Wage Setting Protocols and Labor Market Conditions: Theory and Evidence*

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Abstract

We theoretically and empirically examine how firms' choices of wage-setting protocols respond to labor market conditions. We develop a simple model in which workers can send multiple job applications and firms choose between posting wages and Nash bargaining. Posting a wage allows the firm to commit to lower wages than would be negotiated ex-post, but eliminates the ability to respond to a competing offer, should the worker have one. The model makes predictions about the joint correlation between the application-vacancy ratio, the number of applications per worker, and the incidence of wage posting. We find empirical support for these predictions in a novel dataset from an online job board. Our theory also implies that an increase in labor market competition may manifest itself through the incidence of wage posting rather than a change in the posted wages themselves; and that labor market regulations such as pay transparency laws have redistributive equilibrium effects by disproportionately benefiting workers with few applications.

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1 Introduction

How wages are determined in frictional labor markets, and how wage determination depends on economic conditions, are classic yet unsettled economic questions. According to one prominent paradigm, wages are posted by firms, and workers select the best offer when faced with multiple offers (e.g. as in Burdett and Judd, 1983; Burdett and Mortensen, 1998). According to another classic paradigm, wages are not committed to, but instead determined by bilateral bargaining between workers and their employers (e.g. as in Pissarides, 2000), and possibly renegotiated in case of receiving a competing job offer (e.g. as in Cahuc et al., 2006). These strands of the literature have operated in parallel and, with a few exceptions discussed below, have largely been separate.

This parallel but separate treatment of distinct wage-setting protocols is restrictive. As shown by Hall and Krueger (2012), job postings with and without explicit wages coexist in the data. Moreover, there may well be important interactions between firms who post wages and firms who don't. For example, the wages firms choose to post may be affected by how many firms opt to not commit to wages. Morevoer, the decision of whether to post a wage at all may depend on the wages posted by other firms. The coexistence of wage-setting modes raises important substantive questions. What determines the choice of wage-setting protocol? How does it vary with labor market conditions? Are the effects of economic shocks or policies amplified or dampened by firms' choices to post or bargain? Finally, some policy questions, such as the effects of pay transparency laws that effectively require firms to post explicit wage offers, are hard to address altogether without a framework with endogenous wage-setting protocols. To a large extent, the big obstacle to making progress in these dimensions has been the availability of data. We contribute to this effort by theoretically analyzing a simple model with endogenous wage-setting protocols; and by empirically confirming the model's predictions in a novel dataset with information on the number of applications submitted by job seekers, as well as on firms' wage-posting choices.

Theory. The theory is a simple static random search model[1] that introduces the possibility of wage bargaining into the classic framework of Burdett and Judd (1983). Firms post vacancies, at a cost, and decide whether or not to commit to a wage. Workers send either one or two applications each, and may therefore obtain multiple offers. The novelty is that not all firms may have chosen to post a wage. If all the job offers obtained by a worker come

[1]As emphasized by Cheremukhin and Restrepo-Echavarria (2020), obtaining co-existence of wage posting and bargaining in a directed-search environment is quite challenging. Our motivation here is to analyze the implications of co-existing wage-setting protocols in a deliberately simple environment. We discuss this further in section 3.2.

from firms who posted a wage, the worker simply chooses the best offer, as in Burdett and Judd (1983). If a firm has not posted a wage, and it is the worker's only match, the worker and firm engage in Nash bargaining, as in the classic Pissarides (2000) model. If a worker has two offers, one from a firm who posted a wage and the other from a firm who did not, the latter firm bargains with the worker, with the worker's outside option being accepting the posted wage of the former firm. Finally, if the worker has met two firms and neither firm has posted a wage, the two firms engage in Bertrand competition for the worker. Equilibrium is determined such that firms' posting vs. bargaining choices are made optimally. In particular, if some firms post and others bargain, each firm is indifferent between bargaining and posting.

The model incorporates the classic tradeoff for a firm who decides to post a wage: a higher wage results in lower profits conditional on hiring, but leads to a higher probability of hiring, more so if there is a larger chance of the worker having a second offer. The novelty is the tradeoff involved when deciding whether to post a wage at all. On the one hand, by posting a wage, the firm is able to commit to a lower wage than would be negotiated ex-post. On the other hand, by posting (and hence committing to) a wage, the firm eliminates the option to respond to a competing offer ex-post, should the worker have one. This logic suggests that the relative benefits of posting a wage are higher when there is a lower chance that the worker has a competing offer. Because firms' entry – and hence the number of offers – is determined endogenously, the probability that the worker has a competing offer is (in contrast to Burdett and Judd, 1983) dependent on labor market conditions, e.g. productivity. This is the case even though we assume the number of applications per worker is fixed. Instead, it is the probability that an application is successful that varies endogenously.

The model yields results consistent with the above intuition. An increase in productivity raises firms entry and thereby lowers the ratio of applications per vacancy, which is the relevant measure of labor market slack in our model. This raises the probability that an individual application is successful, and therefore the probability that an applicant at a firm has a competing offer. Consistent with the above intuition, firms then have a weaker incentive to post a wage. This leads to the first main result: an increase in productivity lowers the application-vacancy ratio and lowers the fraction of firms who post wages. Our second result has to do with the effect of increasing the fraction of workers who send two rather than one application. This increase in average applications per worker likewise increases the chance that an applicant at a given firm has a competing offer, and therefore also lowers the fraction of firms who post wages.

The second set of results has to do with the effect on the distribution of posted wages. In the classic Burdett and Judd (1983) framework, either an increase in productivity or an increase in the fraction of workers sending two applications would raise posted wages, in the stochastic dominance sense. An increase in productivity raises equilibrium posted wages by raising the firm's benefit from hiring a worker. An increase in the fraction of workers sending two applications raises equilibrium posted wages by increasing the chance that a firm's applicant has a competing offer. Consistent with these results, our model implies that an increase in productivity raises equilibrium posted wages. However, an increase in the fraction of workers sending two applications has no effect on posted wages, as long as firms are indifferent between bargaining and posting. Instead, in response to an increase in expected applications per worker, it is the fraction of firms posting wages that adjusts, rather than the distribution of wages posted by those firms.

Empirical analysis. Our model makes predictions regarding how the fraction of firms advertising an explicit wage, and the average wage they post, varies with labor market conditions, such as productivity and the number of applications per worker. In our empirical analyses, we test the model's predictions using proprietary data from www.Dice.com (henceforth, the DHI data), an online job board.[2] The DHI data links the universe of vacancy postings advertised on Dice.com between January 1, 2012 and December 31, 2017 to all of the job seekers that applied to them. On the job posting side, the data includes detailed information about the job posting, such as the position's job title, location, tax-terms, work-schedule, and – crucially for our analysis – whether the vacancy offers an explicit wage (or wage range) and, if so, the amount offered. On the applicant side, the data includes the candidate's job title, their location, and the exact time-stamp and job ID for each of their applications.

In addition to providing information about the incidence of wage posting and wage levels, the data allow us to construct two key measures of labor market conditions: the application-vacancy ratio and the number of applications per applicant, both if which have model counterparts. Because productivity is not observed directly, we devise an alternative strategy to test the theoretical predictions discussed above. The theory implies that, to the extent that productivity is driving the variation in the data, it would induce a *positive* correlation between the application-vacancy ratio and the fraction of firms posting a wage. It would also induce a negative correlation between the application-vacancy ratio and the average posted wage. On the other hand, if the variation in the data is driven by variation in applications per worker, the model implies that such variation would induce a *negative* correlation between the application-vacancy ratio and the fraction of firms posting a wage, and *no* correlation between the application-vacancy ratio and the average posted wage.

This set of predictions can be taken to the data. The first implication is that a regression

[2] See Davis and Samaniego de la Parra (2019) for a detailed description of the data.

of the incidence of wage posting on both the application-vacancy ratio and a measure of intensive-margin labor market competition (i.e. applications per worker) is expected to produce a positive coefficient on the former and a negative coefficient on the latter. Moreover, omitting applications per worker results in downward omitted variable bias, since the coefficient on the application-vacancy ratio now captures the combined effect of labor demand-side factors (such as productivity) and applications per worker. Turning to the magnitude of posted wages, our results imply that the average posted wage should be negatively associated with the application-vacancy ratio and positively associated with applications per worker, if both variables are controlled for. Omitting applications per worker would introduce upward omitted variable bias. We show that these predictions are confirmed in the empirical analyses.

Implications. Our theory implies that the incidence of wage posting is endogenous and therefore not invariant to labor market conditions. Our empirical analyses confirm this by showing that the incidence of wage posting covaries systematically with labor market slack. This result underscores the importance of modeling the wage-setting choice of a firm, as this wage-setting choice can compound the effects of economic shocks on labor market outcomes. The model with endogenous choice of wage-setting protocol also makes qualitatively different predictions from a model where a particular wage-setting protocol is assumed exogenously. Consider an increase in average applications per worker, which through the lens of our model, can be thought of as a measure of competition between employers. The standard Burdett and Judd (1983) framework implies that such an increase would induce employers to post higher wages and thereby affect workers' realized wages. In our model, an increase in applications per worker certainly affects labor market outcomes, but it does so not through a change in posted wages, but a change in the number of firms who post.

Finally, we use the model to understand the effect of a change in worker bargaining power, and the effect of a pay transparency regulation that requires firms to post. We first obtain a stark invariance result that either an increase in worker bargaining power or a prohibition on bargaining would leave the average expected wage unaffected. In spite of this, a prohibition on bargaining has redistributive effects: it raises the expected wage of workers who send one application, and lowers the expected wage of workers who send two. Intuitively, workers sending multiple applications benefit the most from bargaining by being able to engage employers in Bertrand competition, and therefore lose the most when it is prohibited. More indirectly, such a prohibition on bargaining has an equilibrium effect: it leads posted wages to rise, disproportionately benefiting the workers sending only one application.

2 Relationship to the Literature

A growing foundational literature identifies and analyzes economic environments in which wage posting and wage bargaining can coexist in equilibrium; see e.g. Postel-Vinay and Robin (2004), Michelacci and Suarez (2006), Cheremukhin and Restrepo-Echavarria (2020), Flinn and Mullins (2021), and Doniger (2023). Most of the literature produces co-existence of different wage-setting protocols by relying on heterogeneity (in productivity, market tightness, and/or search costs) and, in most theoretical frameworks, asymmetric information. The novel targeted-search framework of Cheremukhin and Restrepo-Echavarria (2020) stands out within this literature, producing co-existence of both wage-setting protocols without asymmetric information. We rely on the insights from this prior work to identify random search (as in Burdett and Judd (1983)) as a sufficient environment that allows for the co-existence of bargaining and wage posting while maintaining a very simple, homogeneous framework.[3] The specific mechanism in our framework is closest to Doniger (2023) and Flinn and Mullins (2021), who both consider environments with on-the-job search nesting Burdett and Mortensen (1998) and Cahuc et al. (2006). In Doniger (2023), renegotiation in response to competing offers is costly. In Flinn and Mullins (2021), the tradeoff is most similar to ours, though the specific comparative statics results of our model are novel and different. The key difference is where the outside offers come from. Whereas on-the-job search generates competing offers in Flinn and Mullins (2021) similarly to Burdett and Mortensen (1998) and Cahuc et al. (2006), in our framework they come from receiving multiple offers as in Burdett and Judd (1983). We also endogenize the probability of receiving multiple offers, thereby allowing the choice of wage-setting to respond to labor market conditions, leading to both a key feature of our mechanism and key testable predictions. We are able to test the model predictions empirically thanks to a novel dataset in which the number of applications per worker – an object with a clear model counterpart in the Burdett and Judd (1983) framework – can be measured.

On the empirical front, Hall and Krueger (2012) and Brenzel et al. (2014) document the co-existence of wage bargaining and wage posting. Banfi and Villena-Roldan (2018) and Marinescu and Wolthoff (2020) examine whether higher posted wages attract more applicants. We contribute to this literature by showing that the incidence of wage posting covaries systematically with measures of labor market slack. Our paper is also relevant for the literature on labor market competition and its effect on wages. Azar et al. (2020) and Azar et al. (2022) argue that labor market power, as measured by employer concentration, is relevant

[3]Without a framework that nests directed search, we are not equipped to say unequivocally whether the random-search assumption is necessary. However, our analysis illustrates that it is sufficient.

for wages. We view our work as complementary and conceptually related, though distinct: in our analysis, it is the number of applications per worker that serves as a measure of labor market competition; furthermore, we argue that an increase in labor market competition can manifest itself not only (or not necessarily) through the magnitude of posted wages but through the fraction of firms who post wages at all.

3 Theory

3.1 Environment

We consider a static model with an exogenous measure u of workers and a larger measure of firms. Each firm can post one vacancy at cost κ. Workers apply to firms randomly and get matched according to the process described below. A matched worker and firm produce y, where $y > \kappa$, whereas workers and firms that remain unmatched produce zero.

Matching. An exogenous fraction $\alpha \in [0, 1]$ of workers sends out 2 applications each. The remaining fraction $1 - \alpha$ of workers sends out one application each. The total number of applications is therefore

$$s = u(1 - \alpha + 2\alpha) = u(1 + \alpha).$$

There is a meeting technology that matches applications to vacancies bilaterally. If v is the measure of vacancies and s is the measure of applications, the total number of meetings is given by $M(v, s)$, where M is concavely increasing in both arguments, exhibits constant returns to scale, and satisfies $0 < M(v, s) \leq \min\{v, s\}$ as well as $dM(0, s)/dv = dM(v, 0)/ds = 1$. We define the vacancy-unemployment ratio $\theta = v/u$, and the application-vacancy ratio $\lambda = s/v = (1 + \alpha)/\theta$. The probability that a vacancy receives an application is then

$$\frac{M(v, s)}{v} = M\left(1, \frac{s}{v}\right) \equiv q(\lambda).$$

while the probability that an application gets matched to a vacancy is given by

$$\frac{M(v, s)}{s} = M\left(\frac{v}{s}, 1\right) \equiv p(\lambda) = q(\lambda)/\lambda.$$

Our assumptions regarding the meeting function M imply that q is increasing in λ from $q(0) = 0$ to $q(\lambda) \to 1$ when $\lambda \to \infty$. In contrast, p is decreasing in λ from $p(0) = 1$ to $p(\lambda) \to 0$ for $\lambda \to \infty$.

Competition. We next derive the probability, from a firm's point of view, that the worker who has a job offer from that firm also has a competing job offer. Conditional on receiving an application, the probability that the worker sending the application has sent a second application equals $\frac{2\alpha}{1+\alpha}$, and the probability that the latter application is successful is $p(\lambda)$. Therefore, the probability that the worker has another job offer is equal to

$$\psi \equiv \frac{2\alpha}{1+\alpha} p(\lambda). \tag{1}$$

This will be a key object from the firm's standpoint, as it determines the probability that the firm is competing with another firm for the applicant. This probability is increasing in the application-vacancy ratio λ and will therefore be endogenously determined.

Wage setting protocols. Each firm decides whether to post a wage (as in Burdett and Judd, 1983) or to bargain (as in Cahuc et al., 2006). This decision is made before meeting an applicant and hence *before* the firm knows whether its applicant has a competing offer. The wage in the match is then determined as follows. If the worker only has one job offer, and that firm posted a wage w, the worker either accepts or rejects. If the worker has only one job offer from a firm that chose to bargain, the worker receives βy, and the firm receives $(1-\beta)y$, where $\beta \in (0,1]$ is worker's bargaining weight.[4] Next, consider a worker with two job offers. If both firms have posted a wage, the worker picks the firm with the higher wage and becomes employed at that wage, with the other firm remaining vacant; if both firms posted the same wage, one is chosen at random. If both firms have chosen to bargain, the two firms engage in Bertrand competition over the worker, which drives the wage to y (and the worker randomly picks one of the firms). Finally, if one of the firms has chosen to bargain and the other has posted a wage of w, then the wage with the first firm is decided by Nash bargaining, with worker bargaining weight β and the outside option being the wage w at the second firm. The outcome is that the worker is employed by the bargaining firm at the wage of $\beta y + (1-\beta)w$.

3.2 Discussion of modeling choices

We model the coexistence of wage posting and wage bargaining in a random search framework building on Burdett and Judd (1983). The choice of a random-search framework is motivated by the goal of integrating wage posting and wage bargaining while keeping the framework as simple as possible. As mentioned above, the literature on the microfoundations of wage-setting protocols demonstrates that generating both bargaining and posting

[4]We restrict attention to positive β as it is easy to show that all firms will bargain when $\beta = 0$.

requires complex theoretical frameworks with either heterogeneity (e.g., Cheremukhin and Restrepo-Echavarria (2020)) or information asymmetries (e.g., Doniger (2023)). The goal of our paper is not to theoretically rationalize the co-existence of both wage-setting protocols in a particularly novel way (and indeed, our mechanism is close to the one in Flinn and Mullins (2021), with the key difference being that we use Burdett and Judd (1983) as the starting point rather than Burdett and Mortensen (1998)). Instead, the goal of our paper is to examine how variation in labor market conditions (specifically, in the degree of competition for applicants across employers) affects a firm's incentives to post and commit to a wage instead of bargaining as well as on the resulting wages they pay. Given this goal, we use a random search model in order to generate testable predictions from the simplest theoretical framework that can sustain both wage-setting protocols.

Our static model is deliberately kept very parsimonious and designed to transparently illustrate the role of the key element, namely the endogenous choice of bargaining vs. posting.[5] Apart from the endogenous wage-setting protocol, the model also departs from the original Burdett and Judd (1983) framework by assuming free entry. This assumption implies that the probability of a worker facing a competing offer is endogenous: this probability depends not only on the probability that the worker sent a second application (which is assumed exogenous here) but also on the chance that the second application is successful.[6] This model feature is generates an endogenous link between economic conditions (e.g. more firms entering due to higher productivity) and the competitiveness of the labor market, which in turn affects the incentives to post a wage.

Similarly to the baseline Burdett and Judd (1983) model, we assume that workers can send either one or two applications. An implication is that a change in the fraction of workers sending two applications is equivalent to a change in the average number of applications, which will be one of the key independent variables in our empirical analysis. The model could, however, be extended to allow for an arbitrary stochastic number of applications, without changing the main economic mechanism at play. In such a richer framework, there would be extensive and intensive margins of competition for applicants. Changes in the mean number of competing applications submitted by job seekers could be driven by either changes in the share of workers sending more than one application, or in the share of workers that submit more than one application (i.e., the same share of workers sending 3 rather than 2 applications).

While a full theoretical examination of such a framework is beyond the scope of this

[5] The model can be extended in a number of ways, e.g. allowing for an arbitrary stochastic number of applications, without changing the main results and intuition.

[6] For a similar feature, see Kaplan and Menzio (2016).

paper, we provide the following two intuitive observations, relevant for the interpretation of the model's predictions. First, with search frictions in converting applications to offers, the extensive-margin channel – the share of workers sending more than one application – is likely to be of primary importance for firms. As we show below, the key object relevant for the firm when deciding whether to bargain is ψ, the probability that a job seeker the firm meets has a competing offer. This object is a function of the probability that any application is successfully matched to an employer, and the mean number of competing applications. We posit that the share of workers with more than one application has more of a first-order effect on this probability than the application intensity of the multi-application workers. To make this point, consider comparing two scenarios with the same mean number of applications. In the first, most workers send two applications; in the second, most workers send one application and a small fraction of workers send a large number N. From the firm's perspective, an increase in N to $N + 1$ has a minimal effect on ψ since these workers likely have a competing offer already. Meanwhile, an increase in the share of workers that sends two applications in the first scenario so that the mean number of applications is fixed will have a first order effect on the pool of applicants with competing offers (mediated by the matching rate).[7]

Second, while the magnitude differs, the direction of the effect of increasing the mean number of applications on employers' choice of wage setting protocol is likely the same, whether the change is driven by an increase in the share of multiple-applications job seekers or by a rise in the number of applications these job seekers send. In this sense, conditional on the share of workers sending more than one application, the effect of an increase in the number of applications these job seekers submit has an analogous effect to an increase in α in our model. Ultimately, both channels will operate through their effect on ψ, the probability that the applicant facing the firm has another offer. In summary, a model with an arbitrary possible number of applications, and hence allowing for both extensive and intensive margin, would likely lead to similar economic mechanisms. In the empirical analysis of section 4, we will verify that our results are robust to defining α based on the share of job seekers with multiple applications or the mean number of competing applications.

3.3 Equilibrium

An individual firm chooses whether or not to post a vacancy, and – conditional on posting a vacancy – whether or not to post a wage. Let λ be the application-vacancy ratio, let $\phi \in [0, 1]$ be the endogenous fraction of firms that choose to post a wage, and let $F(w)$

[7]In this sense, conditional on mean applications, employers prefer a world with low shares of job seekers that submit many applications, relative to one with a high share of job seekers that submit two.

be the probability distribution of posted wages among them. When making its individual decisions, a firm takes these objects as given.

Consider a firm posting a wage w. Conditional on having an applicant, there are two ways for the firm to obtain a payoff $y - w$. First, with probability $1 - \psi$, the applicant does not have another offer. Second, with probability ψ, the applicant has another offer. In that case, the firm will succeed to hire the worker if the competing firm posts a wage that is lower than w (where ties in the posted wage are broken randomly). Hence, the expected payoff of a firm posting w is

$$\Pi(w) = -\kappa + q(\lambda)\left[1 - \psi + \psi\phi \int \left(\mathbf{1}_{w'<w} + \frac{1}{2}\mathbf{1}_{w'=w}\right) dF(w')\right](y - w). \tag{2}$$

Naturally, the firm will choose the wage that maximizes $\Pi(w)$, and thus obtain a payoff

$$\Pi = \max_{w} \Pi(w). \tag{3}$$

Next, consider a firm that decides to bargain. Conditional on having an applicant, there are again a few possibilities. With probability $1 - \psi$, the applicant has no other offer and the firm obtains a payoff $(1-\beta)y$. With probability $\psi(1-\phi)$, the applicant has a competing offer from a bargaining firm. In this case, Bertrand competition implies that the worker extracts the entire surplus, leaving a zero payoff for either firm. Finally, with probability $\psi\phi$, the applicant has a competing offer from a firm that has posted some wage $w \sim F(w)$. In this case, the bargaining firm hires the worker and obtains a payoff $(1 - \beta)(y - w)$. Hence, the expected payoff for a firm that decides to bargain equals

$$\Pi_b = -\kappa + q(\lambda)\left[(1 - \psi)(1 - \beta)y + \psi\phi(1 - \beta)\int (y - w)\, dF(w)\right]. \tag{4}$$

We can then define an equilibrium for this economy as follows:

Definition 1. *An equilibrium consists of a number λ, a distribution F, a number $\phi \in [0, 1]$, and numbers Π_b and Π such that*

1. *Π_b, and Π satisfy (4) and (3) with $\Pi(w)$ given by (2);*
2. *$\Pi(w) = \Pi$ for every w in the support of F;*
3. *ϕ satisfies*

$$\phi \begin{cases} = 0 & \textit{if } \Pi_b > \Pi, \\ \in (0, 1) & \textit{if } \Pi_b = \Pi, \\ = 1 & \textit{if } \Pi_b < \Pi. \end{cases} \tag{5}$$

4. Free entry holds: $\max\{\Pi_b, \Pi\} = 0$.

To characterize the equilibrium, we first consider the equilibrium wage-setting choices of firms taking as given the application-vacancy ratio λ. We then use the free-entry condition to determine the equilibrium value of λ.

We first rule out the case where all the firms bargain. Intuitively, if a firm's applicant has a competing offer from a firm that has chosen to bargain, the former firm gets zero profits. Thus, if all firms bargain, then an individual firm only obtains a positive payoff if it has an applicant with no other offers. However, the firm's chances of meeting and hiring such an applicant are exactly the same if it posts a wage of zero instead, which is a profitable deviation. The following lemma formalizes this result.

Lemma 1. *Given $\beta > 0$, there does not exist an equilibrium in which all the firms bargain.*

Proof. See Appendix A.1. □

Given that some firms post wages, the next step in our equilibrium derivation is to characterize the distribution $F(w)$. This part of the analysis closely follows the standard logic of Burdett and Judd (1983), adjusted for the fact that some firms might bargain in equilibrium. Specifically, a firm must be indifferent between any two wages in the support of the wage distribution. This uniquely pins down the distribution.

Lemma 2. *In any equilibrium, the distribution F of wages among the firms that post wages has connected support $[0, \bar{w}]$ and no mass points, and satisfies*

$$F(w) = \frac{1}{\phi} \frac{1-\psi}{\psi} \frac{w}{y-w} \tag{6}$$

and

$$\bar{w} = \frac{\phi\psi}{1-(1-\phi)\psi} y, \tag{7}$$

where $\psi = \frac{2\alpha}{1+\alpha} p(\lambda)$.

Proof. See Appendix A.2. □

Intuitively, suppose that a firm slightly increases its posted wage, leading to lower profits conditional on hiring but, possibly, a greater likelihood of hiring. The strength of the latter depends on the probability ψ that the firm's applicant has a competing offer. This is the standard result of Burdett and Judd (1983), captured by the presence of ψ in (6) and (7). The novelty is that the profit increase from increasing one's posted wage also depends on the likelihood that the competing offer is from a firm that posted a wage, which is captured by the presence of ϕ in the expressions (6) and (7). If fewer firms post wages, then an individual wage-posting firm is less likely to attain the worker whether it raises its wage or not.

Having characterized the distribution of posted wages, we are then in a position to consider the fraction of firms that choose to post a wage rather than bargain.

Lemma 3. *In equilibrium, all firms post, i.e.* $\phi = 1$, *if and only if*

$$\psi \leq 1 - \exp\left(-\frac{\beta}{1-\beta}\right). \tag{8}$$

Otherwise, the equilibrium fraction of firms that post equals

$$\phi = \frac{(1-\psi)}{\psi}\left[\exp\left(\frac{\beta}{1-\beta}\right) - 1\right] < 1. \tag{9}$$

Proof. See Appendix A.3. □

A firm's decision whether to post a wage depends on the worker's bargaining power β and the (endogenous) probability ψ that a firm is competing with another firm for its applicant. The benefit from posting a wage is the ability to commit to a lower wage than would be bargained ex post; this benefit is stronger if the worker bargaining power is high. On the other hand, the cost of posting a wage consists of the inability to respond to a competing offer should the worker has one; this cost is stronger if the likelihood of a competing offer is high. These two forces determine the firm's incentive to post rather than bargain. Furthermore, from the perspective of an individual firm, the aforementioned cost of posting a wage is strongest if the competing offer is likely to be from a firm who *also posts*: otherwise, the ability to respond to the competing offer would be irrelevant. In equilibrium, it follows that one of two cases occur. The equilibrium may be such that a firm strictly prefers to post even when every other firm posts, so that $\phi = 1$; this occurs if worker bargaining power is sufficiently high. Alternatively, the equilibrium may feature a positive fraction of bargaining firms, such that a firm is just indifferent between bargaining and posting.

The final step in the equilibrium derivation is to characterize the application-vacancy ratio. The following lemma does this and establishes uniqueness of the equilibrium.

Lemma 4. *A unique equilibrium exists. The equilibrium application-vacancy ratio solves*

$$-\kappa + q(\lambda)\left[1 - \frac{2\alpha}{1+\alpha}p(\lambda)\right]y = 0. \tag{10}$$

Proof. See Appendix A.4. □

The intuition is straightforward. By Lemma 1, any equilibrium features some wage posting. Lemma 2 then implies that the equilibrium payoff of any firm entering the market must equal the payoff of a firm posting the wage of zero. Since such a firm only hires if the applicant has no competing offer, we immediately obtain λ from (10). The equilibrium λ, in turn, pins down the equilibrium ϕ via Lemma 3 and the wage distribution via Lemma 2. Substituting $\psi = \frac{2\alpha}{1+\alpha}p(\lambda)$ in Lemma 3, we then obtain a complete equilibrium characterization. If the λ solving (10) is such that $\frac{2\alpha}{1+\alpha}p(\lambda) \leq 1 - \exp\left(-\frac{\beta}{1-\beta}\right)$, the equilibrium has $\phi = 1$, and the wage distribution is given by

$$F(w) = \frac{1-\psi}{\psi}\frac{w}{y-w}, \tag{11}$$

where $\psi = \frac{2\alpha}{1+\alpha}p(\lambda)$. The worker bargaining power is so high that all the firms post wages in equilibrium; the economy behaves like a standard Burdett and Judd (1983) economy, except for the addition of free entry. On the contrary, if the λ solving (10) is such that $\frac{2\alpha}{1+\alpha}p(\lambda) > 1 - \exp\left(-\frac{\beta}{1-\beta}\right)$, the equilibrium has

$$\phi = \frac{1-\psi}{\psi}\left[\exp\left(\frac{\beta}{1-\beta}\right) - 1\right] < 1, \tag{12}$$

and the wage distribution is now given by

$$F(w) = \left[\exp\left(\frac{\beta}{1-\beta}\right) - 1\right]^{-1}\frac{w}{y-w}, \tag{13}$$

where again $\psi = \frac{2\alpha}{1+\alpha}p(\lambda)$. In this case, a positive measure of firms decide not to post a wage. A firm is indifferent between posting and bargaining, and, conditional on posting, indifferent among all the wages in the support of the wage distribution.

3.4 Comparative Statics

Our focus is on generating model-based comparative statics predictions regarding the behavior of the application-vacancy ratio λ, the incidence of wage posting ϕ and the distribution of posted wage F, which can then be confronted with data. We first examine comparative statics with respect to y, which can be thought of as a shock to labor demand. We then contrast them with the effect of α, which can be thought of as measuring the degree of competition in the labor market (e.g., a shock to labor supply).

Lemma 5. *An increase in productivity y*

1. *decreases the application-vacancy ratio λ.*

2. *decreases the fraction ϕ of firms that post a wage, as long as $\phi < 1$.*

3. *increases the distribution of posted wages in a first-order stochastic dominance way.*

Proof. See Appendix A.5. □

A higher value of productivity encourages entry by firms, lowering the equilibrium application-vacancy ratio λ. With fewer applications per vacancy, the probability $p(\lambda)$ that an application results in a job offer then increases, as evident from (10). For a firm making a job offer to an applicant, it then becomes more likely that the applicant has a competing offer. This make it less attractive to post a wage, all else equal. As a result, the equilibrium fraction ϕ of firms that post a wage decreases, as long as the economy is already in the interior region where $\phi < 1$, as evident from (12). Finally, the increase in productivity has a direct effect on the wage distribution via either (11) or (13).

We next examine the effect of an increase in α, the fraction of workers sending two applications.

Lemma 6. *An increase in the fraction α of workers sending two applications*

1. *increases the application-vacancy ratio λ.*

2. *decreases the fraction ϕ of firms that post a wage, as long as $\phi < 1$.*

3. *increases the distribution of posted wages in a first-order stochastic dominance way if $\phi = 1$, but has no effect on wages if $\phi < 1$.*

Proof. See Appendix A.6. □

An increase in α directly increases the probability ψ that an applicant has a competing offer. Although the effect is mitigated by a reduction in entry, Lemma 6 establishes that the direct effect dominates; competition for workers rises, making wage posting less attractive and thereby reducing ϕ. Of particular interest is the effect of α on the posted wage distribution. If all the firms are posting wages ($\phi = 1$), the marginal effect of an increase in α is to raise wages, in the first-order stochastic dominance sense. This well-known and intuitive result from Burdett and Judd (1983) is evident from (11): an increase in competition for workers raises wages. The interior ($\phi < 1$) case, illustrated by absence of α in the expression in (13), is the less intuitive result, which runs quite contrary to the standard Burdett and Judd (1983) logic. All else equal, an increase in α raises a firm's marginal benefit from raising its wage as it essentially makes its labor supply more elastic. However, at the same time, an increase in α lowers the fraction of firms who post wages, thereby lowering an individual firm's marginal benefit from raising its wage. These two effects exactly cancel each other out. Note that this does not mean that labor market competition has no effect on workers' wage prospects: it will affect realized wages both directly, by enabling workers to sample more offers on average, and indirectly, by inducing more firms to bargain. The novelty is that the change in firm's behavior manifests through a lower probability of posting wages, not through the wages that they post conditional on doing so.

3.5 Predictions

Taking stock of the above comparative statics, we formulate model-based predictions to be taken to data. We focus on the interior case ($\phi < 1$) as this will be the empirically relevant case in the data. As explained below, our data allow measuring the application-vacancy ratio (λ) in a local labor market and, less trivially, a proxy for expected labor market competition (α). We can also directly observe the fraction of firms posting wages, and on the wages they post.

The main model predictions that we wish to test relate to the correlations between the application-vacancy ratio (λ) and the fraction of firms posting a wage, as well as between λ and the average posted wage. Since λ is itself endogenous, a regression of the incidence of wage posting or of offered wages on λ cannot be interpreted as measuring a causal effect. However, the model has predictions not just about the correlation between wage posting and λ, but also on how this correlation varies when controlling for the expected competition for applicants (α). Therefore, reduced-form regressions between the prevalence of wage posting and wage posting levels on the applicant-vacancy ratio (i.e., λ) and how it changes when controlling for the expected number of competing applications (α) are informative for testing the model's comparative statics predictions, despite not measuring causal effects.

The key idea is the following. The variation in the endogenous object λ is driven by a combination of labor supply and labor demand shocks (i.e., changes in α, or changes in y and κ). When variation in λ is due to a labor demand shock (e.g., higher y), Lemma 5 implies a *positive* correlation between the application-vacancy ratio and the fraction of firms posting a wage, keeping labor supply factors (e.g., the number of applications per job seeker) fixed. Meanwhile, Lemma 6 implies that variation in α would induce a *negative* correlation between the application-vacancy ratio and the fraction of firms posting a wage. Therefore, failing to control for the number of applications submitted per job seeker when analyzing the impact of λ on wage posting propensity suffers from negative omitted variable bias. Indeed, (12) implies that, all else equal, the fraction of firms posting a wage is positively associated with λ and negatively associated with α. Thus, a regression of the fraction of posting firms on both λ and α would be expected to produce a positive coefficient on the former and a negative coefficient on the latter. Moreover, omitting α would likely introduce a negative bias in the coefficient on α. This is the first testable prediction.

Next, consider the model-implied correlation between the application-vacancy ratio (λ) and the magnitude of posted wages, conditional on posting a wage. Lemma 5 and Lemma 6 imply that the average posted wage is, all else equal, directly increasing in y and, for the case $\phi < 1$, not directly dependent on α. Since (10) gives λ as decreasing in y and increasing in α, a regression of the average posted wage on both λ and α would be expected to produce a negative coefficient on the former and a positive coefficient on the latter. That is, keeping α fixed, a higher λ is indicative of a lower y (hence, the negative coefficient in the coefficient for λ). Meanwhile, a higher α for a given λ is informative of a higher y (hence, the positive coefficient). Moreover, omitting α from this regression would again result in omitted variable bias, which is likely to be upwards as long as α and λ are positively correlated. This is the second testable prediction. The two predictions form the basis of our empirical strategy for indirectly testing the model-implied comparative statics results.

4 Empirical Evidence

In this section, we test the model's predictions regarding the correlation between applications per vacancy, incidence of wage posting, and the magnitude of posted wages. In particular, we focus on the direction of these correlations and how they change when controlling for the number of competing applications submitted by the job seeker pool. Confronting these predictions with empirical evidence requires data on the fraction of firms posting an explicit wage, as well as the magnitude of the wages posted by those firms. It also requires measuring the empirical counterparts of λ, the average applications per vacancy, and α, which is proxied for by the average applications per job seeker. First, we describe the data used in these analyses. Then, we explain how we construct empirical analogs to the objects λ and α. Next, we develop our empirical specification and present the estimation results.

4.1 Data Description

We use proprietary data from www.Dice.com (henceforth, the DHI data),[8] an online job board that focuses on technology sectors, software development, other computer-related occupations, engineering, financial services, business and management consulting, and other careers that require technical skills.[9] The DHI data links the universe of vacancy postings

[8] DHI Group Inc owns and operates the Dice.com platform.

[9] Dice.com has operated in the US since 1996, first, offering services to recruiting and staffing firms and later (1999) expanding to direct hire employers.

advertised on Dice.com between January 1, 2012 and December 31, 2017 to all of the job seekers that applied to them. It includes detailed information about the job posting, including the position's job title, location, tax-terms, work-schedule, and, crucially for our analysis, whether the posting offers an explicit wage (or wage range) and, if so, the amount offered.[10] We can also observe the exact time-stamp when the job posting was first visible to potential applicants, the number of seconds it was visible to applicants on each day, and the number of views and applications it received on each day that it was active on the platform. On the applicant side, we can observe the candidate's job title, their location, and the exact time-stamp and job for each of their applications.[11,12]

We focus on jobs posted by "Direct Hire" employers. These refer to companies that hire workers to add to their payroll or directly employ as contractors, as opposed to jobs posted by recruitment or staffing firms, who post jobs to find candidates for other firms to consider hiring or leasing. These employers represent 82 percent of all job posting accounts and a third of posted vacancies on the Dice.com platform. This sample restriction allows us to focus on jobs in which the employer and job poster coincide with the organization that determines the wage setting protocol. We further restrict the sample to jobs in the United States (which represent 99% of all jobs). The Dice.com platform allows employers to post, remove, and re-post the job several times. We focus on cases where the employer posts the job and then permanently removes it from the platform within 31 days after first posting. Three-fourths of vacancies on Dice.com exhibit this posting pattern.[13]

For each posting, employers can choose to include information on the job's compensation. This variable is a free-text field in which employers can include specific values or ranges of wages/salaries, information about benefits, bonuses, etc. The field can also be left blank or include generic terms like "Negotiable," "Competitive," or "Market-based." There is also variation in terms of the frequency of the reported compensation, that is, a job posting may state that it pays $50 per hour or $96,000 per year.[14]

[10] The "tax-terms" variable includes information about the contract type (i.e., 1099 or W2) and the hiring modality (i.e., corporation-to-corporation or corporation-to-individual). Work schedule specifies whether the job is part-time, full-time, remote, and at times whether there are travel requirements.

[11] The applicant's job title is self-reported and may refer to the current, most recent, or desired job title. Based on conversations with DHI, our understanding is that many of the applicants that are active on the platform search on-the-job, so the job title likely reflects current positions. However, we cannot distinguish between those who are and are not employed.

[12] See Davis and Samaniego de la Parra (2019) for detailed variable description and summary statistics.

[13] Focusing on jobs with this "standard" posting pattern allows us to likely identify job postings that advertise a single position. In contrast, jobs that are posted and remain active for long periods of time likely reflect recurring hiring needs and multiple job openings. See Davis and Samaniego de la Parra (2023) for additional information on standard postings and the distribution of posting duration.

[14] Under a full-time, 8-hour day, 5-day week schedule with 48 workweeks per year, these hypothetical jobs have equivalent pay.

We construct three variables using the information available on compensation. The first variable is an indicator ($wage\_posted_{i,j,t}$) equal to 1 if job posting i posted by employer j with first-active date on calendar month t includes a specific pay level or range, and zero if it does not. For the second variable, conditional on $wage\_posted_{i,j,t} = 1$, we calculate the natural logarithm of the wage or the midpoint of the range ($\ln(w_{i,j,t})$).[15] The third variable is categorical and describes the pay rate frequency. While there are six possible values to this variable (daily, hourly, weekly, biweekly, monthly, and annual), job postings with per hour and per year pay frequencies account for 58 percent and 39 percent of all jobs that offer an explicit wage.[16] We trim the bottom and top 5 percent of wages within each pay frequency.

We define a labor market using job function-location categories. We distinguish between 52 locations, consisting of the 49 states in the continental United States, D.C., Puerto Rico and all others. Job functions refer to occupational categories such as "Programmer, or "Consultant." We group postings into 56 different job functions categories.[17] This categorization covers 93% of all standard job postings in the Dice.com platform. We further restrict our sample to job markets with at least 10 distinct employers and 10 distinct job postings in each month of the sample period. Our baseline sample includes 721,274 job postings, posted by 1,636 employers across 129 job markets.[18]

Table 1 presents descriptive statistics for the sample. Job postings receive on average 2.5 applications per day and are active (i.e., receiving applications) for 9.8 days. Out of all jobs in the sample, 10.5 percent explicitly mention a numeric pay. Conditional on offering an explicit pay, 38 percent of jobs mention a range. When posting a range, the mean difference between the lower and the upper bound is 17 percent for postings with hourly rates and 26 percent for postings with annual rates. Moreover, the midpoint for jobs that announce pay ranges is in line with the mean pay offered by jobs that mention a point value instead of a range. The mean labor market (defined as location $\times$ job function) has 93 active job postings per month and 47 distinct employers recruiting workers. Job seekers submit on average 9.2 applications during each search spell and the average spell lasts 9.5 days.

[15] Results are robust to using the lower bound of the range. Our results are also robust to excluding job postings that include a pay range instead of a point value. See Appendix Table B.4.

[16] 5% of jobs offer an explicit wage but do not specify the pay rate frequency. For these jobs, we impute the frequency using the distribution of wages across the six pay rate frequency categories.

[17] A job posting can mention more than one job function in its description. We classify postings based on the first job function mentioned in the job title.

[18] Importantly, the set of job seekers include only those individuals applying to the jobs in our selected sample. However, we consider all the applications submitted by these individuals, including those to jobs that we exclude from the sample. This allows us to correctly capture employers' competition for job seekers, across all job postings regardless of whether the competing jobs meet our sample criteria.

Table 1: Descriptive Statistics (Means and Std. Dev.)

Panel A: Across Job Postings (N = 721,274)					
Applications per Day	Views per Day	Posting duration	% Jobs w/ posted pay level	% Jobs w/ posted pay range	
2.5	19.3	9.8	6.5	4.0	
(8.6)	(19.7)	(8.7)			
Panel B: Across Job Postings Conditional on Posting Pay Level; (N= 46,673)					
Hourly Freq.			Annual Freq.		
% Jobs	Mean Pay		% Jobs	Mean Pay	
75.2	57.3		24.6	99,212.9	
	(14.1)			(26,533.0)	
Panel C: Across Job Postings Conditional on Posting Pay Range; (N= 29,011)					
Hourly Freq.			Annual Freq.		
	Mean Pay			Mean Pay	
% Jobs	Lower	Upper	% Jobs	Lower	Upper
46.8	52.5	61.9	52.6	84,634.2	108,184.9
	(15.7)	(18.4)		(25,467.6)	(29,939.2)
Panel D: Across Labor Markets (Job Function × Location; N= 129)					
Monthly Job postings	Firms	Applications	Job Seekers	Competing offers ($\bar{\alpha}$)	Applications/Vacancies (λ)
93.2	47.6	1,156.2	1,169.1	17.5	8.7
(98.7)	(34.5)	(2,328.4)	(714.5)	(13.1)	(7.5)
Panel E: Across Job Seekers' Search Spells (N = 1,977,718)					
Applications	N Spells	Spell duration			
9.2	1.9	9.5			
(57.3)	(1.8)	(21.1)			

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Standard deviations reported in parenthesis.

4.2 Application-Vacancy Ratio and Labor Market Competition

In the data, we directly observe the application-vacancy ratio for each labor market in each time period. Let A_i be the total number of applications received by posting i. Let $\mathbf{I}[i \in l \times f, t]$ be an indicator function equal to 1 if job posting i belongs to job market $l \times f$ and was first active on the platform in month t. The application-vacancy ratio, $\lambda_{l \times f,t}$ is given by:

$$\lambda_{l \times f,t} = \frac{\sum_{i=1}^{I} A_i \times \mathbf{I}[i \in l \times f, t]}{\sum_{i=1}^{I} \mathbf{I}[i \in l \times f, t]} \tag{14}$$

A firm's decision whether to post a wage or bargain with a worker upon matching depends on whether the employer believes the worker has a competing offer. As shown in equation (1), the probability of a competing offer depends on the application-vacancy ratio and on the probability that the worker submitted other applications (α).[19] We next describe how we construct an empirical analog to α using a measure of the expected number of competing applications submitted by each job posting's applicants.

In the data, we can directly observe the applications submitted by each job seeker.[20] To calculate the number of competing applications for each job posting, we first construct search spell as the set of applications that are likely simultaneously considered for each job seeker. We define a search spell as the set of all applications submitted by a job seeker such that the length of time between applications is less than 30 days.[21] The average applicant has 2 search spells on the platform with a mean duration of 9.5 days. On an average search spell, job seekers submit 9 applications. Let $A_{-i,k}$ be the number of applications submitted to job postings, different from posting i but within the same search spell, by job seeker k (who has also applied to i). Then, the mean number of competing applications submitted by the set of job seekers that applied to job posting i is given by:

$$\alpha_i = \frac{\sum_{k=1}^{A_i} A_{-i,k}}{A_i} \tag{15}$$

[19] In the theoretical model, we assume α is an exogenous parameter. Arguably, the optimal number of applications submitted by optimizing job seekers may vary. Importantly, in a related paper Samaniego de la Parra (2023) finds that, while labor market tightness affects the type of job postings that job seekers target, the number of applications submitted is not significantly affected.

[20] Some job seekers are flagged by the platform or self-identify as "3rd-party applicants." These applications are difficult to track to a single job seeker and we therefore exclude them when calculating the number of competing applications submitted by the average job seeker.

[21] The results are robust to using 15 or 45 days as the maximum time between applications when defining search spells.

Employers do not directly observe the number of competing applications submitted by the job seekers that apply to their job postings before determining whether to offer an explicit wage. Instead, the firm considers the number of competing applications submitted by the mean job seeker applying to postings in the same labor market.

$$\bar{\alpha}_{l\times f,t} = \frac{\sum_{i=1}^{I} \alpha_i \times \mathbf{I}[i \in l \times f, t]}{\sum_{i=1}^{I} \mathbf{I}[i \in l \times f, t]} \tag{16}$$

The theoretical model developed in section 3 assumes that an exogenous share of job seekers, α, submits two applications while the rest submit one. As we discussed previously, a more flexible specification could define α as the average number of potential competing applications and separately consider the effect increasing the share of job seekers that submit more than one application versus keeping this share fixed but increasing the number of applications submitted by each job seeker that submits more than one. In our sample, more than half (55%) of all job seekers submit only one concurrent application. Figure B.1 in the Appendix shows that conditional on submitting more than one, a third of job seekers submit exactly two. For the empirical analyses that follow, we use equation 16 as the baseline measure of expected competing applications. Our results are robust to instead measuring expected competition for applicants ($\bar{\alpha}$) as the share of applicants with more than one application (see Tables B.1 and B.2 in the Appendix).

4.3 Empirical Analysis

We consider two outcome variables: first, each firm's decision to post an explicit wage for a given job posting, and second, the chosen wage level, conditional on choosing to post a wage. It is important to emphasize that the estimated coefficients do not have a causal interpretation, since both the left-hand-side and the right-hand-side variables of the regressions are endogenous. Instead, the goal of the empirical exercise is to evaluate the model's predictions regarding the sign of the correlation between queue length (λ) and firms' behavior, and how this correlation changes when controlling for fluctuations in labor-supply factors, specifically, the number of competing offers submitted by job seekers (α).

Wage posting. As discussed in section 3.5, the model implies an ambiguous correlation between the application-vacancy ratio (λ) and the incidence of wage posting (ϕ) if the number of applications submitted by job seekers (α) is not controlled for. The reason for the ambiguity is that shocks to α or to the expected surplus of a match imply different signs for the correlations between λ and ϕ. An empirical analysis of the effects of λ on the incidence of wage posting that does not control for α includes the combined effect of both sources of variation.

In our baseline empirical specification, we model the wage posting propensity for a job i at firm j in month t as a function of the log applications-vacancy ratio in the job's labor market ($\ln\left(\lambda_{l\times f(i),t}\right)$) and the log expected number of competing offers ($\ln\left(\bar{\alpha}_{l\times f(i),t}\right)$):

$$wage\_posting_{i,j,t} = \beta_1 \ln\left(\lambda_{l\times f(i),t}\right) + \beta_2 \ln\left(\bar{\alpha}_{l\times f(i),t}\right) + \epsilon_{i,j,t}, \tag{17}$$

where the dependent variable, $wage\_posting_{i,j,t}$ is an indicator function equal to 1 if job i at firm j active on month t includes a numeric pay (a level or range) and 0 otherwise. We first consider a model in which we constrain β_2 to equal 0, then we constrain β_1 to equal 0, and finally we estimate an unrestricted model which includes both regressors. Our main focus is to analyze the sign of β_1 in the restricted and the unrestricted model.

Our baseline specification includes no fixed effects, thereby allowing for variation in productivity across firms, across time, and across local labor markets (locations and job functions). Therefore, β_1 denotes the percentage point difference in the probability of posting a wage across markets or periods where the applications-to-vacancy ratio is 1 percent higher, keeping the number of applications per job seeker fixed. Comparing β_1 in a model where β_2 equals zero to the coefficient in an unrestricted model illuminates the effect of controlling for changes in the expected number of competing applications among the job seekers across labor markets.

Table 2: Labor Market Conditions and the Probability of Wage Posting

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\lambda)$	0.003		0.007	0.006		0.010
	(0.001)		(0.001)	(0.001)		(0.001)
$\ln(\bar{\alpha})$		-0.002	-0.007		-0.003	-0.007
		(0.001)	(0.001)		(0.001)	(0.001)
Observations	721,274	721,274	721,274	703,741	703,741	703,741
R-squared	0.000	0.000	0.000	0.502	0.502	0.502
Firm × Time	N	N	N	Y	Y	Y
absorbed FE				62,287	62,287	62,287
singletons				17,533	17,533	17,533

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Standard errors are clustered at the labor market × time level where a labor market is defined by job function and location.

Columns (1) through (3) in table 2 present the baseline results for the incidence of wage posting. Consistent with the model's predictions, column (3) shows that the coefficient on λ is positive when controlling for α, and the coefficient on α itself is negative. Failure to control for α means that the regression coefficient on λ includes both the positive effect of the applications-to-vacancy ratio on wage posting and the negative effect from variation induced by α. As expected in a classical case of downward omitted variable bias, the coefficient in column (1) is thus smaller than in column (3).

In our baseline specification, we consider jobs that mention a pay range as part of the "wage posting" (i.e., not bargaining) group. Arguably, posting a wage range enables firms to bargain with job seekers who have another offer. The posted range is a commitment to bargain up to the upper bound of the range. In this sense, posting a wage range can be thought of as an intermediate point between "full" wage bargaining and strict commitment to a single wage value. Using a multinomial logit model that distinguishes between posting a wage, a wage range, or no numeric pay information, we show that our empirical results are not driven by firms posting wage ranges (see table B.3 in the Appendix). Panel B shows that it is the likelihood of posting a specific (single value) wage that is positively correlated with queue length and negatively correlated with the expected competition for applicants (measured as the number of applications submitted by the average applicant). Table B.4

in the Appendix further shows that our results are robust to excluding jobs that offer pay ranges from the sample.

Our stylized theoretical model abstracts from many other sources of heterogeneity across firms and markets that would generate variation in wage posting propensity. Columns (4) to (6) in table 2 extend our baseline specification to include firm × time fixed effects.[22] Again, and consistent with the model's comparative statics, wage posting incidence is positively correlated with the applications-vacancy ratio, and more so when controlling for the expected number of competing applications submitted by the job seeker pool. The purpose of this robustness exercise is two-fold. First, by adding these fixed effects we control for potential confounding factors outside the model that may affect the incidence of wage posting (e.g. some firms are more likely to bargain, or bargaining is more prevalent at some periods in time, for institutional reasons outside the theory). Second, the robustness checks illustrate that a significant part of the variation in the incidence of wage posting, and its correlation with the applications-vacancy ratio, is driven by within firm-time variation, in contrast with the notion of wage setting protocols being determined as a firm-level strategy.[23]

Offered wages. We next consider the log offered wage, conditional on wage posting. We focus on job postings that mention per hour rates. [24] Our baseline specification is

$$\ln\left(wage_{i,j,t}|wage\_posting_{i,j,t}\right) = \delta_1 \ln\left(\lambda_{l\times f(i),t}\right) + \delta_2 \ln\left(\bar{\alpha}_{l\times f(i),t}\right) + \epsilon_{i,j,t}. \tag{18}$$

As before, we initially constrain δ_1 or δ_2 to equal zero, and then we estimate the unrestricted model to compare how the direction of the correlation between the market's applications-vacancy ratio and firms' offered wages when including or excluding the impact of competing applications. We also examine how adding the estimated fixed effects from equation (17) affects these coefficients.[25]

[22] Our results are also robust to simultaneously including function × time, location × time, and firm × time fixed effects.

[23] Our results are also robust to adding firm-job-function-time fixed effects, indicating that much of the variation in wage setting protocols occurs within firms even when hiring for similar occupations.

[24] Table B.5 in the Appendix shows the results with postings that offer per year pay.

[25] We rely on the estimated fixed effects from the wage posting linear probability model in equation (17) to account for firms' self-selection into posting a wage or not as a function of λ, α and other firm time-varying factors.

Table 3: Labor Market Conditions and Posted Hourly Wages

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\lambda)$	-0.005		-0.034	-0.005		-0.034
	(0.002)		(0.003)	(0.002)		(0.003)
$\ln(\bar{\alpha})$		0.021	0.046		0.021	0.046
		(0.002)	(0.003)		(0.002)	(0.003)
N	44,268	44,268	44,268	43,102	43,102	43,102
R-squared	0.000	0.004	0.010	0.002	0.006	0.012
$\widehat{Firm \times Time}$	N	N	N	Y	Y	Y

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Sample includes postings that explicitly mention a wage per hour rate. For postings with a numerical posted wage but without a rate frequency, we input the frequency based on the distribution of wages with explicitly payment frequency. We trim that bottom and top 5% of the hourly wages (after imputing frequencies). For columns (4) through (6), we use the firm-time fixed effects estimated in equation (17) as controls. Bootstrapped standard errors with 1,000 replications are clustered at the labor market $\times$ time level where a labor market is defined by job function and location.

Table 3 presents the results of estimating (18). Consistent with the model's comparative statics, there is a negative association between λ and the average level of posted wages. Further, the coefficient on α is positive, controlling for λ. The theory predicts that an increase in α, all else equal, should increase λ and leave wages unaffected. The interpretation of the coefficient is therefore that it is measuring the effect of an increase in α with a countervailing increase in productivity so as to keep λ fixed, which would increase posted wages. In summary, the empirical evidence in both Table 2 and Table 3 is consistent with our model's predictions.

If posting a wage range is indicative of employers' willingness to bargain with a worker, then we would expect the size of the range to vary with competition for job seekers. Consistent with this interpretation, we find that conditional on posting a range, the width of the range increases with the expected number of competing applications submitted by the job's applicants and decreases with queue length (see Table 4). Moreover, the increase in the size of the posted wage range is due to an increase in both the lower and the upper bounds. An increase in the expected number of potential outside offers (proxied by the average number of competing applications controlling for queue length) increases both the upper and the lower bound of the wage range, but the magnitude of the effect is larger on the upper bound hence the width of the range increases (see Table 5).

Table 4: Labor Market Conditions and Posted Wage Range Size

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\lambda)$	0.007		-0.029	-0.069		-0.104
	(0.009)		(0.011)	(0.015)		(0.017)
$\ln(\bar{\alpha})$		0.037	0.056		0.024	0.062
		(0.009)	(0.011)		(0.012)	(0.013)
N	12,432	12,432	12,432	9,925	9,925	9,925
R-squared	0.000	0.002	0.003	0.495	0.494	0.497
Firm × Time FE	N	N	N	Y	Y	Y

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Sample includes postings that explicitly mention a wage range per hour. The dependent variable is the natural logarithm of the difference between the upper and lower bounds of the posted range. For postings with a numerical posted wage but without a rate frequency, we input the frequency based on the distribution of wages with explicitly payment frequency. We trim that bottom and top 5% of the hourly wages (after imputing frequencies). Standard errors are clustered at the labor market × time level where a labor market is defined by job function and location.

Table 5: Labor Market Conditions and Posted Wage Ranges' Bounds

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: ln(lower bound)						
ln(λ)	-0.006		-0.035	-0.004		-0.031
	(0.004)		(0.005)	(0.004)		(0.006)
ln($\bar{\alpha}$)		0.022	0.047		0.021	0.033
		(0.004)	(0.006)		(0.004)	(0.006)
N	12,117	12,117	12,117	10,864	10,864	10,864
R-squared	0.000	0.004	0.009	0.005	0.008	0.013
Panel B: ln(upper bound)						
ln(λ)	-0.001		-0.034	0.000		-0.029
	(0.004)		(0.005)	(0.004)		(0.006)
ln($\bar{\alpha}$)		0.028	0.052		0.027	0.047
		(0.004)	(0.006)		(0.004)	(0.006)
N	12,336	12,336	12,336	11,057	11,057	11,057
R-squared	0.000	0.006	0.011	0.002	0.007	0.010
$\widehat{\text{Firm} \times \text{Time}}$ FE	N	N	N	Y	Y	Y

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Sample includes postings that explicitly mention a wage range per hour. The dependent variable is the natural logarithm of the lower bound for panel A and the upper bound for panel B. Less than 2 percent of the sample posts a lower bound for the range of $0 and hence are excluded from the lower bound regressions. The results are robust to using an inverse hyperbolic sine transformation instead of the natural logarithm to keep these postings in the sample. For postings with a numerical posted wage but without a rate frequency, we input the frequency based on the distribution of wages with explicitly payment frequency. We trim that bottom and top 5% of the hourly wages (after imputing frequencies). For columns (4) through (6), we use the firm-time fixed effects estimated in equation (17) as controls. Bootstrapped standard errors with 1,000 replications are clustered at the labor market $\times$ time level where a labor market is defined by job function and location.

5 Implications

Our model serves as a parsimonious benchmark for understanding how endogenous choice of wage-setting protocol affects the wage distribution. In particular, it lets us examine the effects of policies such as pay transparency laws – policies that are hard to conceptualize in frameworks where either bargaining or posting is imposed from the start. To illustrate some of these implications, in this section we derive the model's predictions for average realized wages and their distribution, and analyze their response to policies impacting the equilibrium share of bargaining firms.

We begin by observing that pay transparency laws that force firms to post wages[26] is equivalent, in our setting, to a sufficiently high increase in workers' bargaining power. This follows from an earlier result, Lemma 3, which shows that no firms bargain if (8) holds, i.e., if $\frac{2\alpha}{1+\alpha}p(\lambda) \leq 1 - \exp\left(-\frac{\beta}{1-\beta}\right)$. The equilibrium λ is uniquely pinned down by the free entry condition (10) and does not depend on β; the inequality in (8), which is a necessary and sufficient condition to rule out bargaining in equilibrium, therefore amounts to a lower bound on β given other exogenous parameters. This observation implies that prohibiting bargaining altogether amounts to a sufficient rise in β. In the subsequent analysis, we therefore conduct comparative statics of equilibrium outcome variables with respect to β, understanding that a high-enough rise in β results in no bargaining (i.e., all firms posting) and is therefore a particular way of thinking about pay transparency laws.

We highlight two main implications from the analysis below. First, the simple framework used here turns out to make stark predictions for the effects of an increase in worker bargaining power on *average* wages: the mean wage is invariant to policies that prohibit bargaining or change the workers' bargaining power parameter. Second, while these policies leave average wages unchanged, there are nonetheless winners and losers among workers. Specifically, a pay transparency policy that forces all firms to post wages redistributes away from workers sending multiple applications and toward workers sending only one.[27]

We first derive the average realized wage, which is distinct from the both the average posted wage and the average realized wage among workers matched with jobs with posted wages, since some matches have their wages bargained in equilibrium. The average realized wage nonetheless admits a simple expression:

Lemma 7. *The average realized wage equals*

$$w_{avg} = \frac{\alpha p(\lambda)}{1 + \alpha - \alpha p(\lambda)} y, \tag{19}$$

[26]Forcing firms to post wages is equivalent to prohibiting them from bargaining under our interpretation of wage posting as commitment to the posted wage.

[27]Our model assumes the share of workers that submit more than one application is exogenous. Endogenizing this decision is outside the scope of this project. Instead, our goal in this section is to provide initial insights into the partial equilibrium distributional consequences of policies that impose restrictions of firms' wage-setting protocols.

where the equilibrium application-vacancy ratio λ *solves* (10).

Proof. See Appendix A.7. □

The intuition for this simple expression is as follows. First, the output of every match y is exogenously given. Second, firms are indifferent across all the posted wages in the support of the posted wage distribution and, in an equilibrium where some firms bargain, they are also indifferent between bargaining and posting. It follows that the average expected profit to a firm from a match is uniquely pinned down and equal to the profit of a firm posting the wage of zero. Since the average wage equals the difference between the match output and the average profit, it is also uniquely pinned down.

The labor share of income in this economy therefore equals $\frac{\alpha p(\lambda)}{1+\alpha-\alpha p(\lambda)}$, which is the same as in the special case of Burdett and Judd (1983), where all the firms post. Hence, the labor share depends on α and λ, but does not depend on β (either directly or indirectly), since bargaining power does not change λ, as seen from (10); nor would it change in response to a blanket prohibition on bargaining, which, as mentioned above, is equivalent to a sufficiently high rise in β and reduces the model back to Burdett and Judd (1983). The reason is that, in response to a change in worker bargaining power or a prohibition on bargaining, our model predicts a shift in the posted wage distribution such that average firm profits, and hence average realized wages, remain unchanged.

The result that a change in the bargaining power leaves the average wage unaffected is clearly a stark one. This is a consequence of the simplicity of the model used here, and is best viewed as a benchmark for the implications of policies that affect the share of firms that bargain with workers without impacting firm entry or the share of workers that submit multiple applications. Similarly to the classic Burdett and Judd (1983) framework, the wage distribution in this homogeneous-firm model results from a mixed-strategy equilibrium in which firms are indifferent across wages. A change in bargaining power, and the resulting change in the share of firms that post wages, preserves the indifference condition and thus leaves expected profits and wages unaffected.[28]

Next, we examine whether different workers are differentially impacted by a change in worker bargaining power or a prohibition on bargaining. Despite leaving the average expected wage over all job seekers unaffected, such an event has *distributional* effects. The next result shows that an increase in worker bargaining power increases the expected wage of job seekers

[28] A richer model, such as the heterogeneous-agent environment of Cheremukhin and Restrepo-Echavarria (2020) or a dynamic model with on-the-job search such as Flinn and Mullins (2021), may lead to different implications for the realized average wage. We see the detailed exploration of the robustness of this indifference finding as an interesting extension best left for future research. Our framework, which nests the classic Burdett and Judd (1983) and inherits its indifference condition, is best seen as a useful benchmark.

who send one application. It is then immediate that it decreases the expected wage of workers who apply twice.

Lemma 8. *Consider β such that $\frac{2\alpha}{1+\alpha}p(\lambda) > 1 - \exp\left(-\frac{\beta}{1-\beta}\right)$, where λ is the solution to (10). An increase in β does not affect the average expected wage, but increases the expected wage conditional on sending only one application.*

Proof. See Appendix A.8. □

The intuition for the result is that a higher bargaining power disproportionately benefits workers who find themselves, ex post, with one offer. To see this, consider first a worker with one offer from a *bargaining* firm: such a worker receives a wage of βy, which directly depends on bargaining power. On the other hand, consider a worker with two offers, *both* from bargaining firms; such a worker receives a wage of y, since the two firms engage in Bertrand competition, and therefore is less concerned about the value of β. Since workers who send one application are more likely to have only one offer relative to those that send two, these workers disproportionately benefit from an increase in β. However, this intuition is clearly only part of the economic mechanism, since some of the offers are from posting rather than bargaining firms. The overall effect of an increase in bargaining power is more complex. First, it directly increases the wage of workers with one offer from a bargaining firm, as just discussed. Second, it leads to an upward shift of the equilibrium distribution of posted wages. This shift, likewise, disproportionately benefits workers with only one offer and therefore, from an ex ante standpoint, benefits workers sending only one application. Third, an increase in worker bargaining power increases the fraction of firms who post rather than bargain; the effect of this is ambiguous, but our result in 8 shows that it never outweighs the other two.

A corollary of the above result concerns the effect of a "pay transparency" policy, which prohibits bargaining. As we have discussed above, and as we show formally in the proof, this amounts to raising β to a high enough level that firms choose not to bargain in equilibrium. As a result, we have

Corollary 1. *A prohibition on bargaining leaves the average expected wage unchanged, but raises the expected wage of workers sending only one application.*

Proof. See Appendix A.9. □

In other words, a prohibition on bargaining redistributes income to workers with relatively few competing job opportunities, at the expense of workers with relatively many. The intuition is similar to the above discussion concerning the effect of increasing worker bargaining power. Workers with two offers ex post are the most hurt by a prohibition on bargaining: they gain the most from bargaining by being able to engage employers in a bidding war. It then follows that workers sending two applications are ex ante hurt by the prohibition on bargaining. The more sophisticated point is that workers with relatively few competing offers benefit *indirectly* from a prohibition on bargaining, because average *posted* wages rise in equilibrium as a result of this prohibition.

6 Concluding Remarks

We have extended a simple search model to incorporate a choice between wage posting and wage bargaining and shown theoretically that the model yields clean predictions about how this choice of wage-setting protocol responds to labor market conditions. Despite its simplicity, our theory yields predictions regarding co-movement of key outcome variables that are consistent with empirical evidence from a rich novel dataset. We believe that the theoretical and empirical analysis here provides new insights on wage setting. In particular, our theoretical results imply that the choice of wage-setting protocol can act as an additional mechanism through which shocks transmit to labor market outcomes, and that regulations on bargaining can have equilibrium effects through the distribution of posted wages.

The contribution here has been of a qualitative nature. A full quantitative analysis of a model with endogenous wage-setting protocols, which would likely require a dynamic model, is a natural step forward in this line of research.

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Appendix A Proofs

A.1 Proof of Lemma 1

Suppose all firms choose to bargain, i.e. $\phi = 0$. In that case, firms' payoff equals Π_b, which reduces to

$$-\kappa + q(\lambda)(1-\psi)(1-\beta)\,y. \tag{20}$$

If a firm deviates to posting a wage of zero, it will obtain a payoff $\Pi(0)$, which equals

$$-\kappa + q(\lambda)(1-\psi)\,y. \tag{21}$$

Clearly, (21) is strictly larger than (20) given that $\beta > 0$. $\square$

A.2 Proof of Lemma 2

The proof that F has no mass points and connected support on $[0, \bar{w}]$ closely resembles Burdett and Judd (1983), We therefore keep the discussion of these results brief and refer to their paper for additional detail. Subsequently, we derive the expressions for F and $\bar{w}$ as given in (6) and (7), respectively.

As a first result, note that no firm will post a wage w equal to y in equilibrium, since that would lead to a payoff of $-\kappa < 0$ with certainty, which is worse than staying out of the market. Suppose then that F has a mass point at some wage $w_0 \in [0, y)$. The probability that a firm posting w_0 competes for an applicant with another firm posting the same wage is $q(\lambda)\psi\phi\mathbb{P}[w = w_0]$ which is strictly positive. In that case, the firm hires the worker with probability 1/2. A profitable deviation then exists: increasing the wage by $\varepsilon \to 0$ leads to a certain hire in this scenario, increasing the payoff by $\frac{1}{2}q(\lambda)\psi\phi\mathbb{P}[w = w_0](y - w_0) > 0$, while only marginally changing payoffs in other scenarios. Hence, mass points in F cannot be part of an equilibrium.

Next, suppose that F has a gap in its support, denoted by (w_1, w_2). A firm offering w_2 then has an incentive to deviate to a wage between w_1 and w_2 which would not affect the firm's probability of hiring, but would increase the payoff conditional on hiring. Note that this argument also rules out a lower bound of the support of F that is strictly larger than zero. Hence, F must have connected support on $[0, \bar{w}]$ for some $\bar{w} \in (0, y)$.

The above results imply that $\Pi(w)$ reduces to

$$\Pi(w) = -\kappa + q(\lambda)\left[1 - \psi + \psi\phi F(w)\right](y - w). \tag{22}$$

To derive F, note that firms must be indifferent between all wages in its support. That is, $\Pi(w) = \Pi(0)$ for any $w \in [0, \bar{w}]$, where $\Pi(0) = -\kappa + q(\lambda)(1-\psi)y$. Solving for F yields equation (6). The upper bound of the support of F, as given in (7), then readily follows from the fact that $F(\bar{w}) = 1$. □

A.3 Proof of Lemma 3

To derive the equilibrium value of ϕ, note that Lemma 2 implies that

$$\begin{aligned}\psi\phi \int (y-w)\, dF(w) &= (1-\psi)y \int_0^{\bar{w}} \frac{1}{y-w}\, dw \\ &= -(1-\psi)y \log\left(\frac{1-\psi}{1-(1-\phi)\psi}\right).\end{aligned}$$

Substituting this into (4) implies that the payoff for a bargaining firm equals

$$\Pi_b = -\kappa + q(\lambda)(1-\psi)(1-\beta)y \left[1 - \log\left(\frac{1-\psi}{1-(1-\phi)\psi}\right)\right]. \tag{23}$$

This expression is strictly increasing in ϕ, i.e. the more firms post, the more attractive it becomes for an individual firm to bargain. It is easy to see that $\Pi_b < \Pi(0)$ when $\phi = 0$. An interior solution for ϕ then requires that $\Pi_b > \Pi(0)$ when $\phi = 1$, which is the case if and only if

$$\beta < \frac{-\log(1-\psi)}{1-\log(1-\psi)} \in [0,1]. \tag{24}$$

In that case, the equilibrium value of ϕ follows from $\Pi_b = \Pi(0)$, which is equivalent to equation (9). □

A.4 Proof of Lemma 4

By Lemma 3, any equilibrium features some wage posting. Lemma 2 then implies that the equilibrium payoff of any firm entering the market must equal

$$\Pi(0) = -\kappa + q(\lambda)\left[1 - \frac{2\alpha}{1+\alpha} p(\lambda)\right] y, \tag{25}$$

which must equal zero in equilibrium. Taking the derivative of (25) with respect to λ yields

$$\frac{d\Pi(0)}{d\lambda} = q'(\lambda)\left[1 - \frac{2\alpha}{1+\alpha} p(\lambda)\right] y - q(\lambda) \frac{2\alpha}{1+\alpha} p'(\lambda)\, y. \tag{26}$$

Since $q'(\lambda) > 0$ and $p'(\lambda) < 0$, this derivative is strictly positive, i.e. $\Pi(0)$ is strictly increasing in λ. Note further that $\Pi(0) = -\kappa < 0$ when $\lambda = 0$ and $\Pi(0) \to y - k > 0$ when $\lambda \to \infty$. Hence, there exists a unique value of λ such that the free entry condition is satisfied. □

A.5 Proof of Lemma 5

By Lemma 4, the free entry condition can be written as $\Pi(0) = 0$ where $\Pi(0)$ satisfies (25). Taking the derivative with respect to y yields

$$\frac{d\Pi(0)}{dy} = \frac{d\Pi(0)}{d\lambda}\frac{d\lambda}{dy} + q(\lambda)\left(1 - \frac{2\alpha}{1+\alpha}p(\lambda)\right) = 0.$$

As discussed around equation (26), the derivative of $\Pi(0)$ with respect to λ is strictly positive, which implies that $d\lambda/dy < 0$. The lower value of λ increases $\psi = \frac{2\alpha}{1+\alpha}p(\lambda)$. This, in turn, decreases ϕ as long as $\phi < 1$, as follows from (9).

To consider the effect of the productivity shock on the distribution of posted wages, note that when ϕ is interior, we can use (9) to rewrite (6) as

$$F(w) = \frac{1}{\exp\left(\frac{\beta}{1-\beta}\right) - 1}\frac{w}{y-w}. \tag{27}$$

Hence, while there is a direct effect of y on $F(w)$, there is no indirect effect that operates through ϕ or ψ. The direct effect of y on $F(w)$ is negative, since

$$\frac{dF(w)}{dy} = -\left[\exp\left(\frac{\beta}{1-\beta}\right) - 1\right]^{-1}\frac{w}{(y-w)^2}. \tag{28}$$

That is, when ϕ is interior, wages are increasing in a first-order stochastic dominance way.

When $\phi = 1$, the wage distribution is given by (11). Hence,

$$\frac{dF(w)}{dy} = \frac{\partial F(w)}{\partial \psi}\frac{d\psi}{d\lambda}\frac{d\lambda}{dy} + \frac{\partial F(w)}{\partial y}. \tag{29}$$

Since $\frac{\partial F(w)}{\partial \psi} < 0$, $\frac{d\psi}{d\lambda}\frac{d\lambda}{dy} > 0$ and $\frac{\partial F(w)}{\partial y} < 0$, it follows that wages are again increasing in a first-order stochastic dominance way. Burdett and Judd (1983)

A.6 Proof of Lemma 6

By Lemma 4, the free entry condition can be written as $\Pi(0) = 0$ where $\Pi(0)$ satisfies (25). Taking the derivative with respect to α yields

$$\frac{d\Pi(0)}{d\alpha} = \frac{d\Pi(0)}{d\lambda}\frac{d\lambda}{d\alpha} - q(\lambda)\, y \frac{\partial\psi}{\partial\alpha} = 0.$$

As discussed around equation (26), the derivative of $\Pi(0)$ with respect to λ is strictly positive. Further, $\partial\psi/\partial\alpha > 0$. Hence, $d\lambda/d\alpha > 0$. Next, consider the total effect of α on ψ, including the effect that operates through the application-vacancy ratio. We get

$$\begin{aligned}\frac{d\psi}{d\alpha} &= \frac{\partial\psi}{\partial\alpha} + \frac{d\psi}{d\lambda}\frac{d\lambda}{d\alpha} \\ &= \frac{\partial\psi}{\partial\alpha}\left[\frac{d\Pi(0)}{d\lambda} + \frac{d\psi}{d\lambda} q(\lambda)\, y\right]\left(\frac{d\Pi(0)}{d\lambda}\right)^{-1}.\end{aligned}$$

This expression has the same sign as

$$\frac{d\Pi(0)}{d\lambda} + \frac{d\psi}{d\lambda} q(\lambda)\, y = q'(\lambda)(1-\psi)y > 0.$$

That is, the probability ψ that an applicant has a competing offer is increasing in α. This, in turn, decreases ϕ as long as $\phi < 1$, as follows from (9).

For the wage distribution, we again distinguish between $\phi = 1$ and $\phi < 1$. When $\phi < 1$, the wage distribution is given by (27), which is clearly independent of α. In contrast, when $\phi = 1$, the wage distribution is given by (11), which depends on α through ψ. In particular, the increase in ψ that results from the increase in α increases wages in a first-order stochastic dominance way. □

A.7 Proof of Lemma 7

Total hires are equal to

$$h = u[1+\alpha-\alpha p(\lambda)]p(\lambda) = v[1+\alpha-\alpha p(\lambda)]\frac{q(\lambda)}{1+\alpha}. \tag{30}$$

Total welfare, by linearity, can be written as the sum of total worker utility and firms' utility:

$$-\kappa v + hy = -\kappa v + u[1+\alpha-\alpha p(\lambda)]p(\lambda)w_{avg} + vq(\lambda)J_{avg}, \tag{31}$$

where w_{avg} is the worker's expected wage conditional on being hired, and J_{avg} is the firm's expected profit (gross of vacancy costs) conditional on meeting a worker. We calculate J_{avg} from the free entry condition (10), together with the fact that any equilibrium wage-setting choice yields the same profit as posting a wage of zero:

$$J_{avg} = \frac{\kappa}{q(\lambda)} = \frac{1 + \alpha - 2\alpha p(\lambda)}{1+\alpha} y \tag{32}$$

Substituting (30) and (32) into (31) and using $v = \lambda(1+\alpha)u$ gives (19). □

A.8 Proof of Lemma 8

Letting λ be the solution the the free entry condition (10) and $\psi = \frac{2\alpha}{1+\alpha} p(\lambda)$, note that λ and ψ are independent of β. Define $\beta^*(\psi)$ as the unique solution to

$$\psi = 1 - \exp\left(\frac{\beta^*}{1-\beta^*}\right)$$

We are interested in the parameter range $\beta \in (0, \beta^*(\psi))$, since in this range $\phi < 1$ in equilibrium, i.e. a positive fraction of firms bargain. In this case, we know that the fraction of posting firms is given by

$$\phi = \frac{1-\psi}{\psi}\left[\exp\left(\frac{\beta}{1-\beta}\right) - 1\right], \tag{33}$$

and the distribution of posted wages is determined by

$$F(w) = \left[\exp\left(\frac{\beta}{1-\beta}\right) - 1\right]^{-1} \frac{w}{y-w}, \tag{34}$$

and the support of F is $[0, \bar{w}]$ with

$$\bar{w} = 1 - \exp\left(\frac{\beta}{1-\beta}\right) \tag{35}$$

Now, consider the workers who send only one application. Conditional on being hired, their expected wage is

$$w_1 = \phi \int_0^{\bar{w}} w \mathrm{d}F(w) + (1-\phi)\beta y \tag{36}$$

From (34) and (35), we derive

$$\int_0^{\bar{w}} w \mathrm{d}F(w) = y \left[1 - \left(\exp\left(\frac{\beta}{1-\beta}\right) - 1 \right)^{-1} \frac{\beta}{1-\beta} \right] \tag{37}$$

Substituting (33) and (37) into (36), we get $w_1 = \omega_1 y$, where

$$\omega_1 = \beta + \frac{1-\psi}{\psi} \left[(1-\beta) \left(\exp\left(\frac{\beta}{1-\beta}\right) - 1 \right) - \frac{\beta}{1-\beta} \right] \tag{38}$$

Next, we show that ω_1 is increasing in β for $\beta \in (0, \beta^*(\psi)]$. Differentiating (38) with respect to β, we get

$$\frac{\partial \omega_1}{\partial \beta} = 1 + \frac{1-\psi}{\psi} \left[\frac{\beta}{1-\beta} \exp\left(\frac{\beta}{1-\beta}\right) + 1 - \frac{1}{(1-\beta)^2} \right] \tag{39}$$

If $\frac{\beta}{1-\beta} \exp\left(\frac{\beta}{1-\beta}\right) + 1 - \frac{1}{(1-\beta)^2} > 0$, then clearly $\frac{\partial \omega_1}{\partial \beta} > 0$. On the other hand, suppose that $\frac{\beta}{1-\beta} \exp\left(\frac{\beta}{1-\beta}\right) + 1 - \frac{1}{(1-\beta)^2} < 0$. Since $\beta < \beta^*(\psi)$ by assumption, we have

$$\frac{1-\psi}{\psi} < \left[\exp\left(\frac{\beta}{1-\beta}\right) - 1 \right]^{-1}$$

and therefore

$$\begin{aligned} \frac{\partial \omega_1}{\partial \beta} =& 1 + \frac{1-\psi}{\psi} \left[\frac{\beta}{1-\beta} \exp\left(\frac{\beta}{1-\beta}\right) + 1 - \frac{1}{(1-\beta)^2} \right] \\ >& \left[\exp\left(\frac{\beta}{1-\beta}\right) - 1 \right]^{-1} \left[\frac{\beta}{1-\beta} \exp\left(\frac{\beta}{1-\beta}\right) + 1 - \frac{1}{(1-\beta)^2} \right] \\ =& \frac{(1-\beta)\exp\left(\frac{\beta}{1-\beta}\right) - 1}{(1-\beta)^2 \left[\exp\left(\frac{\beta}{1-\beta}\right) - 1 \right]} > 0. \end{aligned} \tag{40}$$

This proves that w_1 is increasing in β on the interval $\beta \in (0, \beta^*(\psi)]$.

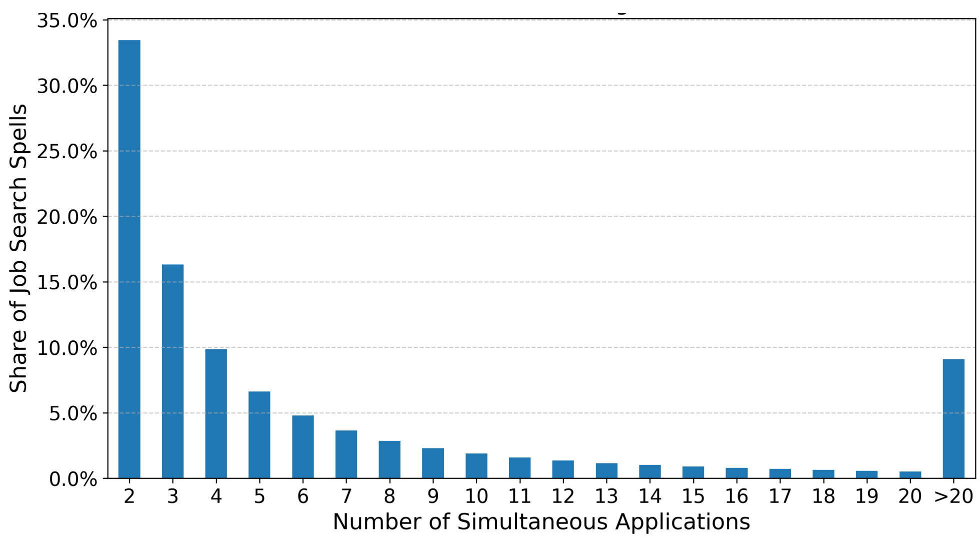
□

A.9 Proof of Corollary 1

As discussed in the text, prohibiting bargaining is equivalent to exogenously setting $\beta = \beta^*(\psi)$. By assumption, this amounts to a rise in β. By Lemma 7, this leaves the average received wage unchanged. By Lemma 8, the expected wage for workers who send only one application increases. □

Appendix B Additional Empirical Analyses

Figure B.1: Job Seekers Distribution by Number of Applications - Conditional on Submitting 2 or More



Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: This figure shows the conditional distribution of job search spells for 1st party applicants by number of submitted applications, conditional on having more than 1 application. A search spell is defined as the set of applications where the elapsed time between the current and prior application is 30 days or less. 1st party applications refer to applications submitted by job seekers on their own behalf (as opposed to 3rd party applications that are submitted by recruiting or staffing firms, for example.

Table B.1: Labor Market Conditions and the Probability of Wage Posting - Alternative α definition

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\lambda)$	0.003		0.003	0.007		0.007
	(0.001)		(0.001)	(0.001)		(0.001)
$\bar{\alpha}$		-0.103	-0.120		-0.112	-0.122
		(0.037)	(0.038)		(0.023)	(0.023)
Observations	721,274	721,274	721,274	703,689	703,689	703,689
R-squared	0.000	0.000	0.000	0.497	0.497	0.497
Firm × Time	N	N	N	Y	Y	Y
absorbed FE				61,814	61,814	61,814
singletons				17,585	17,585	17,585

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: $\bar{\alpha}$ is the mean share of job seekers that submit more than one applications. To calculate this value, we first calculate for each job posting, the share of its applicants that apply to at least one other job. We then calculate the average of this share across all job postings in the same labor market defined as stateXjob functionXmonth cells. Standard errors are clustered at the labor market × time level where a labor market is defined by job function and location.

Table B.2: Labor Market Conditions and Posted Hourly Wages - Alternative α definition

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\lambda)$	-0.005		-0.009	-0.005		-0.008
	(0.002)		(0.002)	(0.002)		(0.002)
$\bar{\alpha}$		1.356	1.400		1.336	1.377
		(0.089)	(0.090)		(0.090)	(0.091)
N	44,268	44,268	44,268	43,156	43,156	43,156
R-squared	0.000	0.009	0.010	0.002	0.011	0.011
$\widehat{\text{Firm} \times \text{Time}}$	N	N	N	Y	Y	Y

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Sample includes postings that explicitly mention a wage per hour rate. For postings with a numerical posted wage but without a rate frequency, we input the frequency based on the distribution of wages with explicitly payment frequency. We trim that bottom and top 5% of the hourly wages (after imputing frequencies). For columns (4) through (6), we use the firm-time fixed effects estimated in equation (17) as controls. When estimating the fixed effects, we use the expected share of job seekers that submit more than one application to measure $\bar{\alpha}$. $\bar{\alpha}$ is the mean share of job seekers that submit more than one applications. To calculate this value, we first calculate for each job posting, the share of its applicants that apply to at least one other job. We then calculate the average of this share across all job postings in the same labor market defined as stateXjob functionXmonth cells. Bootstrapped standard errors with 1,000 replications are clustered at the labor market $\times$ time level where a labor market is defined by job function and location.

Table B.3: Labor Market Conditions and the Probability of Posting a Wage Range or a Single Wage Value Relative to Bargaining

Panel A: Posted Wage Range			
$\ln(\lambda)$	-0.064		-0.006
	(0.017)		(0.021)
$\ln(\alpha)$		-0.098	-0.094
		(0.017)	(0.020)
Panel B: Posted Wage (Single Value)			
$\ln(\lambda)$	0.090		0.124
	(0.011)		(0.014)
$\ln(\alpha)$		0.033	-0.057
		(0.012)	(0.016)
Observations	721,274	721,274	721,274
Firm $\times$ Time FE	N	N	N

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: This table shows the estimated coefficients from a multinomial logit with three possible outcomes: posting a wage range, posting a single wage value, and not posting any numeric value for pay (the baseline outcome). Standard errors are clustered at the labor market $\times$ time level where a labor market is defined by job function and location.

Table B.4: Labor Market Conditions and the Probability of Wage Posting (excluding jobs with pay range)

	(1)	(2)	(3)	(4)	(5)	(6)
ln(λ)	0.006		0.008	0.006		0.009
	(0.001)		(0.001)	(0.001)		(0.001)
ln($\bar{\alpha}$)		0.002	-0.003		-0.001	-0.005
		(0.001)	(0.001)		(0.001)	(0.001)
N	692,263	692,263	692,263	674,676	674,676	674,676
R-squared	0.000	0.000	0.000	0.450	0.450	0.451
FirmXTime	N	N	N	Y	Y	Y
absorbed FE				60,332	60,332	60,332
singletons				17,587	17,587	17,587

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: This analysis excludes job postings that mention a pay range. Standard errors are clustered at the labor market $\times$ time level where a labor market is defined by job function and location.

Table B.5: Labor Market Conditions and Posted Annual Wages

	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\lambda)$	0.014		-0.041	0.012		-0.041
	(0.004)		(0.005)	(0.004)		(0.005)
$\ln(\bar{\alpha})$		0.062	0.091		0.061	0.089
		(0.004)	(0.006)		(0.004)	(0.006)
N	24,600	24,600	24,600	23,592	23,592	23,592
R-squared	0.001	0.023	0.030	0.006	0.027	0.033
$\widehat{Firm \times Time}$	N	N	N	Y	Y	Y

Source: Own calculations using the DHI Vacancy and Application Flow Database (2012-2016).

Notes: Sample includes postings that explicitly mention an annual pay rate. For postings with a numerical posted wage but without a rate frequency, we input the frequency based on the distribution of wages with explicitly payment frequency. We trim that bottom and top 5% of the hourly wages (after imputing frequencies). For columns (4) through (5), we use the firm-time fixed effects estimated in equation (17) as controls. Bootstrapped standard errors with 1,000 replications are clustered at the labor market × time level where a labor market is defined by job function and location.