The Art of Compromising: Voting with Interdependent Values and the Flag of the Weimar Republic

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Abstract

We compare sequential, binary voting schemes conducted by privately informed agents with interdependent preferences. In addition to two "extreme" positions on the left and on the right, we consider the effect of a compromise alternative. The Anglo-Saxon amendment procedure (AV) always selects the (complete-information) Condorcet winner. In contrast, the continental successive procedure (SV) does not. This holds because AV allows gradual learning about the preferences of both leftists and rightists, while SV only allows one-directional learning at each step. In addition, under SV the agenda that puts the "extreme" alternative with ex ante higher support last elects the Condorcet winner with a higher probability than the agenda that puts that alternative first. We illustrate our main findings with the vote on the flag of the Weimar republic.

1 Introduction

Sequential, binary voting schemes are used by almost all democratic legislatures to decide among more than two alternatives (see Rasch [2000]). In this paper we generalize the associated, standard voting model by introducing interdependent preferences. Since others' signals are private information, each agent is ex ante uncertain about her own preferred alternative.

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The voting process gradually reveals and aggregates information, and agents respond to new information by adjusting their voting strategy. Our main results show how commonly used voting procedures - that are outcome-equivalent under complete information or under private values - yield starkly different outcomes if values are interdependent. The new phenomena arise because different voting rules induce different dynamic processes of information aggregation and learning. In particular, we show that the Anglo-Saxon voting by amendment always selects the complete information Condorcet winner, while this is not the case for the Central-European successive voting procedure.

We model a situation where ex ante opinions are dichotomous and cross the traditional left-right party lines.[1]Several privately informed agents have single-peaked preferences over three alternatives, and each agent's peak is determined by his/her own signal and by the mean signal of others. In addition to the two "extreme" positions on the "left" and on the "right", we consider a compromise alternative whose location may be endogenous.[2] The interdependence of preferences is what makes the compromise salient, whereas the compromise alternative would never be elected in our model under a private values assumption.

We focus here on two most frequently used sequential, binary procedures: all English-speaking democracies, several Scandinavian countries and Switzerland regularly use the *amendment* voting procedure (AV) where alternatives are considered two-by-two, and where the majority winner advances to the next stage, as in an elimination tournament. In contrast, most continental European parliaments (including the EU parliament) use the *successive* voting procedure (SV) where alternatives are put to vote, one after another, until one of them gets a majority. Moreover, we consider *convex* agendas where each of the binary Yes/No votes in the sequence must be among two subsets of options such that each of them covers a well-defined, coherent segment of positions in the respective ideological spectrum (see the Literature Review below for a justification of this choice).[3]

[1]For example, in the German 2017 vote to legalize same-sex marriage, the main Government party, the CDU, was split with 225 MPs against vs. 75 in favor. The CDU and their leader Angela Merkel, who voted against, were defeated since all other parties voted in favor. Similarly, in 2019 Prime Minister Boris Johnson lost his first two crucial votes in parliament because many in his own party voted with the opposition in order to avoid a no-deal Brexit.

[2]For example, during March 2019 the UK parliament struggled to identify and elect a compromise deal between the "hard" Brexit demanded by a large faction of the Tories, and the "soft" version, closer in spirit to economically remaining in the EU, supported by Labour and other smaller parties.

[3]The amendment procedure will satisfy convexity if the pairing is such that the most "extreme" alternatives compete against each other at each round of voting. The successive procedure will satisfy convexity if, at each stage, the considered alternative is one of the two most extreme ones.

An example of a convex agenda formation rule is given by the long-standing practice of the German parliament and its Weimar precursor:

> "if several proposals are made to the same subject, then the first vote shall be on the farthest-reaching proposal. Decisive is the degree of deviation from status quo."

In contrast, when the U.S. Congress, say, takes a decision involving, say, the status quo, a proposed change and an amendment to that change, the status quo is usually put up to vote at the second, final stage independnetly of its ideological position. If the status quo is an extreme—more to the "left" or to the "right" relative to the other two alternatives—this procedure is not convex.

Our first main result is that, for any constellation of parameters, the amendment procedure with a convex agenda has an equilibrium that always selects the (complete-information) Condorcet winner. In contrast, the successive procedure does not always possess such an equilibrium even if convex agendas are used. The reason is that the amendment procedure—that considers two alternatives at a time—allows bi-directional learning about the preferences of both leftists and rightists, while the successive procedure only allows one-directional learning at each stage. Thus, these two procedures are **not** equivalent once preferences are interdependent, in stark contrast to the private values case (see Literature Review below).

A second main result is that, under the successive procedure, the two possible convex agendas (that are also equivalent under a private values assumption) are **not** anymore equivalent with interdependent values: the agenda that puts the "extreme" alternative with ex ante higher support last elects the Condorcet winner with a higher probability than the agenda that puts that alternative first. The reason is, again, connected to the direction of learning: putting the alternative with ex ante higher support first on the ballot risks of "hastily" giving up that alternative in some cases: voters rally around the compromise before anything new has been learned about the number of opponents. Indeed, if the number of opponents is relatively small, the Condorcet winner is the foregone extreme alternative, rather than the chosen compromise. Such an undesirable outcome is less likely if the first alternative on the ballot is the more extreme one with less ex ante support. This result fits well the well-documented practice to consider last on the agenda the Government's proposal that, supposedly it has a higher ex ante chance of getting a majority (see also the agenda in the Weimar flag case studied below).

Finally, we analyze the optimal location of the compromise.[4] We first identify the main

[4]For example, the quest for a Brexit compromise continued via an "indicative voting" process, designed to

forces motivating this: (i) to elect an alternative that is superior to those already on the table or (ii) to insure against the election of another, worse alternative. The optimal location of the compromise is shown to finely depend on several important parameters such as the size and ideology of the ex ante expected majority and the degree of interdependence in preferences, but, importantly, also on the underlying voting procedure.

We conclude the paper with an illustration: we describe the Weimar Flag Controversy where a compromise flag—a combination of the pre-WWI German Reich's flag on the one side, and the flag mostly associated with the progressive 1830 and 1848 revolutions on the other—was ultimately selected following a process where learning about the position of others affected the voting behavior and outcome.

1.1 Related Literature

Enelow [1983] contains an early model of optimal compromise location under an amendment procedure with a non-convex agenda. His model is neither game-theoretic nor otherwise micro-founded: the (numerical) results depend on the agenda setter's exogenously given beliefs about the probabilities of various outcomes.

Following the pioneering work by Farquharson [1969], almost the entire literature on binary, sequential voting assumed that agents are completely informed about the preferences of others (see Miller [1977], McKelvey and Niemi [1978] and Moulin [1979], among others, for early important contributions). Under complete information, the associated extensive form games are amenable to analysis by backward induction: voters can, at each stage, foresee which alternative will be finally elected, essentially reducing each decision to a vote among two alternatives. If a simple majority is used at each stage, then, whenever it exists, a Condorcet winner is selected by sophisticated voters, independent of the particular structure of the binary voting tree, and independent of its agenda.[5]

An early analysis of strategic, sequential voting under incomplete information with private values is Ordeshook and Palfrey [1988]. They constructed Bayesian equilibria for an *amendment* procedure with three alternatives and with preference profiles that potentially lead to a Condorcet paradox. Gershkov, Moldovanu and Shi [2017] (GMS hereafter) analyzed voting by qualified majority in SV via a model where agents' preferences are single-peaked and

establish which one out of at least 8 suggested compromises might get a majority (none did).

[5] If a Condorcet winner does not exist, then a member of the Condorcet cycle is elected. The influence of agenda manipulations has been emphasized by Ordeshook and Schwartz [1987], Austen-Smith [1987] and, more recently by Barbera and Gerber [2017]. Apesteguia et al. [2014] axiomatically characterize SV and AV under complete information,

follow the private values paradigm.[6] In their study, the order in which alternatives are put to vote follows the order defining single-peakedness (or its reverse). Kleiner and Moldovanu [2017] generalized the GMS results to the class of all sequential, binary procedures with a *convex* agenda. Recall that in a binary, sequential procedure each vote is taken by (possibly qualified) majority among two, not necessarily disjoint, subsets of alternatives. Convexity says that if two alternatives a and c belong to the left (right) subset at a given node, then any alternative b such that $a < b < c$ (in the ideological order governing single-peakedness) also belongs to the left (right) subset.

Under single-peaked, private values preferences, Kleiner and Moldovanu showed that sincere voting constitutes an ex post perfect equilibrium in any voting game derived from a sequential, binary voting tree with any convex agenda.[7]. An important corollary is that, if simple majority is used at each stage of the voting tree, the associated equilibrium outcome is always the complete information Condorcet winner. Thus, all sequential binary voting trees with convex agendas and all information policies are **equivalent** under single-peaked, private values preferences, and this theory cannot discriminate among them.

There are only a few papers that study voting models with more than two alternatives and with interdependent values (note that interdependence generalizes the more ubiquitous assumption of common values).[8] Gruener and Kiel [2004] and Rosar [2015] analyze static voting mechanisms in a setting where agents have interdependent preferences, focusing on the mean and the median mechanisms. Moldovanu and Shi [2013] analyze voting in a dynamic setting where multi-dimensional alternatives appear over time and where voters are only partially informed about some aspects of the alternative. Piketty [2000] studies a two-period voting model where a large number of agents care about the decisions taken at both stages. As in our model, voting at the first stage reveals information about preferences that is relevant at the second stage. Piketty concludes that electoral systems should be designed to facilitate efficient communication, e.g. by opting for two-round rather than one-round systems—this

[6]Their focus was on finding the welfare maximizing procedure. This is achieved by varying the thresholds needed for the adoption of each alternative.

[7]In other words, voters cannot gain by manipulating their vote, regardless of their beliefs about others' preferences, and regardless of the information disclosure policy along the voting sequence. Under a mild refinement, this equilibrium is unique.

[8]Dekel and Piccione [2000] analyzed sequential voting with interdependent values in a model with only two alternatives: sequentiality is with respect to individual voting. They showed that, although the history of the first votes should intuitively affect the behavior of the later voters, equilibrium conditioning on pivotality leads voters to ignore the revealed history. Ali and Kartik [2012] displayed other equilibria where voters do take into account the observed history.

is congruent with the kind of multi-stage procedures observed in legislatures and discussed in this paper.

Martin and Vanberg [2014] empirically test several models of legislative compromise in coalition governments, and conclude that these tend to be positions that average opinions in coalitions rather than representing, say, the view of the median coalition member. Ezrow et al. [2011] conducted an analysis of political parties in 15 Western European democracies from 1973 to 2003 and showed that the larger, mainstream parties tend to adjust their positions on the Left-Right spectrum in response to shifts in the position of the mean voter, while being less sensitive to policy shifts of their own supporters. The opposite pattern holds for smaller, niche parties. Chappel et al. [2004] studied the Federal Open Market Committee's detailed voting patterns on monetary policy, and test the hypothesis that the chairman's preferred policy is a weighted average of her own and the other members' signals – the same functional form as the one adopted here.[9],[10]

Finally, our model and results are pertinent to voting in other committee settings. For example, Posner and Vermeulen [2016] note that a more or less evenly split decision by several judges, or by a jury, may be logically incompatible with a conviction based on guilt "beyond reasonable doubt". They propose a dynamic voting procedure where members learn about the positions of others and adjust their opinion, and also argue that a formal procedure where the revealed numbers of supporters for each option speak for themselves is better than an informal, hard to quantify deliberation.

The rest of the paper is organized as follows: In Section 2 we describe the social choice model, the calculation of the Condorcet winner with interdependent preferences, and the considered voting procedures. In Section 3 we analyze voting in the continental successive procedure, and compare the outcomes under various agendas. Section 4 considers the Anglo-Saxon amendment procedure, and compares its outcome to the one under the successive procedure. Section 5 studies the optimal location of the compromise alternative. In Section 6 we describe the voting process that determined the flag of the Weimar republic. Section 7 concludes. In Appendix A we briefly describe two basic probabilistic tools employed in several arguments that involve a large number of voters. All proofs are collected in Appendix B.

[9] There are twelve members, and the chairman's weight on his own signal is estimated to be between 0.15 and 0.20. Chappel et al. take their cue from an earlier study by Yohe [1966] who writes "...there is also no evidence to refute the view that the chairman adroitly detects the consensus of the committee, with which he persistently, in the interests of Systems harmony, aligns himself."

[10] They also estimate the opposite influence of the chairman on members.

2 A Model of Compromise

2.1 The Basic Features

There are $2n+1$ voters who collectively choose among three alternatives: L (left), C (compromise) and R (right). Let x_a denote the "location" of alternative a, $a \in \{L, C, R\}$, on a left-right ideological spectrum. The locations of alternatives L and R are exogenously given and normalized to be $x_L = -1$ and $x_R = +1$, while the location of the compromise, $x_C \in [-1, 1]$ may be chosen endogenously, e.g., in order to maximize some goal.

Before voting, each agent i, $i = 1, ..., 2n+1$, obtains a signal $s_i \in \{-1, 1\}$. Signals $\{s_i\}_{i=1}^{2n+1}$ are assumed to be i.i.d., and we let $p \in (0, 1)$ denote the ex ante probability of drawing signal -1. Hence, voters with signal -1 are in an ex ante minority if and only if $p < 1/2$.

We denote by $\widetilde{n}_{-1}$ the random variable representing the number of voters with signals -1 and by n_{-1} its realization. The realized number of voters with signal $+1$ is denoted by $n_{+1} = 2n+1-n_{-1}$. The expected number of voters with signal -1 is $\mathbb{E}\left[\widetilde{n}_{-1}\right] = (2n+1)\,p$.

Each voter, $i = 1, ..., 2n+1$, has an "ideal" location y_i for the elected alternative. Voter i's ideal point depends both on her own private signal s_i and also on the mean of all other voters' private signals s_j, $j \neq i$. Let $\gamma_{-1}, \gamma_1 \in \left[\frac{1}{2n+1}, 1\right]$ denote the weight that voters with signal -1 and $+1$ assign to their own signal, respectively. The ideal location $y_i\left(s_i, s_{-i}\right)$ for voter i is

$$y_i\left(s_i, s_{-i}\right) = \gamma_i s_i + \frac{1-\gamma_i}{2n} \sum_{j \neq i} s_j, \tag{1}$$

where $\gamma_i = \gamma_{-1}$ if $s_i = -1$ and $\gamma_i = \gamma_1$ if $s_i = +1$. Note that, in order to avoid a complex model with more than two types determining preferences, we assumed that the degree of interdependence in the preferences is determined by the obtained signal.

Thus, preferences are assumed here to be *interdependent*, and the weight γ_i on own signal s_i, captures the level of interdependence. A special case is the one where all agents share a common $\gamma = \gamma_1 = \gamma_{-1}$. Then $\gamma = 1$ yields the private values case (no interdependence), while $\gamma = \frac{1}{2n+1}$ yields the pure common values case where, ex post, all voters share the same ideal point.

If alternative $a \in \{L, C, R\}$ is elected, the utility of voter i with ideal point y_i is given by $u\left(x_a, y_i\right)$ where $u(\cdot, y_i)$ is single-peaked at, and symmetric around $x_a = y_i$. In particular, any utility function $u\left(x_a, y_i\right)$ that is monotonically decreasing in the absolute value of the difference between y_i and x_a is feasible. We use the cardinal representation only for welfare comparisons, while all other results, such as equilibrium constructions, are solely based on ordinal information.

For some results we assume that the number of voters is large, and we use Hoeffding's Inequality and the Gärtner-Ellis Large Deviations Theorem (see Appendix A for details).

2.2 The Condorcet Winner: Interdependence and Compromise

An alternative is the *complete information Condorcet winner* if it is the Condorcet winner when all voters' types are public information. For any given realization of signals, the assumed preferences are here single-peaked according to the left-right natural order L, C, R (or R, C, L). Therefore, the Condorcet winner always exists.

To compute the Condorcet winner, note first that the ideal point of voter i with signal $+1$ can be written as

$$y_i(+1, s_{-i}) = \gamma_1 + \frac{1-\gamma_1}{2n}(-n_{-1} + (2n - n_{-1})) = 1 - (1-\gamma_1)\frac{n_{-1}}{n}.$$

In the private values case where $\gamma_1 = 1$, such a voter has a peak on alternative R. If $\gamma_1 < 1$, the peak monotonically shifts to the left as the number of voters with the opposite signal increases. Let

$$k = \frac{n}{2}\frac{1-x_C}{1-\gamma_1} \tag{2}$$

and observe that, if k is an integer and if $n_{-1} = k$, then voters with signal $+1$ are indifferent between alternatives C and R, because their peak is then given by

$$1 - (1-\gamma_1)\frac{n}{2}\frac{1-x_C}{1-\gamma_1}\frac{1}{n} = \frac{1}{2}(1 + x_C),$$

exactly half-way between 1 and x_C. Thus, if $n_{-1} \leq n$, the voters with signal $+1$ form a majority, and the Condorcet winner is given by

$$CW = \begin{cases} R & \text{if} \quad n_{-1} \leq \lceil k \rceil - 1, \\ C & \text{if} \quad n_{-1} > \lceil k \rceil - 1 \end{cases} \tag{3}$$

where $\lceil z \rceil$ denotes the smallest integer no less than a real number z. In this case, C can be the Condorcet winner only if $k \leq n$ which is equivalent to $\gamma_1 \leq \frac{1}{2}(1 + x_C)$. If the majority group n_{+1} puts a relatively high weight on own signal, the Condorcet winner is always R.

Similarly, the ideal point of a voter i with signal -1 can be written as

$$y_i(-1, s_{-i}) = -\gamma_{-1} + \frac{1-\gamma_{-1}}{2n}(n_{+1} - (2n - n_{+1})) = -1 + (1-\gamma_{-1})\frac{n_{+1}}{n}.$$

In the private values case where $\gamma_{-1} = 1$, such a voter has a peak on alternative L. If $\gamma_{-1} < 1$, then the peak monotonically shifts to the right as the number of voters with the opposite signal increases. Let

$$\kappa = \frac{n}{2}\frac{1+x_C}{1-\gamma_{-1}} \tag{4}$$

and observe that, if κ is an integer and if $n_{+1} = \kappa$, then voters with signal -1 are indifferent between L and C, because their peak is given by

$$-1 + (1 - \gamma_{-1})\frac{n_{+1}}{n} = -1 + (1 - \gamma_{-1})\frac{n}{2}\frac{1 + x_C}{1 - \gamma_{-1}}\frac{1}{n} = \frac{1}{2}\left(-1 + x_C\right),$$

exactly half way between -1 and x_C. If $n_{-1} \geq n + 1$, then voters with signal -1 form a majority, and the Condorcet winner is

$$CW = \begin{cases} L & \text{if} \quad n_{+1} \leq \lceil \kappa \rceil - 1, \\ C & \text{if} \quad n_{+1} > \lceil \kappa \rceil - 1 \end{cases}. \tag{5}$$

In this case, C can be the Condorcet winner only if $\kappa \leq n$ which is equivalent to $\gamma_{-1} \leq \frac{1}{2}\left(1 - x_C\right)$. If the majority group n_{-1} puts a high enough weight on own signal, then the Condorcet winner is always L.

To conclude, if the voters whose signal is in majority put a high enough weight on the opinion of others (given a fixed compromise location x_C), they will prefer the compromise alternative if and only if the number of voters with the opposite signal exceeds a certain threshold. As we shall see below, the cutoffs κ and k defined above play an important role also in the construction of strategic voting equilibria.

2.3 The Voting Procedures

A strategy profile is an *ex post equilibrium* if, given that all other agents follow their equilibrium strategies, each voter plays a best-response for all signal realizations. We study and compare equilibria of two main voting procedures.

1. Successive voting (SV): alternatives are ordered according to an agenda, say $[L,\ \{C, R\}]$. With this agenda, voters first decide by simple majority to accept, or to reject alternative L. If L is accepted, voting ends. Otherwise, voters decide whether to accept alternative C. Alternative C is accepted if it has majority support and R is accepted otherwise. The general results of GMS and Kleiner and Moldovanu (2017) for **private values** and single-peaked preferences imply that this procedure yields the Condorcet winner in an ex post and sincere voting equilibrium if the agenda is either $[L,\ \{C, R\}]$ or $[R,\ \{C, L\}]$. We focus here on these *convex* agendas, where the alternative put to vote in the first stage is one of the two "extreme" alternatives. Sincere voting need not be an equilibrium for the agenda that starts by voting on the compromise C, and, even under private values, this agenda may not elect the Condorcet winner

2. Voting by amendment (AV): alternatives are ordered according to an agenda, say $[\{L, C\}, \{C, R\}]$. Voters vote first by simple majority between alternatives L and R. If $L(R)$ is chosen, then at the second stage voters decide by simple majority between C and $L(R)$. The general results of Kleiner and Moldovanu (2017) for **private values** imply that this procedure also yields a Condorcet winner in a sincere equilibrium, while this is not necessarily the case for an agenda where the middle alternative C is one of the alternatives considered at the first step.

As we show below, the introduction of interdependent values yields quite different insights from the above.

3 Successive Voting: One-Directional Vote Shifting

We start with successive voting, and focus on the information policy that reveals the margin of victory at the first stage. The derived strategies remain an equilibrium even if individual voting behavior is reported, as long as we focus on type-symmetric equilibria where all voters with the same signal behave in the same way.[11]

3.1 Vote-Shifting Equilibrium

We first introduce an important phenomenon, *vote shifting*, as a response to information disclosure and interdependent values: at the second stage, some voters may want to condition their behavior on the voting outcome of the first stage since this past result conveys valuable information about the signals of other agents (that directly affect their own preferences here).

Consider agenda $[L,\ \{C, R\}]$ and the following strategy profile: Voters with signal -1 vote in favor of L in the first stage and in favor of C at the second stage; Voters with signal $+1$ vote against L in the first stage, and in the second stage vote for C if L received at least $\lceil k \rceil$ votes in the first stage, and vote against C otherwise. We denote this profile by $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$, where the first component denotes the strategy of voters with signal -1 at stage 1 (L_1) and at stage 2 (C_2), and the second component analogously denotes the strategy of voters with signal $+1$ at the two stages. The symbol $\neg$ denotes voting **against** the respective alternative.

[11] One can also consider the minimal information policy where, if the second stage is reached, voters know only that the first stage alternative did not get the support of a majority. For both SV and AV, the policy of revealing the margin of defeat at the first stage is better than the minimal information policy in electing the Condorcet winner. The formal analysis of the minimal information policy is available from authors upon request.

Remarkably, the same cutoff k defined in (2) that appeared in the non-strategic determination of a Condorcet winner plays a role in the strategic analysis below: it is chosen such that, when there are k voters with signal -1, voters with signal $+1$ are indifferent between C and R. Intuitively, k is increasing in γ_1 and decreasing in x_C. That is, vote-shifting will be more likely when voters with signal $+1$ care more about other voters' private information (lower γ_1), or when the compromise is located closer to R.

With interdependent values, the successive voting procedure with convex agenda $[L,\ \{C,R\}]$ critically relies on vote shifting to dynamically discover the Condorcet winner. This discovery process need not be always successful, contrasting the private values case.

Proposition 1 *Consider SV with agenda* $[L,\ \{C,R\}]$.

(i) If $\gamma_{-1} \geq \frac{1}{2}(1-x_C)$, *then the strategy profile* $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ *constitutes an equilibrium that always selects the complete information Condorcet winner. If, in addition,* $\gamma_1 \leq \frac{1}{2}(1+x_C)$, *actual vote-shifting may occur in equilibrium.*

(ii) If $\gamma_{-1} < \frac{1}{2}(1-x_C)$, *then there is no equilibrium that always results in the selection of the full information Condorcet winner.*

The main reason for the failure to select the Condorcet winner is that no information may be revealed in the first vote on L: if $\gamma_{-1} < \frac{1}{2}(1-x_C)$, voters may unanimously reject L even though L may be the Condorcet winner. This cannot happen in the private values setting. As we shall see below, this defect cannot occur in the amendment procedure with a convex agenda that always reveals information about votes received by **both** L and R at the first stage.

The results for agenda $[R,\ \{C,L\}]$ are analogous: consider the strategy profile

$$(\neg R_1C_2 \text{ if} \geq \kappa,\ R_1C_2)$$

where, if alternative R receives at least $\lceil \kappa \rceil$ votes in the first stage, voters with signal -1 shift and vote for C in the second stage. The cutoff κ in a vote-shifting equilibrium is the same cutoff (4) used to determine the complete information Condorcet winner. In order to have effective vote shifting in equilibrium, we need $\kappa \leq n$ which is equivalent to $\gamma_{-1} \leq \frac{1}{2}(1-x_C)$.

3.2 Agendas and Their Likelihood of Selecting the Condorcet Winner

Since, by Proposition 1, the selection of the complete information Condorcet winner is not guaranteed, we now compare the two convex agendas $[L, \{C,R\}]$ and $[R,\ \{C,L\}]$ according to their respective likelihood of selecting the Condorcet winner. We find that the agenda where the alternative with ex ante higher support is put to vote last is superior,. This agrees well with observed practice in many parliaments.

For the comparison, we need an intuitive and consistent method of selecting equilibria for each possible parameter constellation: the multiplicity of voting equilibria is a standard problem in voting games, and pivotality considerations alone are not sufficient for equilibrium selection. We base our selection criterion, and hence our comparison, on the following concept:

Definition 1 *A voter's strategy in SV is sincere if, at each stage in the process, and conditional on all available information, the voter approves the current alternative if it yields the highest expected payoff among the remaining ones, and rejects it otherwise. A strategy is sincere if all agents use sincere strategies. A strategy profile is semi-sincere if one type of voters votes sincerely, but not both.*

If the equilibrium strategy is not sincere, legislators may have difficulties explaining their behavior to constituents. This feature often constrains opportunistic equilibrium behavior and is the subject of a large literature in Political Science (see Fenno [1978]). Semi-sincerity is needed here for a few special cases where sincere equilibria do not exist.

In this subsection (and some of the later subsections), we also assume the following:

Assumption A Ex ante, voters with signal -1 are in minority (i.e., $p < 1/2$), and weakly prefer L to R.

Assumption A has two parts. The first part, $p < 1/2$ is without loss of generality. The second part assumes that, ex ante, there is indeed a conflict of interest between the two types of voters (otherwise the situation is trivial). Formally, it requires that

$$-\gamma_{-1} + \frac{1-\gamma_{-1}}{2n}\left(2n\left(1-p\right) - 2np\right) \leq 0 \Leftrightarrow \gamma_{-1} \geq \frac{1-2p}{2\left(1-p\right)}.$$

Let us define cutoffs γ_1^* and γ_{-1}^* as follows:

$$\begin{aligned}\gamma_{-1}^* &\equiv \frac{1}{2}\left(1-x_C\right) + \frac{1-2p}{2\left(1-p\right)}\frac{1}{2}\left(1+x_C\right), \\ \gamma_1^* &\equiv \frac{1}{2}\left(1+x_C\right) - \frac{1-2p}{2p}\frac{1}{2}\left(1-x_C\right).\end{aligned}$$

Then, voters with signal -1 ex ante prefer L to C if and only if $\gamma_{-1} \geq \gamma_{-1}^*$ and voters with signal $+1$ ex ante prefer R to C if and only if $\gamma_1 \geq \gamma_1^*$.

We compute the sincere/semi-sincere equilibria in the Appendix. Based on Proposition 6 and Tables 2 and 3 there, the possible outcomes under the two agendas are:

		$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1+x_C}{2})$	$\gamma_1 \in (\frac{1+x_C}{2}, 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \frac{1-x_C}{2})$	$n_{-1} \geq n+1$	(C, C)	(C, C)	$\left(C, \begin{cases} C \text{ if } n_{+1} \geq \lceil \kappa \rceil \\ L \text{ if } n_{+1} < \lceil \kappa \rceil \end{cases}\right)$
	$n_{-1} \leq n$	(C, C)	(C, C)	(R, R)
$\gamma_{-1} \in (\frac{1-x_C}{2}, \gamma_{-1}^*)$	$n_{-1} \geq n+1$	(C, C)	(C, C)	(C, L)
	$n_{-1} \leq n$	(C, C)	(C, C)	(R, R)
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$n_{-1} \geq n+1$	(L, L)	(L, L)	(L, L)
	$n_{-1} \leq n$	$\left(\begin{cases} C \text{ if } n_{-1} \geq \lceil k \rceil \\ R \text{ if } n_{-1} < \lceil k \rceil \end{cases}, C\right)$	$\left(\begin{cases} C \text{ if } n_{-1} \geq \lceil k \rceil \\ R \text{ if } n_{-1} < \lceil k \rceil \end{cases}, C\right)$	(R, R)

Table 1: Equilibrium outcomes under the two agendas

The first and second component in each cell denote the potential outcomes under agendas $[L, \{C, R\}]$ and $[R, \{C, L\}]$, respectively.[12]

The outcome tally in Table 1, and an application of the Hoeffding inequality yield:

Proposition 2 *Suppose that Assumption A holds.*

(i) If $\gamma_1 > \frac{1+x_C}{2}$ and $\gamma_{-1} < \gamma_{-1}^*$, then $[R, \{C, L\}]$ selects the Condorcet winner with a higher probability than $[L, \{C, R\}]$, but this probability decays exponentially to zero as n grows.

(ii) If $\gamma_1 < \frac{1+x_C}{2}$ and $\gamma_{-1} > \gamma_{-1}^*$, then $[L, \{C, R\}]$ selects the Condorcet winner with a higher probability than $[R, \{C, L\}]$. Moreover, this probability remains significantly different from zero as n grows.

(iii) If $\gamma_1 > \frac{1+x_C}{2}$ and $\gamma_{-1} > \gamma_{-1}^*$, or if $\gamma_1 < \frac{1+x_C}{2}$ and $\gamma_{-1} < \gamma_{-1}^*$, the two agendas are outcome equivalent.

Interestingly, agenda $[L, \{C, R\}]$ dominates $[R, \{C, L\}]$ not because voters with signal $+1$ shift their votes to C in the second stage when there is sufficiently strong support for L in the first stage (in this case, the outcomes are identical under two agendas), but because, under $[L, \{C, R\}]$, these voters choose **not** to shift to C when there is not enough support for L in the first stage. This case is captured by part (ii) in the above proposition and is illustrated by the blue cells in Table 1.

Intuitively, if both types of voters put a high enough weight on own signal ($\gamma_{-1} > \gamma_{-1}^*$, $\gamma_1 > \frac{1}{2}(1 + x_C)$), no vote-shifting occurs in equilibrium, and the same extreme alternative

[12] The slight asymmetry between the agendas is due to the assumption that $p < 1/2$: while this is without loss of generality per-se, it obviously implies that $\gamma_{-1}^* > \frac{1}{2}(1 - x_C)$ and $\gamma_1^* < \frac{1}{2}(1 + x_C)$.

(either L or R) is chosen under both agendas. On the other hand, if both types of voters care a lot about other voters' signals ($\gamma_{-1} < \gamma^*_{-1}$, $\gamma_1 < \frac{1}{2}(1+x_C)$), they all vote in the sincere/semi-sincere equilibrium against the first extreme alternative on the ballot, and then all vote for the compromise. This gives part (iii).

To understand parts (i) and (ii), note from Table 1 that the equilibrium outcomes of the two agendas may differ only when the voter type that has the ex post majority puts a high enough weight on the signals of others ($n_{-1} \geq n+1$ and $\gamma_{-1} < \gamma^*_{-1}$ in part (i) – the red cells in Table 1, or $n_{-1} \leq n$ and $\gamma_1 < \frac{1}{2}(1+x_C)$ in part (ii) – the blue cells in Table 1). In those cases, the agenda that puts first the extreme alternative with ex post less support induces the ex post majority voters to condition their second stage votes on the first stage outcome, and thus always selects the Condorcet winner. To induce information revelation in the first stage, however, the ex post minority voters must put a high enough weight on their own signal ($\gamma_1 > \frac{1+x_C}{2}$ in part (i) and $\gamma_{-1} \geq \gamma^*_{-1}$ in part (ii)) so that they will vote for the first extreme alternative in the ballot. Since $p < 1/2$ by Assumption A, alternative R has more ex ante support than alternative L. As the number of voters grows, with probability approaching one, alternative R also has more ex post support than alternative L. Therefore, $[L,\ \{C,R\}]$ dominates $[R,\ \{C,L\}]$.

For a further illustration, consider the special case where $\gamma_{-1} = \gamma_1 = \gamma$. If $\gamma^*_{-1} \leq \frac{1}{2}(1+x_C)$, the scenario described in part (i) never occurs, and thus $[R,\ \{C,L\}]$ never dominates $[L,\ \{C,R\}]$. The condition $\gamma^*_{-1} \leq \frac{1}{2}(1+x_C)$ is equivalent to

$$x_C \geq \hat{x}_C \equiv \frac{1-2p}{3-2p}.$$

The lower bound $\hat{x}_C$ is between 0 and 1/3 when voters with signal +1 have an ex ante majority ($p < 1/2$). If $x_C \geq \hat{x}_C$, then alternative L can be intuitively considered as the most "extreme" alternative among the three. The above Proposition suggests that it is better to vote on such an alternative first, confirming the standing practice in many parliaments that use the successive procedure.

4 The Amendment Procedure: Bidirectional Vote Shifting

We now proceed to AV, where voters choose between L and R at the first stage, and where, at the second stage, they choose between C and the winner of the first stage. This is the only possible convex agenda for AV with three alternatives. As in SV, we assume that the information regarding the margin of victory at the first round is disclosed before the second-stage vote. Recall that a voter with signal +1 prefers C to R if at least $\lceil k \rceil$ voters have signal

-1, and that a voter with signal -1 prefers C to L if at least $\lceil \kappa \rceil$ voters have signal $+1$. Consider the following strategy profile denoted by Π:[13]

1. Voters with signal -1 vote for L in the first stage. In the second stage they vote for C in a vote R vs. C; in a vote L vs. C they vote for C if and only if at least $\lceil \kappa \rceil$ voters voted for R in the first stage;

2. Voters with signal $+1$ vote for R in the first stage. In the second stage they vote for C in a vote L vs. C; in a vote R vs. C they vote for C if and only if at least $\lceil k \rceil$ voters voted for L in the first stage.

Proposition 3 *The strategy profile* Π *is an equilibrium under AV with a convex agenda, and the complete information Condorcet winner is always elected.*

Thus, for **any** parameter values there is an equilibrium of AV with bi-directional (potential) shifting that always elects the Condorcet winner. It follows that AV is superior in this respect to SV (Proposition 1),

For a welfare comparison, note that the first stage voting in AV fully reveals the voters' signals, so the second stage voting is under complete information. For SV, we focus on agenda $[L, \{C, R\}]$.[14] As we argue below, AV dominates SV if the first stage voting under $[L, \{C, R\}]$ reveals n_{-1}, or the first stage voting is not revealing and R is elected. The welfare comparison is ambiguous and depends on n_{-1} only if the first stage voting of SV is not revealing and C is elected.

(i) If alternative L is elected in the first stage of $[L, \{C, R\}]$, then AV eventually elects either L or C; in the latter case, AV improves welfare relative to SV because it allows voters with signal -1 to shift their votes to C when they prefer C to L (voters with signal $+1$ are also better off since they prefer C to L).

(ii) If alternative L is rejected by a split vote in the first stage of $[L, \{C, R\}]$, then the second stage equilibrium play is also under complete information in SV. In this case, the two procedures are equivalent.

(iii) If alternative L is unanimously rejected in the first stage and if R is elected under $[L, \{C, R\}]$, then AV improves welfare because it allows voters with signal $+1$ to shift to

[13] Note that Π is the unique sincere equilibrium if $\frac{1}{2} - \frac{1}{2}\frac{\gamma_{-1}}{1-\gamma_{-1}} \leq p \leq \frac{1}{2} + \frac{1}{2}\frac{\gamma_1}{1-\gamma_1}$. The first stage voting is sincere because voters with signal -1 ex ante prefer L to R, and voters with signal $+1$ ex ante prefer R to L. Voting at the second stage is under complete information, and hence also sincere.

[14] The comparison for the other convex agenda $[R, \{C, L\}]$ is analogous.

the compromise alternative, and because vote shifting benefits both types of voters when it occurs.

(iv) If alternative L is unanimously rejected in the first stage and if C is elected under $[L, \{C, R\}]$, the comparison is ambiguous, and it depends on the realization of n_{-1}. If $n_{-1} \in [\lceil k \rceil, 2n + 1 - \lceil \kappa \rceil]$ so that C is the Condorcet winner, then the outcomes of SV and AV coincide. When C is not the Condorcet winner, SV dominates AV if n_{-1} is close to either $\lceil k \rceil$ or $2n + 1 - \lceil \kappa \rceil$, and the reverse is true otherwise.[15]

To conclude, the above findings—strong rationales in favor of AV—starkly contrast the results under complete information or under private values, where the two procedures are always equivalent under a convex agenda and single-peaked preferences.

5 The Location of the Compromise

So far we have assumed that x_C, the location of the compromise alternative C on the ideological scale, is exogenous. We consider now varying its location.

5.1 Electable Compromises

We first prove a key Lemma that identifies the different compromise locations **effectively** leading to its election under SV and AV, respectively. For a given n and for a given realization of signals, let $x_C^L(n_{+1})$ denote the compromise location such that, **ex post**, voters with signal -1 are indifferent between L and C:

$$\frac{-1 + x_C^L(n_{+1})}{2} = -\gamma_{-1} + \left(1 - \gamma_{-1}\right) \frac{1}{2n} \left(n_{+1} - (2n - n_{+1})\right),$$

which yields

$$x_C^L(n_{+1}) = 2(1 - \gamma_{-1}) \frac{n_{+1}}{n} - 1.$$

For a given n, voters with signal -1 are **ex ante** indifferent between L and C if the compromise is located at

$$x_C^L = 4\left(1 - \gamma_{-1}\right)(1 - p) - 1.$$

[15] To see this, suppose that $n_{-1} \geq n + 1$ and $n_{-1} > 2n + 2 - \lceil \kappa \rceil$. Then L is the Condorcet winner, and it is always elected under AV. On the one hand, SV is welfare superior to AV if n_{-1} is close to $2n + 2 - \lceil \kappa \rceil$, because voters with signal -1 are almost indifferent between L and C, but voters with signal $+1$ strictly prefer C to L. On the other hand, AV is welfare superior to SV if n_{-1} is close to $2n + 1$, because then almost all voters strictly prefer L to C. As a result, there must exist a cutoff $n_{-1}^* \in (2n + 2 - \lceil \kappa \rceil, 2n + 1)$ such that SV dominates AV if $n_{-1} \in (2n + 2 - \lceil \kappa \rceil, n_{-1}^*)$ and the reverse is true if $n_{-1} \in (n_{-1}^*, 2n + 1]$. The case where $n_{-1} < \lceil k \rceil$ is analogous.

For voters with signals $+1$ we analogously define:

$$x_C^R(n_{-1}) = 1 - 2(1-\gamma_1)\infty\frac{n_{-1}}{n}, \text{ and } x_C^R = 1 - 4(1-\gamma_1)p.$$

Lemma 1 *(i) Consider SV with agenda $[L, \{C, R\}]$, and suppose that Assumption A holds.*

a. If $n_{-1} \geq n+1$, then compromise C is elected if and only if $x_C \leq x_C^L$. Otherwise, alternative L gets elected.

b. If $n_{-1} \leq n$, then compromise C is elected if and only if $x_C \in \left[-1+2\gamma_1,\ x_C^L\right] \cup \left[x_C^R(n_{-1}), 1\right]$. Otherwise, alternative R gets elected.

(ii) Consider AV.

a. If $n_{-1} \geq n+1$, then compromise C is elected if and only if $x_C \leq x_C^L(n_{+1})$. Otherwise, alternative L gets elected.

b. If $n_{-1} \leq n$, then compromise C is elected if and only if $x_C \geq x_C^R(n_{-1})$. Otherwise, alternative R gets elected.

If the voters with signal -1 form an ex post majority, C will be elected under both voting procedures if it close enough to L so that even those voters find it more attractive than L (i.e., if $x_C \leq x_C^L$ under SV, and if $x_C \leq x_C^L(n_{+1})$ under AV).[16]

If voters with signal $+1$ form an ex post majority, C will be elected under AV only if it is sufficiently close to R so that voters with signal $+1$ ex post prefer C to R (i.e., if $x_C \geq x_C^R(n_{-1})$). In contrast, C will be elected under SV in more cases: it will be elected if $x_C \geq x_C^R(n_{-1})$, but it can also get elected even if $x_C < x_C^R(n_{-1})$. In particular, C is elected under SV if $-1+2\gamma_1 \leq x_C \leq \min\left\{x_C^L,\ x_C^R(n_{-1})\right\}$. In this case, the equilibrium profile is $(\neg L_1 C_2, \neg L_1 C_2)$, and C is elected with unanimous support because no information is released after the first stage voting. The difference arises because the strategic voting at both stages of SV (and the resulting information disclosure) depends on the compromise location, while in AV only the second stage voting behavior is affected by the compromise location.

5.2 Optimal Location

An optimal compromise location will clearly depend on the underlying goal, on the expected numbers of voters with the various signals and on the interdependence parameters. Whatever

[16] This is calculated from an ex ante perspective under SV, and from an ex post perspective under AV.

the underlying goal is, the main constraint on the optimal location is that, in order to be effective, the compromise must also be elected with positive probability.[17]

The above Lemma implies that the employed voting procedure also influences the optimal location - this is the main new insight in this part. To illustrate this, let us assume that an agenda setter (for example, the Government) can determine, prior to voting, the location of C. The decision where to place the compromise is not trivial because the electorate has here two factions with two different signals, and hence it incorporates diverse preferences.

Throughout the sequel, we assume that the agenda setter has a single-peaked utility function that is maximized at some alternative $x^* \in [-1, 1]$, and that it locates the compromise in order to maximize the expected utility it derives from the elected alternative.[18]

1) We first assume that $x^* \in (-1, 1)$. The agenda setter will propose here a policy because her first-best alternative hasn't been put forward yet. For example, x^* could be the policy position that maximizes the expected utility of the members of the majority party, or the expected utility of all voters, etc. The key observation is that the location must be chosen such that the compromise is elected with high probability. Therefore, the agenda setter should set $x_C = x^*$ if x^* belongs to the set of electable compromises (characterized in Lemma 1), and otherwise choose among the electable compromise locations the one that is closest to x^*.

Proposition 4 *Assume that* $\max\{\gamma_{-1}, \gamma_1\} < 1$ *and that the agenda setter has a utility function that is symmetric around its peak* $x^* \in (-1, 1)$*.*

(i) Suppose that Assumption A holds, and that $-1+2\gamma_1 \neq x_C^L$*. Let* $X_C \equiv [-1+2\gamma_1\ , x_C^L] \cup [x_C^R, 1]$ *denote the set of compromise locations that get elected under SV with agenda* $[L, \{C, R\}]$ *and suppose that* $\min_{x \in X_C} |x - x^*|$ *has a unique solution. Then, the optimal compromise location under SV satisfies*

$$\lim_{n\to\infty} x_C(n) = \arg\min_{x \in X_C} |x - x^*| .$$

(ii) Under AV, the optimal compromise location satisfies

$$\lim_{n\to\infty} x_C(n) = \begin{cases} \max\{x^*, x_C^R\} & \text{if } p < 1/2 \\ \min\{x^*, x_C^L\} & \text{if } p > 1/2 \end{cases}$$

[17]Recall Theresa May's frustration from her continued failure to pass a negotiated Brexit compromise through Parliament, although her Government party, the Tories, possessed, together with an allied, small North-Irish party, a theoretical majority.

[18]The empirical analysis of Martin and Vanberg [2014] suggests that, at least in coalition governments, the most likely compromise is an average of the positions of the represented parties.

2) Suppose now that $x^* = 1$. That is, the goal of the agenda setter is to elect an alternative that is as close as possible to R.[19] Since R is already on the table, the only reason for the agenda setter to propose a compromise in this case is to prevent alternative L from being elected. From his point of view, the compromise would ideally be elected if voters with signal -1 have a majority (it prevents then outcome L), but not be elected if voters with signal $+1$ have a majority (so that their ideal policy R gets elected then). The location of the compromise must therefore take into account the precise conditions under which the compromise will be elected.

Proposition 5 *Suppose that the agenda setter chooses the location of the compromise $x_C(n)$ in order to maximize the expected location of the elected alternative, i.e., $x^* = 1$.*

(i) Consider SV with agenda $[L,\ \{C, R\}]$ and assume that Assumption A holds. The optimal compromise C is located at

$$x_C(n) = \begin{cases} \textit{just below } -1 + 2\gamma_1 \textit{ or at } x_C^L & \textit{if } \ x_C^L \geq -1 + 2\gamma_1 \\ \textit{at } x_C^L & \textit{if } \ x_C^L < -1 + 2\gamma_1 \end{cases}.$$

(ii) Under AV, the optimal compromise location satisfies

$$\lim_{n\to\infty} x_C(n) = \begin{cases} \min\{-1 + 2\gamma_1\ ,\ 1 - 2\gamma_{-1}\} & \textit{if } \ p < 1/2 \\ x_C^L & \textit{if } \ p > 1/2 \end{cases}.$$

Under SV, the optimal compromise location for a traditionalist agenda setter is often determined by the position $x_C = x_C^L$ that makes voters with the **opposite** signal ex ante indifferent between the compromise and their own traditional position: this is the highest compromise that will still be elected if those voters do have a majority. Sometimes it is even better to choose a location further to the left: rather than appealing to voters with the opposite signal, such a move makes the compromise less attractive for voters with signal $+1$ and increases the chance that R will be elected when they have a majority.

Under AV, the second stage voting is under complete information, so the realization (rather than the expectation) of n_{-1} determines whether the compromise gets elected. The ex ante optimal compromise location will therefore depend on the exact probability distribution. But, we can explicitly determine the optimal location for large populations. In that case, if $p > 1/2$ then voters with signal -1 have a majority with high probability. The largest

[19] This may be the case, for example, if its constituent base supports that position more strongly than the legislators themselves.

compromise these voters are willing to elect is then close to x_C^L, which is therefore optimal. The situation is more subtle if $p < 1/2$. Then, the optimal location must then take into account the value provided by the compromise (conditional on being elected), the likelihood that it gets elected if voters with signal -1 have a majority (in which case it protects against the election of L), and the likelihood that it gets elected if voters with signal $+1$ have a majority (in which case it prevents R from being elected). Using tools from large deviation theory, we show that the last channel dominates for large n: the optimal compromise satisfies $x_C \leq -1+2\gamma_1$ so that it will never be elected by a majority of voters with signal $+1$. For the compromise to be elected (at least sometimes) when voters with signal -1 have a majority, it must satisfy $x_C \leq 1 - 2\gamma_{-1}$. We show that, conditional on $n_{-1} \geq n+1$, the location $x_C = 1-2\gamma_{-1}$ gets elected with probability approaching 1, and the limit optimal compromise is therefore $x_C = \min\{-1+2\gamma_1 \ , \ 1-2\gamma_{-1}\}$.

Remark 1 *Let us compare the optimal compromise location under the two procedures for large populations with* $x^* = 1$. *In both procedures, the optimal compromise is chosen low enough (*$x_C \leq -1+2\gamma_1$*) so that it will not be elected if voters with signal* $+1$ *have a majority, but it is likely to be elected if voters with signal* -1 *have a majority. For the latter,* $x_C \leq x_C^L$ *is sufficient in SV, but* $x_C \leq 1-2\gamma_{-1}$ *is required in AV. If* $p < 1/2$ *and* $\gamma_{-1} < 1$*, then* $x_C^L = 4\left(1-\gamma_{-1}\right)(1-p) - 1 > 1 - 2\gamma_{-1}$. *Therefore, if* $p < 1/2$ *and* $\gamma_{-1} < 1$*, the agenda setter will choose a larger compromise under SV, and it would strictly prefer SV to AV whenever* $1-2\gamma_{-1} < -1+2\gamma_1$.

6 Case Study: The Flag of the Weimar Republic

The flag was one of the most contested issues during the Weimar Republic.[20] The principal argument was between the Black-Red-Gold (BRG) flag and the Black-White-Red (BWR) flag. The BRG flag was associated with progressive, anti-monarchistic ideas, while BWR were the official colors of the Reich in the period 1871-1919, and, significantly, already from 1867, the flag adorning of North-German Confederation's fleet.

The 421 seats in the Weimar National Constitutional Assembly were divided among various parties as follows:

[20]The flag controversy reflected, in compressed form, the entire century preceding Weimar. See Winkler [1993]. Article 3, defining defining the flag was the only article of the Constitution (out of 181!) whose outcome was determined by open roll-calls where individual votes were registered.

Party	**SPD**	**Z**	**DDP**	DNVP	USPD	DVP	BBB	DHP	SHBLD	BL
Seats	163	91	75	44	22	19	4	1	1	1

Table 4: The division of seats among parties

SPD, the left-leaning social democrats constituted the main party of the ruling coalition. Z(entrum) and DDP were centrist parties, also in the government coalition (in **bold**). DNVP and DVP were right-leaning conservative parties, both in the opposition. USPD, the independent social democrats were to the left of the SPD, and were also in opposition.[21]

6.1 The Proposed Flags

The Assembly considered 4 alternative proposals with respective support that crossed several party lines, leading to genuine uncertainty about the outcome of a vote:

1. BRG. This was the government's proposal, considered the "main" alternative. It was submitted to the Constitutional Committee on February 21, 1919 but it was subsequently adjusted at the initiative of the SPD to include a possible later determination of a flag for the fleet.

2. BWR. This was supported by the two opposition right-conservative parties DNVP and DVP, and by conservative factions of the centrist parties in the ruling coalition, Z and DDP.

3. R. Red, the color of the Socialist International, was supported by the more radical left, the USPD.[22]

4. BRG/BWR. This was the compromise arrangement: BRG as national colors, together with an adjusted BWR flag with BRG *canton* for the fleet.[23] The compromise was proposed by members of both Z and DDP.

The clear left-right ideological order was

$$R - BRG - BRG/BWR - BWR$$

[21]The other 4 very small parties in opposition, BBB, DHP, SHBLD and BL mostly represented regional interests.

[22]This proposal was also adjusted to include a provision about a future, possibly different flag for the fleet.

[23]A canton is a small flag within a flag, usually at the NW corner.

6.2 The Voting Outcome and Its Analysis

The standard decision making procedure of the Assembly was successive voting. Mentioning the "voting on the farthest alternative first" logic, the agenda-setting *Elders' Council* suggested[24]

$$\mathbf{A} : 1)\ R \longrightarrow 2)\ BWR \longrightarrow 3)\ BRG/BWR \longrightarrow 4)\ BRG$$

The first substantial vote was thus on R.[25] The outcome of the vote on R was widely anticipated, and this also explains why it was not deemed necessary to conduct a roll-call in that case.[26] We thus focus focus on the remaining agenda

$$\mathbf{A}' : 2)\ BWR \longrightarrow 3)\ BRG/BWR \longrightarrow 4)\ BRG$$

Voting was by roll-call, where each individual vote was carefully registered. BWR was also defeated: 111 members voted in favor, 190 members voted against, and 6 abstained. Finally, BRG/BWR was accepted: 211 voted in favor, 90 against and 1 abstained. According to the rules of the successive procedure, the remaining proposal BRG was not put to vote anymore.

This situation precisely fits the set-up analyzed in the theoretical part: we show below how vote-shifting can explain the observed outcome. Note also that the government's proposal BRG, that presumably had the highest ex ante support is put to here to vote last, as suggested by our theory. The following table displays the results in disaggregated form:[27]

[24] This is the method proposed by Trendelenburg, 1850, and Tecklenburg, 1914, for cases where the proposals are on both sides of the "main" alternative, taken here to be the Government's position. See Protocols [1919].

[25] A conservative member proposed a non-convex agenda that was defeated by simple majority before the substantial vote

[26] It is probable that only members of USPD—who had proposed R and who presumably had their peak on it—voted in its favor, while all other parties— who had peaks to the right—voted against.

[27] 111 members missed both votes, most of them from the ruling coalition. For simplicity, we do not list here the few incomplete profiles where one roll-call was missed, nor the few profiles that contained abstentions. Adding all these, a total of 17 (out of which 8 voted Yes on the compromise), does not change the result or its interpretation.

	$Y-Y$	$Y-N$	$N-N$	$N-Y$
SPD	0	0	0	106
Z	10	1	0	49
DDP	21	19	0	14
DNVP	0	33	0	0
USPD	0	0	18	0
DVP	1	16	0	0
BBB	0	0	0	1
DHP	0	0	1	1
BVP	0	0	0	1
Total	**32**	**69**	**19**	**172**

Table 5: The outcome of the roll-calls

1. 106 out of the 107 present SPD members voted N-Y.[28] This is consistent with either a peak on the compromise BRG/BWR, or with an initial peak on BRG together with a subsequent shift to BRG/BWR caused by the relatively large and vocal support for BWR.[29] Fixing the behavior of all other actors, adding 106 No votes of SPD members would have led to a clear rejection of the compromise and the likely election of the Government's proposal BRG. The omission to do so therefore suggests that these voters shifted their vote from BRG to the compromise BRG/BWR. The interdependent component of their preferences was clearly expressed in the Government's willingness to adjust its initial proposal to allow for a later determination of a fleet flag. Hence, we conclude that, after observing more than 100 votes in favor of BWR—about one third of the total—the members of the SPD most likely shifted their votes from an initial peak BRG to the compromise BRG/BWR.

2. All 18 present members of USPD voted N-N, consistent with sincere voting given a presumed peak on R.

3. A large majority of members of Z voted N-Y (49), while 7 abstained/missed the first vote and voted Y at the second. This is, again, consistent with either a peak on BRG/BWR, or with a peak on BRG and a subsequent shift after the BWR vote. But, the fact that BRG/BWR was formally proposed by this party, points to the first alternative. 10

[28] One member missed the first vote, and voted Y on the second.

[29] Note that, given the chosen agenda, this shift is immaterial for the behavior in the first two votes on R and BWR.

other members of Z voted Y-Y, which is consistent with sincere voting and a peak on BWR.

4. The DDP party was also split: 21 members voted Y-Y, consistent with a peak on BWR,[30] while 14 of its members voted N-Y, which, as we saw above, is consistent with a peak on BRG/BWR.

5. 49 out of the 50 present members of the right-wing, conservative parties DNVP and DVP, , voted Y-N,[31] They were joined by 20 members of the coalition (19 DDP and 1 Z). All these voters had a presumed peak on BWR and were expected to vote Y-Y, since BRG/BWR was their preferred alternative after the defeat of BWR. But they didn't, and their choice of seemingly **dominated** action cannot be explained by looking at the voting game in isolation. Therefore, we advance here a more speculative explanation: the radical conservative voters wanted to signal their unwillingness to compromise on the flag, thus lending credibility to their threat of rejecting the entire constitution because of it. This explanation is a twist on Fenno's "home-style" hypothesis.[32] Home-style—the need to justify behavior to constituents—is invoked to explain seemingly sub-optimal behavior (such as sincere voting) instead of behavior that exploits each strategic opportunity. Here sincere voting was in fact optimal but, at the last binary vote, it delivered the wrong signal. It is also very likely that the conservatives anticipated the vote shifting by the SPD, and hence believed that the compromise will be adopted anyhow, rendering their otherwise risky signaling behavior costless.

Remark 2 *We check the above outcome for consistency in light of the theoretical considerations. Consider SV with agenda* $[R,\ \{C, L\}]$ *and the profile* $(\neg R_1C_2 \text{ if} \geq \kappa, R_1C_2)$ *which is an equilibrium if* γ_1 *is relatively high, i.e., when right-leaning voters are close to weighing solely their own signal. This assumption certainly fits well their total unwillingness to compromise. We obtain an estimate of* γ_{-1}*, the weight on own signal of the left leaning members. For the shifting parameter, we obtain that* $\kappa = \frac{n}{2}\frac{1+x_C}{1-\gamma_{-1}} \leq 111$ *(since 111 voters voted in favor of BGR). Observing that* $n \approx 159$ *(since about 319 voters participated in the vote), this yields* $\gamma_{-1} \leq 0.3 - 0.7x_C$*. Note that the compromise location* x_C *of the cantoned flag BRG/BWR is best thought to satisfy* $-1 < x_C \leq 0$*: it was definitely closer in spirit to the left alternative BRG (main flag) than to right alternative BWR (canton). The weighted average compromise*

[30] Another 2 members of this party voted Y-Miss.

[31] The remaining member voted Y and then missed the second vote.

[32] See Fenno [1978], Denzau, Riker and Shepsle [1985], and Austen-Smith [1992].

in the spirit of the coalitional analysis of Martin and Vanberg [2014] would be about -0.52 *in this case. Setting* $x_C = -0.5$ *yields* $\gamma_{-1} \leq 0.65$*, a high degree of interdependence, explaining the governing coalition's willingness to compromise. Finally, the cantoned flag was most probably the Condorcet winner* CW *since*

$$n + 1 = 160 < n_{-1} \simeq 201 < 208 \leq 2n + 1 - \lceil \kappa \rceil,$$

where we estimated n_{-1} *by counting the legislators that voted against* BWR*.*

The short history of the Weimar Republic was marred by political instability and government falls, often connected to the flag. The BWR flag was restored by the Nazis immediately after taking power in 1933, and later another official flag, personally designed by Hitler, combining the BWR colors and the Swastika was added.[33]

BRG is again the flag of the re-united Germany, while the BWR colors are often used as a surrogate for illegal Nazi symbols. Demonstrators opposing Corona regulations attempted to storm the Reichstag in 2020 while adorning huge BWR flags. Subsequently, several German states completely banned its use.

7 Conclusion

Under complete information and single-peaked preferences all sequential binary procedures and all agendas are equivalent: if simple majority is used at each step, the Condorcet winner is always elected. The situation changes under incomplete information and a private values assumption: the Condorcet winner is elected by any sequential, binary procedure under any convex agenda, but this may not be true if the agenda is not convex. Assuming interdependent preferences, our present results allowed us to differentiate among various voting procedures and among convex agendas pertaining to the same procedure. Our results explain the emergence of compromises and describes the forces that determine their location on the ideological spectrum.

These insights may be used to explain a variety of observed phenomena in real-life voting situations. For example, in a case, the German Bundestag considered a reform of Paragraph 219a, the law governing the advertising of abortion procedures. The "extreme" alternatives were: 1) keeping the status quo that forbids any such advertising, and includes criminal

[33] This discredited use of the BWR colors led both East and West Germany's to return to the same (!) BRG flag after WWII. At least in West Germany this decision was controversial, and many pleaded for a complete new start (see Die Zeit, 1949). East Germany added a communist emblem in 1959.

charges against doctors that do so, and 2) scraping this paragraph altogether. Both the ruling coalition and the opposition contain parties on the left and on the right, and were split on this question. A compromise was forged that allows doctors and hospitals to advertise that they perform abortions, but does not allow them to provide further information about the methods, etc... The sequential voting agenda started with the two motions that wanted to scrap the law altogether. After these were defeated, the compromise was elected by a large majority.

On the other hand, Theresa May's inability to get her Brexit selected by the UK parliament points to a non-optimal choice of compromise, one that does not respect the majority opinion in that divided house.

8 Appendix A: Probabilistic Tools

For several comparison results we consider the case where the number of voters is large. For this purpose, we use two well-known probabilistic tools, the *Hoeffding inequality* and the *Gärtner-Ellis Large Deviation Principle* that allow us to approximate the probabilities with which both typical and atypical realizations of random variables deviate from their mean. The approximations are very precise for large democracies that often have more than 500 members of parliament.[34]

Definition 2 *A random variable X is σ-subgaussian if for all $t \in \mathbb{R}$ there is $\sigma > 0$ such that its moment generation function $\mathbb{E}(e^{tX})$ satisfies $\mathbb{E}\left[e^{t(X-\mathbb{E}[X])}\right] \leq e^{\sigma^2 t^2/2}$.*

A Bernoulli random variable $X \sim \text{Bernoulli}(p)$ is σ-subgaussian with $\sigma = 1/2$. A binomial random variable $X \sim B(N, p)$, the sum of N independent Bernoulli random variables, is $\sqrt{N}/2$-subgaussian. Any σ-subgaussian random variable X satisfies the *Hoeffding bounds*: for all $t \geq 0$,

$$\Pr\{X - \mathbb{E}[X] \geq t\} \leq e^{-t^2/\left(2\sigma^2\right)}, \tag{6}$$

and

$$\Pr\{X - \mathbb{E}[X] \leq -t\} \leq e^{-t^2/\left(2\sigma^2\right)}. \tag{7}$$

We shall repeatedly use (6) or (7) to bound (tail) probabilities since the random variable $\widetilde{n}_{-1}$, the number of agents with signal -1, is $\sqrt{2n+1}/2$-subgaussian. For example, applying

[34] To get an idea about the involved numbers, consider $n = 250$ which yields 501 voters. Assume that $p^L_{-1} = p^R_{-1} = 0.45$ which gives an expected value for the number of -1 signals of 225, a minority. The probability of nevertheless having a majority – at least 251 voters – with this signal is then less than 1% !

inequality (6) to $\widetilde{n}_{-1}$ yields

$$\Pr\left\{\widetilde{n}_{-1} - \mathbb{E}\left[\widetilde{n}_{-1}\right] \geq t\right\} \leq e^{-2t^2/(2n+1)}. \tag{8}$$

By setting $t = n + 1 - \mathbb{E}\left[\widetilde{n}_{-1}\right]$, we can rewrite the above inequality as

$$\Pr\left\{\widetilde{n}_{-1} \geq n+1\right\} \leq e^{-2(n+1-\mathbb{E}[\widetilde{n}_{-1}])^2/(2n+1)} \simeq e^{-(n+\frac{3}{2})(1-2p)^2}. \tag{9}$$

In other words, if voters with signal -1 are in an ex ante minority (i.e., $p < 1/2$), the probability that they are in an ex post majority (i.e., $n_{-1} \geq n+1$) decays exponentially to zero as n grows.

We now the state the *Large Deviation Principle* due to Gärtner and Ellis (see for example Ellis [2006]) that will be used in the proof of Proposition 5 below:

Theorem 1 *Suppose that X_n , $n \in \mathbb{N}$, is a family of real-valued random variables such that*

$$\Lambda(t) := \lim_{n\to\infty} \left(\frac{1}{n} \log \mathbb{E}\left[e^{ntX_n}\right]\right)$$

exists, and is finite for all $t \in \mathbb{R}$. If Λ is differentiable, then, for all Borel sets A such that the closure of A equals the closure of its interior, it holds that

$$\lim_{n\to\infty} \frac{1}{n} \log \Pr(X_n \in A) = -\inf_{x\in A} I(x) \tag{10}$$

where $I(x) = \sup_{t\in\mathbb{R}}[xt - \Lambda(t)]$ is the Fenchel-Legendre transform of Λ.[35]

9 Appendix B: Proofs

9.1 Proofs of Proposition 1

(i) We first show that the profile $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ is an equilibrium. We only need to consider signal realizations such that an individual voter is pivotal at a given stage.

Consider first voters with signal -1. They play a best response by voting for C in the second stage because their ideal point is at most

$$-\gamma_{-1} + \frac{1-\gamma_{-1}}{2n}(2n) = 1 - 2\gamma_{-1} \leq x_C.$$

Voter i with signal -1 is pivotal in the first stage if (1) there are $\lceil k \rceil - 1$ other voters having signal of -1 (in this case, voter i is pivotal between C and R), or if (2) there are exactly n

[35]The function Λ, the logarithm of the moment generating function, is also known as the *cumulant generating function.*

other voters having signal of -1 (in this case, voter i is pivotal between L and C). The ideal point of voter i for the first case is

$$-\gamma_{-1}+\frac{1-\gamma_{-1}}{2n}\left(-\left(\lceil k\rceil-1\right)+\left(2n-\lceil k\rceil+1\right)\right)=-\gamma_{-1}+\frac{1-\gamma_{-1}}{n}\left(n-\lceil k\rceil+1\right).$$

The ideal point for the second case is $-\gamma_{-1}$. Therefore, in the first case, voter i with signal -1 will vote for L in the first stage if

$$-\gamma_{-1}+\frac{1-\gamma_{-1}}{n}\left(n-\lceil k\rceil+1\right)\leq\frac{1}{2}\left(x_C+1\right)\Leftrightarrow\gamma_{-1}\geq\frac{n+2-2\lceil k\rceil-nx_C}{4n-2\lceil k\rceil+2} \tag{11}$$

Since $-2\lceil k\rceil+2\leq 0$ and $\gamma_{-1}\geq\frac{1}{2}\left(1-x_C\right)$ by assumption, we get

$$\gamma_{-1}\geq\frac{1-x_C}{2}\geq\frac{n-nx_C-2\lceil k\rceil+2}{2n-2\lceil k\rceil+2}\geq\frac{n-2\lceil k\rceil+2-nx_C}{4n-2\lceil k\rceil+2}.$$

Therefore, condition (11) is always satisfied. In the second case, since $\gamma_{-1}\geq\frac{1}{2}\left(1-x_C\right)$, voter i with signal -1 will vote for L in the first stage.

Consider next voters with signal $+1$. By the definition of cutoff k, they play a best response in the second stage by voting for C if and only if at least $\lceil k\rceil$ voters support L in the first stage. Voter i with signal $+1$ is pivotal in the first stage if (1) there are $\lceil k\rceil-1$ voters having signal of -1 (in this case, voter i is pivotal between C and R), or if (2) there are exactly n voters having signal of -1 (in this case, voter i is pivotal between L and C). Therefore, for the first case, voter i plays a best response by voting against L in the first stage if

$$\gamma_1+\frac{1-\gamma_1}{2n}\left(-\left(\lceil k\rceil-1\right)+\left(2n-\lceil k\rceil+1\right)\right)\geq\frac{1}{2}\left(1+x_C\right),$$

which always holds since $\lceil k\rceil-1\leq k$ and k satisfies by definition

$$\gamma_1+\frac{1-\gamma_1}{2n}\left(-k+\left(2n-k\right)\right)=\frac{1}{2}\left(1+x_C\right).$$

For the second case, since her ideal point $\gamma_1\geq\frac{1}{2}\left(x_C-1\right)$, voter i plays a best response by voting against L in the first stage.

. To see that the complete information Condorcet winner will be selected under this strategy profile, note that. if at least $n+1$ voters have signal -1, L will be selected and this is the preferred alternative for voters with signal -1 since $\gamma_{-1}\geq\frac{1}{2}\left(1-x_C\right)$. If at least $n+1$ voters have signal $+1$, L will be rejected in the first stage. In the second stage, agents are, essentially, completely informed about signals of others, and C gets elected if and only if voters with signal $+1$ prefer C to R given the realized preferences.

Finally, note that if $k>n$, then, whenever L receives at least $\lceil k\rceil$ votes in the first stage, L is chosen and thus vote shifting from voters with signal $+1$ never occurs in equilibrium.

In order for vote shifting to possibly occur in equilibrium, we must have $k \leq n$, which is equivalent to $\gamma_1 \leq \frac{1}{2}(1 + x_C)$.

(ii) To obtain a contradiction, suppose there is a pure strategy profile that always selects the Condorcet winner, and let σ denote the corresponding profile of actions for the first stage. For each voter i this yields a mapping $\sigma_i : \{-1, +1\} \to \{L, \neg L\}$. Observe that, for any signal realization such that $n_{-1} = n + 1$, L is not the Condorcet winner: the ideal point of voters with signal -1 is then $-\gamma_{-1}$, which is closer to x_C than to -1 because $\gamma_{-1} < \frac{1}{2}(1 - x_C)$.

Suppose first that there are at most n voters whose strategy satisfies $\sigma_i(-1) = \neg L$. Consider a signal realization such that these voters have signal $+1$, and such that $n_{-1} = n+1$. Then L gets selected even though it is not the Condorcet winner because all voters that have signal -1 vote for L. We conclude that, for at least $n + 1$ voters, the profile σ must satisfy $\sigma_i(-1) = \neg L$. But this implies that L will not be selected even if all voters have signal -1, in which case L is the Condorcet winner, a contradiction.

Finally, since there is no pure strategy profile that always selects the Condorcet winner, there can also be no mixed strategy profile that always selects the Condorcet winner.

9.2 Proof of Proposition 2

We first combine sincerity and pivotality considerations to characterize the sincere and semi-sincere equilibria under $[L,\ \{C, R\}]$:

Proposition 6 *Consider SV with agenda* $[L,\ \{C, R\}]$ *and suppose that Assumption A holds.*

(i) The profile $(\neg L_1C_2, \neg L_1C_2)$ *is a sincere equilibrium if* $\gamma_{-1} \leq \gamma^*_{-1}$ *and* $\gamma_1 \leq \gamma^*_1$*. This is the unique such equilibrium for any* γ_{-1} *and* γ_1 *in these ranges such that* $\gamma_{-1} \neq \gamma^*_{-1}$ *and* $\gamma_1 \neq \gamma^*_1$*.*

(ii) The profile $(\neg L_1C_2, \neg L_1\neg C_2)$ *is a sincere equilibrium if* $\gamma_{-1} \leq \gamma^*_{-1}$ *and* $\gamma_1 \geq \frac{1}{2}(1 + x_C)$*. This is the unique such equilibrium for any* γ_{-1} *and* γ_1 *in these ranges such that* $\gamma_{-1} \neq \gamma^*_{-1}$*.*

(iii) The profile $(L_1C_2, \neg L_1C_2$ *if* $\geq k)$ *is a sincere equilibrium if* $\gamma_{-1} \geq \gamma^*_{-1}$*. This is the unique such equilibrium for any* γ_{-1} *and* γ_1 *in these ranges such that* $\gamma_{-1} \neq \gamma^*_{-1}$*, and such that* k *is not an integer.*

(iv) There is no sincere equilibrium if $\gamma_{-1} \in [\frac{1}{2n+1}, \gamma^*_{-1})$ *and* $\gamma_1 \in (\gamma^*_1, \frac{1}{2}(1 + x_C))$*. The profile* $(\neg L_1C_2, \neg L_1C_2)$ *forms a semi-sincere equilibrium: no voter is ever pivotal, and voters with signal* -1 *use a sincere strategy.*

Proof. Sincerity Considerations: Consider the first stage voting. By Assumption A, voters with signal -1 weakly prefer L to R ex ante. These voters prefer C to L ex anter if

$$-\gamma_{-1} + (1 - \gamma_{-1})(1 - 2p) \geq \frac{1}{2}(-1 + x_C)$$

which is equivalent to $\gamma_{-1} \leq \gamma_{-1}^*$. Voters with signal $+1$ prefer R to L ex anter if $\gamma_1 + (1 - \gamma_1)(1 - 2p) \geq 0$, which is always satisfied. Therefore, it is a sincere strategy for voters with signal $+1$ to always vote against L, and it is a sincere strategy for voters with signal -1 to vote for L if and only if $\gamma_{-1} \geq \gamma_{-1}^*$.

For the second stage, we need to consider two possible cases:

(a) $\gamma_{-1} \leq \gamma_{-1}^*$: all voters vote against L in the first stage, and no information about n_{-1} is revealed. By Assumption A, voters with signal -1 prefer L to R ex anter, and because $\gamma_{-1} \leq \gamma_{-1}^*$ they also ex ante prefer C to L. As a result, these voters prefer C to R ex ante. Voters with signal $+1$ prefer R to C ex anter if

$$\gamma_1 + (1 - \gamma_1)(1 - 2p) \geq \frac{1}{2}(1 + x_C)$$

which is equivalent to $\gamma_1 \geq \gamma_1^*$. Therefore, if $\gamma_{-1} \leq \gamma_{-1}^*$, it is a sincere strategy for voters with signal -1 to always vote for C, while for voters with signal $+1$ it is sincere to vote against C if and only if $\gamma_1 \geq \gamma_1^*$.

(b) $\gamma_{-1} \geq \gamma_{-1}^*$: not all voters vote against L and n_{-1} is revealed. In this case, voters with signal -1 always prefer C to R, because $\gamma_{-1}^* \geq \frac{1}{4}(1 - x_C)$ and because a voter with signal -1 prefers C to R in the situation where she is the lone -1 voter if

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(2n) \leq \frac{1}{2}(1 + x_C) \Leftrightarrow \gamma_{-1} \geq \frac{1}{4}(1 - x_C).$$

By the definition of the cutoff k, voters with signal $+1$ prefer R to C in the second stage if and only if $n_{-1} \geq k$. Therefore, if $\gamma_{-1} \geq \gamma_{-1}^*$, it is a sincere strategy for voters with signal -1 to always vote for C, while voters with signal $+1$ vote sincerely against C if and only if $n_{-1} \geq k$.

To summarize, in the first stage, it is a sincere strategy for voters with signal -1 to vote in favor of L if and only if $\gamma_{-1} \geq \gamma_{-1}^*$, and for voters with signal $+1$ to always vote against L. In the second stage, sincerity requires that (1) if $\gamma_{-1} \leq \gamma_{-1}^*$, voters with signal $+1$ vote for C if and only if $\gamma_1 \leq \gamma_1^*$ while voters with signal -1 always vote for C, and that (2) if $\gamma_{-1} \geq \gamma_{-1}^*$, voters with signal $+1$ vote for C if and only if $n_{-1} \geq k$ while voters with signal -1 always vote for C. As a result, the profiles $(\neg L_1 C_2, \neg L_1 C_2)$, $(\neg L_1 C_2, \neg L_1 \neg C_2)$, and $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ are the unique sincere profiles corresponding to the cases ($\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \gamma_1^*$), ($\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \geq \gamma_1^*$), and ($\gamma_{-1} \geq \gamma_{-1}^*$), respectively. Moreover, if k is not an integer, and if $\gamma_{-1} \neq \gamma_{-1}^*$, and $\gamma_1 \neq \gamma_1^*$ the sincere strategies are uniquely defined.

Equilibrium Considerations: In profiles $(\neg L_1C_2, \neg L_1C_2)$ and $(\neg L_1C_2, \neg L_1\neg C_2)$, all voters vote against L in the first stage, so that we only need to consider pivotality at the second stage. Conditional being on pivotal between R and C, voters with signal $+1$ prefer C if and only if $\gamma_1 \leq \frac{1}{2}(1+x_C)$, while voters with signal -1 prefer C for all γ_{-1}. It follows from the proof of Proposition 1 that profile $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ is an equilibrium if $\gamma_{-1} \geq \frac{1}{2}(1-x_C)$. Note that, by Assumption A we obtain $\gamma_1^* \leq \frac{1}{2}(1+x_C)$ and $\gamma_{-1}^* \geq \frac{1}{2}(1-x_C)$. Therefore, $(\neg L_1C_2, \neg L_1C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \gamma_1^*$, $(\neg L_1C_2, \neg L_1\neg C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \geq \frac{1}{2}(1+x_C)$, and $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ is a sincere equilibrium if $\gamma_{-1} \geq \gamma_{-1}^*$.

Finally, when $\gamma_{-1} < \gamma_{-1}^*$ and $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C)]$, the non-existence of sincere equilibrium is due to a conflict between sincerity and pivotality. Sincerity requires that both type of voters vote against L at the first stage and voters with signal -1 vote for C at the second stage. Therefore, no new information would be revealed by the vote at the first stage. If $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C)]$, then, based on ex ante information, voters with signal $+1$ prefer R to C. But, conditional on pivotality, this preference is reversed. Hence, sincere voting suggests that voters with signal $+1$ vote against C, but pivotality requires that they vote in favor of C. Thus, sincere voting by voters with signal $+1$ cannot be part of an equilibrium in this case. ■

The table below summarizes sincere/semi-sincere equilibria for agenda $[L, \{C, R\}]$, where the strategy profile in quotation marks is semi-sincere, while all other strategy profiles are fully sincere.[36]

	$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C))$	$\gamma_1 \in (\frac{1}{2}(1+x_C), 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$	$(\neg L_1C_2, \neg L_1C_2)$	"$(\neg L_1C_2, \neg L_1C_2)$"	$(\neg L_1C_2, \neg L_1\neg C_2)$
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$	$(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$	$(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$

Table 2: Sincere/Semi-sincere equilibria for agenda $[L, \{C, R\}]$

[36] We take (half-)open intervals to exclude the cutoff points $\gamma_1^*, \gamma_{-1}^*, (1+x_C)/2$ and $(1-x_C)/2$ so that the respective sincere/semi-sincere equilibrium is unique.

For the alternative convex agenda $[R,\ \{C,L\}]$, we can analogously obtain:[37]

	$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C))$	$\gamma_1 \in (\frac{1}{2}(1+x_C), 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$	$(\neg R_1 C_2, \neg R_1 C_2)$	"$(\neg R_1 C_2, \neg R_1 C_2)$"	$(\neg R_1 C_2$ if $\geq \kappa, R_1 C_2)$
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$(\neg R_1 \neg C_2, \neg R_1 C_2)$	"$(\neg R_1 \neg C_2, \neg R_1 C_2)$"	$(\neg R_1 C_2$ if $\geq \kappa, R_1 C_2)$

Table 3: Sincere/Semi-sincere equilibria for agenda $[R, \{C, L\}]$

Again, the profiles within quotation marks are semi-sincere equilibria for the given parameter values: $(\neg R_1 C_2, \neg R_1 C_2)$ is an equilibrium because no voter is ever pivotal and voters with signal -1 vote sincerely. Profile $(\neg R_1 \neg C_2, \neg R_1 C_2)$ is an equilibrium because no one is pivotal in the first stage: conditional on n other voters having signal -1, a voter with signal -1 prefers L to C and a voter with signal $+1$ prefers C to R. Voters with signal -1 play a sincere strategy.

Now we are ready to prove Proposition 2.

Proof of Proposition 2. If $\gamma_1 > \frac{1+x_C}{2}$ and $\gamma_{-1} < \gamma_{-1}^*$, we see from Table 1 that the outcomes of the two agendas differ only when (1) $\gamma_{-1} \in [\frac{1}{2n+1}, \frac{1-x_C}{2})$ and $n_{-1} > 2n+1-\lceil\kappa\rceil$, or when (2) $\gamma_{-1} \in \left(\frac{1-x_C}{2}, \gamma_{-1}^*\right)$ and $n_{-1} \geq n+1$. In both cases, C is chosen under $[L,\ \{C,R\}]$ and L is chosen under $[R,\ \{C,L\}]$. Moreover, whenever L is chosen under $[R,\ \{C,L\}]$, it is the Condorcet winner. In both cases, $n_{-1} \geq n+1$ and $[R,\ \{C,L\}]$ selects the Condorcet winner while $[L,\ \{C,R\}]$ does not. The event $\{\widetilde{n}_{-1} \geq n+1\}$, however, has a vanishing probability as n grows. To see this, recall that $p < 1/2$ by Assumption A. It follows from (9) that the probability of the event $\{\widetilde{n}_{-1} \geq n+1\}$ decays exponentially to zero as n grows:

$$\Pr\{\widetilde{n}_{-1} \geq n+1\} \leq e^{-2(n+1-\mathbb{E}[\widetilde{n}_{-1}])^2/(2n+1)} \simeq e^{-(n+\frac{3}{2})(1-2p)^2}. \tag{12}$$

Therefore, as n grows, the advantage of $[R,\ \{C,L\}]$ becomes negligible.

If $\gamma_1 < \frac{1+x_C}{2}$ and $\gamma_{-1} > \gamma_{-1}^*$, the outcomes of the two agendas differ only if $n_{-1} \leq \lceil k\rceil - 1$. In this case, $[L, \{C,R\}]$ elects R while $[R, \{C,L\}]$ elects C. Whenever R is chosen under $[L, \{C,R\}]$, it is the Condorcet winner. Therefore, agenda $[L, \{C,R\}]$ selects the Condorcet winner with a higher probability than agenda $[R,\ \{C,L\}]$. We can set $t = \lceil k\rceil - \mathbb{E}[\widetilde{n}_{-1}]$ in (8) to obtain

$$\Pr\{\widetilde{n}_{-1} \geq \lceil k\rceil\} \leq e^{-2(\lceil k\rceil-\mathbb{E}[\widetilde{n}_{-1}])^2/(2n+1)} \simeq e^{-(n+\frac{3}{2})\left(\frac{1}{2}\frac{1-x_C}{1-\gamma_1}-2p\right)^2}. \tag{13}$$

Therefore, if $\mathbb{E}[\widetilde{n}_{-1}] \leq \lceil k\rceil - 1$ (and thus $p < \frac{1}{4}\frac{1-x_C}{1-\gamma_1}$ in the limit), then the event $\{\widetilde{n}_{-1} \geq \lceil k\rceil\}$ has a probability exponentially decaying to 0 as n grows. Equivalently, the event $\{\widetilde{n}_{-1} \leq \lceil k\rceil - 1\}$

[37] The formal characterization of sincere equilibria under $[R, \{C, L\}]$ is analogous to Proposition 6, and is thus omitted.

has a probability exponentially approaching one, and the advantage of $[L, \{C, R\}]$ can be significant.

For the other parameter values of γ_1 and γ_{-1}, the two convex agendas are outcome equivalent. ∎

9.3 Proof of Proposition 3

We focus on voters with signal -1. The arguments for voters with signal $+1$ are analogous.

We start with the second stage. If L wins at the first stage, a voter with signal -1 prefer C to L if at least $\lceil \kappa \rceil$ voters have signal $+1$; otherwise she prefers L to C. Given the strategy profile Π, all first-stage votes for R come from voters with signal $+1$. Therefore, voters with signal -1 play a best response. If R wins the first stage, voters with signal -1 are not pivotal in the second stage and therefore play a best response.

In the first stage, voter i with signal -1 is pivotal if (1) there are exactly n other voters with signal -1, or if (2) there are exactly $\lceil k \rceil - 1$ other voters with signal -1 and $\lceil k \rceil \leq n$, or if (3) there are exactly $\lceil \kappa \rceil - 1$ others with signal $+1$ and $\lceil \kappa \rceil \leq n$.

In case (1), voter i likes alternative R the least. If she prefers L to C, then the worst feasible alternative when L wins the first stage is weakly better than the best possible alternative when R wins the first stage, so voting for L is a best response. If she prefers C to L, she will get her preferred alternative by voting for L in the first stage and by voting for C in the second stage.

In case (2), voter i is pivotal between C and R, and voting L is a best response if her ideal point satisfies

$$-\gamma_{-1} + \frac{1-\gamma_{-1}}{2n} \left(-\lceil k \rceil + 1 + 2n - \lceil k \rceil + 1\right) \leq \frac{1}{2}\left(1 + x_C\right).$$

Using the definition of $\lceil k \rceil$, a sufficient condition for this inequality is

$$-\gamma_{-1} + \frac{1-\gamma_{-1}}{2n} \left(-\frac{n(1-x_C)}{1-\gamma_1} + 2n + 2\right) \leq \frac{1}{2}\left(1 + x_C\right).$$

Note that $k \leq n$ implies $\gamma_1 \leq \frac{1}{2}\left(1 + x_C\right)$. The left-hand side is therefore decreasing in both γ_1 and γ_{-1}, and is equal to $\frac{1}{2}\left(1 + x_C\right)$ when $\gamma_{-1} = \gamma_1 = \frac{1}{2n+1}$. Since $\gamma_{-1} \geq \frac{1}{2n+1}$ and $\gamma_1 \geq \frac{1}{2n+1}$, the above inequality always holds. We conclude that voting for L is a best response.

In case (3), voter i is pivotal between L and C. By the definition of κ, she prefers L if there are $\lceil \kappa \rceil - 1$ others with signal $+1$. Voting for L is therefore a best response.

For the Condorcet claim, suppose that alternative R wins in the first stage. Then the number of voters with signal $+1$ is at least $n + 1$. Hence, once all private information becomes public, any voter with signal $+1$ prefers R over L, and therefore L is not the full information Condorcet winner. At the second stage, the number of voters with signal -1 is public information, and hence the full information Condorcet winner gets elected. The argument is analogous if L wins in the first stage.

9.4 Proof of Lemma 1

(i). We first prove the claim for SV. Suppose that $n_{-1} \geq n+1$. Then C gets elected if and only if $\gamma_{-1} \leq \gamma^*_{-1}$ (see Table 3). By definition, -1 voters are ex ante indifferent between L and C if $\gamma_{-1} = \gamma^*_{-1}$ or if $x_C = x_C^L$. Therefore, $\gamma_{-1} \leq \gamma^*_{-1}$ is equivalent to $x_C \leq x_C^L$.

Suppose next that $n_{-1} \leq n$. Then C gets elected either if (1) $\gamma_{-1} \leq \gamma^*_{-1}$ and $\gamma_1 \leq \frac{1+x_C}{2}$ or if (2) $\gamma_{-1} \geq \gamma^*_{-1}$, $\gamma_1 \leq \frac{1+x_C}{2}$ and $n_{-1} \geq k$ (see Table 3). As shown above, $\gamma_{-1} \leq \gamma^*_{-1}$ is equivalent to $x_C \leq x_C^L$. Also, $\gamma_1 \leq \frac{1+x_C}{2}$ is equivalent to $x_C \geq 2\gamma_1 - 1$. Hence, case (1) applies if and only if $2\gamma_1 - 1 \leq x_C \leq x_C^L$. Case (2) applies if and only if $x_C \geq x_C^L$, $x_C \geq 2\gamma_1 - 1$, and $n_{-1} \geq k$ (which is equivalent to $x_C \geq x_C^R(n_{-1}) = 1 - 2(1-\gamma_1)\frac{n_{-1}}{n}$). Since $n_{-1} \leq n$, we have $2\gamma_1 - 1 \leq x_C^R(n_{-1})$, and therefore, alternative C is chosen in case (2) if $x_C \geq \max\left\{x_C^L,\ x_C^R(n_{-1})\right\}$. Finally, since $n_{-1} \leq n$, we have $x_C^R(n_{-1}) \geq 2\gamma_1 - 1$, and thus the set of implementable compromise locations

$$\left[-1+2\gamma_1\ ,\ x_C^L\right] \cup \left[\max\left\{x_C^L\ ,\ x_C^R(n_{-1})\right\},1\right]$$

can be rewritten as

$$\left[-1+2\gamma_1\ ,\ x_C^L\right] \cup \left[x_C^R(n_{-1})\ ,\ 1\right].$$

(ii). Consider now AV, and recall that it always selects the Condorcet winner. Alternative C is the Condorcet winner if either $n_{-1} \geq n+1$ and $n_{+1} \geq \kappa(x_C)$, or if $n_{+1} \geq n+1$ and $n_{-1} \geq k(x_C)$, where $\kappa(x_C) = \frac{n}{2}\frac{1+x_C}{1-\gamma_{-1}}$ and $k(x_C) = \frac{n}{2}\frac{1-x_C}{1-\gamma_1}$. Rearranging the terms, we obtain that C gets elected if $n_{-1} \geq n+1$ and $x_C \leq x_C^L(n_{+1})$ or if $n_{-1} \leq n$ and $x_C \geq x_C^R(n_{-1})$.

9.5 Proof of Proposition 4

(i). Consider SV and suppose Assumption A holds. For any x in the interior of X_C, the probability that x gets elected converges to 1. To see this, note that, for any $t > 0$, it follows from (7) that

$$\Pr\left\{x_C^R(\tilde{n}_{-1}) - x_C^R \geq t\right\} = \Pr\left\{2(1-\gamma_1)\left(2p - \frac{\tilde{n}_{-1}}{n}\right) \geq t\right\}$$
$$= \Pr\left\{\frac{\tilde{n}_{-1}}{n} - 2p \leq -\frac{t}{2(1-\gamma_1)}\right\} \leq e^{-\frac{2t^2}{4(2n+1)(1-\gamma_1)^2}}.$$

Therefore, any compromise $x > x_C^R$ is elected with probability approaching 1 as n grows. By analogous arguments, any compromise $x \in (-1 + 2\gamma_1\ ,\ x_C^L)$ is elected with probability approaching 1. For $x \notin X_C$, however, the probability of x being elected converges to 0.

We assume by contradiction that $\lim_{n\to\infty} x_C\,(n) \neq \arg\min_{x\in X_C} |x - x^*|$. Suppose first that $x^* \in X_C$, but there exists $\varepsilon > 0$ such that $x_C(n) > x^* + \varepsilon$ for infinitely many n (the argument is analogous if $x_C(n) < x^* - \varepsilon$ for infinitely many n). Then, there exists $x \in int(X_C)$ that is sufficiently close to x^* such that the utility of the agenda setter if x gets elected is strictly higher than those obtained when $x^* + \varepsilon$ is elected or R are elected. Since the probability that x gets elected converges to 1, and since the agenda setter's utility is single-peaked, we conclude that, for n large enough, it is strictly better to propose x than to propose any compromise above $x^* + \varepsilon$. Since $x_C(n)$ is optimal by assumption, this yields a contradiction.

Suppose now that $x^* \notin X_C$ and that $\arg\min_{x\in X_C} |x - x^*|$ is a singleton, denoted by x'. To obtain a contradiction, suppose $x_C(n) > x' + \varepsilon$ or $x_C(n) < x' - \varepsilon$ for infinitely many n, and let $x \in int(X_C)$ and sufficiently close to x'. Then x gets elected with probability approaching 1 and, conditional on being elected, provides strictly greater utility compared to $x' + \varepsilon$ and compared to R. It follows that, for n large enough, it is strictly better to propose compromise x than to propose compromise $x_C(n)$, a contradiction.

(ii). Consider now AV, and observe that, if $p < 1/2$, any $x > x_C^R$ gets elected with probability approaching 1, and any $x < x_C^R$ gets elected with probability approaching 0. Similarly, if $p > 1/2$, any $x < x_C^L$ gets elected with probability approaching 1, and any $x > x_C^L$ gets elected with probability approaching 0.

Assume that $p < 1/2$, and suppose that there exists $\varepsilon > 0$ such that $x_C(n) < \max\{x^*,\ x_C^R\} - \varepsilon$ for infinitely many n. The probability that the compromise $x \equiv \max\{x^*,\ x_C^R\} + \delta$ with $\delta > 0$ gets elected approaches 1. If $x^* \geq x_C^R$ and if δ is small enough, then, conditional on being elected, the compromise x provides strictly higher utility than either $\max\{x^*,\ x_C^R\} - \varepsilon$ or R . For n large enough, it is therefore strictly better to propose compromise x than to propose $x_C(n)$. If $x^* < x_C^R$, the probability that $x_C(n)$ gets elected approaches 0, and it is again better to propose compromise x for sufficiently small δ, a contradiction.

Suppose now that there exists $\varepsilon > 0$ such that $x_C(n) > \max\{x^*,\ x_C^R\} + \varepsilon$ for infinitely many n. The probability that the compromise gets elected along this subsequence approaches 1. Similarly, the probability that the compromise $\max\{x^*,\ x_C^R\} + \frac{\varepsilon}{2}$ gets elected also approaches 1. Since the utility gain of compromise $\max\{x^*,\ x_C^R\} + \frac{\varepsilon}{2}$ compared to compromise $x_C(n)$ is strictly positive and remains bounded away from 0, we conclude that compromise

$\max\{x^*,\ x_C^R\} + \frac{\varepsilon}{2}$ is strictly better than $x_C(n)$ for n large enough, a contradiction. Therefore, $\lim_{n\to\infty} \max\{x^*,\ x_C^R\}$ if $p < 1/2$. The arguments are similar for $p > 1/2$.

9.6 Proof of Proposition 5

For the proof of Proposition 5-(ii), we need the following Lemma:

Lemma 2 *Let $p < b < c$. Then*

$$\lim_{n\to\infty} \left(\frac{\Pr(\frac{\tilde{n}_{-1}}{2n} \geq c)}{\Pr(\frac{\tilde{n}_{-1}}{2n} \geq b)} \right) = 0.$$

Proof. Let X_i be a Bernoulli random variable with success probability p, and assume all random variables are independently distributed. We apply the Gaertner-Ellis Theorem to the family of random variables of the form $\frac{1}{2n+1}[\sum_{i=1}^{2n+1} X_i]$, $n \in \mathbb{N}$. Note that the cumulant generating function is

$$\Lambda(t) = \lim_{n\to\infty} \frac{1}{2n+1} \log[\mathbb{E}\left[e^{tX_i}\right]^{2n+1}] = \log\left\{1 - p + pe^t\right\}$$

This shows that $\Lambda(t) < \infty$ for all t, and that Λ is convex and twice differentiable. The Gaertner-Ellis theorem implies that the family of random variables $\frac{\tilde{n}_{-1}}{2n+1} = \frac{1}{2n+1}[\sum_{i=1}^{n+1} X_i]$, $n \in \mathbb{N}$, satisfies the large deviation principle (10) with rate function $I(x) = \sup_{t\in\mathbb{R}} xt - \Lambda(t)$.

To understand the properties of $I(x)$, fix an arbitrary x and note that the function $xt - \Lambda(t)$ is concave and differentiable. For each real x, denote by t_x the maximizing t in the definition of I. The maximizer t_x must satisfy the first-order condition:

$$x = \Lambda'(t_x) = \frac{pe^{t_x}}{1 - p + pe^{t_x}}$$

and

$$\Lambda'(0) = p > 0$$

Since $\Lambda''(t) > 0$, we obtain that $t_x > 0$ for $x > p$. By the envelope theorem, we obtain that the rate function I is strictly increasing for $x > p$ since $I'(x) = t_x > 0$.

We conclude that

$$\Pr(\tilde{n}_{-1} \geq an) = e^{-2nI(a)+o(n)}$$

for all $a > p$, and that

$$\lim_{n\to\infty} \left(\frac{\Pr(\frac{\tilde{n}_{-1}}{2n} \geq c)}{\Pr(\frac{\tilde{n}_{-1}}{2n} \geq b)} \right) = \lim_{n\to\infty} (e^{-2n[I(c)-I(b)]+o(n)}) = 0$$

where the last equality follows because $p < b < c$ and because $I(x)$ is strictly increasing for $x > p$. ■

Proof of Proposition 5. (i). SV. Note that Assumption A implies $x_C^L \leq 1$. Also, observe that $n_{-1} \leq n$ implies $-1 + 2\gamma_1 \leq x_C^R(n_{-1})$.

(a). If $-1 + 2\gamma_1 \leq x_C^L$, then, by Lemma 1, we can set x_C just below $-1 + 2\gamma_1$ to get C elected if $n_{-1} \geq n + 1$, and get R elected if $n_{-1} < n + 1$. On the other hand, setting $x_C = x_C^L$ will always get C elected. No other compromise location can be optimal: Setting x_C substantially below $-1 + 2\gamma_1$ is dominated by setting it just below $-1 + 2\gamma_1$. Setting x_C between $-1 + 2\gamma_1$ and x_C^L is dominated by setting $x_C = x_C^L$, while setting x_C above x_C^L is dominated by setting it just below $-1 + 2\gamma_1$.[38]

(b). If $x_C^L < -1 + 2\gamma_1$, then Lemma 1 implies that if we set $x_C = x_C^L$, then if $n_{-1} \geq n + 1$ alternative C gets elected, while if $n_{-1} < n + 1$ alternative R gets elected. Any higher compromise location is worse because such a compromise will never be elected if $n_{-1} \geq n + 1$. Any lower compromise location is worse because it will be elected in the same instances, but will provide lower utility conditional on being elected.

(ii). AV

(a) Assume first that $p > 1/2$. Suppose, by contradiction, that there exists $\varepsilon > 0$ such that the optimal compromise satisfies $x_C(n) < x_C^L - \varepsilon$ for infinitely many n, and consider the corresponding subsequence. We argue that, for n large enough, the compromise $x_C^L - \varepsilon/2$ is strictly better than $x_C(n)$ because its location is further to the right, and because it still gets elected with probability approaching 1. Note that $\lim_{n\to\infty} \Pr\{n_{+1} \geq n + 1\} = 0$ since $p > 1/2$. Observe also that

$$\lim_{n\to\infty} \Pr\{\kappa(x_C^L - \frac{\varepsilon}{2}) \leq \tilde{n}_{+1} \leq n\} = 1$$

since $\kappa(x_C^L - \frac{\varepsilon}{2}) = n\left[2(1-p) - \frac{\varepsilon}{4(1-\gamma_{-1})}\right]$. This yields

$$\Pr\{\kappa(x_C^L - \frac{\varepsilon}{2}) \leq \tilde{n}_{+1}\} = 1 - \Pr\{-\frac{\varepsilon}{4(1-\gamma_{-1})} > \frac{\tilde{n}_{+1}}{n} - 2(1-p)\}.$$

[38] Setting x_C just below $2\gamma_1 - 1$ is not dominated by $x_C = x_C^L$, because R is elected when x_C is set just below $2\gamma_1 - 1$ and $n_{-1} \geq n+1$. Hence, the optimal location depends on the probability of a realized majority of voters with signal -1. It is optimal to locate the compromise at x_C^L if

$$x_C^L \geq \Pr(\tilde{n}_{-1} \geq n+1) \cdot (2\gamma_1 - 1) + \Pr(\tilde{n}_{-1} \leq n) \cdot 1.$$

By Hoeffding's inequality, the last expression converges to 1 as n grows. The compromise $x_C^L - \varepsilon/2$ will therefore be elected with probability approaching 1, and it dominates compromise $x_C(n)$ for n large enough. This contradicts the assumed optimality of $x_C(n)$.

If there exists $\varepsilon > 0$ such that $x_C(n) > x_C^L + \varepsilon$ for infinitely many n, then the probability that L is elected converges to 1 along this subsequence, which is therefore dominated by choosing a compromise location just below x_C^L - such a compromise will be elected with probability approaching 1. We conclude that the optimal compromise converges to x_C^L.

(b) Assume now that $p < 1/2$, and let $x_C(n)$ denote the optimal compromise. The proof is divided in several steps:

Step 1: For all n, it holds that $x_C(n) \leq 1-2\gamma_{-1}$. If $x_C(n) > 1-2\gamma_{-1}$ then the compromise would never be elected if $n_{-1} \geq n+1$. It is then strictly better to choose $x_C(n) < 1-2\gamma_{-1}$ instead.

Step 2: For any $\varepsilon > 0$, $x_C(n) \leq -1 + 2\gamma_1 + \varepsilon$ for all n large enough. Assume, by contradiction, that there exists $\varepsilon > 0$ such that $x_C(n) > -1 + 2\gamma_1 + \varepsilon$ for infinitely many n, and consider the corresponding subsequence of compromise locations. We compare below any compromise $x_C \in (-1+2\gamma_1+\varepsilon\ ,\ 1-2\gamma_{-1})$ with the compromise location $-1+2\gamma_1$, and show that $-1+2\gamma_1$ is superior, which contradicts the optimality of $x_C(n)$.

The resulting outcomes differ in the following events: (a) x_C gets elected but a compromise located at $-1+2\gamma_1$ does not; (b) both compromises get elected; (c) compromise $-1+2\gamma_1$ gets elected but x_C does not.

By Lemma 1, event (a) can only occur if $n_{+1} \geq n+1$, in which case R gets elected if the compromise is located at $-1+2\gamma_1$. Therefore, compromise $-1+2\gamma_1$ is strictly better than x_C in event (a). Event (c) can only occur if $n_{-1} \geq n+1$, hence L gets elected in event (c) if the compromise is located at x_C. Therefore, compromise $-1+2\gamma_1$ is also strictly better than x_C in event (c). In event (b) both compromises get elected; hence, compromise x_C is strictly better in event (b) because $x_C > -1+2\gamma_1$. To show that, in expectation, compromise $-1+2\gamma_1$ is better than compromise x_C it therefore suffices to show that the probability of event (a) divided by the probability of event (b) grows without bound as n grows. The probability of event (a) is

$$\Pr\{n+1 \leq \tilde{n}_{+1} \leq 2n+1-k(x_C)\} \tag{14}$$

and that the probability of event (b) is

$$\Pr\{\kappa(x_C) \leq \tilde{n}_{+1} \leq n\}$$

Let $\beta = \frac{k(2\gamma_1 - 1 + \varepsilon)}{n}$ where $k(x) = \frac{n}{2}\frac{1-x}{1-\gamma_1}$ and note that β does not depend on n. Since the function k is decreasing, $k(-1+2\gamma_1) = n$ and $x_C > -1 + 2\gamma_1 + \varepsilon$ imply that $\frac{k(x_C)}{n} \leq \beta < 1$. Since $\tilde{n}_{-1} + \tilde{n}_{+1} = 2n+1$, the probability of event (i) satisfies

$$\Pr\{n+1 \leq \tilde{n}_{+1} \leq 2n+1-k(x_C)\} \geq \Pr\{\tilde{n}_{-1} \geq \beta n\} - \Pr\{\tilde{n}_{-1} \geq n+1\}.$$

Also, the probability of event (b) satisfies

$$\Pr\{\kappa(x_C) \leq \tilde{n}_{+1} \leq n\} \leq \Pr\{\tilde{n}_{-1} \geq n+1\}.$$

Hence,

$$\frac{\Pr\{\text{event (a)}\}}{\Pr\{\text{event (b)}\}} \geq \frac{\Pr\{\tilde{n}_{-1} \geq \beta n\}}{\Pr\{\tilde{n}_{-1} \geq n+1\}} - 1.$$

Since $p < 1/2$, Lemma 2 implies that the term on the left side grows without bound as n goes to infinity.

Step 3: $x_C(n)$ converges to $\min\{-1+2\gamma_1\ ,\ 1-2\gamma_{-1}\}$. Steps 1 and 2 imply then that $\limsup_n\{x_C(n)\} \leq \min\{-1+2\gamma_1\ ,\ 1-2\gamma_{-1}\}$. Suppose now that there exists $\varepsilon > 0$ such that $x_C(n) < \min\{-1+2\gamma_1\ ,\ 1-2\gamma_{-1}\} - \varepsilon$ for infinitely many n, and consider the corresponding subsequence. Let $x_1 := \min\{-1+2\gamma_1\ ,\ 1-2\gamma_{-1}\} - \frac{\varepsilon}{2}$. Neither $x_C(n)$ nor x_1 gets elected if $n_{-1} \leq n$, and we can focus on the event $n_{-1} \geq n+1$. Note that x_1 gets elected whenever $n_{-1} \geq n+1$ and n_{-1} is sufficiently close to n.

Since $p < 1/2$ it follows from Lemma 2 that for all $\delta \in (0,1)$, $\lim_{n\to\infty} \Pr\{\frac{\tilde{n}_{-1}}{n} > 1+\delta | \frac{\tilde{n}_{-1}}{n} > 1\} = 0$. This allows us to conclude that the probability with which x_1 gets elected, conditional on $n_{-1} \geq n+1$, approaches 1, and the same holds for $x_C(n)$. Since x_1 is strictly better conditional on being elected, the compromise location x_1 is strictly superior to $x_C(n)$, which yields a contradiction. Therefore, $\liminf_n\{x_C\} \geq \min\{-1+2\gamma_1\ ,\ 1-2\gamma_{-1}\}$. ■

References

[2012] Ali, Nageeb and Kartik, Navin (2012), "Herding with Collective Preferences," *Economic Theory* **51** (3), 601-626.

[2014] Apesteguia, J., Ballester, M. A., & Masatlioglu, Y. (2014). "A Foundation for Strategic Agenda Voting," *Games and Economic Behavior* **87**, 91-99.

[1987] Austen-Smith, David (1987), "Sophisticated Sincerity: Voting over Endogenous Agendas," *American Journal of Political Science* **81**(4): 1323-1330.

[1992] Austen-Smith, David (1992), "Explaining the Vote: Constituency Constrains on Sophisticated Voting," *American Journal of Political Science* **36**: 68-95.

[2017] Barbera, Salvador and Gerber, Anke (2017), "Sequential Voting and Agenda Manipulation," *Theoretical Economics* **12**: 211-247.

[2004] Chappell Jr, H. W., McGregor, R. R., and Vermilyea, T. (2004), *Committee decisions on monetary policy: Evidence from historical records of the Federal Open Market Committee.* MIT Press.

[2000] Dekel, Eddie and Piccione, Michele (2000), "Sequential Voting Procedures in Symmetric Binary Elections," *Journal of Political Economy* **108**(1), 34-55.

[1985] Denzau, Arthur, Riker, William and Shepsle, Kenneth (1985), "Farquharson and Fenno: Sophisticated Voting and Home Style," *American Political Science Review* **79**: 1117-1134.

[2004] Dutta, Bhaskar, Jackson, Matthew O., and Le Breton, Michel (2004), "Equilibrium Agenda Formation," *Social Choice and Welfare Welfare* **23**: 21-57.

[2006] Ellis, R.S. (2006), The Theory of Large Deviations and Applications to Statistical Mechanics. In: *Lectures for international seminar on extreme events in complex dynamics.*

[1983] Enelow, J. M. (1983), "A New theory of Congressional Compromise". *American Political Science Review*, **78**(3), 708-718.

[2011] Ezrow, L., De Vries, C., Steenbergen, M., and Edwards, E. (2011), "Mean Voter Representation and Partisan Constituency Representation: Do Parties Respond to the Mean Voter Position or to Their Supporters?" *Party Politics*, **17**(3), 275-301.

[1969] Farquharson, Robin (1969), *The Theory of Voting*, Yale University Press, New Haven.

[1978] Fenno, Richard F. (1978). *Home Style*, Little Brown, Boston.

[2017] Gershkov, Alex, Moldovanu, Benny and Shi, Xianwen (2017), "Optimal Voting Rules," *Review of Economic Studies* **84** (2): 688-717.

[2004] Grüner, H. P., and Kiel, A. (2004), "Collective decisions with interdependent valuations," *European Economic Review*, 48(5), 1147-1168.

[2017] Kleiner, Andreas and Moldovanu, Benny (2017), "Content-Based Agendas and Qualified Majorities in Sequential, Binary Voting," *American Economic Review* **107** (6): 1477-1506.

[2014] Martin, L. W., and Vanberg, G. (2014), "Parties and Policymaking in Multiparty Governments: The Legislative Median, Ministerial Autonomy, and the Coalition Compromise," *American Journal of Political Science*, **58**(4), 979-996.

[1978] McKelvey, Richard D. and Niemi, Richard G. (1978), "A Multi-Stage Game Representation of Sophisticated Voting for Binary Procedures," *Journal of Economic Theory* **18**: 1-22.

[1977] Miller, Nicholas R. (1977), "Graph Theoretic Approaches to the Theory of Voting," *American Journal of Political Science* **24**: 68-96.

[2013] Moldovanu, B., and Shi, X. (2013), "Specialization and Partisanship in Committee Search," *Theoretical Economics*, **8**(3), 751-774.

[1979] Moulin, Hervé (1979), "Dominance Solvable Voting Schemes," *Econometrica* **47**: 1337–1351.

[1988] Ordeshook, Peter C. and Palfrey, Thomas R. (1988), "Agendas, Strategic Voting, and Signaling with Incomplete Information," *American Journal of Political Science* **32**(2): 441-463.

[1987] Ordeshook, Peter C. and Schwartz, Thomas (1987), "Agendas and the Control of Political Outcomes," *American Political Science Review* **81**(1): 179-200.

[2000] Piketty, T. (2000), "Voting as communicating," *Review of Economic Studies*, **67**(1), 169-191.

[2016] Posner, Eric A., and Adrian Vermeulen (2016), "The Votes of Other Judges," *The Georgetown Law Journal* **105** 159.

[1919] Protocols of the Weimar National Assembly, Sessions 44-45, July 2-3 1919, in *Verhandlungen des Reichtages*, Band **327**: 1919-1920.

[2000] Rasch, Björn E. (2000), "Parliamentary Floor Voting Procedures and Agenda Setting in Europe," *Legislative Studies Quarterly* **25**(1), 3-23.

[2015] Rosar, F. (2015), "Continuous Decisions by a Committee: Median versus Average Mechanisms," *Journal of Economic Theory*, **159**, 15-65.

[1914] Tecklenburg, Adolf (1914), "Die parlamentarische Beschlussfassung," *Jahrbuch für öffentliches Recht* **8**, 75-99.

[1850] Trendelenburg, Adolf (1850), *Über die Methode bei Abstimmungen*, Berlin: Gustav Bethge.

[1993] Winkler, Heinrich A. (1993), *Weimar 1918-1933*, München: C. H. Beck.

[1966] Yohe, W. P. (1966), "A study of federal open market committee voting, 1955-64," *Southern Economic Journal*, 396-405.