Information Acquisition in Interdependent Value Auctions*

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Abstract

We consider an auction environment with interdependent values. Each bidder can learn her payoff type through costly information acquisition. We contrast the socially optimal decision to acquire information with the equilibrium solution in which each agent has to privately bear the cost of information acquisition.

In the context of the generalized Vickrey-Clarke-Groves mechanism, we establish that the equilibrium level exceeds the socially optimal level of information with positive interdependence. The individual decisions to acquire information are strategic substitutes. The difference between the equilibrium and the efficient level of information acquisition is increasing in the interdependence of the bidders' valuations and decreasing in the number of informed bidders.

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1 Introduction

1.1 Motivation

In the vast literature on auctions, surprisingly few papers have focussed explicitly on costly information acquisition. This is somewhat puzzling given the close connections between auctions and price formation processes in competitive markets. Milgrom (1981) uses a multi-unit Vickrey auction model to illustrate the possible coexistence of costly information and efficient information aggregation. The connections to the rational expectations equilibrium have been since worked on extensively but the issue of information acquisition has received substantially less attention. In our view, the questions relating to socially optimal information acquisition remain open for a large class of auction models.

Consider, for example, the situation in which a number of established companies bid for the takeover of a target company. At the outset, all bidders are symmetrically informed but before submitting a bid, each bidder may hire a consulting firm to assess the value of the target company. The value of the target company may depend on the extent to which the asset (or activity) of the target firm matches that of the acquiring firms. In addition, if the bidders are competing with each other in the (product) market, then the quality of the match between target firm and firm i also matters for firm j. In this paper, we analyze the bidders' private incentives to acquire information in such *interdependent* value models and establish a comparison between the equilibrium level and the socially optimal level of information acquisition.

If the auction designer has a utilitarian welfare objective and valuations are *private*, it is transparent to see that the agents have the correct incentives to acquire information in a socially optimal manner. In the Vickrey auction, individual payoffs, when viewed as functions of own type only, coincide with the sum of payoffs to all players (up to the addition of a constant). As a result, the private incentives coincide with those of the planner, as established by Stegeman (1996).

However, if the valuations are interdependent, the private incentives to acquire information differ systematically from those of the social planner. Maskin (1992) and Bergemann and Välimäki (2002), among others, have shown that given the decisions by the other bidders, an individual bidder has too strong incentives to acquire information if the valuations are positively dependent. Since these earlier contributions considered only individual decisions, they could not compare the decentralized equilibrium level of information acquisition with the socially optimal level. In this paper, we analyze equilibrium information acquisition in a model with binary information decisions where the bidders' true payoff types are initially unknown and each bidder may observe a costly private signal revealing her true payoff type. In a linear model with positive dependence, we show that:

(i) the private value of information exceeds the social value of information everywhere,

(ii) the private value of being informed is decreasing in the number of informed bidders.

Property (i) confirms the results in Bergemann and Välimäki (2002), and property (ii) means that bidders' information decisions are strategic substitutes. Property (ii) insures that the pure strategy equilibrium in the game of information acquisition is unique and that the local comparison can be extended to the equilibrium comparison. That is, with positive dependence, more bidders become informed in the equilibria of the information acquisition game than in the social planner's solution.

Our basic model is a single-unit auction where bidders acquire information about their true valuations simultaneously prior to the auction stage. We assume that information acquisition is covert, and as a result, the allocation mechanism cannot be conditional on information acquisition decisions. In the main section of the paper, we assume that the bidders' valuations are linear in their own signals as well as their opponents' signals. A generalized Vickrey-Clarke-Groves (VCG) mechanism is used to allocate the object. Given the independence of the bidders' types, the extension of the revenue equivalence theorem in Jehiel and Moldovanu (2001) implies that the same expected payoffs result in any auction with an efficient allocation rule. In our symmetric linear single-unit auction model, these payoffs coincide with the payoffs of the more familiar ascending price auction.[1]

In our model, the bidders' types are assumed to be independent across individual bidders. The main reason for this assumption is a theoretical one. With independence, the socially optimal allocation mechanism is quite simple and, furthermore, it is detail-free in the sense of Dasgupta and Maskin (2000), i.e. it does not depend on the distributions of individual valuations. In models with correlated types (including affiliated values), insights from the previous literature following Cremer and McLean (1985) and Cremer and McLean (1988) suggest that it is possible to construct mechanisms that align individual incentives to acquire

[1]Dasgupta and Maskin (2000) shows more generally how to construct efficient detail-free indirect mechanisms where the bidders submit contingent bids.

information with the societal ones (see Obara (2008)). Such mechanisms, however, depend on the fine details of the statistical dependence between types. If we restrict attention to a specific auction format such as the ascending price auction that has an equilibrium in ex post incentive compatible strategies, our results can be extended to cover models with correlated types as well. With correlated types, however, revenue equivalence does not hold and different efficient mechanisms yield different expected payoffs to the bidders and therefore also different incentives to acquire information at the ex ante stage.

The third main modeling assumption in our model is that information acquisition is a binary choice. The cost of becoming informed can be interpreted as the cost of inspecting the item for sale. Without inspection, bidding would be based on prior information only. While it is clear that in some applications the amount of information (or the rigor of the inspection) can be varied, it is less clear how this should be modeled. The general model used in Bergemann and Välimäki (2002) assumes that signals of different informativeness can be purchased at the ex ante stage at a cost that is increasing in the informativeness. At this level of generality, it appears difficult to compare the individual and social incentives in a definitive manner. The main source of the difficulty is to establish a link between an abstract and statistical notion of informativeness and the game theoretic property of strategic substitutes. On the other hand, if we were to consider a particular parametric model (such as the normal learning model), then we expect that the current results continue to hold qualitatively.

We consider pure strategy equilibrium as well as mixed strategy equilibria of this game of information acquisition. For both types of equilibria, information decisions of bidders are strategic substitutes and the equilibria feature socially *excessive* information acquisition. Moreover, the difference between equilibrium level of information and efficient level of information diminishes as the strength of positive dependence weakens or as the number of informed bidders increases.

Since information acquisition is assumed to be a binary decision, the pure strategy equilibrium may be asymmetric: in equilibrium some bidders acquire information while others do not. In this case, we show that more bidders become informed in equilibrium than in a planner's solution. For the mixed strategy equilibrium, we focus on the symmetric one. The relevant comparison in the information acquisition game is the problem where the planner chooses the same probability of becoming informed for all bidders. By this choice we can concentrate solely on the informational externalities in the problem rather than the coordination problems arising due to mixing. Again, our results show that the equilibrium probability of information acquisition exceeds the socially optimal probability. In the working paper version, Bergemann, Shi, and Valimaki (2007), we show that the basic insights of the single-unit auction carry over to a model where multiple objects are sold but bidders have unit demands.

Furthermore, we extend our positive results to a nonlinear model which nests the additive and multiplicative specifications as special cases. We confirm both property (i) and (ii). However, it is difficult to obtain similar results in a general nonlinear setting. The main reason is the following. In nonlinear models, the ranking of two agents' valuations, depends in general on the signal realization of a third agent. Thus, the third agent's decision to acquire information produces socially valuable information for the allocation decision between the first two agents. This is possible even in situations where the third agent does not receive the object and the private value of her information is zero. Consequently, this may lead to the reversal of ranking between private value of information and social value of information. That is, property (i) may be violated.

In the paper, we also present an example to show why property (ii) is important for our positive results. In the example, bidders' valuations are positively dependent and individual incentives to gather information are higher than social incentives. But information decisions are strategic complements, and thus violate property (ii). We show that the equilibrium level of information may be insufficient compared to the social optimal one. We finally discuss how our analysis of the auction environment with positive interdependence can be extended to the case with negative interdependence.

1.2 Related Literature

Grossman and Stiglitz (1980) propose a standard rational expectations model to address a fundamental issue in economics: How does the market adjust to new information. In their model, ex ante identical and uninformed market traders can acquire information at a cost and use it to make a profit by trading a risky asset. The fluctuation of prices makes private information (partially) revealed to uninformed traders. They conjecture that if information is costly, then equilibrium market prices cannot fully reveal private information. Furthermore they argue that costless information is not only sufficient, but also necessary for efficient market. Another important conclusion in the paper is that information acquisition decisions are strategic substitutes: the more individuals are informed, the less valuable is the information.[2] Since Grossman and Stiglitz (1980) analyze a model with pure common values, the question of socially efficient information acquisition does not arise.

While a large number of papers analyze information aggregation in large markets *given* endowed or acquired information (among them Milgrom (1981), Pesendorfer and Swinkels (1997) and Jackson (2003)), other equally important questions are largely unaddressed. Does equilibrium information acquisition coincide with the socially optimal level? What types of models generate excessive information in equilibrium and what types of models lead to insufficient investment in information? These are the focus of the current paper. In the papers on information aggregation, either the object has pure common value or the common component of the object is the same for everyone, and hence the socially optimal level of information is always zero for allocation purposes. Thus, these specifications are not suitable for our analysis. The specifications contained in the current paper introduce a more general positive dependence into the model, and thus allow us to compare equilibrium level of information to the social efficient level of information.

A recent strand of literature has studied the incentives for information acquisition in specific auction formats. Stegeman (1996) shows that first and second price independent private value auctions result in the same and importantly efficient incentives for information acquisition. Matthews (1984) analyzes information acquisition in a first-price auction with pure common values and investigates how the seller's revenue varies with respect to the amount of acquired information and whether the equilibrium price fully reveals bidders' information. Finally, Persico (2000) shows that the incentive to acquire information is stronger in the first-price auction than in the second-price auction if bidders' valuations are affiliated. Since the latter two papers are not directly concerned with efficient allocation mechanisms, they say little about the overall level of information acquisition either from the social welfare or the equilibrium point of view.

Some of the issues that we are interested in also arise in the context of costly entry into auctions. In these models, it is typically assumed that a bidder learns her true valuation for the object for sale upon paying an entry fee to the auction. In French and McCormick

[2]In a recent paper, Chamley (2007) proposes a model of financial markets in which the decision to acquire information are strategic complements rather than substitutes.

(1984) and Levin and Smith (1994), an auction with pure private or pure common values is analyzed. With an exogenously fixed entry cost, entry is shown to be efficient for private values and excessive for pure common values. In Levin and Smith (1994) the focus is on the determination of the optimal entry fee so as to maximize revenue. In our model, valuations are interdependent and the information acquisition is in general not at the socially optimal level. This raises the possibility that a social planner could increase the welfare by setting a positive entry fee in our setting as we discuss in the conclusion.

This paper is organized as follows. Section 2 sets up the model. Section 3 presents the pure strategy analysis of the single unit auction with a linear payoff structure. Section 4 derives the mixed strategy equilibrium in this environment. Section 5 extends the analysis to nonlinear payoff environments. Section 6 discusses the role of the strategic substitute property and illustrates how the analysis can be adapted to the case of negative interdependence. Finally, Section 7 concludes. The proofs of all results are relegated to the appendix.

2 Model

We consider an auction setting with a single object for sale and I bidders. The true value of the object to bidder i is given by

$$u_i(\theta_i, \theta_{-i}) = \theta_i + \alpha \sum_{j \neq i} \theta_j. \tag{1}$$

The parameter α is a measure of interdependence.[3] With $\alpha > 0$ the interdependence is positive, and with $\alpha < 0$ the model displays negative interdependence. If $\alpha = 0$, then the model is one of private values and if $\alpha = 1$, then the model is of pure common values. We shall initially concentrate on the case of positive interdependence and discuss in Section 6 how the analysis can be extended to the case of negative interdependence. Each agent i has a quasilinear utility:

$$u_i(\theta) - t_i$$

where $t_i \in R$ is a monetary transfer.

[3] In an earlier version of the paper we considered a normalized specification given by: $u_i(\theta_i, \theta_{-i}) = (1-\alpha)\theta_i + \alpha (1/(I-1)) \sum_{j \neq i} \theta_j$ and the results are qualitiatively similar. With this specification, the range of possible valuations for bidder i is independent of the number of bidders, but now the single-crossing condition depends on the number of bidders.

Initially, each bidder i only knows that the payoff relevant types $\{\theta_j\}_{j=1}^{I}$ are independently drawn from a common distribution F with support $\left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}_+$. The distribution F has an associated density f and a mean valuation:

$$\mu \triangleq \mathbb{E}\left[\theta_i\right].$$

Each bidder i can acquire information about her payoff relevant type θ_i at a positive cost $c > 0$. The decision to acquire information is a binary decision. If bidder i acquires information, then she observes the realization of θ_i; otherwise her information is given by the prior distribution F.

We consider the possibility of informational efficiency in an efficient allocation mechanism, namely the generalized Vickrey-Clarke-Groves mechanisms (see Maskin (1992) and Dasgupta and Maskin (2000)). A necessary and sufficient condition for the implementability of the efficient allocation is that $u_i\left(\theta_i, \theta_{-i}\right)$ satisfies the single crossing property. In the current linear setting, the single crossing condition is equivalent to $\alpha \leq 1$.

We denote by y_i the highest signal among bidders other than i, that is,

$$y_i = \max_{j \neq i} \theta_j.$$

The generalized Vickrey-Clarke-Groves (VCG) mechanism is defined by an allocation rule

$$q_i\left(\theta_i, \theta_{-i}\right) = \begin{cases} 1 & \text{if} \quad \theta_i > y_i, \\ 0 & \text{if} \quad \theta_i < y_i, \end{cases} \tag{2}$$

which specifies the probability of winning conditional on the reported type profile θ. In case of a tie among the highest types, the winner is chosen among the tied bidders with equal probability. The associated payment rule

$$t_i\left(\theta_i, \theta_{-i}\right) = \begin{cases} u_i\left(y_i, \theta_{-i}\right) & \text{if} \quad \theta_i \geq y_i, \\ 0 & \text{if} \quad \theta_i < y_i, \end{cases} \tag{3}$$

specifies the payment of bidder i. The winning bidder makes a payment which is equal to her value of the object conditional on tying the payoff type of the second highest bidder. Consequently, the equilibrium payoff for a bidder i at type profile θ_i is

$$q_i\left(\theta\right) u_i\left(\theta\right) - t_i\left(\theta\right) = \begin{cases} \theta_i - y_i & \text{if} \quad \theta_i \geq y_i, \\ 0 & \text{if} \quad \theta_i < y_i. \end{cases}$$

In the generalized VCG mechanism, the payment of agent i is independent of the report of agent i conditional on the allocation and the equilibrium is an *ex post* Bayesian Nash equilibrium (see Dasgupta and Maskin (2000)).

With the linear specification of payoff types given by (1), the expected value of an uninformed bidder is the same as that of an informed bidder with a true payoff type $\theta_i = \mu$. The direct revelation mechanism therefore does not have to account for informed and uninformed bidders separately. In section 5 we shall look at environments in which the valuation function is not linear in the types θ_i and then an uninformed agent will have to report her entire distributional information rather than the mean μ only.

3 Pure Strategy Equilibrium

We first establish the socially optimal information policy. Subsequently, we analyze the equilibrium information policies of the agents in the generalized VCG mechanism. Initially we focus our attention on the pure strategy equilibrium and consider the mixed strategy equilibrium in the next section.

3.1 Socially Efficient Policy

The bidders are ex ante identical. The socially optimal policy has to weigh the benefits of increasing information against the cost of additional information. The social value of additional information arises from the possibility of identifying an agent with a higher valuation. As the number of informed agents increases, it becomes increasingly unlikely that an additionally informed agent will have a valuation exceeding the highest valuation among the currently informed agents. The optimal number of informed agents will therefore depend on the prior distribution and the cost of information acquisition. We denote the set of informed agents by $\{1, ..., m\}$ and the remaining set of uninformed agents by $\{m+1, ..., I\}$. The agent m is the *marginally* informed agent. We denote by θ_h the *highest* payoff type among the $(m-1)$ informed bidders, and denote the informed bidder with payoff type θ_h as bidder h.

It is straightforward to characterize the expected social gain of the m-th informed agent. The information of the m-th agent improves the social efficiency if and only if the information is pivotal for the allocation decision. By the single crossing condition, the information of agent m is pivotal if and only if it leads the planner to allocate the object to agent m. If the payoff type of agent m exceeds the types of all other agents, then it has to be larger than the payoff types of all informed agents $\{1, ..., m-1\}$ as well as all uninformed agents $\{m+1, ..., I\}$. Without loss of generality, we may assume that if the object is optimally assigned to an uninformed agent, then it is assigned to agent I.

The expected social gain of informing agent m, denoted by Δ_m^*, is then given by:

$$\Delta_m^* = \mathbb{E}_\theta \left[\left(u_m(\theta) - u_h(\theta) \right) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) + \left(u_m(\theta) - u_I(\theta) \right) \cdot \mathbf{1}(\theta_m \geq \mu > \theta_h) \right], \tag{4}$$

where the indicator function $\mathbf{1}(\cdot)$ is defined by:

$$\mathbf{1}(A) = \begin{cases} 1 & \text{if} \quad \text{event } A \text{ is true}, \\ 0 & \text{if} \quad \text{event } A \text{ is false}. \end{cases}$$

If bidder m becomes informed, then she may either win against bidder h or bidder I, respectively. The size of the expected gain from the improved allocation is the corresponding difference in the valuations between bidder m and the current winner. We can use the linear structure of the valuation $u_i(\theta)$ to rewrite (4) as:

$$\mathbb{E}_\theta \left[\left((\theta_m - \theta_h) + \alpha(\theta_h - \theta_m) \right) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) + \left((\theta_m - \mu) + \alpha(\mu - \theta_m) \right) \cdot \mathbf{1}(\theta_m \geq \mu > \theta_h) \right].$$

With the linear payoff structure, the expected gain from information depends only on θ_m and the highest payoff type among the remaining agents, θ_h or μ. We denote by y_m the highest payoff type among all agents exclusive of m:

$$y_m = \max_{j \neq m} \theta_j = \max\{\theta_h, \mu\}. \tag{5}$$

We can write Δ_m^* in a more compact form as:

$$\Delta_m^* = (1-\alpha)\, \mathbb{E}_{\theta_m, y_m} \left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m) \right]. \tag{6}$$

The case of $m = I$ is slightly different. If the last bidder I is informed, then indeed all bidders are informed. The information of bidder I now becomes pivotal in two different circumstances: (i) $\theta_h > \mu$ and (ii) $\theta_h < \mu$. In the first case, bidder I did not get the object without additional information, but may now receive the object. In the second case, bidder I did get the object without information, but may now fail to get the object if her true payoff type turns out to be below θ_h.[4] The expected social gain from information is then given by:

$$\Delta_I^* = \mathbb{E}_\theta \left[\left(u_I(\theta) - u_h(\theta) \right) \cdot \mathbf{1} \left(\theta_I \geq \theta_h \geq \mu \right) + \left(u_h(\theta) - u_I(\theta) \right) \cdot \mathbf{1} \left(\mu > \theta_h > \theta_I \right) \right]. \quad (7)$$

By using the linear structure of the model, we can rewrite (7) to obtain:

$$\Delta_I^* = (1-\alpha)\, \mathbb{E}_{\theta_I, \theta_h} \left[\left(\theta_I - \theta_h \right) \cdot \mathbf{1} \left(\theta_I \geq \theta_h \geq \mu \right) + \left(\theta_h - \theta_I \right) \cdot \mathbf{1} \left(\mu > \theta_h > \theta_I \right) \right]. \quad (8)$$

We denote the socially optimal decision to acquire information for the m-th agent by $s_m^* \in \{0, 1\}$. Then we can state the social efficient policy as follows.

Proposition 1 (Social Efficient Policy)

1. *The socially efficient policy s_m^* is given by:*

$$s_m^* = \begin{cases} 0 & \text{if} \quad \Delta_m^* < c, \\ 1 & \text{if} \quad \Delta_m^* \geq c. \end{cases}$$

2. *Δ_m^* is strictly decreasing in m and α for all m.*

The social gain from an additional informed bidder is positive when the information is pivotal with positive probability. When more bidders are informed, it is less likely that the newly informed bidder has a payoff type higher than those of her opponents. Therefore, the gross social gain from information acquisition is decreasing in the number of informed bidders.

With positive dependence, the efficiency loss from misallocation is lower than in an environment with private values. A larger α, which represents a higher positive interdependence, leads to a smaller loss from the misallocation due to the imperfect information. In consequence the social gain from information acquisition is smaller when bidders' valuations are more positively dependent.

[4] The formulae of the marginal gain of an informed bidder m are different for $m < I$ and $m = I$. The diference arises as for interior m a low realization of the marginal bidder always results in the reassignment of the object to another uninformed bidder. In contrast, with a low realization of the last bidder, the object will be reassigned to an informed bidder with signal realization above that of the last bidder but below the mean.

3.2 Equilibrium Policy

We now consider the private incentives of the agent to acquire information in the generalized VCG mechanism. We maintain the notation and identify bidder m as the *marginal* bidder to acquire information. We denote the expected private gain of agent m to become informed by $\widehat{\Delta}_m$. As in the socially optimal program, we assume that if an uninformed agent is assigned the object, then it is assigned to the last agent I. If there are multiple uninformed agents, then the resulting monetary transfer will leave agent I indifferent between receiving and not receiving the object. With this convention, it is again useful to treat the case of $m < I$ and $m = I$ separately. The marginal bidder m gains from gathering information if and only if she wins the object with the information. In the generalized VCG mechanism, the m-th bidder's net gain from information is:

$$\widehat{\Delta}_m = \mathbb{E}_\theta \left[\left(u_m \left(\theta_m, \theta_{-m} \right) - u_m \left(y_m, \theta_{-m} \right) \right) \cdot \mathbf{1}(\theta_m \geq y_m) \right],$$

where we defined y_m earlier in (5) as the highest payoff type among all bidders other than m. We can use the linear payoff structure (1) to rewrite $\widehat{\Delta}_m$:

$$\widehat{\Delta}_m = \mathbb{E}_{\theta_m, y_m} \left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m) \right]. \tag{9}$$

We compare the private gain with the social gain of information as described by (9) and (6), respectively. If bidder m's information is not pivotal, then both the private value and the social value of information about the payoff type θ_m are equal to zero. On the other hand, when bidder m's signal is pivotal, then the private gain from information about θ_m is given by $(\theta_m - y_m)$, but the social gain from information is only $(1 - \alpha)(\theta_m - y_m)$. The difference between the private and the social gains stems from the requirement of incentive compatibility. If agent m is to report truthfully, then she can only be asked to pay a monetary transfer equal to the lowest possible type at which the planner would be indifferent between assigning and not assigning the object to bidder m. In particular for all payoff types above her pivotal type, the monetary transfer has to stay constant, yet her private benefit increases at the rate of 1. In contrast, in the social program, the marginal benefit from an increase in the payoff type of agent m is given by $(1 - \alpha)(\theta_m - y_m)$ as a higher payoff type of agent m would already be beneficial at the rate α even if agent m is not assigned the object. Thus the social benefit of an increase in the payoff type of agent m is moderated by the interdependence of the valuations.

As we discussed in the context of the socially efficient policy, the description of the bidder's gain from information is somewhat different in the case of $m = I$. We have:

$$\begin{aligned}\widehat{\Delta}_I &= \mathbb{E}_\theta\left[\left(u_I\left(\theta_I, \theta_{-I}\right) - u_I\left(\theta_h, \theta_{-I}\right)\right) \cdot \mathbf{1}\left(\theta_I \geq \theta_h \geq \mu\right)\right] \\ &\quad + \mathbb{E}_\theta\left[\left(u_I\left(\theta_h, \theta_{-I}\right) - u_I\left(\theta_I, \theta_{-I}\right)\right) \cdot \mathbf{1}\left(\mu > \theta_h > \theta_I\right)\right],\end{aligned}$$

and using the linear payoff structure we get:

$$\widehat{\Delta}_I = \mathbb{E}_{\theta_I, \theta_h}\left[\left(\theta_I - \theta_h\right) \cdot \mathbf{1}\left(\theta_I \geq \theta_h \geq \mu\right) + \left(\theta_h - \theta_I\right) \cdot \mathbf{1}\left(\mu > \theta_h > \theta_I\right)\right]. \tag{10}$$

The first term represents bidder I's gain by winning the object from the informed bidder h when her payoff type is higher than the payoff type of the remaining agents. The second term represents bidder I's gain by avoiding to pay more for the object than it is worth to her given that her true payoff type is lower than θ_h.[5] By analogy, we refer to the equilibrium decision of agent m to acquire information by $\widehat{s}_m \in \{0, 1\}$.

Proposition 2 (Equilibrium Policy)

1. *The equilibrium information policy is given by*

$$\widehat{s}_m = \begin{cases} 0 & \text{if } \widehat{\Delta}_m < c, \\ 1 & \text{if } \widehat{\Delta}_m \geq c. \end{cases}$$

2. *$\widehat{\Delta}_m$ is strictly decreasing in m and constant in α for all m.*

The private decisions of the agents to acquire information are again strategic substitutes: bidder i is less willing to become informed as more of her opponents are informed. When one more opponent gets informed, a bidder's expected gain from information acquisition is reduced in two ways. First, her chance of winning is lower. Second, conditional on winning, her expected gain from winning is lower. Thus, if there are more informed opponents, a bidder's incentives to acquire information are lower. This property ensures that the game of information acquisition has an essentially unique pure strategy equilibrium for any given level of information cost.

[5] As before, we assume that the object is always assigned to bidder I if an uninformed bidder has the highest value. This specific tie-breaking rule is without loss of generality in the social problem *and* in the equilibrium problem. If there are more than one uninformed bidders, then the transfer price in the generalized VCG mechanism is exactly equal to the opportunity cost of giving the object to an uninformed bidder, i.e. the expected value of the object to an uninformed bidder. Thus the net utility of an uninformed bidder is zero for any efficient tie-breaking rule.

3.3 Welfare Comparison

We can now contrast the information decisions in the social and the equilibrium program by comparing the marginal gains of information given by

$$\{\Delta_m^*\}_{m=1}^I \text{ and } \{\widehat{\Delta}_m\}_{m=1}^I .$$

We use the strategic substitute property to guarantee the uniqueness of the equilibrium.

Proposition 3 (Welfare Analysis)
For all m,

1. *The marginal gains of information satisfy:*

$$\widehat{\Delta}_m > \Delta_m^*;$$

2. *The equilibrium information acquisition is (weakly) socially excessive;*
3. *The difference $\widehat{\Delta}_m - \Delta_m^*$ is strictly increasing in α;*
4. *The difference $\widehat{\Delta}_m - \Delta_m^*$ is strictly decreasing in m.*

With positive dependence, the equilibrium gain for a bidder to acquire information is higher than the social gain. This result, together with the strategic substitute property, implies that given any level of information cost, more bidders get informed in equilibrium than desired in the social optimum. The discrepancy between equilibrium and social policy decreases as more bidders are informed and as the positive interdependence weakens.

The statement about the comparison between the private value and the social value of information, $\widehat{\Delta}_m > \Delta_m^*$, is weaker than what is actually established in the proof of the proposition. The first part of the above proposition says that *on average* the social value of information is lower than the private value of information. In fact, the social value of information is lower than the private value of information at *every possible profile of payoff types.* To see this, notice that when bidder m's signal is not pivotal, both social value and private value of information about θ_m is zero. On the other hand, when bidder m's signal is pivotal, then the private gain from information about θ_m is $(\theta_m - y_m)$, but the social gain from information is $(1-\alpha)(\theta_m - y_m)$, which is smaller as long as $\alpha > 0$. The results in Proposition 3 are consistent with Theorem 3 in Bergemann and Välimäki (2002). The case with positive α corresponds to their case (ii), while their case (i) corresponds to our case with negative α.

The current results for the single unit auction generalize naturally to the case of multi-unit auctions with the associated generalized VCG mechanism. The precise statements and proofs for the multi-unit case are stated in the appendix of working paper version, see Bergemann, Shi, and Valimaki (2007), of this paper.

Our analysis also applies to the following two-component specification that is extensively studied in the literature (see Pesendorfer and Swinkels (2000) and Jackson (2003) among others):

$$u_i(\theta) = \theta_0 + \theta_i,$$

where θ_0 and θ_i are independently distributed random variables, representing a public and a private component to agent i, respectively. In this bivariate model, we may think about two alternative models of information acquisition with identical qualitative conclusions. If each bidder has to make separate decisions regarding information about the private and the common component, then it is as if the bidder faces a pure private value model with respect to the first decision and faces a pure common value model with respect to the second decision. Our earlier results for the generalized VCG mechanism now apply componentwise, i.e. we will observe efficient information acquisition with respect to the private type and excessive information acquisition with respect to the common component. Alternatively, if the bidder can only observe a signal about the joint realization of the private and common component, say about the sum, $\theta_0 + \theta_i$, then the incentives are jointly determined by the private and common components. As the private element leads to an efficient decision and the common element to excessive information acquisition, the sum of these effects leads again to excessive information acquisition, just as predicted by our interdependent value model.

4 Mixed Strategy Equilibrium

So far we restricted our attention to the analysis of pure strategy equilibria and we showed that the pure strategy equilibrium is unique up to permutations of the ex ante identical bidders. Similarly, a socially efficient decision is always a deterministic policy. The pure strategy equilibrium is frequently an asymmetric equilibrium in that some bidders acquire and some bidders do not acquire information even though the bidders are ex ante identical.

It may be difficult to see how this type of coordination might be achieved in a one-shot game. As a result, it is of interest to also analyze the symmetric mixed strategy equilibria of the game.

The mixed strategy equilibrium leads almost by definition to a socially inefficient information policy since the agents frequently fail to coordinate their decision. In order to facilitate a comparison with a socially optimal solution, we consider the problem of a planner who is restricted to choose an anonymous and hence symmetric information policy across all agents. Under this restricted efficiency criterion we conclude, as in our earlier analysis, that the equilibrium level of information acquisition will be higher than the socially efficient level.

In the current model, the cost of information acquisition is known and identical to all the bidders. It is natural to extend the model to allow for differential costs of information acquisition. If the cost of information acquisition were private information, then we can think of the mixed strategy equilibrium here as the limit equilibrium of a model with private information about the cost of information acquisition. By a standard argument first suggested by Harsanyi (1973), the mixed strategy equilibrium here can then be purified by a model with private information about the cost of information.

We restrict our analysis here to the case of moderate information cost so that the bidders acquire information with a probability strictly between 0 and 1. The case of very low and very high information costs would of course lead to degenerate mixed strategies and the only complication would come from the qualification to weak inequalities rather than equalities in the equilibrium conditions. We denote by $\sigma^* \in (0,1)$ the socially optimal probability of acquiring information, and by $\widehat{\sigma} \in (0,1)$ the equilibrium probability of acquiring information. With slight abuse of notation, we define the expected gain from information for a representative agent by $\Delta^*(\sigma)$:

$$\Delta^*(\sigma) \triangleq \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} \Delta_m^*.$$

We recall that Δ_m^* is the expected social gain from an additional informed bidder when $(m-1)$ bidders have already acquired information. The expected gain $\Delta^*(\sigma)$ represents the expected social gain from information when the planner is required to choose a single probability σ of acquiring information for all bidders. Similarly, we define $\widehat{\Delta}(\sigma)$ as

$$\widehat{\Delta}(\sigma) \triangleq \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} \widehat{\Delta}_m,$$

where $\widehat{\Delta}_m$ is the individual gain for bidder m to acquire information when $(m-1)$ bidders are already informed.

In the symmetric mixed strategy equilibrium, individual bidders must be indifferent between acquiring information and staying uninformed. The expected relative gain from acquiring information must be equal to the cost of information. Thus, the equilibrium policy $\widehat{\sigma}$ must satisfy the following condition:

$$\widehat{\Delta}(\widehat{\sigma}) - c = 0. \tag{11}$$

Similarly, for the socially efficient policy σ^*, the expected social gain from information must be equal to the information cost:

$$\Delta^*(\sigma^*) - c = 0. \tag{12}$$

Proposition 4 (Strategic Substitutes)
The gains from information, $\widehat{\Delta}(\sigma)$ and $\Delta^*(\sigma)$, are strictly decreasing in σ.

The private and the social gain of a bidder from information acquisition are thus decreasing as the probability of other buyers' being informed increases. That is, the decisions to acquire information remain strategic substitutes when we allow for probabilistic policies. It also ensures that there are unique probabilities σ^* and $\widehat{\sigma}$ that satisfy equilibrium conditions (11) and (12), respectively.

Proposition 5 (Excessive Information Acquisition)
For all $\sigma^* \in (0,1)$, $\sigma^* < \widehat{\sigma}$.

Thus the bidders have a higher probability of acquiring information in equilibrium than in the social optimum. Proposition 4 and Proposition 5 extend the previous results in the pure strategy analysis to the symmetric mixed strategy equilibrium.

5 Nonlinear Interdependence

We now investigate to what extent the results obtained in the linear payoff model generalizes to nonlinear environments.[6] We introduce a new condition, referred to as no-crossing property, which guarantees that the ranking of any two bidders is unaffected by the payoff type of a third bidder. We now consider general nonlinear valuation functions given by:

$$u_i : \left[\underline{\theta}, \overline{\theta}\right]^I \to \mathbb{R}.$$

We maintain a symmetric framework across agents. Specifically we require that for all i and j and all payoff type profiles θ and θ', if θ' is a permutation of θ and $\theta_i = \theta'_j$, then

$$u_i\left(\theta\right) = u_j\left(\theta'\right).$$

We also maintain the single crossing condition in order to guarantee the truthful implementation of the efficient allocation by means of the generalized VCG mechanism:

$$\theta_i \geq \theta_j \Leftrightarrow u_i\left(\theta\right) \geq u_j\left(\theta\right). \tag{13}$$

Finally, the positive interdependence in the nonlinear setting simply requires that

$$\frac{\partial u_i\left(\theta\right)}{\partial \theta_j} > 0, \ \ \forall i, j, \ \forall \theta.$$

Clearly, the earlier linear payoff model belongs to the environment considered here. The symmetry assumption is restrictive but natural. The single-crossing property is necessary to implement the efficient allocation. The positive dependence is necessary to guarantee that the individual returns from information exceed the social returns from information. Within this setting, we introduce the no crossing property.[7] Without loss of generality, we label the set of informed bidders by $\{1, ..., m\}$.

Assumption 1 (No-Crossing Property)
The valuations $\{u_i\left(\theta\right)\}_{i=1}^{I}$ satisfy the no-crossing property if for all m and all $i, j \neq m$:

$$\exists \theta_m \text{ s.t. } \mathbb{E}\left[u_i\left(\theta\right)|\theta_1, ..., \theta_m\right] > \mathbb{E}\left[u_j\left(\theta\right)|\theta_1, ..., \theta_m\right] \Rightarrow \forall \theta_m, \ \mathbb{E}\left[u_i\left(\theta\right)|\theta_1, ..., \theta_m\right] > \mathbb{E}\left[u_j\left(\theta\right)|\theta_1, ..., \theta_m\right].$$

[6]We would like to thank our discussant, Tim van Zandt, who asked us to further develop the nonlinear environment and who suggested the no crossing condition.

[7]A slightly stronger version of this condition is imposed in Bergemann and Välimäki (2002) to establish their Theorem 3.

It is easy to verify that the earlier linear payoff model satisfies the no-crossing property. The no-crossing property requires that the ranking of the expected payoff of two alternatives, i versus j, is constant across all payoff types θ_m of agent m. If all agents are informed about their payoff types and $m = I$, then the no-crossing property is automatically satisfied by the single crossing condition (13), which simply states that the binary ranking of the alternatives i and j is determined exclusively by their respective payoff types θ_i and θ_j. The no-crossing property condition extends the uniformity of the binary ranking to the comparison of two bidders i and j, where i is informed about her payoff type and j is not informed about her payoff type. If the no-crossing property is violated, then the information of agent m may be socially valuable in determining the allocation between i and j without agent m ever getting the object. But if agent m does not receive the object, then she will have very weak private incentives to acquire information even though it would be socially valuable. In consequence, the ranking between the social incentive and the private incentive to acquire information may be reversed.

An example of a valuation function with the no-crossing property is given by:

$$u_i(\theta) = \phi(\theta_i + \alpha \sum_{j \neq i} \theta_j),$$

where ϕ is an increasing function. It is further apparent that the no-crossing condition is valid for any multiplicative or additive separable specifications of the valuation functions. The no-crossing condition has an important implication.

Lemma 1
If the no-crossing property is satisfied, then for all m and all $i, j \neq m$, if

$$\mathbb{E}[u_i(\theta) | \theta_1, ..., \theta_{m-1}] > \mathbb{E}[u_j(\theta) | \theta_1, ..., \theta_{m-1}],$$

then

$$\mathbb{E}[u_i(\theta) | \theta_1, ..., \theta_m] > \mathbb{E}[u_j(\theta) | \theta_1, ..., \theta_m].$$

In words, the ranking between bidder i and j after bidder m gets informed is the same as the ranking between them before bidder m becomes informed. The definitions of the marginal gains from information, given by $\widehat{\Delta}_m$ and Δ_m^*, extend in the natural way to the nonlinear environment. The next proposition shows that the private gain from acquiring information will be higher than the social gain from information.

Proposition 6 (Excessive Incentives)
If the no-crossing property is satisfied, then $\widehat{\Delta}_m > \Delta_m^*$ *for all* m.

Next we need to demonstrate that the private decisions of the bidders to acquire information will remain strategic substitutes in the nonlinear environment, provided that the no-crossing property is satisfied.

Proposition 7 (Strategic Substitutes)
If the no-crossing property is satisfied, then $\widehat{\Delta}_m < \widehat{\Delta}_{m-1}$ *for all* $m < I$.

The current result is stated and proved only for $m < I$. While we suspect that it extends to the final bidder, we have not been able to prove this result in the nonlinear setting. We should point out, that as long as the number of potential bidders is large (for example, if entry is close to free), and the cost of acquiring information is not negligible, then the issue of $m = I$ will be irrelevant in equilibrium as in equilibrium there will always be informed and uninformed bidders as $m \ll I$. Proposition 6 and 7 can still enable us to get the uniqueness of the equilibrium, and thus the equilibrium information acquisition will be excessive in comparison to the social optimum.

The no-crossing property imposes a substantial restriction on the form of the payoff functions. If the payoff functions are neither additively nor multiplicatively separable, then the no-crossing property might be violated. This is easily shown with the following example of three bidders, $i \in \{1, 2, 3\}$. The valuation of bidder i is given by:

$$u_i(\theta) = (1 + \theta_i)^2 + (\theta_j + \theta_k)\,\theta_j \theta_k. \tag{14}$$

The payoff types θ_i are assumed to be independently drawn from the uniform distribution on the unit interval. Clearly, the model is symmetric, satisfies the single-crossing property and displays positive interdependence. Within the example given by (14) it is now easy to show that the ranking of an informed agent, say 1 and an uniformed agent, say 3, is affected by the payoff type of agent 2. We omit the calculations.

6 Discussion

The analysis of the auction environment began with positive interdependence of the payoff types. We established that the private and the social decisions to acquire information are strategic substitutes and that the private returns from information exceed the social returns from information. Consequently, it might be plausible to deduce that the driving force behind the results is the positive interdependence of the payoff types. In this section, we show that positive interdependence is not sufficient to make information acquisition decisions strategic substitutes. In order to obtain the result on excessive information acquisition, a separate argument for the strategic substitute property is needed.

So far, our analysis considered only the case of positive interdependence. We now show how the analysis naturally extends to a setting with negative interdependence. With negative interdependence, the bidders' decisions to acquire information remain strategic substitutes, but the comparison between private and social gains from information is reversed and consequently the equilibrium information acquisition is socially *insufficient*.

6.1 Strategic Complements

With positive interdependence, Bergemann and Välimäki (2002) show that the individual bidders will have socially excessive incentives to acquire information given the information decision of the remaining agents. The results in Bergemann and Välimäki (2002) are thus about a local property of the decision of agent i in the sense that the decision of the remaining agents are kept constant. In particular, the characterization of the individual decision may not transfer to the equilibrium decisions of the agents. We now provide an example of positive interdependence with the property that the decisions to acquire information are strategic complements rather than strategic substitutes. Despite the positive interdependence, an equilibrium of the game will display a lower level of information acquisition than the social equilibrium.

Suppose there are two bidders, $i \in \{1, 2\}$, competing for a single object. The payoff structure is the linear payoff structure of the previous sections:

$$u_i\left(\theta_i, \theta_j\right) = \theta_i + \alpha\theta_j,$$

but we now allow for negative payoff types. The social planner can either allocate the object to bidder 1 or 2 or decide not to allocate the object at all. In particular it is efficient not to assign the object if the expected valuation of both bidders is below zero.

For concreteness, we consider $\alpha = 0.5$ and assume that the payoff types θ_i are independently drawn from the uniform distribution with the support given by $[-5, 1]$. It is now easy to verify that the decisions to acquire information are strategic complements. If bidder j stays uninformed, then it is efficient not to assign the object to bidder i for any realization of her payoff type. In consequence, the value of information for agent i is zero if agent j does not acquire information. On the other hand, if agent j does acquire information, then a positive realization by both agents may lead to the assignment of the object to agent i and hence there is positive value of information.[8]

Therefore, consistent with the analysis of Bergemann and Välimäki (2002), the private gain is weakly higher than the social gain from information. But the analysis of the individual decision of agent i does not necessarily translate into a corresponding equilibrium characterization. For example, we can show by elementary computations that if the cost of information is $c = 1/100$, then the efficient policy requires that both bidders acquire information. However, due to the strategic complementarity, there are now two pure strategy equilibria for the game of information acquisition: one in which both bidders remain uninformed, the other one in which both bidders become informed. It is therefore possible that the two bidders fail to coordinate on the efficient equilibrium and stay uninformed. The key for the failure of the equilibrium characterization is that the private gain from information is increasing in the number of informed bidders, that is, information decisions are strategic complements.

6.2 Negative Interdependence

Until now, our analysis assumed that the bidders' valuations are positively dependent. We now adapt the argument to the case with negative interdependence. For example, consider

[8]We note that the negative payoff types are not necessary to generate the strategic complementarity in the information decision among the agents. A similar result could be generated in an asymmetric three bidder model with positive types. If we were to introduce a third bidder with a strictly positive valuation, then exactly the same argument would go through.

the linear model in Section 2:

$$u_i(\theta_i, \theta_{-i}) = \theta_i + \alpha \sum_{j \neq i} \theta_j,$$

but with $\alpha < 0$. Analogous to our previous analysis, one can show that

$$\Delta_m^* = (1 - \alpha) \widehat{\Delta}_m, \text{ for } m = 1, ..., I.$$

Therefore, given $\alpha < 0$, the social gain from information acquisition is now higher than the private gain from information. Furthermore, we can verify that both Δ_m^* and $\widehat{\Delta}_m$ remain strictly decreasing in m. In other words, the individual decisions to acquire information remain strategic substitutes in both the socially efficient policy and the equilibrium policy. Hence, the equilibrium information acquisition is now socially *insufficient*. Similarly, the analysis for the nonlinear model in Section 5 is extended to the case of negative interdependence by reversing the relevant inequalities.

7 Conclusion

In a model with interdependent valuations, the equilibrium level of information acquisition differs from the socially efficient level. This paper shows that in many specifications of the model, information acquisition is excessive in equilibrium. This opens up a number of new questions. How could a planner correct the incentives? If information acquisition is covert, it is not easy for the planner to change the cost of information as the uninformed bidders can always pretend to be the informed bidders. While participation fees reduce the bidders' expected payoffs, they do at the same time discourage uninformed bidders from participating. This may result in suboptimal allocations. Another possibility to correct the private incentives could be to assign the object at random between the highest bidders if the bids (and hence the valuations) are relatively close. The welfare losses from such a distorted allocation would be small since it would only happens if the valuations do not differ by much. On the other hand, the payments of all winning bids are increased and hence the expected payoffs decrease as a result of such a policy.

The current paper primarily considers ex post efficient mechanisms. Since there are close connections between efficient mechanisms and optimal (revenue maximizing) mechanisms, some of the results here have direct implications for revenue maximizing mechanisms as well. Consider first the case when the seller controls the access to information and hence the relevant participation constraints for the bidders are the ex ante participation constraints. Then we can use the insights of Levin and Smith (1994) to show that the revenue maximizing mechanism in certain auction environment with endogenous entry also leads to an ex post efficient allocation. In particular, with positive interdependence, an appropriately chosen entry fee may correct private incentives and help induce the socially optimal number of bidders in equilibrium.[9]

However, our model of covert information acquisition is more in line with a model where the seller does not control the access to the information source and information decisions are decentralized. In such models, the participation constraint is effective at the interim stage. In this environment, it may not be feasible to charge an entry fee and the seller may have to use reserve prices to align incentives. In general, the optimal mechanism will no longer coincide with the efficient mechanism. In particular, we expect that the informational efficiency in auctions will depend critically on the underlying distribution of valuations as shown by Shi (2007) in a private value environment.

Finally, in our model, the supply side of the model is fixed. It would be interesting to study information acquisition in large double auctions where the equilibrium price is determined by competing buyers and sellers.

[9]If bidders make information decisions sequentially, Cremer, Spiegel, and Zheng (2008) show that a sequential mechanism can extract full surplus and induce efficient level of information acquisition.

8 Appendix

The appendix contains the proofs of all the results presented in the main body of the paper. We first state and establish a lemma that will be used in proving both Proposition 1 and 2. Specifically, we will use the first part of the lemma to show the monotonicity of the gains from information in the number of informed bidders for $m < I$, and use the second part of the lemma to extend the monotonicity to the last bidder with $m = I$.

Lemma 2

1. *For all* m,

$$\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right]$$

is strictly decreasing in m.

2. *For all* m,

$$\mathbb{E}_{\theta_m, \theta_h}\left[(\theta_h - \theta_m) \cdot \mathbf{1}\left(\mu > \theta_h > \theta_m\right) - (\theta_m - \mu) \cdot \mathbf{1}\left(\theta_m \geq \mu > \theta_h\right)\right] < 0.$$

Proof of Lemma 2. We denote by G_m and g_m the cumulative distribution and density of θ_h, respectively. Since bidder h has the highest signal among $(m-1)$ informed bidders, we have the usual order statistics with

$$G_m\left(\theta_h\right) = F^{m-1}\left(\theta_h\right)$$

and

$$g_m\left(\theta_h\right) = (m-1)\, F^{m-2}\left(\theta_h\right) f\left(\theta_h\right).$$

For part 1, we observe that

$$\begin{aligned}
& \mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right] \\
= & \int_{\mu}^{\overline{\theta}} \int_{\mu}^{\theta_m} \left(\theta_m - \theta_h\right) g_m\left(\theta_h\right) f\left(\theta_m\right) d\theta_h d\theta_m + \int_{\mu}^{\overline{\theta}} \int_{\underline{\theta}}^{\mu} \left(\theta_m - \mu\right) g_m\left(\theta_h\right) f\left(\theta_m\right) d\theta_h d\theta_m \\
= & \int_{\mu}^{\overline{\theta}} \left(-\left(\theta_m - \mu\right) F^{m-1}\left(\mu\right) + \int_{\mu}^{\theta_m} F^{m-1}\left(\theta_h\right) d\theta_h\right) f\left(\theta_m\right) d\theta_m + \int_{\mu}^{\overline{\theta}} \left(\theta_m - \mu\right) F^{m-1}\left(\mu\right) f\left(\theta_m\right) d\theta_m \\
= & \int_{\mu}^{\overline{\theta}} \left[\int_{\mu}^{\theta_m} F^{m-1}\left(\theta_h\right) d\theta_h\right] f\left(\theta_m\right) d\theta_m.
\end{aligned}$$

The first equality follows by the definition of y_m, and the second equality is a result of integration by parts. It is easy to see from the last expression that $\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right]$ is strictly decreasing in m.

For part 2, we note that

$$\begin{aligned}
& \mathbb{E}_{\theta_m,\theta_h}\left[(\theta_h-\theta_m)\cdot\mathbf{1}(\mu>\theta_h>\theta_m)-(\theta_m-\mu)\cdot\mathbf{1}(\theta_m\geq\mu>\theta_h)\right] \\
= \; & \int_{\underline{\theta}}^{\mu}\int_{\underline{\theta}}^{\theta_h}(\theta_h-\theta_m)\,f(\theta_m)\,g_m(\theta_h)\,d\theta_m d\theta_h-\int_{\underline{\theta}}^{\mu}\int_{\mu}^{\overline{\theta}}(\theta_m-\mu)\,f(\theta_m)\,g_m(\theta_h)\,d\theta_m d\theta_h \\
= \; & \int_{\underline{\theta}}^{\mu}\left[\int_{\underline{\theta}}^{\theta_h}(\theta_h-\theta_m)\,f(\theta_m)\,d\theta_m-\int_{\mu}^{\overline{\theta}}(\theta_m-\mu)\,f(\theta_m)\,d\theta_m\right]g_m(\theta_h)\,d\theta_h \\
< \; & \int_{\underline{\theta}}^{\mu}\left[\int_{\underline{\theta}}^{\mu}(\mu-\theta_m)\,f(\theta_m)\,d\theta_m-\int_{\mu}^{\overline{\theta}}(\theta_m-\mu)\,f(\theta_m)\,d\theta_m\right]g_m(\theta_h)\,d\theta_h \\
= \; & 0.
\end{aligned}$$

Thus, the proof is complete.■

Proof of Proposition 1. From the social point of view, bidder m should acquire information if and only if the social expected gain Δ_m^* is higher than the information cost c. That is,

$$s_m^*=\begin{cases}0 & \text{if} \quad \Delta_m^*<c \\ 1 & \text{if} \quad \Delta_m^*\geq c\end{cases}.$$

The result $\Delta_1^*>\Delta_2^*>\cdots>\Delta_{I-1}^*$ follows from (6) and part 1 of Lemma 2. In order to prove $\Delta_{I-1}^*>\Delta_I^*$, note that

$$\begin{aligned}
\Delta_I^* &= (1-\alpha)\,\mathbb{E}_{\theta_I,\theta_h}\left[(\theta_I-\theta_h)\cdot\mathbf{1}(\theta_I\geq\theta_h\geq\mu)+(\theta_h-\theta_I)\cdot\mathbf{1}(\mu>\theta_h>\theta_I)\right] \\
&= (1-\alpha)\,\mathbb{E}_{\theta_I,y_I}\left[(\theta_I-y_I)\cdot\mathbf{1}(\theta_I\geq y_I)\right] \\
&\quad +(1-\alpha)\,\mathbb{E}_{\theta_I,\theta_h}\left[(\theta_h-\theta_I)\cdot\mathbf{1}(\mu>\theta_h>\theta_I)-(\theta_I-\mu)\cdot\mathbf{1}(\theta_I\geq\mu>\theta_h)\right] \\
&< (1-\alpha)\,\mathbb{E}_{\theta_I,y_I}\left[(\theta_I-y_I)\cdot\mathbf{1}(\theta_I\geq y_I)\right] \\
&< (1-\alpha)\,\mathbb{E}_{\theta_{I-1},y_{I-1}}\left[(\theta_{I-1}-y_{I-1})\cdot\mathbf{1}(\theta_{I-1}\geq y_{I-1})\right] \\
&= \Delta_{I-1}^*.
\end{aligned}$$

The first inequality follows from part 2 of Lemma 2 and the second inequality follows by part 1 of Lemma 2. The fact that Δ_m^* is strictly decreasing in α follows from expression (6) and (8).■

Proof of Proposition 2. From bidder m's point of view, the optimal information decision is to acquire information if and only if $\widehat{\Delta}_m\geq c$. That is,

$$\widehat{s}_m=\begin{cases}0 & \text{if} \quad \widehat{\Delta}_m<c \\ 1 & \text{if} \quad \widehat{\Delta}_m\geq c\end{cases}.$$

The fact that $\widehat{\Delta}_1 > \widehat{\Delta}_2 > \cdots > \widehat{\Delta}_{I-1}$ follows from part 1 of Lemma 2. Notice that

$$\begin{aligned}
\widehat{\Delta}_I &= \mathbb{E}_{\theta_I,\theta_h}\left[(\theta_I-\theta_h)\cdot\mathbf{1}\left(\theta_I\geq\theta_h\geq\mu\right)+(\theta_h-\theta_I)\cdot\mathbf{1}\left(\mu>\theta_h>\theta_I\right)\right]\\
&= \mathbb{E}_{\theta_I,y_I}\left[(\theta_I-y_I)\cdot\mathbf{1}(\theta_I\geq y_I)\right]\\
&\quad+\mathbb{E}_{\theta_I,\theta_h}\left[(\theta_h-\theta_I)\cdot\mathbf{1}\left(\mu>\theta_h>\theta_I\right)-(\theta_I-\mu)\cdot\mathbf{1}\left(\theta_I\geq\mu>\theta_h\right)\right]\\
&< \mathbb{E}_{\theta_I,y_I}\left[(\theta_I-y_I)\cdot\mathbf{1}(\theta_I\geq y_I)\right]\\
&< \mathbb{E}_{\theta_{I-1},y_{I-1}}\left[(\theta_{I-1}-y_{I-1})\cdot\mathbf{1}(\theta_{I-1}\geq y_{I-1})\right]\\
&= \widehat{\Delta}_{I-1}.
\end{aligned}$$

The first inequality follows from part 2 of Lemma 2 and the second inequality follows by part 1 of Lemma 2. From expression (9) and (10), it is easy to see $\widehat{\Delta}_m$ is constant in α.■

Proof of Proposition 3. Comparing expression (6) and (8) with (9) and (10), we have

$$\Delta_m^* = (1-\alpha)\,\widehat{\Delta}_m, \text{ for all } m. \tag{15}$$

Since $0 < \alpha \leq 1$, $\Delta_m^* < \widehat{\Delta}_m$. This result, together with the fact that both sequences $\{\Delta_m^*\}$ and $\{\widehat{\Delta}_m\}$ are strictly decreasing, implies that in equilibrium, (weakly) more bidders are informed in equilibrium than in the socially optimal solution.

¿From (15), we obtain

$$\widehat{\Delta}_m - \Delta_m^* = \alpha\widehat{\Delta}_m.$$

By Proposition 2, $\widehat{\Delta}_m$ is constant in α and decreasing in m. Therefore, $\widehat{\Delta}_m - \Delta_m^*$ is strictly increasing in α and strictly decreasing in m.■

Proof of Proposition 4. Recall that $\widehat{\Delta}_m$ is strictly decreasing in m, and

$$\widehat{\Delta}\left(\sigma\right) \triangleq \sum_{m=1}^{I}\binom{I-1}{m-1}\sigma^{m-1}\left(1-\sigma\right)^{I-m}\widehat{\Delta}_m.$$

Notice that if $\sigma_1 > \sigma_2$ then a binomial distribution generated by the probability of success σ_1 first-order stochastically dominates a binomial distribution generated by σ_2 (see Example 6.A.2 in Shaked and Shanthikumar (1994), p.172-173). This fact, together with the monotonicity of $\widehat{\Delta}_m$, implies that $\widehat{\Delta}\left(\sigma\right)$ is decreasing in σ. The monotonicity of $\Delta^*\left(\sigma\right)$ in σ can be proved analogously.■

Proof of Proposition 5. Recall that σ^* and $\widehat{\sigma}$ are defined by the following two conditions:

$$\begin{aligned} c &= \sum_{m=1}^{I} \binom{I-1}{m-1} (\sigma^*)^{m-1} (1-\sigma^*)^{I-m} \Delta_m^*, \\ c &= \sum_{m=1}^{I} \binom{I-1}{m-1} \widehat{\sigma}^{m-1} (1-\widehat{\sigma})^{I-m} \widehat{\Delta}_m. \end{aligned}$$

Since $\widehat{\Delta}_m > \Delta_m^*$ for all m, we have

$$\sum_{m=1}^{I} \binom{I-1}{m-1} (\sigma^*)^{m-1} (1-\sigma^*)^{I-m} \widehat{\Delta}_m > \sum_{m=1}^{I} \binom{I-1}{m-1} \widehat{\sigma}^{m-1} (1-\widehat{\sigma})^{I-m} \widehat{\Delta}_m.$$

Then $\sigma^* < \widehat{\sigma}$ can be deduced by applying Proposition 4.■

Proof of Lemma 1. The assumption $\mathbb{E}\left[u_i(\theta)\,|\theta_1,...,\theta_{m-1}\right] > \mathbb{E}\left[u_k(\theta)\,|\theta_1,...,\theta_{m-1}\right]$ implies that there exists a $\widehat{\theta}_m$ such that

$$\mathbb{E}\left[u_i(\theta)\,|\theta_1,...,\theta_{m-1},\widehat{\theta}_m\right] > \mathbb{E}\left[u_k(\theta)\,|\theta_1,...,\theta_{m-1},\widehat{\theta}_m\right]$$

But by the no-crossing property, we must have

$$\mathbb{E}\left[u_i(\theta)\,|\theta_1,...,\theta_{m-1},\theta_m\right] > \mathbb{E}\left[u_k(\theta)\,|\theta_1,...,\theta_{m-1},\theta_m\right] \text{ for all } \theta_m,$$

which completes the proof. ■

Proof of Proposition 6. To simplify notation, let J denote the set of informed bidders $\{1,...,m-1\}$ and θ_J denote the vector $\{\theta_j\}_{j\in J}$. We need to show that, for any bidder $m \notin J$, her private gain is lower than the social gain from information about θ_m. Conditional on the realization of θ_J, there are two possible scenarios. First, conditional on θ_J, the social planner awards the item to $i \neq m$. Second, conditional on θ_J, the social planner awards the item to m and bidder i is the runner-up. Let z_m be the value at which

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J,\theta_m = z_m\right] = \mathbb{E}\left[u_i(\theta)\,|\theta_J,\theta_m = z_m\right].$$

That is, z_m is the cutoff type such that bidder m will overtake bidder i if bidder m has a type realization higher than z_m. By Lemma 1, we need to focus on bidder i and m only.

Case 1: Conditional on θ_J, the social planner awards the item to $i \neq m$. If $\theta_m \leq z_m$, bidder i will retain the object, and both the private value and the social value of information about θ_m is equal to zero. If $\theta_m > z_m$, then bidder m wins the object and her VCG payment is $\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right]$. Therefore, the private gain from information for bidder m is

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] - \mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right],$$

which is larger than the social gain from information

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] - \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right],$$

because

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right] < \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right]$$

by the assumption of positive dependence.

Case 2: Conditional on θ_J, the social planner awards the item to m. Let bidder i be the second highest bidder when bidder m is informed. If $\theta_m \geq z_m$, bidder m retains the object and both the private value and the social value of information are equal to zero. If $\theta_m < z_m$, bidder i wins the object. The gain to bidder m is then the foregone loss from paying more than the true value of the object to m. Her private gain is given by

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right] - \mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right],$$

which is higher than the social value of information:

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right] - \mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right],$$

since by Assumption 2:

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right] > \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right].$$

Thus, in both cases, the private gain from information is higher than the social gain from information. Therefore, $\widehat{\Delta}_m < \Delta^*_m$ for all m. ■

Proof of Proposition 7. Again let J denote the set of informed bidders $\{1, ..., m-1\}$ and θ_J denote the vector $\{\theta_j\}_{j \in J}$. Fix J such that $I - \#\{j | j \in J\} \geq 2$, that is, there are at least two uninformed bidders. We need to show that a bidder i's private gain from information when bidder $m \notin J$ is uninformed is lower than her private gain when bidder m becomes informed. Conditional on θ_J only, there are five possible cases:

Case 1: The social planner awards the object to $k \neq \{i, m\}$.

Case 2: The social planner awards the object to m, and the second highest bidder is $k \neq i$.

Case 3: The social planner awards the object to m, and the second highest bidder is i.

Case 4: The social planner awards the object to i, and the second highest bidder is $k \neq m$.

Case 5: The social planner awards the object to i, and the second highest bidder is m.

Given the result of Lemma 1, it is easy to see that for Case 1 and Case 2 bidder m's information decision has no effect on bidder i's expected gain from information. So we only need to analyze the remaining three cases.

For Case 3, bidder m wins and the runner up is i. Since there are at least two uninformed bidders (including m), there must be another uninformed bidder tied with m. So bidder i's payoff will be zero regardless of bidder m's information decision.

For Case 4, bidder i must be an informed bidder. Let $\theta_{J\backslash i}$ denote the vector of signals of all bidders in J other than i, and let z_i solve

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i}, \theta_i = z_i\right] = \mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i}, \theta_i = z_i\right]$$

That is, z_i is the critical type at which bidder i has a weakly higher valuation than bidder k conditional on $\theta_{J\backslash i}$. Then bidder i's expected gain from information is the difference

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i}, \theta_i\right] - \mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i}, z_i\right].$$

In order to compare bidder i's private gains with and without bidder m's information, it is convenient to introduce an auxiliary benchmark. We will show that bidder i's expected payoff without bidder m's information is the same as her payoff in the auxiliary benchmark. Suppose the social planner has access to bidder m's private information, but only treats it as an exogenous signal. In particular, bidder m cannot be the winner or the runner up based on her information. Then by Lemma 1, bidder i will still win the object and the runner up is still bidder k. However, i's payment will depend on the realization of θ_m. Let $\widehat{z}_i$ solve

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i}, \theta_i = \widehat{z}_i, \theta_m\right] = \mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i}, \theta_i = \widehat{z}_i, \theta_m\right].$$

Then bidder i's expected gain is given by

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i}, \theta_i, \theta_m\right] - \mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i}, \widehat{z}_i, \theta_m\right].$$

We claim that $\widehat{z}_i = z_i$. To see this, suppose $\widehat{z}_i$ be the solution for some θ'_m. That is,

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i}, \widehat{z}_i, \theta'_m\right] = \mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i}, \widehat{z}_i, \theta'_m\right].$$

By no-crossing property assumption, we have for all θ_m,

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\setminus i}, \widehat{z}_i, \theta_m\right] = \mathbb{E}\left[u_k(\theta)\,|\theta_{J\setminus i}, \widehat{z}_i, \theta_m\right].$$

Therefore, $\widehat{z}_i = z_i$. Then it follows from the law of iterated expectation that bidder i's expected payoff with bidder m uninformed is the same as her expected payoff in the auxiliary benchmark.

Once we have established the payoff equivalence between the equilibrium payoff and the payoff in the auxiliary benchmark given an uninformed bidder m, it only remains to consider the case with an informed bidder m. If

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] > \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right],$$

then bidder m wins the object, so the private gain for bidder i is equal to zero. Compared to the auxiliary benchmark, bidder i is worse off. If

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right] > \mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] > \mathbb{E}\left[u_k(\theta)\,|\theta_J, \theta_m\right],$$

then bidder m becomes the second highest bidder, and bidder i's payment will be based on bidder m's valuations. Thus bidder i pays more and consequently her payoff decreases. Finally, if

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] < \mathbb{E}\left[u_k(\theta)\,|\theta_J, \theta_m\right],$$

then bidder i wins and the runner up is still bidder k, so bidder i's payoff remains unchanged. In summary, bidder i's expected gain from information is lower when bidder m gets informed.

Finally, in Case 5, bidder i wins and the runner up is bidder m. Again since there are at least two uninformed bidders, there must be another uninformed bidder $j \neq m$. We can redefine the runner up as bidder j, and then we return to Case 4.

Therefore, in all five cases, when bidder m gets informed, bidder i's expected gain from information either stays the same or decreases and the proof is complete.■

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