Journal of Financial Services Research 25:1 5–23, 2004

Liquidity Pools, Risk Sharing, and Financial Contagion

LAWRENCE SÁEZ
London School of Economics

XIANWEN SHI
Yale University

Abstract

This paper reevaluates the Allen–Gale (2000) analysis of interbank deposits to explain financial contagion. This paper modifies the pecking order of asset liquidation developed in Allen–Gale, which is essential in fragility analysis. Furthermore, we also provide a claim structure called ''liquidity pool'' that can both achieve risk sharing and prevent financial contagion across regions when asymmetric information about bank assets is absent. This model can partly explain why bank panics reduced substantially after the founding of the Fed and the role of IMF in regional financial crises.

Key words: Financial intermediation, risk sharing, financial contagion.

1. Introduction

Consumers always encounter *ex ante* liquidity uncertainty in their daily life. They cannot satisfactorily forecast the supply and demand of future liquidity, and often suffer from unexpected liquidity shocks. Therefore, formal institutional arrangements that smooth liquidity fluctuation can improve consumers' welfare greatly. As argued by Diamond (1984), banks are also valuable providers of monitoring services since they act as delegated monitors for small investors and thus avoid the duplication of monitoring costs. Therefore, the ability to provide liquidity insurance and monitoring services to individuals is cited as one justification for the existence of depository institutions.[1]

When there are idiosyncratic shocks that affect the liquidity demand of consumers, banks act as a liquidity pool to achieve optimal risk sharing. However, the function of liquidity provision is also the source of potential financial fragility of banks. Bryant (1980) and Diamond and Dybvig (D–D) (1983) formalized the relationship between liquidity provision and bank runs. They demonstrated that bank deposit contracts provide more optimal allocations than market trade contracts, but correspondingly banks are susceptible

1 See Freixas and Rochet (1997, chapter 2).

to bank runs that are self-fulfilling prophecies. In a bank run, the rationality of individual depositors induces the irrationality of all depositors as a group.[2]

The D–D model led to a re-examination of several unresolved issues in financial intermediation, such as the mechanism forming a bank run, and the role of government deposit insurance and suspension of convertibility in preventing bank runs. Likewise, the D–D model also fostered critiques that followed several lines of argument. First, some authors have questioned the theoretical justification that the federal government should provide deposit insurance (Dowd, 1993).[3] Cooper and Ross (1991) argued that the D–D model's deposit insurance could have adverse effects on monitoring incentives. The presentation in the D–D model of the role of certain banks also led to some critiques. For instance, Huo and Yu (1994) questioned whether the D–D model contained bank runs at all. Von Thadden (1998) argued that whenever a market investment opportunity is completely reversible, deposit contracts cannot provide any insurance against liquidity risks.

In retrospect, another clear weakness in the D–D model was that the banking industry was treated as a representative bank. It mainly concerns itself with individual bank runs, and the whole banking industry is regarded as a unique entity. However, in reality there are many types of banks in different regions or countries and problems arising within the context of such heterogeneity may be different from those found in the D–D model. Difficulty in one bank may cause depositors to suspect that the whole banking industry itself is experiencing difficulties. Therefore, partial regional bank runs may spread to the rest of the country or even cross borders. As shown in subsequent studies, the number and size of banks in a banking system is critical. For instance, Temzelides (1997) argued that with multiple banks, the probability of a panic is inversely correlated to the number of banks. In the Temzelides model, though, banks act in isolation. The contagion sparked by a bank run in the same geographic location inevitably extends to the rest of the population.

The effects of minor bank runs, particularly in transition and developing economies, should not be underestimated. Allen and Gale (1994) showed that a small aggregate liquidity shock could produce significant asset-price volatility, particularly with limited market participation. In light of these potentially devastating effects, the causes of bank

2 In the equilibrium of a bank run, all depositors lose confidence on the solvency of the bank. According to Diamond and Dybvig (1983: 404) ''[t]he shift in expectations . . . could depend on almost everything, consistent with the apparently irrational observed behavior of people running on banks.'' In this case, all depositors withdraw deposits from the bank, even though some of them would be better off holding onto their deposits until expiration, when the possibility of bank failure is excluded. From the perspective of individual depositors, it is rational for them to liquidate their deposits, otherwise they can get nothing. However, from the perspective of all depositors, their competition for deposits can force a healthy bank into bankruptcy, which in turn makes their overall welfare worse off. Therefore, the behavior of the depositors running on a healthy bank is irrational as a whole.

3 The role of deposit insurance was developed earlier in Bryant (1980) and Bryant and Wallace (1980). Bryant (1980) and Bryant and Wallace (1980) differ from the D–D model in that they suspect that the costs of illiquidity may foster asymmetric distribution of risk. Faced with this signal extraction problem, deposit insurance eliminates incentives for agents to seek socially unwasteful information in the presence of undiversifiable systemic risk.

runs and financial contagion have been analyzed at length in the mounting literature on financial contagion.

The thrust of the analysis by Gorton (1985, 1989), Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Calomiris (1989), and Alonso (1996), is that the mechanism resulting in contagion is primarily information-based. The asymmetry of information about a bank's assets makes them susceptible to bank runs. For instance, Jacklin and Bhattacharya (1988) argued that banks runs are triggered by suboptimal asymmetric information between the banks' knowledge about its depositor's liquidity needs and the depositor's information about the banks' assets. We also view that the contagion effects of asymmetric information are important. However, we build upon previous efforts by claiming that whenever individual banks have crossholding interbank deposits, financial contagion is possible even though asymmetric information about the bank assets is absent.

Allen and Gale (2000) introduced the notion of interbank deposits to explain financial contagion.[4] They showed that banks holding interregional claims can insure against liquidity preference shocks in a decentralized setting, even when a central bank is absent. But whenever there is aggregate uncertainty, this arrangement is vulnerable to financial contagion, especially when the financial claims structure is incomplete. According to their model, whenever there is aggregate uncertainty, bank runs in one region may spread to other regions due to excess long asset liquidation triggered by the depreciation of interbank deposits on the disturbed banks. In order to analyze the fragility of interbank claim structures, they compared the liquidation cost of different assets and proposed a pecking order for banks to liquidate their assets. Namely they argued that banks ought to sell short assets before interbank deposits, and interbank deposits before long-term assets. Such an order plays a vital role in their analysis of contagion.

Allen and Gale's proposed order of asset liquidation is tenable in most circumstances, but in some particular cases it may not hold and their subsequent contagion analysis needs some modification. In this article, we provide an improvement over the Allen and Gale model by addressing a possible anomaly. When a disturbed bank becomes bankrupt, but the liquidity gap is rather small, then the bank holding deposits on the disturbed bank may sacrifice itself—liquidating its own long assets before liquidating the interbank deposits on the disturbed bank—if the interbank deposits depreciation is large enough. After this modification, the chance of financial contagion is reduced slightly. Building upon this approach, we then advance an alternative model of the liquidity pecking order.

Moreover, we find that financial contagion can be avoided to an even greater extent through exogenous international cooperation. For this purpose we develop the concept of a liquidity pool. A liquidity pool is defined as a claim structure where banks are indirectly connected by holding bank deposits on a commercial pool. As we will suggest later, our analysis has far-reaching policy implications because the operation mechanism of the

4 The interbank market discussed here is different from the market defined in Bhattacharya and Gale (1987). In Bhattacharya and Gale (1987), the price of interbank deposits is determined by the interaction of supply and demand. Here the price of interbank deposits is *ex ante* determined before the liquidity shock.

liquidity pool without a formal central bank most resembles international monetary organizations, such as the International Monetary Fund (IMF).

This article will be arranged as follows. In section 2, we will first provide the basic framework of our model built upon some relevant conclusions made in Allen and Gale (2000). Our modification to the liquidating order will then be developed in section 3. In section 4 we will provide the framework of the market structure of a liquidity pool. We also outline the parameters of a money center. Finally, in section 5, we offer some preliminary conclusions and guidance for future research.

2. Allen and Gale reconsidered

2.1. The basic framework of the model

Consider an economy with one good and a continuum of consumers (agents or depositors). The model consists of three periods: a planning period $t = 0$, and two subsequent consumption periods, $t = 1$ and $t = 2$. All consumers are endowed with one unit of good at period 0, which they may want to consume at periods $t = 1$ or $t = 2$. Two kinds of investment technologies are also assumed: a storage technology and a long-term technology. One good invested in the storage technology at period t produces one unit good at period $t + 1$. However, the long-term technology is illiquid. One good invested in the long-term technology at period $t = 0$, produces a return of R $(R > 1)$ at period 2, but produces r $(0 < r < 1)$ at period 1, because the investment must be liquidated with some cost. Therefore, storage technology represents liquid assets and long-term technology represents illiquid assets.

In common with Allen and Gale (2000), the above economy is constituted by four *ex ante* identical regions, labeled A, B, C, and D, respectively. Each region has two kinds of *ex ante* identical consumers: early consumers and later consumers. They are subject to the typical D–D preferences: with some probability ω they are early consumers; with probability $(1 - \omega)$ they are late consumers. The utility of early consumers is $u(c_1)$ while that of the later consumers is $u(c_2)$. Therefore, *ex ante* all consumers have the same expected utility at period 0. A given consumer's type is not observable, so late consumers can always pretend to be early consumers. To make the analysis explicit, the utility function $u(\cdot)$ is assumed to be continuously differentiable, strictly increasing, and strictly concave. Then the expected utility function can be expressed as follows:

$$U(c_1, c_2) = \omega u(c_1) + (1 - \omega)u(c_2). \tag{1}$$

Different regions have different ratio of early consumers. The value of ω in two regions is high, denoted by ω_H, and the value in other regions is low, denoted by ω_L. The value of ω is randomly realized, depending on the nature. There are two states, s_1 and s_2, each with probability of 0.5. The corresponding realizations of the liquidity preference are given in table 1.

Consider a single region, for example, region A. Consumers in this region cannot predict future liquidity demand; banks in this region can help consumers smooth their liquidity fluctuations. Nevertheless, the percentage of early consumers in this region can be high or

***Table 1.* Regional liquidity shocks**
Different regions have different ratio (ω) of early consumers. The value of ω is randomly realized, depending on the nature. ω can take two values, high (ω_H) or low (ω_L). There are two states s_1, and s_2, each with probability of 0.5. In state s_1, the ratio of early consumers is high in region A and C, and low in region B and D. In state s_2, the ratio of early consumers is low in region A and C, and high in region B and D.
Source: Allen and Gale, 2000: 8.

Region State	A	B	C	D
s_1	ω_H	ω_L	ω_H	ω_L
s_2	ω_L	ω_H	ω_L	ω_H

low, hence banks in this region alone cannot get rid of this risk. Therefore, every realization of the percentage is an aggregate shock for this region. Applying the D–D analysis to this single region, we can conclude that optimal risk sharing cannot be achieved due to the aggregate liquidity shock. That is, when the four regions are disconnected, they all suffer regional liquidity shocks. However, as shown below, if there is a central planner who can transfer liquidity across regions, then optimal risk sharing defined in the D–D analysis can be achieved easily.

2.2. Interbank deposits and optimal risk sharing

Here we give out the description of the optimal question faced by central planner, a situation that we will use as a benchmark. In our model, all the regions are identical at period 0 and the central planner can transfer liquidity across regions at period 1. Hence, at period 0, we can treat banks in each region as a representative bank, whose ratio of early consumers is $\gamma = (\omega_H + \omega_L)/2$.[5] Suppose the representative bank invests x on long-term assets and y on short-term assets. Then the optimal problem becomes:

$$\begin{aligned} &\max_{x,y} \gamma u(c_1) + (1-\gamma)u(c_2) \\ &\text{s.t:} \quad x + y \leq 1 \\ &\qquad \gamma c_1 \leq y \\ &\qquad (1-\gamma)c_2 \leq Rx. \end{aligned} \tag{2}$$

The above analysis is also applicable to a situation where a central planner is absent, but where regions are connected by some other linkages. In this case, decentralized decisions can still achieve optimal risk sharing, just as in the case where the central planner is present. Following on Allen and Gale (2000), we also link four regions by interbank deposits. According to the way of interbank deposits cross holding, the claim structure differs from the complete structure to the incomplete structure. Figure 1(a) depicts a

5 For the sake of simplicity, the terms ''Bank A'' and ''Banks in region A'' have been used interchangeably in this study.

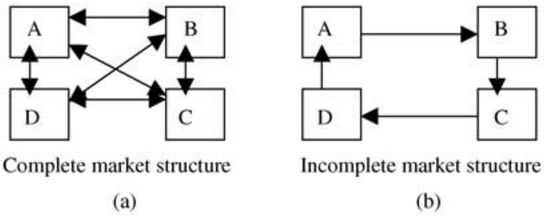

Figure 1. **Two market structures.** A, B, C, and D are the four *ex-ante* identical regions (or representative regional banks) in the economy. In figure (a), each region *exchanges* deposits with every another region at date 0. For example, region A holds deposits on region B, C, and D, and B, C and D hold the *same* amount of deposits on A as well. In contrast, the connection between regions in figure (b) is one-way. Region A holds deposits on region B, but B does not hold deposits on A. In the absence of aggregate uncertainty, the amount of interbank deposits needed to achieve first best allocation in figure (a) is only half of those needed in figure (b). Thereafter, we use two-way arrows to denote deposits exchange and one-way arrows to denote one-way deposits.

complete market structure and figure 1(b) represents an incomplete structure. If there are only two states, s_1 and s_2, then there is no aggregate uncertainty, optimal risk sharing can be achieved by both market structures, complete or incomplete.

In the complete market, each region holds deposits z on every other region. Hence, there is a two-way connection in the form of interbank deposits between any two regions. For example, region A holds deposits z on B, C, D, while B, C, and D also each hold deposits z on region A. In contrast, in an incomplete market, such as the one that we have shown in figure 1(b), the connection is one-way; therefore more complex connections between some regions are absent. Hence, region A only holds deposits on region B, B only holds deposits on C, and so on. For the sake of parsimony, there is no connection between A and C, or B and D in our analysis. Here we will simply show that we can achieve optimal risk sharing in the incomplete market structure by holding interbank deposits. Just as indicated in figure 1(b) and table 1, the liquidity demand of each region is negatively connected with that of adjacent regions and each region holds $z = (\omega_H - \gamma) > 0$ on one of its adjacent regions. For example, in state s_1, region A has a high liquidity demand; its adjacent regions B and D have low-liquidity demand; and A hold z interbank deposits on B. At period 1, regions with low liquidity demand will retain the deposits they hold while regions with high liquidity demand will liquidate their interbank deposits to satisfy the demand of early consumers.

Consider banks in a region with high liquidity demand. At period 1, they have y units of short-term assets and z interbank deposits to satisfy liquidity demand of $\omega_H c_1$. At period 2, they then have x units of long-term assets to satisfy liquidity demand of $(1 - \omega_H)c_2 + zc_2$. Therefore, their constraint for optimal allocation becomes:

$$\begin{aligned} x + y &\leq 1 \\ \omega_H c_1 &\leq y + zc_1 \\ (1 - \omega_H)c_2 + zc_2 &\leq Rx. \end{aligned}$$

Substituting $z = (\omega_H - \gamma)$ into the above constraints, we can see above constraints are

***Table 2.* Regional liquidity shocks with perturbation**
In this table, we add an additional state $\bar{s}$, the state with aggregate liquidity shock, and assign zero probability to this state. In state $\bar{s}$, each region has the previous average liquidity demand $\gamma = (\omega_H + \omega_L)/2$ except for region A, where the average liquidity demand is $\gamma + \varepsilon$, where $\varepsilon > 0$. State s_1 or s_2 still occurs with half probability and the rest of the table is the same as table 1.
Source: Allen and Gale, 2000: 16.

Region State	A	B	C	D
s_1	ω_H	ω_L	ω_H	ω_L
s_2	ω_L	ω_H	ω_L	ω_H
$\bar{s}$	$\gamma + \varepsilon$	γ	γ	γ

the same as those in problem (2). The same conclusion is also applied to banks with low-liquidity demand.

In sum, if there is no aggregate uncertainty, then optimal risk sharing can be achieved without the risk of financial contagion, even in the context of an incomplete interbank deposits market. However, when aggregate liquidity shock is present, even with negligible probability, financial contagion is possible, especially in an incomplete interbank deposits market.

3. Modification for the pecking order: The selfless sacrifice

3.1. The pecking order in the Allen–Gale model

Allen and Gale (2000) introduced a model of regional liquidity shock with perturbation. As can be gleaned from table 2, they introduce state $\bar{s}$, to represent the state in which the aggregate liquidity demand is higher than the system's ability to supply, and the demand of liquidity in the disturbed region is $\gamma + \varepsilon$. In addition, they assumed that such perturbation happens with zero probability.

Allen and Gale combined the market structure illustrated in figure 1(b) with the liquidity shocks indicated in table 2. In this way, it is always optimal for early consumers to withdraw deposits at period 1. Late consumers will withdraw their deposits at period 2, especially if holding deposits until their expiration can bring them higher consumption level. Otherwise, they will liquidate their deposits at period 1. At period 1, when the bank cannot meet the demand of depositors, all bank assets are liquidated and split equally among all depositors.[6]

In order to maximize the interest of all depositors, the bank must liquidate its assets with the lowest cost in order to meet the demand of depositors at period 1. According to Allen and Gale, the liquidation cost of short assets, interbank deposits and long assets is 1, c_2/c_1

6 The assumption here is different from the sequential service constraint assumption (namely a first come, first served) implicit in the D–D model.

and R/r, respectively. If the liquidation cost of long assets is large enough, in other words, r is sufficiently small, then the following condition holds:

$$1 < \frac{c_2}{c_1} < \frac{R}{r}. \tag{3}$$

If condition (3) holds, then the bank should observe the following liquidity pecking order: short assets, deposits, long assets.[7] According to the pecking order, when bank A is hit by a liquidity shock, bank D will not liquidate its long assets before redeeming its deposits on bank A.

The pecking order is essential in analyzing the fragility of the financial claim structure in Allen and Gale (2000). Only with the pecking order can we clearly know whether the bank in consideration is in the state of being *solvent*, *insolvent* or *bankrupt* as defined in their paper. They argue that:

> ''The bank is said to be *solvent* if it can meet the demands of every depositor who wants to withdraw . . . by using only its liquid assets, that is, the short asset and the deposits in other regions. The bank is said to be *insolvent* if it can meet the demands of its deposits but only by liquidating some of the long asset. Finally, the bank is said to be *bankrupt* if it cannot meet the demands of its depositors by liquidating all its assets.''[8]

Their liquidity order forms an essential base of the robustness analysis in the Allen–Gale paper.

3.2. Modified pecking order

Allen and Gale's conclusion holds in most circumstances. But here we would like to point out that the liquidity cost of interbank deposits is not generally c_2/c_1. When some banks become bankrupt, the unit value of interbank deposits on these disturbed banks is lower than c_1, hence, the cost for other banks to liquidate deposits is larger than c_2/c_1. If we denote the unit deposits value on the representative bank in region i at period 1 as q^i, then we can generalize the liquidity cost of deposits (LCD) on bank A as follows:

$$\text{LCD} = c_2/q^{\text{A}}, \text{ where } \begin{cases} q^{\text{A}} = c_1, & \text{if bank A does not become bankrupt;} \\ q^{\text{A}} < c_1, & \text{if bank A becomes bankrupt.} \end{cases}$$

When bank A becomes bankrupt, the liquidation value of interbank deposits on bank A is lower than c_1. As long as the liquidation value q^{A} is sufficiently low, the liquidation cost of interbank deposits can be much higher than c_2/c_1.

Furthermore, Allen and Gale neglect the possibility that, in some circumstances, liquidating long assets and transferring them to bank A may help A overcome difficulties.

7 Allen and Gale (2000: 17).

8 *Ibid*. Italics in the original citation.

With the liquidity transfer from bank D, bank A can assure its later consumers not to withdraw deposits early, thus avoiding excess long assets liquidation and recovering the value of interbank deposits to c_1. If we let *DEP* denote the interbank deposit depreciation avoided by liquidating unit long assets, then the actual cost to liquidate long asset is not R/r, but R/DEP. If the latent deposit depreciation due to bankruptcy is sufficiently high, then the actual liquidation cost of long assets may be much smaller than R/r.

Therefore, in some circumstances, condition (3) still holds but it is no longer relevant. Instead, the pecking order of asset liquidation is determined by following condition:

$$\frac{c_2}{q^{\mathrm{A}}}(\text{interbank deposits}) > \frac{R}{DEP}(\text{long assets}) > 1(\text{short assets}). \tag{4}$$

Consequently, the pecking order proposed by Allen and Gale should be replaced by the following pecking order: short assets, long assets, and then interbank deposits.

However, we should point out that here we do not take into account the free-rider problem and liquidity allocation problem between banks in the same region. When region D, for example, has more than two banks, and each of them holds deposits on banks in the disturbed region A, then coordination between banks in region D is important. In addition, the allocation of additional liquidity between banks in the disturbed region A is also important. These situations are interesting, but in our simple model, they are too complex to handle. Here we simply assume that banks in the same region can reach unanimous agreement and select a representative bank to bargain with representative banks in other region. In our setting, it is appropriate to consider each region as a country, and the national governments assume the representative role in cross-countries bargaining.

Next we will investigate the specific condition for our modified pecking order to hold. Consider the following situation. The representative bank in region A has an excess liquidity demand and becomes bankrupt, but the liquidity gap is rather small and the loss due to deposit depreciation is rather large. Formally, if we define $b(\omega) \equiv r[x - ((1-\omega)c_1/R)]$ as the bank's buffer, namely the maximal amount of liquidity that can be attained by liquidating the long assets without causing a bank run. In addition, the liquidity gap of the disturbed bank is broadly defined as the gap between the excess liquidity demand εc_1 and the bank's buffer $b(r+\varepsilon)$, denoted as $g(\varepsilon) = \varepsilon c_1 - b(r+\varepsilon)$. Then, the above exception case can be described as

$$\varepsilon c_1 \geq b(\gamma + \varepsilon) \text{ and } z(c_1 - \bar{q}^{\mathrm{A}}) \geq \lambda R. \tag{5}$$

Where λ denote the minimum of long assets needed to be liquidated to meet the liquidity gap, that is $\lambda = g(\varepsilon)/r$. $\bar{q}^i$ denotes the up bound of the value of interbank deposits on the bankrupt bank, and $\bar{q}^{\mathrm{A}} = (y + rx + zc_1)/(1+z)$.[9]

As long as condition (5) holds, our modified pecking order holds, and the bank will liquidate short assets before long assets, and long assets before interbank deposits. Condition (5) has two implications. First, the liquidity gap is positive and the representative bank A become bankrupt, so the liquidating cost of interbank deposits is

9 We can derive $q^{\mathrm{A}} = (y + rx + zq^{\mathrm{B}})/(1+z)$ and $q^{\mathrm{A}} \leq \bar{q}^{\mathrm{A}} = (y + rx + zc_1)/(1+z)$.

c_2/q^A, where q^A is less than c_1. Second, for the representative bank in region D, it has two choices: to help or not to help. If it leaves bank A alone, it must suffer a loss of $z(c_1 - \bar{q}^A)$ caused by the depreciation of deposits. On the other hand, if it liquidates λR long assets and transfers them to help the disturbed bank, then it suffers a loss of the long asset liquidated, but the deposits depreciation is avoided.

Condition (5) says that the depreciation of interbank deposits is higher than the cost to prevent disturbed banks from bankruptcy. In this case, banks in adjacent region will liquidate their own long asset before redeeming interbank deposits. Therefore, we conclude that the pecking order proposed by Allen and Gale is violated.

If we replace λ with $\lambda = g(\varepsilon)/r$ in condition (5), we can get

$$\varepsilon c_1 \geq b(\gamma + \varepsilon) \text{ and } z(c_1 - \bar{q}^A) \geq g(\varepsilon)R/r. \tag{6}$$

However, we know that $\bar{q}^A = (y + rx + zc_1)/(1 + z)$, and according to the budget constraint of the first-best allocation we have $\gamma c_1 = y$ and $(1-\gamma)c_2 = Rx$.[10] If we substitute all the aforementioned equations into condition (6), then we can arrive at

$$\varepsilon \geq \left(1 - \frac{r}{R}\right)^{-1} r\left(x - \frac{(1-\gamma)c_1}{R}\right) = \frac{r}{R}\left(1 - \frac{r}{R}\right)^{-1}(1-\gamma)\left(\frac{c_2}{c_1} - 1\right), \tag{7a}$$

and

$$\varepsilon \leq \left(1 - \frac{r}{R}\right)^{-1}\left[\frac{r}{R}(1-\gamma)\left(\frac{c_2}{c_1} - 1\right) + \left(\frac{r}{R}\right)^2\left(\frac{z}{1+z}\right)(1-\gamma)\left(\frac{R}{r} - \frac{c_2}{c_1}\right)\right]. \tag{7b}$$

Moreover, if we let

$$\underline{\varepsilon} = \frac{r}{R}\left(1 - \frac{r}{R}\right)^{-1}(1-\gamma)\left(\frac{c_2}{c_1} - 1\right),$$

then we can define

$$\begin{aligned}\bar{\varepsilon} &= \left(1 - \frac{r}{R}\right)^{-1}\left[\frac{r}{R}(1-\gamma)\left(\frac{c_2}{c_1} - 1\right) + \left(\frac{r}{R}\right)^2\left(\frac{z}{1+z}\right)(1-\gamma)\left(\frac{R}{r} - \frac{c_2}{c_1}\right)\right] \\ &= \underline{\varepsilon} + \left(1 - \frac{r}{R}\right)^{-1}\left(\frac{r}{R}\right)^2\left(\frac{z}{1+z}\right)(1-\gamma)\left(\frac{R}{r} - \frac{c_2}{c_1}\right).\end{aligned}$$

Therefore, the condition for the modified pecking order to be held is $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The length of the interval is

$$L = \left(1 - \frac{r}{R}\right)^{-1}\left(\frac{r}{R}\right)^2\left(\frac{z}{1+z}\right)(1-\gamma)\left(\frac{R}{r} - \frac{c_2}{c_1}\right),$$

10 Remember that we assume the state $\bar{s}$ occurs with zero probability, so the budget constraint of for the optimal allocation is the same as before at period 0.

which describes the probability of the violation behavior of adjacent banks, and is always positive.

From the definition of interval length, we can see that the incentive of adjacent banks to sacrifice is determined by several variables: z, r/R, γ, and c_2/c_1. The more closely related that regional banks are, the larger the value of z. Consequently, an adjacent bank has a stronger incentive to violate the pecking order to rescue its neighbors. The ratio r/R reflected the difference between the liquidation and long-term returns. If the liquidation cost of long-term assets is very large relative to its natural return of R, then the resulting ratio of r/R is very small. Hence, the incentive is small. Similarly, γ represents the average ratio of early consumers. Thus, the larger the value of γ, then more likely the probability that a rescue has occurred. Finally, the interval length is negatively correlated with the ratio of c_2/c_1. The larger the ratio is, the less likely the modified pecking order will hold (see Appendix A for an illustrative example of the modified pecking order).

3.3. The selfless sacrifice and self-salvation

As we have discussed above, when condition (5) is satisfied, it pays for banks in region D to liquidate λ long assets and transfer to banks in region A in order to meet the liquidity gap. After receiving the additional liquidity, banks in region A can reassure their later consumers not to pre-liquidate deposits. Therefore, the long asset liquidation in region A is largely avoided and the value of interbank deposits hold by banks in region D is recovered to c_1.

Note that banks in region D cannot require more than deposits z even though they liquidate λ long assets, so it seems that they selflessly sacrifice themselves to help others go through financial distress. But if we notice that the expected gain $z(c_1 - \bar{q}^A)$ from liquidation is larger than the cost of liquidation, it is in their interest to violate the pecking order.

Sometimes the sacrifice behavior can even turn into self-salvation, only if we add the following conditions (8) and (9):

$$z(c_1 - \bar{q}^A) > b(\gamma), \tag{8}$$

and

$$(x - \lambda)R \geq (1 - \gamma)c_1. \tag{9}$$

Condition (8) ensures that if banks in region A become bankrupt, then banks in region D will also become bankrupt. Thus financial contagion is spread. However, condition (9) can ensure that liquidating λ long assets will not trigger bank run. We can change condition (9) into condition (10):

$$g(\varepsilon) < b(\gamma). \tag{10}$$

If we combine condition (8) and (10), we can get

$$g(\varepsilon) < b(\gamma) < z(c_1 - \bar{q}^A). \tag{11}$$

Equation (11) represents two conditions under which the selfless sacrifice will become self-salvation. First, the liquidity gap is smaller than the bank buffer. So the adjacent bank has the ability to sacrifice without threatening its own survival. Second, the bank buffer is lower than the loss that results from interbank deposits depreciation. In other words, if it does not help its neighbor to avoid bankruptcy, then the depreciated value of interbank deposits on the disturbed bank will make itself bankrupt.

Applying such an approach to banks in region C and B, the possibility of financial contagion is further reduced as illustrated by the following case. If banks in region A become bankrupt due to direct perturbation and condition (5) holds, but the liquidity gap $g(\varepsilon)$ satisfies

$$b(\gamma) < g(\varepsilon) < 2b(\gamma). \tag{12}$$

Then banks in region D will observe our modified pecking order and become bankrupt. Obviously, the liquidity gap of the representative bank in region D is less than $b(\gamma)$. From the perspective of banks in region C, the tenets of condition (5) also hold, so it will liquidate its long assets to help banks in region D. Therefore, bank in region A, C, and D will liquidate part of their long assets simultaneously and avoid mutual financial contagion. If we further take bank B into consideration, the financial contagion can be avoided as long as $g(\varepsilon)$ is lower than $3b(\gamma)$ and condition (5) holds.[11] Such coordination is absent in the Allen–Gale analysis.[12]

3.4. Summary

As we have shown here, only when the liquidity shock is sufficiently large ($\varepsilon > \overline{\varepsilon}$), can we apply the basic structure of the Allen and Gale model to financial contagion analysis. When the liquidity shock ε is located somewhere in the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, our modified pecking order works most optimally and financial contagion can be largely prevented. Whenever conditions (5) and (11) are satisfied, banks in region D will liquidate their long assets to help bank A. By undertaking such a course of action, banks in region D also save themselves from bankruptcy.

From above analysis, we can yield the following conclusions:

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11 We can also derive the largest ε^0 for $g(\varepsilon) < 3b(\gamma)$ to hold. It is easy to verify that ε^0 is much larger than $\overline{\varepsilon}$ with common parameters. Hence, if coordination between banks in different regions is possible, financial contagion is absent when $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$.

12 The coordination can be expressed in game theoretic terms as an n-person cooperative game. In such a game it is assumed that all the players coordinate their strategies in order to maximize their total payoff. Likewise, utility is transferable between the members of the coalition. In our model, all non-disturbed banks would share the rescue responsibility to help the disturbed bank. In the presence of coordination, each bank only has to liquidate one third of long assets necessary to meet the liquidity gap, thus greatly reducing the chance to trigger bank run. Then financial contagion can largely be avoided as long as the liquidity gap is lower than three times the bank's buffer. Whether such coordination is possible needs to be developed elsewhere. As a consequence of uncertain payoffs, multiple imputations are possible, hence threatening the internal instability of the coalition.

- When the liquidity shock $\varepsilon \in (0, \underline{\varepsilon})$, the disturbed banks become insolvent but not bankrupt, and then other banks need not liquidate their long assets and no financial contagion occurs.
- When the liquidity shock $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$, disturbed banks become bankrupt and our modified pecking order holds. If the liquidity gap is higher than bank's buffer $b(\gamma)$, but lower than $3b(\gamma)$, then some kind of coordination could appear automatically. All regional banks will liquidate part of their long assets simultaneously to help others. With common parameters, no financial contagion occurs.
- When the liquidity shock $\varepsilon \in (\overline{\varepsilon}, 1]$, the pecking order proposed by Allen and Gale works, and financial contagion spreads.
- The more closely related that regional banks are, the higher liquidation cost of long assets and the lower the average ratio of early consumers, the more likely banks will violate the pecking order defined in the Allen–Gale model.

4. Liquidity pool

4.1. Risk sharing and the liquidity pool

Just as the risk sharing function of individual bank may lead to bank runs, the risk sharing function of interbank deposits market also induces the conditions under which the risk of financial contagion is probable. After analyzing the alternative interbank markets that follows their fragility analysis, Allen and Gale pointed out that ''[a]n important issue is whether there exist alternatives to interbank deposits that achieve risk sharing in states s_1 and s_2 but avoid contagion in state $\bar{s}$.''[13] They concluded that ''provided that there are *ex ante* contracts between banks that are signed at period 0, a contagion will occur even if contracts are not demand deposits.''[14]

Hence, the question that remains to be answered is whether there is any claim structure that can prevent financial contagion in state $\bar{s}$ while at the same time achieving optimal risk sharing in state s_1 and s_2. While it is true to the specific claim structures indicated in their article, it might need some qualification, particularly if we apply it to a special claim structure and permit a kind of interbank deposits with characteristics different from those introduced in their paper. Here we will give a claim structure called a *liquidity pool*.

We define a liquidity pool as a claim structure where banks are indirectly connected by holding bank deposits onto a common pool. Our definition of a liquidity pool presents us with two options. If we define bank deposits in an identical manner in which they are presented in the Allen–Gale model, then this structure is similar to the completely connected market structure. However, if we introduce interbank deposits of a different nature into this structure, then we can achieve risk sharing as well as contagion proof. In section 4.2 we will investigate the liquidity pool with common interbank deposits. We will

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13 Allen and Gale (2000: 24).

14 *Ibid*., p. 26.

then discuss a liquidity pool with interbank deposits of different kinds in section 4.3. A similar claim structure, which we will call a *money center*, will be analyzed in section 4.4.

4.2. Optimal risk sharing as developed in Allen–Gale model

Let us consider a claim structure illustrated earlier in figure 2(a). At period 0, each bank holds a deposit $z = (\omega_H - \gamma) > 0$ on the pool and the pool holds a deposit z on each bank. So the net effect is zero, and there are no real liquidity creation and transfer. At period 1, each bank adjusts its liquidity position according to realized liquidity demand. Banks with high-liquidity demand will withdraw their deposits from the pool, while banks with low demand will retain the deposits in the pool. On the other hand, in order to maintain balance, the pool will withdraw deposits from bank with low-liquidity demand and will retain deposits on the banks with high-liquidity demand.

At period 1 in state s_1 or s_2, the liquidity demand is $2zc_1$, and the liquidity supply is also $2zc_1$. Therefore, the net position of the pool is zero. At period 2, banks in regions with high-liquidity demand will return zc_2 to the pool, an amount that will be used to pay to banks in regions with low-liquidity demand. It is easy to verify that such market structure can achieve first best allocation when the aggregate liquidity demand is certain. When there is an aggregate liquidity shock, the effect on the banks in such structure is similar to the complete structure analyzed in Allen and Gale (2000).

4.3. Optimal risk sharing and financial contagion proof

Next, we will consider the market structure indicated in figure 1(b). All regions in this claim structure agree that:

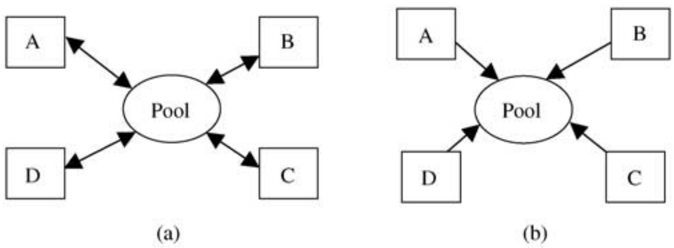

Figure 2. **Liquidity pool.** The four identical regions are indirectly connected through their deposits on the liquidity pool. There is no other connection between any two regions. In figure (a), each *ex-ante* identical region (representative bank) *exchanges* deposits with the liquidity pool. In figure (b), each region holds the same amount of deposits on the liquidity pool, but the pool holds no deposits on any region. Hence, in the absence of aggregate uncertainty the amount of deposits needed to achieve first best allocation in figure (a) is only half of those needed in figure (b).

- At period 0, each region transfer liquidity assets zc_1 to the liquidity pool. It is worth noting here that the liquidity transfer is a net transfer (one-way) and the amount is zc_1 not z.
- At period 1 the liquidity pool can grant contingent loans zc_1 to regions with high-liquidity demand conditioning on the liquidity availability: if every region rushes to withdraw their deposits from the pool, then the liquidity pool will kick one of the regions (the disturbed region) out, and the rest regions will adjust their liquidity position through the pool; otherwise, regions withdrawing deposits from the pool can have additional liquidity loans zc_1.
- At period 2, the regions with high demand will transfer zc_2 to the pool. Then the liquidity pool pays all the money collected to regions with low-liquidity demand. These low liquidity demand regions will get zc_2 from the pool to meet demand of their later consumers.

In states s_1 and s_2, there is no aggregate uncertainty, so banks with low liquidity demand will not withdraw deposits from the pool. Consequently, at period 1, a region with high demand will withdraw its deposits from the pool and be granted an additional liquidity loan with amount of zc_1. Its liquidity demand and supply are $\omega_H c_1$ and $(y + zc_1)$, respectively.

At period 2, though, it must return zc_2 to the pool to pay the loan. Therefore, its liquidity demand and supply are $(1 - \omega_H)c_2 + zc_2$ and Rx, respectively.

In sum, its budget constraints are:

$$\begin{aligned} &\omega_H c_1 \leq y + (\omega_H - \gamma)c_1 \\ &[(1 - \omega_H) + (\omega_H - \gamma)]c_2 \leq Rx. \end{aligned} \tag{13}$$

While the budget constraints of banks with low liquidity demand are:

$$\begin{aligned} &[\omega_L + (\omega_H - \gamma)]c_1 \leq y \\ &[(1 - \omega_L) + (\omega_H - \gamma)]c_2 \leq Rx. \end{aligned} \tag{14}$$

If we substitute $\gamma = (\omega_H + \omega_L)/2$ into above, we can derive the same budget constraints as those in the first best allocation already shown in problem (2). Therefore, first-best allocation can be achieved.

In state $\bar{s}$, banks in the disturbed region A suffered a liquidity shock ε, and the ratio of early consumers in the region is higher than the average level γ. After the state is realized, all other regions can observe the state and know whether the banks in the disturbed region will become bankrupt if they are left alone. If banks in region A are not face with bankruptcy, then regions with low liquidity demand will not withdraw deposits from the pool. They will use these liquidity to grant region A liquidity loans additional to its own deposits. Accordingly, no financial contagion occurs. If all other regions anticipate that the banks in region A will become bankrupt, then all other region (namely B, C, and D) would decline to make loans to region A. In this situation, all of them will withdraw deposits from the liquidity pool. Under the agreement of the liquidity pool, region A will be kicked out of the pool. Hence, the financial distress affecting region A will be isolated from the other healthy regions. Again, no financial contagion occurs. Whether banks in the disturbed region A become bankrupt or not does not influence the solvency of banks in other regions. Therefore, there is no financial contagion in the claim structure of liquidity pool.

4.4. The money center

If we randomly choose one of the four regions as the liquidity pool, a structure that we call a money center (illustrated in figure 3), similar results can be reached although sometimes financial contagion does occur in such a market structure.

For instance, region A is selected as the money center. Every region other than A holds deposits z on region A, but holds no deposits on each other. Money center A does not hold deposits on any other regions. In this claim structure, regions are indirectly connected by depositing on the money center only. At period 0, banks in region A accept deposits from banks in other regions and stipulate the condition and amount of the additional loan that regions with high liquidity demand can get from the money center. At period 1, all banks adjust their liquidity position according to realized liquidity preference. Banks in regions with high liquidity demand will withdraw deposits and get additional loans from region A, while banks with low liquidity demand will retain their deposits in the money center. At period 2, banks with liquidity demand will pay the loans from banks in region A as stipulated in period 0.

One can assume two kinds of interbank deposits: one involving a one-way net liquidity transfers; the other involving no net liquidity transfers. If we adopt the former kind of deposits, indicated as figure 3(a), we must face the paradox: interbank deposits must receive lower return than consumer deposits. In this case, the returns of interbank deposits and consumer deposits in period 1 are 0 and c_1, and in period 2, c_2/c_1 and c_2, respectively. This return differential between different deposits could reflect the insurance fee that the money center bank charges on other regional banks for its promise to provide additional liquidity loans when they face high-liquidity demand. However, such assumption seems to be a little unrealistic. Therefore, the utility of developing a money center with net liquidity transfer has some important limitations that need to be developed in future research.

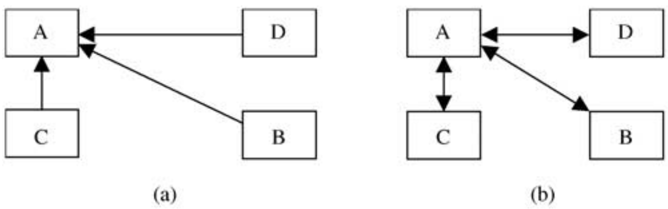

Figure 3. **Money Center.** Suppose region (or representative bank) A is randomly selected as the money center through which the four regions adjust their liquidity position when liquidity shock happens. There is no connection among B, C and D other than the indirect connection through their deposits on the money center A. In figure (a), the rest three regions B, C and D hold deposits on the money center at date 0, but the money center does not hold any deposits on region B, C and D. In figure (b), the money center *exchanges* deposits with each region at date 0, so A holds the same amount of deposits on the other three regions as well.

If the money center adopts deposits as defined in Allen–Gale (shown in figure 3(b)), then the money center structure is very similar to the complete market structure. Since the money center involves much less connection between regions, yet achieves similar risk sharing results stemming from the complete market structure, we can claim that the money center structure is nearer to reality than the complete market structure. But the most important assumption in our liquidity model is that the central bank (or the liquidity pool) can grant liquidity loans contingent to the availability of liquidity. If we adopt interbank deposits as Allen–Gale, such assumption cannot hold. Therefore, financial contagion cannot be completely prevented.

4.5. Summary

From what is discussed above, we can conclude that if banks are connected indirectly and liquidity loans decisions are contingent upon the availability of the liquidity pool, then risk sharing and financial contagion proof can both be achieved. The connecting medium (in this case the liquidity pool) smoothes the uneven liquidity demand and insulates the disturbed region from other regions.

The liquidity pool can be concentrated into a single region, what we have labeled a money center. However, as we have suggested here, this configuration may have some limitations based on the types of deposits.

5. Conclusion

Effective and timely regulation of bank insolvencies is probably one of the most important elements of a banking safety net. As illustrated in the D–D model, deposit contracts can smooth the inter-temporal liquidity fluctuation of consumers. But this risk sharing function can also make banks vulnerable to bank runs. Even more, bank runs can develop into bank panics due to the domino effect that emerges once financial contagion has started. Allen and Gale (2000) successfully extended the D–D model by introducing interbank deposits to explain financial contagion across different regions. They concluded that in an incomplete financial claim structure, a small preference shock in one region might spread to others easily.

In order to illustrate the financial fragility of the interregional claim structure, Allen and Gale proposed a specific pecking order of asset liquidation. From the perspective of simple liquidation cost, their pecking order holds under most circumstances. However, as shown here, when the liquidity gap of the distressed bank is small and the depreciation of interbank deposits is large, related banks might have incentive to violate the order.

In this article we have also addressed the issue of alternative market structures in preventing financial contagion. We demonstrate that banks indirectly connected by liquidity pool can achieve risk sharing and prevent financial contagion. In our model, the liquidity pool acts something like a passive central bank. Although our conclusions are preliminary, our model best corresponds with the empirical research of financial panics in the United States before and after the founding of the Federal Reserve System.

Historically, U.S. banks were providing services limited to specific jurisdiction. Most banks were regional banks, and they were incompletely connected just as the incomplete financial market described in figure 1(b). Therefore, an individual bank run led to unexpected large deposits withdrawals and developed into a typical bank panic.

As described by Miron (1986), Calomiris and Gorton (1991), Mishkin (1991), and others, during the National Banking System period (1863–1913), financial panics were frequent. After the founding of Federal Reserve, the interbank market changed into a structure akin to the liquidity pool presented here. Banks submitted reserves to the central bank, just as the liquidity transfer in our model. Since then, reserve requirements are widely used by central banks as a means to improve monetary control and to act as a safeguard of bank liquidity. The frequency of financial contagion is steadily decreased just as our model predicts.

The model of liquidity pool may also shed some light on the questions about the different experiences with financial contagion in situations were banks are not directly connected. We see the basis for further research on this problem in a number of directions. The presence of the IMF connects banks in different countries indirectly. The role of IMF to prevent financial contagion is very like the role of the liquidity pool when asymmetric information about the bank assets is absent.

Acknowledgments

The authors would like to thank Sumon Bhaumik, Rafiq Dossani, Binu George, Stephen Morris, Matthew Rudolph, Joy Yang, and Zhiyong Yao, for their helpful insights. The authors of this article are solely responsible for any error of fact or interpretation. Xianwen Shi also acknowledges the financial support from the China National Social Science Foundation (grant no. 02CJL007).

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