = G_n(w)$, $G(\underline{w}(w)) = G_{n-1}(\underline{w}(w))$, and that $\int_{\underline{w}(w)}^{w}\frac{y-z}{y-w}g(z)\,dz$ can be rewritten as

$$\int_{\underline{w}(w)}^{w}\frac{y-z}{y-w}g(z)\,dz = \int_{\underline{w}(w)}^{\hat{w}_{n-1}}\frac{y-z}{y-w}g_{n-1}(z)\,dz + \int_{\hat{w}_{n-1}}^{w}\frac{y-z}{y-w}g_n(z)\,dz.$$

Again, a differential equation can be created by multiplying both the left hand side and the right hand side with $y-w$ and taking the first derivative with respect to w. For the left hand side of (23) this results in $\frac{\phi}{\mu_U}(1-\phi-\mu_U)$. For the right hand side we get

$$\frac{d}{dw}G_n(w)(y-w) = g_n(w)(y-w)-G_n(w),$$

$$\frac{d}{dw}G_{n-1}(\underline{w}(w))(y-w) = g_{n-1}(\underline{w}(w))e^{\lambda_E\tau}(y-w)-G_{n-1}(\underline{w}(w)),$$

and

$$\begin{aligned}\frac{d}{dw}\int_{\underline{w}(w)}^{w}(y-z)g(z)\,dz &= (y-w)g_n(w)-e^{\lambda_E\tau}(y-\underline{w}(w))g_{n-1}(\underline{w}(w)).\\ &= (y-w)g_n(w)-e^{2\lambda_E\tau}(y-w)g_{n-1}(\underline{w}(w)).\end{aligned}$$

Hence, we get the following first-order non-homogeneous linear differential equation

$$g_n(w) - \frac{\Delta_E}{y-w} G_n(w) = \frac{\Delta_E}{y-w} \left(\frac{\phi}{\mu_U} (1-\phi-\mu_U) - (1-\phi) G_{n-1}(\underline{w}(w)) \right).$$

The solution is equal to

$$G_n(w) = C_n - \frac{\phi}{\mu_U}(1-\phi-\mu_U) - (1-\phi)\Delta_E (y-w)^{-\Delta_E} \int (y-w)^{\Delta_E - 1} G_{n-1}(\underline{w}(w))\, dw.$$

where the constant C_n follows from the condition $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$. Finally, integration by parts gives

$$G_n(w) = C_n - \frac{\phi}{\mu_U}(1-\phi-\mu_U) + (1-\phi)\left(G_{n-1}(\underline{w}(w)) - (y-w)^{-\Delta_E} \int (y-w)^{\Delta_E} dG_{n-1}(\underline{w}(w)) \right).$$

A.2.3 Derivation of $G_{\hat{n}+2}, \ldots$

Next, consider $\mathcal{Z}(w)$ for $w \in (\hat{w}_{n-1}, \hat{w}_n]$, $n \in \{\hat{n}+2, \hat{n}+3, \ldots\}$. In these intervals the inflow no longer depends on w, since unemployed workers finding a job are always hired at a wage below $\hat{w}_n$. Hence, inflow equals

$$I(w) = u\mu_U.$$

Outflow still depends on w, but again the integral in the right hand side of (14) starts at $\underline{w}(w)$. Hence, the earnings distribution is implied by

$$\frac{u}{1-u}\mu_U = G(w) - (1-\phi) G(\underline{w}(w)) - (1-\phi-\mu_E) \int_{\underline{w}(w)}^{w} \frac{y-z}{y-w} g(z)\, dz. \tag{24}$$

Note that now both $\hat{w}_{U,n}$ and $\hat{w}_{E,n}$ contribute to the cutoff points, implying that there are twice as many intervals as for $w < \hat{w}_{U,1}$. More specifically, there are two cutoff points between any w and $\underline{w}(w)$. Hence, $G(w) = G_n(w)$, $G(\underline{w}(w)) = G_{n-2}(\underline{w}(w))$, and $\int_{\underline{w}(w)}^{w} \frac{y-z}{y-w} g(z)\, dz$ can be rewritten as

$$\int_{\underline{w}(w)}^{w} \frac{y-z}{y-w} g(z)\, dz = \int_{\underline{w}(w)}^{\hat{w}_{n-2}} \frac{y-z}{y-w} g_{n-2}(z)\, dz + \int_{\hat{w}_{n-2}}^{\hat{w}_{n-1}} \frac{y-z}{y-w} g_{n-1}(z)\, dz + \int_{\hat{w}_{n-1}}^{w} \frac{y-z}{y-w} g_n(z)\, dz.$$

Create again a differential equation by multiplying both the left hand side and the right hand side of (24) with $y-w$ and taking the first derivative with respect to w. For the left hand side, this yields $-\frac{u}{1-u} e^{-\delta} \left(1 - e^{-\lambda_U}\right) = -\phi$. On the right hand side we get

$$\frac{d}{dw} G_n(w)(y-w) = g_n(w)(y-w) - G_n(w),$$

$$\frac{d}{dw}G_{n-2}\left(\underline{w}\left(w\right)\right)\left(y-w\right)=g_{n-2}\left(\underline{w}\left(w\right)\right)e^{\lambda_E\tau}\left(y-w\right)-G_{n-2}\left(\underline{w}\left(w\right)\right),$$

and

$$\begin{aligned}\frac{d}{dw}\int_{\underline{w}(w)}^{w}\left(y-z\right)g\left(z\right)dz&=\left(y-w\right)g_n\left(w\right)-e^{\lambda_E\tau}\left(y-\underline{w}\left(w\right)\right)g_{n-2}\left(\underline{w}\left(w\right)\right).\\&=\left(y-w\right)g_n\left(w\right)-e^{2\lambda_E\tau}\left(y-w\right)g_{n-2}\left(\underline{w}\left(w\right)\right).\end{aligned}$$

Hence, we obtain the following first-order non-homogeneous linear differential equation.

$$g_n\left(w\right)-\frac{\Delta_E}{y-w}G_n\left(w\right)=-\frac{\Delta_E}{y-w}\left(\phi+\left(1-\phi\right)G_{n-2}\left(\underline{w}\left(w\right)\right)\right)$$

The solution is equal to

$$G_n\left(w\right)=C_n+\phi-\left(1-\phi\right)\Delta_E\left(y-w\right)^{-\Delta_E}\int\left(y-w\right)^{\Delta_E-1}G_{n-2}\left(\underline{w}\left(w\right)\right)dw,$$

where the constant C_n follows from the condition $G_n\left(\hat{w}_{n-1}\right)=G_{n-1}\left(\hat{w}_{n-1}\right)$. Finally, integration by parts gives

$$G_n\left(w\right)=C_n+\phi-\left(1-\phi\right)\left(G_{n-2}\left(\underline{w}\left(w\right)\right)-\left(y-w\right)^{-\Delta_E}\int\left(y-w\right)^{\Delta_E-1}dG_{n-2}\left(\underline{w}\left(w\right)\right)\right).\tag{25}$$

A.3 Proof of Lemma 7

It is intuitively clear that there are three candidates for the equilibrium strategy of a high type firm.

1. Low Wages: If $\overline{x}_1\left(w\right)$ is sufficiently low compared to $\widetilde{x}(w)$, the high type firms are not affected by the discontinuity in the value function and the entire support will be below $\widetilde{x}(w)<y_1$;
2. Intermediate Wages: If $\overline{x}_1\left(w\right)$ takes an intermediate value relative to $\widetilde{x}(w)$, the support of $F_2\left(w\right)$ will be split into two parts, i.e. $\mathcal{F}_2\left(w\right)=\left(\overline{x}_1\left(w\right),\widetilde{x}(w)\right]\cup\left(y_1,\overline{x}_2\left(w\right)\right]$ with $\overline{x}_2\left(w\right)>y_1$;
3. High Wages: If $\overline{x}_1\left(w\right)>\widetilde{x}(w)$, any wage offer must be larger than y_1, hence $\mathcal{F}_2\left(w\right)=\left(y_1,\overline{x}_2\left(w\right)\right]$.

I analyze each of the cases in more detail.

Low Wages. Suppose that high type firms offer wages on $\mathcal{F}_2\left(w\right)=\left(\overline{x}_1\left(w\right),\overline{x}_2\left(w\right)\right]$ with $\overline{x}_2\left(w\right)<\widetilde{x}(w)$. Then, by lemma 2 we have

$$\begin{aligned}\overline{x}_2\left(w\right)&=e^{-\lambda_2\tau}\overline{x}_1\left(w\right)+\left(1-e^{-\lambda_2\tau}\right)y_2\\&=e^{-\lambda_1\tau}e^{-\lambda_2\tau}w+e^{-\lambda_2\tau}\left(1-e^{-\lambda_1\tau}\right)y_1+\left(1-e^{-\lambda_2\tau}\right)y_2.\end{aligned}$$

Consistency requires $\overline{x}_2\left(w\right)<\widetilde{x}(w)$ which is equivalent to

$$w < y_1 - \left(\frac{1-\rho\left(1-\phi\right)e^{-\lambda\tau}}{1-\rho\left(1-\phi\right)e^{-\lambda_2\tau}} \frac{1}{e^{-\lambda_2\tau}} - 1 \right) \frac{y_2 - y_1}{e^{-\lambda_1\tau}}.$$

Intermediate Wages. For high type firms to offer intermediate wages with split support, we must have that $\bar{x}_1(w) < \widetilde{x}(w)$ but $e^{-\lambda_2\tau}\bar{x}_1(w) + \left(1-e^{-\lambda_2\tau}\right)y_2 > \widetilde{x}(w)$. This implies

$$w > y_1 - \left(\frac{1-\rho\left(1-\phi\right)e^{-\lambda\tau}}{1-\rho\left(1-\phi\right)e^{-\lambda_2\tau}} \frac{1}{e^{-\lambda_2\tau}} - 1 \right) \frac{y_2 - y_1}{e^{-\lambda_1\tau}}$$

but

$$w < y_1 - \left(\frac{1-\rho\left(1-\phi\right)e^{-\lambda\tau}}{1-\rho\left(1-\phi\right)e^{-\lambda_2\tau}} - 1 \right) \frac{y_2 - y_1}{e^{-\lambda_1\tau}}.$$

Equal profit implies

$$F_2\left(x|w\right) = \begin{cases} \frac{1}{\lambda_2\tau}\log\left(\frac{y_2-\bar{x}_1(w)}{y_2-x}\right) & \text{for } x < y_1 \\ F_2\left(y_1|w\right) + \frac{1}{\lambda_2\tau}\log\left(\frac{y_2-y_1}{y_2-x}\right) & \text{for } x > y_1. \end{cases}$$

Therefore, $\bar{x}_2(w)$ equals

$$\bar{x}_2(w) = e^{-\lambda_2\tau(1-F_2(y_1|w))}y_1 + \left(1-e^{-\lambda_2\tau(1-F_2(y_1|w))}\right)y_2.$$

High Wages. For high type firms to offer high wages, we need $\bar{x}_1(w) > \widetilde{x}(w)$, which is equivalent to

$$w > y_1 - \left(\frac{1-\rho\left(1-\phi\right)e^{-\lambda\tau}}{1-\rho\left(1-\phi\right)e^{-\lambda_2\tau}} - 1 \right) \frac{y_2 - y_1}{e^{-\lambda_1\tau}}.$$

A.4 Proof of Lemma 8

To simplify notation, let $\xi = \widehat{\rho} + \delta + \lambda_2$. Then

$$\widetilde{w}_2 = y_1 - \frac{e^{-\xi\tau}}{1-e^{-\xi\tau}} \frac{1-e^{-\lambda_1\tau}}{e^{-\lambda_1\tau}}\left(y_2 - y_1\right).$$

Letting τ go to 0 and applying l'Hospital yields

$$\widetilde{w}_2 = y_1 - \frac{\lambda_1}{\xi}\left(y_2 - y_1\right).$$

The derivative of $\widetilde{w}_2$ with respect to τ equals

$$\frac{\partial \widetilde{w}_2}{\partial \tau} = -\frac{\lambda_1 \left(1-e^{-\xi\tau}\right) - \xi\left(1-e^{-\lambda_1\tau}\right)}{\left(1-e^{-\xi\tau}\right)^2} \frac{e^{-\xi\tau}}{e^{-\lambda_1\tau}} \left(y_2 - y_1\right),$$

which equals 0 if $\lambda_1 = \xi$ and if $\lambda_1 = 0$.

Next, consider the cross-derivative to τ and λ_1:

$$\frac{\partial}{\partial \lambda_1} \frac{\partial \widetilde{w}_2}{\partial \tau} = \left(\frac{e^{-\xi\tau}}{1-e^{-\xi\tau}}\right)^2 \frac{1}{e^{-\lambda_1\tau}} \left(1 + \lambda_1\tau + \frac{1}{e^{-\xi\tau}}\left(-1+\xi\tau-\lambda_1\tau\right)\right).$$

Note that $\frac{\partial}{\partial \lambda_1}\frac{\partial \widetilde{w}_2}{\partial \tau}$ is continuous in λ_1 and that $\frac{\partial}{\partial \lambda_1}\frac{\partial \widetilde{w}_2}{\partial \tau} = 0$ has only one solution, i.e.

$$\lambda_1 = \frac{e^{-\xi\tau} - 1 + \xi\tau}{\tau\left(1-e^{-\xi\tau}\right)}.$$

Further,

$$\lim_{\lambda_1 \to 0} \frac{\partial}{\partial \lambda_1} \frac{\partial \widetilde{w}_2}{\partial \tau} = \left(\frac{1}{1-e^{-\mu\tau}}\right)^2 e^{-\xi\tau}\left(e^{-\xi\tau} - 1 + \xi\tau\right) > 0$$

and

$$\lim_{\lambda_1 \to \infty} \frac{\partial}{\partial \lambda_1} \frac{\partial \widetilde{w}_2}{\partial \tau} = -\infty.$$

Consequently, $\frac{\partial \widetilde{w}_2}{\partial \tau} > 0$ for all $\lambda_1 < \xi$ and $\frac{\partial \widetilde{w}_2}{\partial \tau} < 0$ for all $\lambda_1 > \xi$.

Finally, note that $\widetilde{w}_2$ can be written as

$$\widetilde{w}_2 = y_1 - \frac{1-e^{-\lambda_1\tau}}{e^{\mu\tau}e^{-\lambda_1\tau} - e^{-\lambda_1\tau}} \left(y_2 - y_1\right),$$

after which it is straightforward to show that

$$\lim_{\tau \to \infty} \widetilde{w}_2 = \begin{cases} y_1 & \text{if } \lambda_1 < \xi \\ 2y_1 - y_2 & \text{if } \lambda_1 = \xi \\ -\infty & \text{if } \lambda_1 > \xi. \end{cases}$$

References

Acemoglu, D. and Shimer, R. (1999). Holdups and efficiency with search frictions. *International Economic Review*, 40:827–851.

Albrecht, J. W. and Axell, B. (1984). An equilibrium model of search unemployment. *Journal of Political Economy*, 92(5):824–840.

Albrecht, J. W., Gautier, P. A., and Vroman, S. B. (2006). Equilibrium directed search with multiple applications. *Review of Economic Studies*, 73:869–891.

Arulampalam, W. (2001). Is unemployment really scarring? Effects of unemployment experiences on wages. *Economic Journal*, 111:585–606.

Blackwell, D. (1965). Discounted dynamic programming. *Annals of Mathematical Statistics*, 36(1):226–235.

Bontemps, C., Robin, J.-M., and van den Berg, G. J. (1999). An empirical equilibrium job search model with search on the job and heterogeneous workers and firms. *International Economic Review*, 40:1039–1074.

Bontemps, C., Robin, J.-M., and van den Berg, G. J. (2000). Equilibrium search with continuous productivity dispersion: Theory and non-parametric estimation. *International Economic Review*, 41(2):305–358.

Buchinsky, M. and Hunt, J. (1999). Wage mobility in the United States. *Review of Economics and Statistics*, 81:351–368.

Burdett, K. and Coles, M. G. (2003). Equilibrium wage-tenure contracts. *Econometrica*, 71(5):1377–1404.

Burdett, K. and Judd, K. L. (1983). Equilibrium price dispersion. *Econometrica*, 51:955–969.

Burdett, K. and Mortensen, D. T. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, 39(2):257–273.

Burdett, K., Shi, S., and Wright, R. (2001). Pricing and matching with frictions. *Journal of Political Economy*, 109:1060–1085.

Carrillo-Tudela, C. (2009a). An equilibrium search model when firms observe workers' employment status. *International Economic Review*, 50(584):485–506.

Carrillo-Tudela, C. (2009b). An equilibrium search model with optimal wage-experience contracts. *Review of Economic Dynamics*, 12:108–128.

Carrillo-Tudela, C., Menzio, G., and Smith, E. (2011). Job search with bidder memories. *International Economic Review*, 52(3):639–655.

Delacroix, A. and Shi, S. (2006). Directed search on the job and the wage ladder. *International Economic Review*, 47(2):651–699.

Eeckhout, J. and Kircher, P. (2010). Sorting vs screening - prices as optimal competitive sales mechanisms. *Journal of Economic Theory*, 145:1354–1385.

Farber, H. (1999). *Handbook of Labor Economics*, volume 3, chapter Mobility and Stability: The Dynamics of Job Changes in Labor Market, pages 2439–2483. New Holland, Amsterdam.

Galenianos, M. and Kircher, P. (2009). Directed search with multiple job applications. *Journal of Economic Theory*, 114:445–471.

Galenianos, M., Kircher, P., and Virag, G. (2011). Market power and efficiency in a search model. *International Economic Review*, forthcoming.

Gautier, P. A. and Moraga-Gonzalez, J. L. (2005). Strategic wage setting and coordination frictions with multiple job applications. mimeo.

Gautier, P. A., Moraga-Gonzalez, J. L., and Wolthoff, R. P. (2009). Structural estimation of search intensity: Do non-employed workers search enough? mimeo.

Gautier, P. A. and Wolthoff, R. P. (2009). Simultaneous search with heterogeneous firms and ex post competition. *Labour Economics*, 16:311–319.

Gregory, M. and Jukes, R. (2001). Unemployment and subsequent earnings: Estimating scarring among British men 1984-94. *Economic Journal*, 111:607–625.

Hornstein, A., Krusell, P., and Violante, G. (2011). Frictional wage dispersion in search models: A quantitative assessment. *American Economic Review*, forthcoming.

Jolivet, G., Postel-Vinay, F., and Robin, J.-M. (2006). The empirical content of the job search model: Labor mobility and wage distributions in Europe and the US. *European Economic Review*, 50(4):877–907.

Julien, B., Kennes, J., and King, I. (2000). Bidding for labor. *Review of Economic Dynamics*, 3:619–649.

Julien, B., Kennes, J., and King, I. (2006). Residual wage disparity and coordination unemployment. *International Economic Review*, 47(3):961–989.

Kaas, L. (2010). Variable search intensity in an economy with coordination unemployment. *B.E. Journal of Macroeconomics (Contributions)*, forthcoming.

Kircher, P. (2009). Efficiency of simultaneous search. *Journal of Political Economy*, 117:861–913.

Kletzer, L. (1998). Job displacement. *Journal of Economic Perspectives*, 12:115–136.

Menzio, G. and Shi, S. (2010a). Block recursive equilibria for stochastic models of search on the job. *Journal of Economic Theory*, 145(4):1453–1494.

Menzio, G. and Shi, S. (2010b). Directed search on the job, heterogeneity and aggregate fluctuations. *American Economic Review*, 100(2):327–332.

Menzio, G. and Shi, S. (2010c). Efficient search on the job and the business cycle. mimeo.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of Political Economy*, 105:385–411.

Mortensen, D. T. (2000). *Panel Data and Structural Labour Market Models*, chapter Equilibrium Unemployment with Wage Posting: Burdett-Mortensen Meet Pissarides., pages 281–292. North-Holland, Amsterdam.

Mortensen, D. T. and Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, 61:397–415.

Moscarini, G. and Vella, F. (2008). Occupational mobility and the business cycle. mimeo.

Pissarides, C. A. (1994). Search unemployment with on-the-job search. *Review of Economic Studies*, 61:457–475.

Postel-Vinay, F. and Robin, J.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70:2295–2350.

Shi, S. (2009). Directed search for equilibrium wage-tenure contracts. *Econometrica*, 77:561–584.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Shimer, R. (2007). Reassessing the ins and outs of unemployment. mimeo.

Wolthoff, R. P. (2010). Applications and interviews: A structural analysis of two-sided simultaneous search. mimeo.