Optimal Auctions with Information Acquisition*

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Abstract

This paper studies optimal auction design in a private value setting with endogenous information gathering. We develop a general framework for modeling information acquisition when a seller wants to sell an object to one of several potential buyers, who can each gather information about their valuations prior to participation in the auction. We first demonstrate that the optimal monopoly price is always lower than the standard monopoly price. We then show that standard auctions with a reserve price remain optimal among symmetric mechanisms, but the optimal reserve price lies between the ex ante mean valuation of bidders and the standard reserve price in Myerson (1981). Finally, we show that the optimal asymmetric mechanism softens the price discrimination against "strong" bidders.

KEYWORDS: optimal auctions, information acquisition, rotation order
JEL CLASSIFICATION: C70, D44, D82, D86

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1 Introduction

A typical assumption in the mechanism design literature is that the information held by market participants is exogenous; yet in many real world situations, agents' information about the goods and services being traded is acquired rather than endowed.[1] When information acquisition is endogenous, the selling mechanism proposed by the seller affects not only buyers' incentives to reveal the information they gathered ex post, but also their incentives to acquire information ex ante. Not surprisingly, if the information structure is endogenous, the ex post optimal selling mechanism characterized in Myerson (1981) may not be optimal ex ante.

The purpose of this paper is to study how a seller should design the selling mechanism in an independent private value setting when information acquisition is endogenous and costly for buyers. We first develop a convenient but reasonably general framework for modeling information acquisition in such a setting, where a seller wants to sell an object to one of several potential buyers and where the buyers can each covertly acquire information about their valuations prior to participation.

In the model, the seller can affect the buyers' incentive to gather information through mechanism choice, but she faces the following trade-off: an increase in information raises the potential social surplus – the difference between the highest expected valuation among buyers and her reservation value, but it also raises buyers' private information and thus their information rents. A buyer acquires information by increasing the precision of the signal he receives. After receiving his signal, each buyer forms a posterior estimate of his valuation, which depends on the realization as well as the informativeness of his signal. As more information is acquired, buyers' valuation estimates move *apart*,[2] i.e., the distribution of posterior estimates conditional on a more informative signal is more spread out. The resulting family of distributions of the posterior estimates with different signals is rotation-ordered.[3] We rank the informativeness of signals using this information order, which has the merit of being analytically tractable in addition to generalizing two important information

[1]For example, consumers collect information about the characteristics of products and match this information with their private preferences to determine their valuations before their purchase decision. In a take-over bidding, buyers gather costly information about potential synergies between their own assets and assets of the target firm to determine how much they should bid.

[2]This is an important feature of the independent private value setting. For example, suppose a consumer tries a newly opened restaurant and finds the food spicy. He likes the restaurant more if he loves spicy food, and he likes it less if he dislikes spicy food.

[3]If two signals are rotation-ordered, then the two distributions of posterior estimates generated by these signals cross each other only once. The rotation order was recently introduced by Johnson and Myatt (2006) who show that advertising, marketing and product design lead to a rotation (rather than a shift) of the market demand curve.

technologies widely used in the literature.

We apply this framework to analyze optimal auctions with information acquisition. We start by considering optimal auctions with a single buyer in order to convey the main intuition. In this case, the optimal selling mechanism is to post a (reserve) price, so the seller's problem can also be reinterpreted as a monopoly pricing problem with endogenous information. The seller first chooses the reserve price, knowing that the price affects both the buyer's subsequent information decision and purchase decision. Upon observing the price, the buyer decides how much information to acquire, and whether to buy after the acquired information is revealed. In order to identify the impact of endogenous information on the seller's pricing decision, we compare the optimal reserve price with the *standard reserve price* – a reserve price that the seller would have chosen if the information acquired by the buyer in equilibrium were exogenously endowed.

Since the buyer always prefers a low reserve price, it may seem at first glance that a lower reserve price always gives the buyer a higher incentive to gather information. Yet the buyer's incentives to acquire information depend on his *relative* gain from information acquisition rather than on his *absolute* payoff; indeed, when the reserve price is lower (higher, respectively) than the ex ante mean valuation of the buyer, the marginal value of information to the buyer is increasing (decreasing, respectively) in the reserve price, and thus the buyer will acquire more information as the reserve price moves toward the mean, either from above or from below. On the other hand, for a given price, the seller's revenue is decreasing (increasing, respectively) in the amount of information if the price is below (above, respectively) the mean. Therefore, compared to the standard reserve price, the optimal reserve price in this setting is always adjusted downward in order to provide the buyer with incentives to acquire the right amount of information.

The same observation about the buyer's incentives can be extended to a general setting with many buyers. But the characterization of the optimal mechanism is subtler and more complicated, primarily because our model involves a mixture of moral hazard and adverse selection with *multiple* agents. A feasible mechanism has to provide buyers with the right incentives to collect information in the information acquisition stage (moral hazard) and be incentive compatible in the information revelation stage (adverse selection). We use the standard first-order approach to tackle the moral hazard problem, replacing the information acquisition constraints by the first-order conditions of the buyers' maximization problems (Mirrlees (1999), and Rogerson (1985)).[4] While in the standard moral hazard problem the principal always benefits from higher effort (without accounting for the incentive cost of inducing higher effort), here more information may hurt or benefit the seller, making the

[4]Appendix B provides a set of sufficient conditions under which the first-order approach is valid in our setting.

tradeoff between efficiency and information rent trickier.

For the general setting, this paper focuses first on the symmetric equilibrium in which all buyers acquire the same level of information. Yet we will discuss asymmetric mechanisms in Section 6, where we are able to characterize the optimal (asymmetric) mechanism for a special class of problems. The symmetry restriction is quite natural for many applications because buyers are ex ante symmetric and symmetric mechanisms are easier to implement and less likely to cause legal disputes.[5] We provide sufficient conditions under which more information strictly benefits the seller. Moreover, we show that standard auctions[6] with a reserve price are optimal, but the reserve price has to be adjusted toward the mean valuation to induce buyers to acquire more information. Further, the buyers' incentives to collect information are socially excessive in standard auctions if the reserve price is lower than the mean valuation.

In terms of its broader significance, the analysis illustrates how the optimal mechanism should respond to endogenous information acquisition and strategic interactions among bidders. It also provides useful guidance for designing optimal selling mechanisms when endogenous information is a concern. In addition, our results have important implications for empirical analysis by suggesting an alternative benchmark – the optimal reserve price – for evaluating reserve price policy as compared with the standard reserve price. This may be preferable in situations where information acquisition is important.

The general framework we develop for modeling information acquisition in a private value setting can be applied to mechanism design problems when agents are able to make investments prior to an auction. For instance, Lichtenberg (1988) finds strong evidence of private R&D investment prior to government procurement auctions. In this vein, our framework can be used to investigate how the government should design procurement auctions in order to promote private R&D investment.

The remainder of the paper is organized as follows: Section 2 discusses the related literature, Section 3 introduces the model, and Section 4 studies optimal auctions with a single bidder. Section 5 then presents the analysis of optimal auctions with many bidders, and Section 6 studies asymmetric mechanisms. Section 7 provides some concluding remarks. All omitted proofs are relegated to Appendix A, and a set of sufficient conditions for the first-order approach is presented in Appendix B.

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[5]Nevertheless, this is an important restriction. In principle, the seller may become better off by implementing an asymmetric equilibrium rather than a symmetric one.

[6]In this paper, we use standard auctions to refer to the four commonly used auction formats: first-price sealed-bid auctions, Vickery auctions, English auctions, and Dutch auctions.

2 Related Literature

This paper is related to the growing literature on information and mechanism design.[7] Cremer and Khalil (1992) and Cremer, Khalil, and Rochet (1998a) (1998b) introduce endogenous information acquisition into the Baron and Myerson (1982) regulation model with a *single* agent, and illustrate how the optimal contract has to be modified in order to give the agent incentives to acquire information. Szalay (2009) extends their framework to a setting with continuous information acquisition, and demonstrates that their findings are robust.[8] Although our model shares with theirs a focus on the interim participation constraint, it differs from theirs in two important dimensions: we use rotation-ordered information structures to model information acquisition and we allow for strategic interactions among multiple agents.

Our analysis is also related to research on information acquisition in given auctions. Matthews (1984) studies information acquisition in a common value auction and investigates whether the equilibrium price fully reveals bidders' information. Stegeman (1996) shows that first and second price auctions with independent private values result in the same incentives for information acquisition, while Persico (2000) demonstrates that the incentive to acquire information is stronger in the first-price auction than in the second-price auction if bidders' valuations are affiliated.[9] In contrast, the current paper examines the optimal mechanism that maximizes the seller's revenue, rather than studying information acquisition under given auction formats.[10]

A third strand of related literature studies optimal auctions when the seller controls either access to information sources or the timing of information acquisition. The information order used in the current paper, the rotation order, was first introduced by Johnson and Myatt (2006), who use it to show that the firm's profits are a U-shaped function of the dispersion of consumers' valuations; thus a monopolist will pursue extreme positions, providing either a minimal or maximal amount of information. Eso and Szentes (2007) study optimal auctions in a setting where the seller controls access to information sources, showing that the seller will fully reveal her information and can extract all of the benefit from the released information.[11]

[7] For a broad survey of the literature on information and mechanism design, see Bergemann and Välimäki (2006).

[8] In particular, Szalay (2009) provides a justification for using the first-order approach to analyze the contracting problem with endogenous information. Although we also provide sufficient conditions for the first-order approach in the Appendix, justifying the first-order approach is not the focus of this paper. Moreover, the information order used in this paper is different from the one used in Szalay (2009) and thus his result may not directly apply to our setting.

[9] See Ye (2007) and Compte and Jehiel (2007) for an analysis of information acquisition in dynamic auctions, and Obara (2008) for an analysis of optimal auctions with hidden actions and correlated signals.

[10] Bergemann and Välimäki (2002) also study information acquisition and mechanism design, but their focus is efficient mechanisms.

[11] Bergemann and Pesendorfer (2007) characterize the optimal information structure in optimal auctions,

In these models, the seller makes the information decision, rather than the buyers.

Several papers study the optimal selling mechanism in a setting where buyers make the information decision, but the seller controls the timing of information acquisition. These models (hereafter referred to as "entry models") impose an ex ante participation constraint, so the buyers' information decision is essentially an entry decision. The optimal selling mechanism typically consists of a participation fee followed by a second price auction with no reserve price, with the participation fee being equal to the bidders' expected rent from attending the auction (see for example, Levin and Smith (1994) and Ye (2004)).[12]

In contrast to these papers, where information acquisition is *centralized*, in the sense that the seller can control either access to information sources or the timing of information acquisition, information acquisition in the current paper is *decentralized*: buyers make the information decision, and can acquire information prior to participation. Thus, we impose an interim rather than an ex ante participation constraint.[13] The relationship between our model and the existing literature can be partially summarized in the following table.

	given auction formats	mechanism design approach
centralized information acquisition	optimal disclosure in auctions	entry models
decentralized information acquisition	information acquisition in auctions	our model

Table 1. Our model and related literature

3 The Model

A seller wants to sell a single object to n ex ante symmetric buyers (or bidders), indexed by $i \in \{1, 2, ..., n\}$.[14] Both the seller and buyers are risk-neutral. The buyers' true valuations $\{\omega_i : i = 1, ..., n\}$, unknown ex ante, are independently drawn from a common distribution F. They have a common support which could be a closed interval $[\underline{\omega}, \overline{\omega}]$ or the real line. To ease exposition, in what follows we write the common support as $[\underline{\omega}, \overline{\omega}]$. F has a strictly positive and differentiable density f and mean μ. A buyer with valuation ω_i gets utility u_i if

while Ganuza and Penalva (2010) study the seller's optimal disclosure policy when the information is costly.

[12]Similarly, with an ex-ante participation constraint, Cremer, Spiegel, and Zheng (2003) construct a sequential selling mechanism in which the seller charges a positive entry fee and extracts the full surplus from buyers. See Gershkov and Szentes (2009) for an analysis of optimal sequential mechanism in voting with costly information acquisition.

[13]Cremer, Spiegel, and Zheng (2007) also analyze optimal auctions where buyers can acquire information prior to participation, but the seller, rather than the buyer, pays the information cost.

[14]The analysis can be extended to a multi-unit setting where each buyer has a unit demand.

he wins the object and pays t_i: $u_i = \omega_i - t_i$. The seller's valuation for the object is normalized to be zero.

3.1 Information Structure

Buyer i can acquire a costly signal s_i about ω_i, with $s_i \in [\underline{s}, \overline{s}]$. Signals received by different buyers are independent. Buyer i acquires information by choosing a joint distribution of (s_i, ω_i) from a family of joint distributions $G(s_i, \omega_i; \alpha_i) : [\underline{s}, \overline{s}] \times [\underline{\omega}, \overline{\omega}] \to [0, 1]$, indexed by $\alpha_i \in [\underline{\alpha}, \overline{\alpha}]$. Each fixed α_i corresponds to a statistical experiment, and a signal with higher α_i is more informative, in a sense to be defined shortly. The cost of performing an experiment α_i is $C(\alpha_i)$, which is increasing in α_i. A buyer can conduct the experiment $\underline{\alpha}$ at no cost, so $\underline{\alpha}$ can be interpreted as the endowed signal. As is standard in the literature, we assume that information acquisition is a once-and-for-all decision.

Let $G(\cdot|\omega_i, \alpha_i)$ denote the prior distribution of signal s_i conditional on ω_i and α_i. A buyer who observes a signal s_i from experiment α_i will update his belief about ω_i according to Bayes' rule. Let $v_i(s_i, \alpha_i)$ denote buyer i's revised estimate of ω_i after performing experiment α_i and observing s_i: $v_i(s_i, \alpha_i) \equiv \mathbb{E}[\omega_i | s_i, \alpha_i]$. Throughout the paper we assume that the distributions $\{G(\cdot|\omega_i, \alpha_i)\}$ have the monotone likelihood ratio property (MLRP), so that $v_i(s_i, \alpha_i)$ is increasing in s_i, i.e., a higher s_i leads to a higher posterior estimate, given the information choice α_i (Milgrom (1981)).

To simplify notation, we use v_i to denote $v_i(s_i, \alpha_i)$, and use v to denote the n-vector $(v_1, ..., v_n)$. Occasionally, we also write v as (v_i, v_{-i}), where $v_{-i} = (v_1, ... v_{i-1}, v_{i+1}, ..., v_n)$. Let $H(\cdot, \alpha_i)$ denote the distribution of v_i with corresponding density $h(\cdot, \alpha_i)$, and $H_{\alpha_i}(\cdot, \alpha_i)$ denotes the partial derivative with respect to α_i. The family of distributions $\{H(\cdot, \alpha_i)\}$ have the same mean because $\mathbb{E}_{s_i}[v_i(s_i, \alpha_i)] = \mathbb{E}[\omega_i] = \mu$.

3.2 Timing

The timing of the game is as follows: the seller first proposes a selling mechanism. After observing the mechanism, each buyer decides how much information to acquire. Once the signals are realized, each buyer decides whether to participate; each participating buyer then submits a report about his private information. Finally, an outcome, consisting of an allocation and payments, is realized. Figure 1 summarizes the timing of the game:

——— | ————	——— | ———	———————— | ———————	————— | ————	——— | —— ⟶
seller announces mechanism	buyer i chooses α_i	buyer i observes s_i and decides whether to participate	buyers report private information	outcome is realized

Figure 1. The timing of the game

The payoff structure, the timing of the game, the information structure G and distribution F are assumed to be common knowledge. As is standard in the mechanism design literature, the solution concept is Bayesian Nash equilibrium.

3.3 Mechanisms

In our setting, the buyer's private information is two-dimensional, consisting of the information choice α_i and the realized signal s_i. This suggests that the design problem here is multi-dimensional and could potentially be very complicated. However, similar to Biais, Martimort, and Rochet (2002) and Szalay (2009), one single variable, the posterior estimate $v_i(s_i, \alpha_i)$, completely captures the dependence of buyer i's valuation on the two-dimensional information because the buyers' payoff is linear in v_i. Furthermore, the seller cannot screen the two pieces of information separately.[15]

We can thus invoke the Revelation Principle to focus on the direct revelation mechanisms $\{q_i(v), t_i(v)\}_{i=1}^n$, where $q_i(v)$ denotes the probability of winning the object for bidder i when the vector of reports is v, and $t_i(v)$ denotes bidder i's corresponding payment. Let $Q_i(v_i)$ and $T_i(v_i)$ be the expected probability of winning and the expected payment conditional on v_i, respectively:

$$Q_i(v_i) = \mathbb{E}_{v_{-i}|\alpha_{-i}}[q_i(v)] \quad \text{and } T_i(v_i) = \mathbb{E}_{v_{-i}|\alpha_{-i}}[t_i(v)].$$

The subscript $v_{-i}|\alpha_{-i}$ in the expectation operator is to emphasize that the expectation depends on the information choice α_{-i} of bidder i's opponents. The interim utility of bidder i who has a posterior estimate v_i and reports v_i' is

$$U_i(v_i, v_i') = v_i Q_i(v_i') - T_i(v_i').$$

Define $u_i(v_i) = U_i(v_i, v_i)$ to be the payoff of bidder i who has a posterior estimate v_i and reports truthfully.

A feasible mechanism has to satisfy the individual rationality constraint (IR):

$$u_i(v_i) = U_i(v_i, v_i) \geq 0, \quad \forall v_i \in [\underline{\omega}, \overline{\omega}],$$

and the incentive compatibility constraint (IC):

$$U_i(v_i, v_i) \geq U_i(v_i, v_i'), \quad \forall v_i, v_i' \in [\underline{\omega}, \overline{\omega}]. \tag{IC}$$

[15] To see this, suppose there are two buyers, i and j, with the same posterior estimate $(v_i = v_j)$, but $\alpha_i \neq \alpha_j$. If the seller wants to favor the buyer with α_i, then buyer j can always report to have α_i. Therefore, the posterior estimate v_i is the only variable that the seller can use to screen different buyers.

It is well-known (Myerson (1981)) that the incentive compatibility constraint (IC) is equivalent to the following envelope condition

$$u_i(v_i) = u_i(\underline{\omega}) + \int_{\underline{\omega}}^{v_i} Q_i(x)\, dx, \tag{1}$$

and the monotonicity condition

$$Q_i(v_i) \text{ is weakly increasing in } v_i. \tag{2}$$

Using equation (1), we can simplify the (IR) constraint as

$$u_i(\underline{\omega}) \geq 0. \tag{IR}$$

With endogenous information acquisition, a feasible mechanism also has to satisfy the information acquisition constraint (IA): no bidder has an incentive to deviate from the equilibrium choice α_i^* :

$$\alpha_i^* \in \arg\max_{\alpha_i} \left\{ \mathbb{E}_{v|\alpha_{-i}=\alpha_{-i}^*} \left[u_i(v_i(s_i, \alpha_i)) \right] - C(\alpha_i) \right\}. \tag{IA}$$

Here $\mathbb{E}_{v|\alpha_{-i}=\alpha_{-i}^*} [u_i(v_i(s_i, \alpha_i))]$ is bidder i's expected payoff by choosing α_i conditional on other bidders choosing α_j^*, $j \neq i$.

The seller chooses mechanism $\{q_i(v), t_i(v)\}_{i=1}^n$ and a vector of recommendations of information choices $(\alpha_1^*, ..., \alpha_n^*)$ to maximize her expected sum of payment from all bidders,

$$\pi_s = \mathbb{E}_{v|\alpha_j=\alpha_j^*, \forall j} \sum_{i=1}^n t_i(v),$$

subject to (1), (2), (IR) and (IA), where the expectation is taken conditional on $\alpha_j = \alpha_j^*$ for all j.

3.4 Information Order

In order to analyze a model with general information structures, we need an information order to rank the informativeness of different signals. Since the distribution of v_i, $H(\cdot, \alpha_i)$, is uniquely determined by α_i, we would like to have an information order that directly ranks $H(\cdot, \alpha_i)$. The rotation order, recently introduced by Johnson and Myatt (2006), meets this requirement.

Definition 1 (Rotation Order)
The family of distributions $\{H(\cdot, \alpha_i)\}$ is rotation-ordered if there exists a rotation point v^+ such that

$$H_{\alpha_i}(v_i, \alpha_i) \geq 0 \text{ if } v_i < v^+, \quad \text{and } H_{\alpha_i}(v_i, \alpha_i) \leq 0 \text{ if } v_i > v^+, \tag{3}$$

for all α_i.

Consider two information choices α_i' and α_i'' with $\alpha_i' > \alpha_i''$. Then distribution $H(v_i, \alpha_i')$ dominates distribution $H(v_i, \alpha_i')$ in rotation order if

$$H(v_i, \alpha_i') \geq H(v_i, \alpha_i'') \text{ if } v_i < v^+, \text{ and } H(v_i, \alpha_i') \leq H(v_i, \alpha_i'') \text{ if } v_i > v^+.$$

Graphically, the rotation order requires that two rotation-ordered cumulative distributions cross each other only once, as illustrated in Figure 2. In particular, the distribution $H(v_i, \alpha_i'')$ crosses the distribution $H(v_i, \alpha_i')$ from below, and the density $h(v_i, \alpha_i')$ is more spread out.

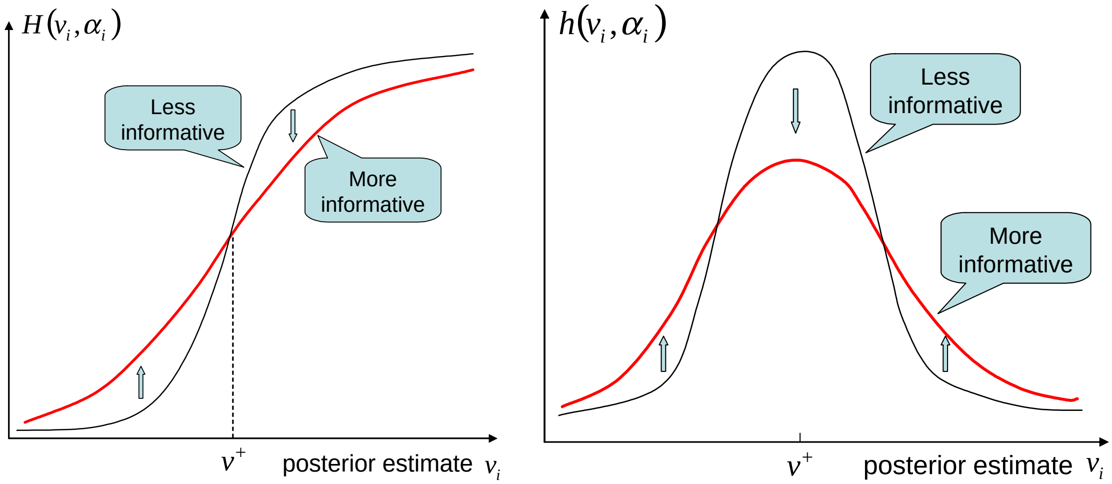

Figure 2. CDFs and PDFs of two rotation-ordered distributions of posterior estimate

The rotation order implies second-order stochastic dominance (see Theorem 3.A.44 in Shaked and Shanthikumar (2007)).[16] Note that bidder i's interim payoff $u(v_i)$ is convex in v_i under any incentive compatible mechanism $\{q_i(v), t_i(v)\}$ (see equation (1)). Therefore, if $\{H(\cdot, \alpha_i)\}$ is rotation-ordered and $\alpha_i' > \alpha_i''$, then a signal with α_i' is more informative than a signal with α_i'' in the sense that α_i' corresponds to a weakly higher expected payoff for bidder i.

Rotation-ordered information structures include two commonly used information technologies in the literature, relevant for later in the paper.

Example 1 (Gaussian Learning)
The buyers' valuations $\{\omega_i\}$ are independently drawn from a normal distribution with mean μ and precision β : $\omega_i \sim N(\mu, 1/\beta)$. Buyer i can observe a signal s_i: $s_i = \omega_i + \varepsilon_i$, where ε_i is independent of ω_i, and $\varepsilon_i \sim N(0, 1/\alpha_i)$. After observing s_i, buyer i forms his posterior estimate of ω_i:

$$v_i(s_i, \alpha_i) \equiv \mathbb{E}(\omega_i | s_i, \alpha_i) = \frac{\beta\mu + \alpha_i s_i}{\alpha_i + \beta}.$$

[16] The reverse is not true: two distributions ordered in terms of second-order stochastic dominance can cross each other more than once.

It follows that the distribution of v_i, $H(v_i, \alpha_i)$, *is normal:* $v_i \sim N(\mu, \sigma^2(\alpha_i))$, *with variance* $\sigma^2(\alpha_i) = \alpha_i / ((\alpha_i + \beta)\beta)$ *increasing in* α_i. *It is easy to verify that*

$$H_{\alpha_i}(v_i, \alpha_i) = -\frac{\beta(v_i - \mu)}{2\alpha_i(\alpha_i + \beta)} h(v_i, \alpha_i), \tag{4}$$

which means that the criterion (3) is satisfied with $v^+ = \mu$ *for all* α_i.

Example 2 (Truth-or-Noise)
The buyers' valuations $\{\omega_i\}$ *are independently drawn from a distribution* F, *and* F *has an increasing hazard rate. Buyer* i *can acquire a costly signal* s_i *about* ω_i. *With probability* $\alpha_i \in [0, 1]$, *the signal* s_i *perfectly matches the true valuation* ω_i, *and with probability* $1 - \alpha_i$, s_i *is noise independently drawn from* F. *This information structure is referred to as "truth-or-noise" in Lewis and Sappington (1994) and Johnson and Myatt (2006). A buyer who observes* s_i *with precision* α_i *will calculate his posterior estimate as:*

$$v_i(s_i, \alpha_i) \equiv \mathbb{E}(\omega_i | s_i, \alpha_i) = \alpha_i s_i + (1 - \alpha_i)\mu.$$

Therefore, we have

$$H(v_i, \alpha_i) = F\left(\frac{v_i - (1 - \alpha_i)\mu}{\alpha_i}\right),$$

and

$$H_{\alpha_i}(v_i, \alpha_i) = -\frac{v_i - \mu}{\alpha_i} h(v_i, \alpha_i). \tag{5}$$

Thus, $H(v_i, \alpha_i)$ *satisfies condition (3) with* $v^+ = \mu$ *for all* α_i.

To ease our exposition, from now on, we will make the following assumption:

Assumption 1 (Rotation Order around μ)
The family of distributions of the posterior estimates $\{H(\cdot, \alpha_i)\}$ *is rotation-ordered and the rotation point is* μ *for all* α_i.

Note that the above assumption does not require the underlying distribution F to be symmetric. For example, for the truth-or-noise technology, the underlying distribution F of ω_i could be convex or concave, but the rotation point is still μ. Throughout the paper, we also assume the family of distributions $\{H(v_i, \alpha_i)\}$ satisfies the following regularity condition which is standard in the mechanism design literature:

Assumption 2 (Regularity)

$$v_i - [1 - H(v_i, \alpha_i)] / h(v_i, \alpha_i) \text{ is strictly increasing in } v_i \text{ for all } \alpha_i \text{ and } v_i.$$

For some of our results, we also impose the following additional assumption:

Assumption 3 (Supermodularity)

$$-H_{\alpha_i}(v_i, \alpha_i)/h(v_i, \alpha_i) \text{ is strictly increasing in } v_i \text{ for all } \alpha_i \text{ and } v_i.$$

This assumption is stronger than the rotation order assumption.[17] It has a natural interpretation in terms of the inverse of $H(v_i, \alpha_i)$: the posterior estimate v_i conditional on the signal s_i being its p-th percentile is supermodular in p and α_i.[18] Ganuza and Penalva (2010) develop an information order called "supermodular precision" based on this stronger requirement and provide additional interpretation. Readers can find in their paper a list of information technologies that satisfy this information order. We can directly verify from expression (4) and (5) that both the Gaussian learning and the truth-or-noise technologies satisfy these three assumptions.

4 Optimal Auctions with One Bidder

We start with a simple model with one buyer, which is a special case of the general model we study later. In this case, posted price mechanisms are optimal,[19] so we can interpret the seller's optimization problem as a monopoly pricing problem with endogenous information.

For a fixed reserve price r, the expected payoff for the buyer with choice α_i is

$$\pi_i(\alpha_i, r) \equiv \int_r^{\overline{\omega}} (v_i - r)\, h(v_i, \alpha_i)\, dv_i - C(\alpha_i).$$

Using integration by parts, we can rewrite it as

$$\pi_i(\alpha_i, r) = \int_r^{\overline{\omega}} [1 - H(v_i, \alpha_i)]\, dv_i - C(\alpha_i).$$

It follows that

$$\frac{\partial^2 \pi_i(\alpha_i, r)}{\partial r \partial \alpha_i} = H_{\alpha_i}(r, \alpha_i). \tag{6}$$

[17] Indeed, the supermodularity assumption, together with the mean-preserving property of our information structures, implies the rotation order.

[18] To see this, define $p \equiv H(v_i(p, \alpha_i), \alpha_i)$. Then $v_i(p, \alpha_i) = H^{-1}(p, \alpha_i)$ and

$$\frac{\partial v_i}{\partial \alpha_i}|_p = -\frac{H_{\alpha_i}(v_i, \alpha_i)}{h(v_i, \alpha_i)}.$$

Thus, the supermodularity assumption is equivalent to the supermodularity of v_i in p and α_i.

[19] As shown in the next section, after incorporating the information acquisition constraint, the seller's objective function will be the Lagrangian specified in (17). If there is only one bidder, it reduces to a simple form similar to the one analyzed in Riley and Zeckhauser (1983). Therefore, their proof of the optimality of the posted price mechanism still applies here.

Given Assumption 1, $\{H(\cdot,\alpha_i)\}$ is rotation-ordered around μ, so $\pi_i(\alpha_i, r)$ is supermodular when $r < \mu$ and is submodular when $r > \mu$. That is, the marginal value information to the buyer is increasing in r when $r < \mu$ and is decreasing in r when $r > \mu$. Topkis's theorem then implies that the buyer's optimal information choice $\alpha_i(r)$ is increasing in r if $r < \mu$ and is decreasing in r if $r > \mu$. In other words, the buyer acquires *more* information when the posted price r is *closer* to μ. It is worth pointing out that the rotation order assumption is necessary and sufficient for this result.

The seller chooses r and a recommendation α^* to maximize her revenue:

$$\max_{r,\alpha^*} \ \{\pi_s(\alpha^*, r) \equiv r(1 - H(r,\alpha^*))\} \tag{7}$$

subject to

$$\alpha^* \in \arg\max_{\alpha_i} \pi_i(\alpha_i, r). \tag{8}$$

Therefore, we have

$$\frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} = -rH_{\alpha_i}(r,\alpha^*). \tag{9}$$

Since $\{H(\cdot,\alpha^*)\}$ is rotation-ordered, $\pi_s(\alpha^*, r)$ is increasing in α^* for all $r > \mu$ and decreasing in α^* for all $r < \mu$.

Before stating our main result in this section, we first define the *standard reserve price* r_{α^*} which we will use as our benchmark.

Definition 2 (Standard Reserve Price)
The standard reserve price r_{α^*} *is defined as*

$$r_{\alpha^*} \in \arg\max_r r[1 - H(r,\alpha^*)].$$

That is, r_{α^*} *solves*

$$r_{\alpha^*} - \frac{1 - H(r_{\alpha^*},\alpha^*)}{h(r_{\alpha^*},\alpha^*)} = 0.$$

Therefore, the standard reserve price r_{α^*} is the solution to the seller's maximization problem (7) for a fixed α^*, without imposing the information acquisition constraint (8). In other words, it is the optimal reserve price that the seller would have charged if the buyer's information α^* were *exogenously* endowed. Given our Assumption 2, r_{α^*} is uniquely defined for each α^*. Let r^* denote the optimal reserve price when the equilibrium level of information α^* is *endogenously* chosen by the buyer, that is, r^* solves the seller's problem (7) subject to the constraint (8). We compare r^* with r_{α^*} to illustrate how the seller's pricing strategy responds to endogenous information or the information acquisition constraint (8) she faces.

Proposition 1 (Monopoly Pricing)
Under Assumption 1 and 2, $r^* \leq r_{\alpha^*}$.

To understand this result, we can decompose the effect of a price increase on the seller's profits in three parts:

$$\frac{d\pi_s}{dr} = \underbrace{1 - H(r, \alpha^*)}_{A} + \underbrace{[-rh(r, \alpha^*)]}_{B} + \underbrace{\left[\frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} \frac{\partial \alpha^*(r)}{\partial r}\right]}_{C}.$$

First, the seller's profits increase given that a trade is made (term A). Second, for a fixed information choice, a price increase reduces the probability of trade (term B). Third, with endogenous information acquisition, a price increase affects the buyer's incentive to acquire information, thereby affecting the probability of trade (term C). The first two terms are standard, while the last one is specific to the setting with endogenous information acquisition.

We now argue that the term C is nonpositive for all r, so that the marginal gain to the seller from raising price r is always smaller than the gain in a setting with exogenous information. To see this, note that if $r > \mu$ then $\partial \pi_s(\alpha^*, r)/\partial \alpha^* \geq 0$, and an increase in r discourages information acquisition: $\partial \alpha^*/\partial r \leq 0$; if $r < \mu$ then $\partial \pi_s(\alpha^*, r)/\partial \alpha^* \leq 0$, and the buyer's incentives to gather information are higher for a higher r: $\partial \alpha^*/\partial r \geq 0$. Therefore, $\partial \pi_s(\alpha^*, r)/\partial \alpha^*$ and $\partial \alpha^*/\partial r$ have the opposite sign for all $r \neq \mu$. If $r = \mu$, $\partial \pi_s(\alpha^*, r)/\partial \alpha^* = 0$. Hence, the term C is nonpositive for all r.

We prove $r^* \leq r_{\alpha^*}$ by showing that $d\pi_s(r)/dr < 0$ for all $r > r_{\alpha^*}$. By the definition of r_{α^*} and Assumption 2, we have, for all $r > r_{\alpha^*}$,

$$1 - H(r, \alpha^*) - rh(r, \alpha^*) = \left(\frac{1 - H(r, \alpha^*)}{h(r, \alpha^*)} - r\right) h(r, \alpha^*) < 0. \tag{10}$$

Since the term C is nonpositive for all r, inequality (10) implies $d\pi_s(r)/dr < 0$ for all $r > r_{\alpha^*}$.

To conclude this section, we point out that the rotation order assumption is not necessary for Proposition 1. Indeed, one can see from (6) and (9) that, even if the information structure is not rotation-ordered, the effect of a price increase on the marginal value of information to the buyer has the opposite sign of the effect of an increase in signal precision on the seller's profits. Therefore, one can show, via the implicit function theorem, that $\partial \pi_s(\alpha^*, r)/\partial \alpha^*$ and $\partial \alpha^*/\partial r$ have the opposite sign and thus the term C is nonpositive. However, this alternative approach requires that the buyer's payoff $\pi_i(\alpha_i, r)$ be concave in α_i for all r, which is not needed when we apply Topkis's theorem.

5 Optimal Auctions with Many Bidders

In the one-bidder model, there is no strategic interaction among bidders, and the simple posted price mechanisms are optimal. In the case of many bidders, the posted-price mechanisms are no longer optimal, and the combination of moral hazard and adverse selection makes the general analysis very complicated.[20] Thus, some restrictions on the structure of model are necessary in order to characterize the optimal selling mechanism.

In this section, we restrict attention to the class of symmetric mechanisms that induce all bidders to acquire the same level of information. We show that, with many bidders, standard auctions are optimal but the reserve price has to be adjusted towards the mean valuation of bidders. In the next section, we drop the restriction on symmetric mechanisms and focus on the case when information acquisition is binary. There, we demonstrate that the optimal asymmetric mechanism softens the price discrimination against (stochastically) strong bidders compared to the case with exogenous information. The analysis in these two sections, together with the analysis in the one-bidder case, helps clarify how the optimal mechanism should respond to endogenous information and strategic interactions among bidders.

In what follows, we first analyze information acquisition in standard auctions, where strategic interactions among bidders play an important role. The result is then used to determine the seller's preference in optimal auctions. Next, we show that standard auctions with an adjusted reserve price are optimal within the class of symmetric mechanisms. We also derive a necessary and sufficient condition under which the bidders' incentives to acquire information are socially excessive.

5.1 Standard Auctions

This subsection analyzes buyers' information decisions and the value of information to the seller in standard auctions with a reserve price. The result obtained here serves to facilitate the analysis of optimal auctions in the next subsection. Since bidders are ex ante symmetric, we focus on the symmetric equilibrium where all bidders acquire the same level of information: $\alpha_j = \alpha^*$ for all j.

Consider the information decision of bidder i in any standard auction. Suppose bidders other than i choose α^* and bid according to some monotone equilibrium bidding function $b^*(\cdot)$. Then bidder i with posterior estimate v_i will bid $b^*(v_i)$, regardless of his information choice α_i, since his investment in information is sunk and covert. Therefore, the bidder i's expected payoff by choosing α_i is given by[21]

$$\begin{aligned}\pi_i(\alpha_i, r) &= H^{n-1}(r, \alpha^*) \int_r^{\overline{\omega}} (v_i - r)\, dH(v_i, \alpha_i) \\ &\quad + \int_r^{\overline{\omega}} \int_x^{\overline{\omega}} (v_i - x)\, dH(v_i, \alpha_i)\, dH^{n-1}(x, \alpha^*) - C(\alpha_i). \qquad (11)\end{aligned}$$

[20]We will elaborate on the nature of the technical difficulty shortly.

[21]The support of v_i, say $\left[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}\right]$, could vary with respect to information choice α_i. However, we can always define $H(v_i, \alpha_i) = 0$ for $v_i \in \left[\underline{\omega}, \underline{\omega}_{\alpha_i}\right]$ and $H(v_i, \alpha_i) = 1$ for $v_i \in [\overline{\omega}_{\alpha_i}, \overline{\omega}]$. Then we can treat the domain of v_i as $[\underline{\omega}, \overline{\omega}]$.

We can apply integration by parts to the second term and simplify $\pi_i(\alpha_i, r)$ into

$$\pi_i(\alpha_i, r) = \int_r^{\overline{\omega}} H^{n-1}(v_i, \alpha^*)[1 - H(v_i, \alpha_i)] dv_i - C(\alpha_i). \tag{12}$$

Hence, we have

$$\frac{\partial^2 \pi_i(\alpha_i, r)}{\partial r \partial \alpha_i} = H^{n-1}(r, \alpha^*) H_{\alpha_i}(r, \alpha_i). \tag{13}$$

Given our rotation order assumption, Topkis's theorem again implies that $\alpha_i(r)$ is increasing in r if $r < \mu$ and decreasing in r if $r > \mu$. Comparing (13) to (6), we can see that the bidder competition does *not* qualitatively affect how a bidder's incentive responds to changes in the reserve price.

In contrast, the seller's preference for information may vary with the number of bidders. In a standard auction with reserve price r and information choice α^*, the seller's revenue is given by

$$\pi_s(\alpha^*, r) = r[1 - H(r, \alpha^*)^n] + \int_r^{\overline{\omega}} \left[1 - nH(v_i, \alpha^*)^{n-1} + (n-1) H(v_i, \alpha^*)^n\right] dv_i.$$

The seller prefers more information if $\partial \pi_s(\alpha^*, r) / \partial \alpha^* > 0$, where

$$\begin{aligned} \frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} &= -nrH(r, \alpha^*)^{n-1} H_{\alpha_i}(r, \alpha^*) \\ &\quad -n(n-1) \int_r^{\overline{\omega}} H(v_i, \alpha^*)^{n-2} [1 - H(v_i, \alpha^*)] H_{\alpha_i}(v_i, \alpha^*) dv_i. \end{aligned} \tag{14}$$

Proposition 2 (Value of Information to the Seller)
Suppose information α^* is exogenously given, and Assumption 1 and 2 hold. Further assume that there exists a $\delta \in (\mu, \overline{\omega})$ such that

$$\min_{v_i \in [\delta, \overline{\omega}]} -\frac{H_{\alpha_i}(v_i, \alpha^*)}{h(v_i, \alpha^*)} = \gamma, \tag{15}$$

for some $\gamma > 0$. Then there exists n^* such that $\partial \pi_s(\alpha^*, r) / \partial \alpha^* > 0$ for all r and $n \geq n^*$.

Condition (15) requires that the upper tail of $H_{\alpha_i}(v_i, \alpha^*)$ be bounded away from zero. It is trivially satisfied if we impose Assumption 3. The intuition for Proposition 2 is as follows: when more information (a higher α^*) is acquired, the variability or spread of $H(\cdot, \alpha^*)$ increases, which has two effects on the seller's revenue. First, it affects the probability of trade, $1 - H(r, \alpha^*)^n$. Second, it also affects the seller's revenue conditional on trade, $\max\{r, V_{n-1,n}\}$, where $V_{n-1,n}$ denotes the second highest order statistic among n samples from distribution $H(\cdot, \alpha^*)$. When $r > \mu$, both effects go in the same direction, so the seller's revenue is higher with a higher α^*. When $r < \mu$, the two effects may go in the opposite direction, but, as long as n is not too small, the probability of $V_{n-1,n} > r$ is close to 1 and the reserve price is almost never binding. Furthermore, for a sufficiently large n, an increase in the variability of $H(\cdot, \alpha^*)$ always increases $\mathbb{E}[V_{n-1,n}]$. Therefore, the second effect is positive and dominates the first one, and a seller would prefer a higher α^*.

It follows that, in standard auctions with $n \geq n^*$, the seller should set the optimal reserve price r^* closer to μ than the standard reserve price r_{α^*} to induce bidders to acquire more information.

5.2 Optimal Symmetric Mechanisms

In this subsection, we restrict our attention to *symmetric* selling mechanisms with $\alpha_1^* = ... = \alpha_n^* = \alpha^*$, and assume $\alpha^* \in (\underline{\alpha}, \overline{\alpha})$. We will show that standard auctions with an appropriately chosen reserve price are optimal.

Using (1) and integration by parts, we can rewrite the information acquisition constraint (IA) as

$$\alpha_i^* \in \arg\max_{\alpha_i} \mathbb{E}_{v_{-i}|\alpha_{-i}=\alpha_{-i}^*} \left\{ \int_{\underline{\omega}}^{\overline{\omega}} [1 - H(v_i, \alpha_i)] \, q_i(v_i, v_{-i}) \, dv_i - C(\alpha_i) \right\}.$$

We adopt the standard first-order approach (Mirrlees (1999), and Rogerson (1985)) to replace it by its first-order condition:

$$-\mathbb{E}_{v_{-i}|\alpha_{-i}=\alpha_{-i}^*} \int_{\underline{\omega}}^{\overline{\omega}} H_{\alpha_i}(v_i, \alpha_i^*) \, q_i(v_i, v_{-i}) \, dv_i - C'(\alpha_i^*) = 0. \tag{16}$$

The first-order approach is valid if the second-order condition of the bidders' optimization problem is satisfied, which we will assume for now and relegate detailed discussions to Section 7 and Appendix B. In principle, the equilibrium information choices could be different for different agents, so there is a system of n first-order conditions, one for each bidder.

We use the Lagrangian approach to incorporate the n first-order conditions. As in the standard moral hazard model, the main difficulty lies in the determination of the sign of the Lagrange multiplier of these first-order conditions (Rogerson (1985)). The seller's maximization problem here is, however, substantially more complicated in three ways. First, we have n agents and n first-order conditions. Second, unlike in the standard moral hazard model where higher effort always benefits the principal if it is costless to induce effort, more information here may hurt the seller, as we can see from the one-bidder case. Finally, the seller has to give bidders not only incentives to acquire information, but also incentives to tell the truth, that is, our model is a mixed model with moral hazard and adverse selection. As such, some restrictions on the model are necessary in order to characterize the optimal selling mechanism.

The symmetric restriction $\alpha_1^* = ... = \alpha_n^* = \alpha^*$ we impose in this subsection helps reduce the system of first-order conditions to a single equation (16). Replacing the incentive constraint by equation (1) and (2), and replacing the (IA) constraint by (16), we can transform the seller's optimization problem from the allocation-transfer space into the allocation-utility space:

$$\max_{q_i, u_i(\underline{\omega}), \alpha^*} \left\{ \mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left[\left(v_i - \frac{1 - H(v_i, \alpha^*)}{h(v_i, \alpha^*)} \right) q_i(v_i, v_{-i}) \right] - n u_i(\underline{\omega}) \right\},$$

subject to

$$0 \leq q_i(v_i, v_{-i}) \leq 1; \sum_{i=1}^{n} q_i(v_i, v_{-i}) \leq 1, \qquad \text{(allocation)}$$

$$Q_i(v_i) \text{ is weakly increasing in } v_i, \qquad \text{(monotonicity)}$$

$$u_i(\underline{\omega}) \geq 0, \qquad \text{(IR)}$$

$$\mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \left[-\frac{H_{\alpha_i}(v_i, \alpha^*)}{h(v_i, \alpha^*)} q_i(v_i, v_{-i}) \right] - C'(\alpha^*) = 0. \qquad \text{(IA-FOC)}$$

It is easy to see that the (IR) constraint must be binding. For now we ignore the allocation constraint and the monotonicity constraint, and verify that they are satisfied later. Then the only remaining constraint is the (IA-FOC) constraint. Let λ denote the Lagrange multiplier for the (IA-FOC) constraint, and write the Lagrangian for the seller's maximization problem as

$$\mathcal{L} = \mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left[\left(v_i - \frac{1 - H(v_i, \alpha^*)}{h(v_i, \alpha^*)} - \frac{\lambda}{n} \frac{H_{\alpha_i}(v_i, \alpha^*)}{h(v_i, \alpha^*)} \right) q_i(v_i, v_{-i}) \right] - \lambda C'(\alpha^*). \qquad (17)$$

A positive λ means that the seller benefits from a deduction in the marginal cost of information. Therefore, the virtual surplus function $J^*(v_i)$ is given by

$$J^*(v_i) = v_i - \frac{1 - H(v_i, \alpha^*)}{h(v_i, \alpha^*)} - \frac{\lambda}{n} \frac{H_{\alpha_i}(v_i, \alpha^*)}{h(v_i, \alpha^*)}. \qquad (18)$$

In order to characterize the optimal symmetric auction, we first need to identify the seller's information preferences – that is, the sign of the Lagrange multiplier λ for the (IA-FOC) constraint. It turns out that this is the difficult part of the analysis. We use the technique in Rogerson (1985) to sign λ: we first relax the (IA-FOC) constraint to an inequality constraint, characterize the optimal solution of the relaxed problem, and then verify that (IA-FOC) constraint is binding in the optimal solution if the seller's revenue in standard auctions is increasing in the level of information.

Lemma 1 (Lagrange Multiplier)
Suppose the first-order approach is valid, and Assumptions 1, 2 and 3 hold. The seller benefits from a reduction in the marginal cost ($\lambda > 0$) if the seller's revenue $\pi_s(\alpha, r)$ in standard auctions is strictly increasing in α for all r.

Note that, under Assumption 1-3 and for a sufficiently large n, the seller's revenue $\pi_s(\alpha, r)$ in standard auctions is strictly increasing in α for all r according to Proposition 2, and thus the seller will prefer more information in the optimal auction setting with endogenous information. The next result shows that any standard auction with an appropriately chosen reserve price is optimal.

Proposition 3 (Optimal Symmetric Mechanism)
Suppose the first-order approach is valid, and Assumptions 1, 2 and 3 hold. If the seller's revenue $\pi_s(\alpha, r)$ in standard auctions is strictly increasing in α for all r, then any standard auction with reserve price r^* is optimal, where $r^* \equiv \min\{r : J^*(r) \geq 0\}$. Moreover, r^* is closer to μ than r_{α^*}. Specifically, if $r_{\alpha^*} < \mu$ then $r_{\alpha^*} \leq r^* < \mu$; if $r_{\alpha^*} = \mu$ then $r^* = \mu$; and if $r_{\alpha^*} > \mu$ then $\mu < r^* \leq r_{\alpha^*}$.

With many bidders, the optimal reserve price r^* is closer to the mean μ than the standard reserve price r_{α^*}, in sharp contrast to the one-bidder case where r^* is always below r_{α^*}. This is because competition among bidders changes the seller's preference over information: as we demonstrate in Proposition 2, when there are many bidders, the seller prefers more information for all levels of reserve prices. In contrast, in the one-bidder case, the seller prefers more information only when the reserve price is above the mean μ.

The identified simple rule of adjusting the reserve price is of practical importance since a key element of the auction design is to determine the reserve price. It provides a useful guidance for the seller to set the optimal reserve price when the bidders' incentives to acquire information are an important concern.

This result also has important implications for empirical analysis. The empirical auction literature has attempted to evaluate the optimality of a seller's reserve price policy from observed bids. Most of these studies assume exogenous information and do not consider the bidders' incentives to acquire information. They use observed bids and equilibrium bidding behavior to recover the distribution of bidders' valuations, and then compare the actual reserve price with the standard reserve price calculated from the estimated distribution. Our results indicate that it may be preferable to use the optimal reserve price, instead of the standard reserve price, as the benchmark when the bidders' incentives to acquire information are important.

Indeed, our analysis may help reconcile the discrepancy between the reserve price predicted by a theory model with exogenous information and the reserve price charged in practice. For example, Paarsch (1997) uses the framework of independent private values to estimate the bidders' valuation distribution for a sample of timber sales held in British Columbia, Canada. The estimated distribution is then used to calculate the reserve price (also known as the upset price in practice). He finds that the actual reserve price is much lower than the calculated reserve price and concludes that "...the Forest Service was too lenient in the setting of the reserve price for timber"(page 352). The independent private values framework is also adopted in Haile and Tamer (2003). They use data from U.S. Forest Service timber auctions to estimate bounds for the bidders' valuation distribution which are then used to construct bounds for the reserve price. Their calculated lower bound is still slightly higher than the average actual reserve price, so they also conclude that the actual reserve price set by U.S Forest Service is well below the optimum.

Our results suggest that the low reserve price policy may be partially justified if information acquisition is important. Indeed, the bidders' incentives to gather information may be important in timber auctions. For example, firms occasionally conduct cruises themselves to obtain information additional to the public timber-cruise report, and they may also have to spend resources to gather and evaluate information regarding their idiosyncratic future demands for end products, contracts for future sales, and inventories of end products and uncut timber from other sales. More importantly, as reported in Paarsch (1997), the mean of the estimated valuation distribution for his sample is negative. Our simple rule then implies that the optimal reserve price is adjusted *downward* toward the (negative) mean valuation. Therefore, it could be ex ante optimal for the Forest Service to deliberately set a low reserve price to induce more information acquisition.

To conclude this subsection, we compare the bidders' incentive to acquire information to the social incentive to acquire information in standard auctions with a reserve price. The social planner chooses α_i to maximize the social surplus

$$\int_0^{\overline{\omega}} \left(1 - H\left(v_i, \alpha_i\right)^n\right) dv_i - nC\left(\alpha_i\right).$$

So the marginal *social* value of information at α_i is

$$-n \int_0^{\overline{\omega}} H_{\alpha_i}\left(v_i, \alpha_i\right) H\left(v_i, \alpha_i\right)^{n-1} dv_i.$$

On the other hand, we can derive the marginal *individual* value of information from equation (12):

$$-\int_r^{\overline{\omega}} H_{\alpha_i}\left(v_i, \alpha_i\right) H\left(v_i, \alpha_i\right)^{n-1} dv_i.$$

Since the social planner has to pay n times the individual information cost, we normalize the social value of information by multiplying $1/n$. The difference between the social and individual gain from acquiring information is

$$\Delta\left(\alpha_i, n\right) = -\int_0^r H_{\alpha_i}\left(v_i, \alpha_i\right) H\left(v_i, \alpha_i\right)^{n-1} dv_i. \tag{19}$$

Under rotation-ordered information structures, if $r < \mu$, then $\Delta\left(\alpha_i, n\right) < 0$. That is, information acquisition in auctions with $r < \mu$ is socially excessive.[22] Thus, we have proved the

[22]However, the rotation order assumption is not necessary for this result.

following result:

Proposition 4 (Informational Efficiency)
Suppose Assumption 1 holds. For standard auctions with reserve price r, there exists $\delta > 0$ such that bidders have socially excessive incentives to acquire information if and only if $r < \mu + \delta$.

When $r = 0$, the bidders' incentive to acquire information coincides with the social optimum, which can be easily seen from equation (19).[23] As r increases, the buyers' incentive to acquire information increases, reaches maximum at $r = \mu$, and declines thereafter. Consequently, there exists a $\delta > 0$ such that the individual incentive to acquire information coincides with the social optimum when $r = \mu + \delta$. Therefore, the bidders' incentive to acquire information is socially excessive when $r \in (0, \mu + \delta)$. For the Gaussian specification, $\delta = \mu$.

6 Optimal Asymmetric Mechanisms

In this section, we derive the optimal mechanisms with *discrete* information acquisition without imposing symmetry restrictions. This is the limiting case when the information cost consists of a lump-sum fixed cost and a very small marginal cost. We assume that bidders are ex ante symmetric and endowed with a signal with precision α_0. Each bidder can opt to receive a signal α_1 that is more informative than signal α_0 in terms of our rotation order, but he has to incur a lump-sum cost k. The distribution of bidder i's posterior estimates v_i is denoted by $H_0(\cdot)$ if he does not acquire information, and $H_1(\cdot)$ if he acquires information. Let h_0 and h_1 denote the corresponding densities. We assume that H_0 and H_1 satisfy the regularity assumption: both $v_i - [1 - H_1(v_i)]/h_1(v_i)$ and $v_i - [1 - H_0(v_i)]/h_0(v_i)$ are strictly increasing in v_i.

Without loss of generality, suppose that the seller wants to induce the first m bidders ($0 \leq m \leq n$) to acquire additional information. The seller sends a recommendation – "acquire information" – to each of the first m bidders, and sends a recommendation – "do not acquire information" to the remaining bidders. After receiving the seller's recommendation, bidders decide whether to follow, and after receiving their signals, bidders form their posterior estimate and report to the seller. By the revelation principle, we can restrict to the direct revelation mechanism $\{q_i(v), t_i(v)\}$. In order to ensure that bidders participate and report truthfully in the second stage, the mechanism must satisfy the standard (IC) and

[23] This is consistent with the results in Bergemann and Välimäki (2002): the individual incentives to acquire information coincide with the social optimum for efficient mechanisms in the private value setting.

(IR) constraints, namely

$$\begin{aligned} u_i(v_i) &= u_i(\underline{\omega}) + \int_{\underline{\omega}}^{v_i} Q_i(x)\,dx \text{ and } Q_i(v_i) \text{ is weakly increasing in } v_i, \text{ for all } i \text{ and } v_i, \\ u_i(v_i) &\geq 0, \text{ for all } i \text{ and } v_i. \end{aligned}$$

Moreover, in order to ensure that bidders follow the seller's recommendation in the first stage, the information acquisition (IA) constraints must be satisfied:

$$\int_{\underline{\omega}}^{\overline{\omega}} u_i(v_i)\,dH_1(v_i) - \int_{\underline{\omega}}^{\overline{\omega}} u_i(v_i)\,dH_0(v_i) \geq k, \text{ for } i \leq m, \tag{20}$$

$$\int_{\underline{\omega}}^{\overline{\omega}} u_i(v_i)\,dH_1(v_i) - \int_{\underline{\omega}}^{\overline{\omega}} u_i(v_i)\,dH_0(v_i) \leq k, \text{ for } i > m. \tag{21}$$

That is, the mechanism has to ensure that the first m bidders have incentives to acquire information and the remaining $(n-m)$ bidders have incentives *not* to acquire information.

Consider an informed bidder $i \leq m$ and an uninformed bidder $j > m$. The distributions of the posterior estimates of bidder i and j are H_1 and H_0, respectively. The standard reserve prices r_0 and r_1 are defined as before:

$$r_0 - \frac{1 - H_0(r_0)}{h_0(r_0)} = 0 \text{ and } r_1 - \frac{1 - H_1(r_1)}{h_1(r_1)} = 0.$$

Since the information structures are rotation-ordered,

$$\begin{array}{lll} H_0(x) \geq H_1(x) & \text{if} & x > v^+ \\ H_0(x) \leq H_1(x) & \text{if} & x < v^+ \end{array},$$

where v^+ is the rotation point.

Now conditional on the posterior estimate $x > v^+$, informed bidder i is stochastically "stronger" than bidder j in the sense that the distribution of v_i first-order stochastically dominates the distribution of v_j, but conditional on $x < v^+$, uninformed bidder j is stochastically "stronger" than bidder i. Myerson (1981) demonstrates that the optimal auction should discriminate against "strong" bidders. Interestingly, endogenous information acquisition reduces the level of ex post discrimination against "strong" bidders, as shown in the following proposition:

Proposition 5 (Asymmetric Optimal Mechanism)
Suppose information acquisition is binary, and H_0 and H_1 are rotation-ordered. Then the optimal reserve price for informed bidders, r_1^*, lies between r_1 and v^+. In the optimal mechanism, the level of price discrimination against "strong" bidders is weaker, compared to Myerson's discriminatory auctions.

The first part of the proposition shows that the optimal rule for adjusting the reserve price identified in Proposition 3 is still valid in this discrete setting. The second part suggests that the seller should soften price discrimination in order to provide bidders with appropriate incentives either to or not to acquire information.

Example 3 (Discriminatory Auctions)
Suppose four bidders compete for an object. Bidders' true valuations are unknown ex-ante and are i.i.d. from $F(x) = \sqrt{x}$ *on* $[0, 1]$*. If a bidder incurs a cost* $k = 0.1$*, he observes his true valuation; otherwise he maintains his prior. Then the optimal selling mechanism is the following: recommend three bidders to become informed, run a second-price auction with* $r_1^* = 0.358$ *among these three bidders, and if unsold, sell the object to the fourth bidder at* $1/3$*. Note that* r_1^* *is between the mean* $(1/3)$ *and the standard reserve price* $r_1 = 4/9$ *in Myerson's discriminatory auctions.*

A key feature of the design problem here is that the information acquisition constraints take the simple form of inequalities. Thus, the Lagrange multipliers associated with these constraints are non-negative, which substantially simplifies the analysis. In contrast, in the general setting with continuous information acquisition, the information acquisition constraints are much more complicated and make the analysis less tractable without imposing symmetry restrictions.

7 Concluding Remarks

Most of the mechanism design literature ignores the influence of the proposed mechanisms on agents' incentives to gather information. In particular, with endogenous information acquisition, the optimal selling mechanism should take into account the bidders' information gathering decision as a response to the proposed mechanism. This paper provides a general framework for studying the mechanism design problem with information acquisition. We solve the problem in three important classes of applications. First, we show that, with a single buyer, the optimal monopoly price is always lower than the standard monopoly price. Second, we show that, with many buyers, standard auctions with an adjusted reserve price can be optimal among symmetric mechanisms. Third, we show that the optimal selling mechanism with binary information acquisition employs a weaker price discrimination against "strong" bidders.

In the paper we use the rotation order to rank signals, which is not necessary for some of our results, for instance, Proposition 1 and 4. Since a buyer's interim payoff is convex under any feasible incentive compatible mechanism, it would be ideal if we order signals according to the second-order stochastic dominance relation. Unfortunately, the relation of second-order stochastic dominance is not strong enough, for example, to establish Proposition 3 and 5. The rotation order, though slightly stronger than second-order stochastic dominance, is analytically more tractable, especially in the case with many bidders.

To facilitate our analysis, we adopt the first-order approach to deal with the bidders' information acquisition constraints. In Appendix B, we provide several sets of sufficient conditions for the first-order approach to be valid. First, it is satisfied if the cost function is sufficiently convex. Second, if the support of $H(\cdot, \alpha_i)$ is invariant with respect to α_i, then a condition analogous to the CDFC condition in the principal-agent literature (Mirrlees (1999), and Rogerson (1985)) is sufficient. Third, we present sufficient conditions for the cases of the Gaussian learning and the truth-or-noise technology, respectively.

As pointed out by Bolton and Dewatripoint (2005), the requirement that the bidders' first-order condition be necessary and sufficient is too strong. All we need is that the replacement of the (IA) constraint by the first-order condition can generate necessary conditions for the seller's original maximization problem. Indeed, for the monopoly pricing setting, we can avoid the first-order approach by applying Topkis's theorem.

Appendix A: Omitted Proofs

Proof of Proposition 2. Recall that

$$\begin{aligned}\frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} &= -nrH(r,\alpha^*)^{n-1} H_{\alpha_i}(r,\alpha^*) \\ &\quad -n(n-1)\int_r^{\overline{\omega}} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i.\end{aligned}$$

Given that the family of distributions $\{H(\cdot,\alpha)\}$ is rotation ordered, it is easy to see that $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > 0$ for $r \geq \mu$. It remains to show that $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > 0$ for $r < \mu$.

If $r < \mu$, we have

$$\begin{aligned}\frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} &= -nrH(r,\alpha^*)^{n-1} H_{\alpha_i}(r,\alpha^*) \\ &\quad -n(n-1)\int_r^{\mu} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i \\ &\quad -n(n-1)\int_\mu^{\overline{\omega}} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i\end{aligned}$$

Since $H(r,\alpha^*)$ is strictly below 1, $nH(r,\alpha^*)^{n-1}$ goes to 0, as n goes to infinity. Therefore, as n increases, the first term vanishes.

The second term can be rewritten as

$$\begin{aligned}& -n(n-1)\int_r^{\mu} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i \\ = \;& -n(n-1)\int_r^{\mu} \left[H(v_i,\alpha^*)^{n-2} - H(v_i,\alpha^*)^{n-1}\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i \\ > \;& -n(n-1)\int_r^{\mu} \left[H(\mu,\alpha^*)^{n-2} - H(r,\alpha^*)^{n-1}\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i \\ = \;& -\left[n(n-1)H(\mu,\alpha^*)^{n-2} - n(n-1)H(r,\alpha^*)^{n-1}\right]\int_r^{\mu} H_{\alpha_i}(v_i,\alpha^*)\,dv_i\end{aligned}$$

Since both $H(\mu,\alpha^*)$ and $H(r,\alpha^*)$ are strictly below 1, both $n(n-1)H(r,\alpha^*)^{n-2}$ and $n(n-1)H(r,\alpha^*)^{n-1}$ go to 0, as n goes to infinity. Therefore, as n increases, the second term also vanishes.

Now consider the last term

$$
\begin{aligned}
& -n(n-1)\int_{\mu}^{\overline{\omega}} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i \\
> & -n(n-1)\int_{\delta}^{\overline{\omega}} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] H_{\alpha_i}(v_i,\alpha^*)\,dv_i \\
= & \; n(n-1)\int_{\delta}^{\overline{\omega}} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right]\left[\frac{-H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)}\right] h(v_i,\alpha^*)\,dv_i \\
\geq & \; n(n-1)\left\{\min_{v_i\in[\delta,\overline{\omega}]}\frac{-H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)}\right\}\int_{\delta}^{\overline{\omega}} H(v_i,\alpha^*)^{n-2}\left[1-H(v_i,\alpha^*)\right] h(v_i,\alpha^*)\,dv_i \\
= & \; n(n-1)\gamma\left[\frac{1}{n-1}H(v_i,\alpha^*)^{n-1}-\frac{1}{n}H(v_i,\alpha^*)^{n}\right]_{\delta}^{\overline{\omega}} \\
= & \; \gamma\left[1-nH(\delta,\alpha^*)^{n-1}+(n-1)H(\delta,\alpha^*)^{n}\right]
\end{aligned}
$$

Since $H(\delta,\alpha^*)$ is strictly between 0 and 1, as n goes to infinity, the last term is no less than the positive constant γ.

Therefore, as n goes to infinity, $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > \gamma > 0$. By continuity, there there exists n^* such that $\partial \pi_s(\alpha^*,r)/\partial\alpha^* > 0$ for all r and $n \geq n^*$.

Proof of Lemma 1. The proof needs the following auxiliary result, which we label as "standard auction lemma". This lemma will be used to prove Proposition 3 as well.

Standard Auction Lemma. Suppose the first-order approach is valid, and Assumptions 1, 2 and 3 hold. Then standard auctions with a reserve price are optimal if the Lagrange multiplier $\lambda \geq 0$.

Proof of the standard auction lemma: Notice that if $\lambda \geq 0$,

$$
J^*(v_i) = v_i - \frac{1-H(v_i,\alpha^*)}{h(v_i,\alpha^*)} - \frac{\lambda}{n}\frac{H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)},
$$

is strictly increasing in v_i under Assumption 1-3. We can then define the reserve price as

$$
r^* = \min\{r : J^*(r) \geq 0\}.
$$

The optimal auctions will assign the object to the bidder with the highest posterior estimate provided his estimate is higher than r^*, that is, they are standard auctions. Thus, standard auctions with a reserve price are optimal. End of proof.

Now we can proceed to prove Lemma 1**.** We adopt the same strategy of Rogerson (1985) by weakening the equality (IA-FOC) constraint to the following inequality constraint:

$$
-\mathbb{E}_{v|\alpha_j=\alpha^*,\forall j}\left[\frac{H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)}q_i(v_i,v_{-i})\right] - C'(\alpha^*) \geq 0.
$$

With the inequality constraint, the corresponding Lagrange multiplier λ is always nonnegative. If we can show that $\lambda > 0$ at the optimal solution of the relaxed program, the (IA-FOC) constraint must be binding in equilibrium, which implies that the optimal solution of relaxed program is also an optimal solution of the original program. Hence, $\lambda > 0$ for the original program.

We can write and simplify the Lagrangian for the relaxed program as

$$L = \mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left[\left(v_i - \frac{1-H(v_i,\alpha^*)}{h(v_i,\alpha^*)} - \frac{\lambda}{n} \frac{H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)} \right) q_i(v_i,v_{-i}) \right] - \lambda C'(\alpha^*).$$

The necessary first-order condition is

$$\begin{aligned} 0 \quad = \quad & \frac{\partial L}{\partial \alpha^*} = \frac{\partial \left[\mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left(v_i - \frac{1-H(v_i,\alpha^*)}{h(v_i,\alpha^*)} \right) q_i(v_i,v_{-i}) \right]}{\partial \alpha^*} \\ & + \lambda \frac{\partial \left[-\frac{1}{n} \mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left(\frac{H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)} q_i(v_i,v_{-i}) \right) - C'(\alpha^*) \right]}{\partial \alpha^*}. \qquad (22) \end{aligned}$$

Since $\lambda \geq 0$, standard auctions are optimal by the standard auction lemma. By our assumption, the seller's revenue in standard auctions,

$$\pi_s(\alpha^*, r) = \mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left(v_i - \frac{1-H(v_i,\alpha^*)}{h(v_i,\alpha^*)} \right) q_i(v_i,v_{-i}),$$

is strictly increasing in α^*. Therefore, the first term on the right hand side of (22) is positive. In order to show $\lambda > 0$, we need to prove that the coefficient of λ in (22) is negative. Note that a standard auction with reserve price r allocates the object to the bidder with the highest valuation that is also higher than r, so we have

$$-\frac{1}{n} \mathbb{E}_{v|\alpha_j=\alpha^*,\forall j} \sum_{i=1}^{n} \left(\frac{H_{\alpha_i}(v_i,\alpha^*)}{h(v_i,\alpha^*)} q_i(v_i,v_{-i}) \right) = -\int_r^{\overline{\omega}} H_{\alpha_i}(v_i,\alpha^*) H(v_i,\alpha^*)^{n-1} dv_i.$$

Therefore, the coefficient of λ in (22) can be rewritten as

$$\begin{aligned} & \frac{\partial \left[-\int_r^{\overline{\omega}} H_{\alpha_i}(v_i,\alpha^*) H(v_i,\alpha^*)^{n-1} dv_i - C'(\alpha^*) \right]}{\partial \alpha^*} \\ = \quad & \underbrace{-\int_r^{\overline{\omega}} \frac{\partial^2 H(v_i,\alpha^*)}{\partial \alpha_i^2} H(v_i,\alpha^*)^{n-1} dv_i - C''(\alpha^*)}_{A} \\ & \underbrace{-\int_r^{\overline{\omega}} H_{\alpha_i}(v_i,\alpha^*)(n-1) H(v_i,\alpha^*)^{n-2} H_{\alpha_i}(v_i,\alpha^*) dv_i}_{B}. \end{aligned}$$

The local second-order condition of bidder i's optimization problem implies that the term A is nonpositive (see expression (12) for bidder i's payoff in a standard auction). The term

B is clearly positive, so the coefficient of λ is negative. Therefore, $\lambda > 0$ at the optimal solution (α^*, q). This implies that the solution to the relaxed program is the same as the one for the original program, and the maximum of the relaxed program can be achieved by the original program. Hence, the Lagrange multiplier $\lambda > 0$ for the original program. ■

Proof of Proposition 3. It follows from Lemma 1 that $\lambda > 0$ under conditions specified in the proposition. The standard auction lemma used in the proof of Lemma 1 then implies that standard auctions are optimal. To complete the proof, we only need to show that r^* is set according to the rule specified in the proposition. There are three cases.

Case 1: $r_{\alpha^*} > \mu$. First we show $r^* \leq r_{\alpha^*}$. By definition of r_{α^*},

$$r_{\alpha^*} - \frac{1 - H\left(r_{\alpha^*}, \alpha^*\right)}{h\left(r_{\alpha^*}, \alpha^*\right)} = 0.$$

Then for all $v_i \geq r_{\alpha^*} > \mu$,

$$J^*\left(v_i\right) \geq -\frac{\lambda}{n} \frac{H_{\alpha_i}\left(v_i, \alpha^*\right)}{h\left(v_i, \alpha^*\right)} \geq 0.$$

The last inequality follows from the rotation order. Therefore, the optimal reserve price $r^* \leq r_{\alpha^*}$.

Next, we show $r^* > \mu$. Suppose $r^* \leq \mu$ by contradiction. Then

$$J^*\left(r^*\right) < -\frac{\lambda}{n} \frac{H_{\alpha_i}\left(r^*, \alpha^*\right)}{h\left(r^*, \alpha^*\right)} \leq 0.$$

The first inequality follows because $r^* < r_{\alpha^*}$, and the second inequality follows from the rotation order. This contradicts the fact the $J^*\left(r^*\right) \geq 0$. Thus, we have shown $\mu < r^* \leq r_{\alpha^*}$

Case 2: $r_{\alpha^*} = \mu$. Then for all $v_i > \mu$,

$$J^*\left(v_i\right) > -\frac{\lambda}{n} \frac{H_{\alpha_i}\left(v_i, \alpha^*\right)}{h\left(v_i, \alpha^*\right)} \geq 0.$$

Therefore, r^* cannot be higher than μ. On the other hand, for all $v_i < \mu$

$$J^*\left(v_i\right) < -\frac{\lambda}{n} \frac{H_{\alpha_i}\left(v_i, \alpha^*\right)}{h\left(v_i, \alpha^*\right)} \leq 0.$$

Therefore, r^* cannot be lower than μ. Therefore, $r^* = r_{\alpha^*} = \mu$.

Case 3: $r_{\alpha^*} < \mu$. Note that for all $v_i < r_{\alpha^*}$,

$$J^*\left(v_i\right) < -\frac{\lambda}{n} \frac{H_{\alpha_i}\left(v_i, \alpha^*\right)}{h\left(v_i, \alpha^*\right)} \leq 0.$$

Therefore, $r^* \geq r_{\alpha^*}$. Furthermore, for all $v_i \geq \mu$,

$$J^*\left(v_i\right) > -\frac{\lambda}{n} \frac{H_{\alpha_i}\left(v_i, \alpha^*\right)}{h\left(v_i, \alpha^*\right)} \geq 0.$$

Thus, $r^* < \mu$. As a result, $r_{\alpha^*} \leq r < \mu$.■

Proof of Proposition 5. Let $\lambda_i \geq 0$ denote the Lagrange multiplier of the information acquisition constraint for bidder i, $i = 1, ..., n$. Then the Lagrangian can be written and simplified as

$$\begin{aligned}\mathcal{L} &= \sum_{i=1}^{m} \left\{ \int_{\underline{\omega}}^{\overline{\omega}} \left[v_i - \frac{1 - H_1(v_i)}{h_1(v_i)} + \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_1(v_i)} \right] Q_i(v_i) \, dH_1(v_i) - \lambda_i k \right\} \\ &\quad + \sum_{i=m+1}^{n} \left\{ \int_{\underline{\omega}}^{\overline{\omega}} \left[v_i - \frac{1 - H_0(v_i)}{h_0(v_i)} - \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_0(v_i)} \right] Q_i(v_i) \, dH_0(v_i) + \lambda_i k \right\}. \quad (23)\end{aligned}$$

The proof of the first part is analogous to the proof of the second part of Proposition 3. Consider the optimal reserve price r_1^* for informed bidders $i \leq m$. Suppose $r_1 > v^+$ and we want to show $v^+ < r_1^* \leq r_1$. For all $v_i > r_1$,

$$J_1(v_i) \equiv v_i - \frac{1 - H_1(v_i)}{h_1(v_i)} + \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_1(v_i)} \geq \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_1(v_i)} > 0.$$

Therefore, $r_1^* \leq r_1$. The inequality $r_1^* > v^+$ follows from the fact that, for all $v_i \leq v^+$,

$$J_1(v_i) < \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_1(v_i)} \leq 0.$$

Now suppose $r_1 < v^+$ and we want to show $r_1 \leq r_1^* < v^+$. Note that for all $v_i \geq v^+$,

$$J_1(v_i) > \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_1(v_i)} \geq 0,$$

and for all $v_i < r_1 < v^+$,

$$J_1(v_i) < \lambda_i \frac{H_0(v_i) - H_1(v_i)}{h_1(v_i)} \leq 0.$$

Thus, we must have $r_1 \leq r_1^* < v^+$.

For the second part, notice that the difference between the virtual surplus functions for bidder $i \leq m$ and bidder $j \geq m+1$ is:

$$J_1(x) - J_0(x) = \left(x - \frac{1 - H_1(x)}{h_1(x)} \right) - \left(x - \frac{1 - H_0(x)}{h_0(x)} \right) + \lambda_i \frac{H_0(x) - H_1(x)}{h_1(v_i)} + \lambda_j \frac{H_0(x) - H_1(x)}{h_0(x)}.$$

Since $\lambda_i, \lambda_j \geq 0$, we have

$$\begin{array}{ll} J_1(x) - J_0(x) \geq \left(x - \frac{1 - H_1(x)}{h_1(x)} \right) - \left(x - \frac{1 - H_0(x)}{h_0(x)} \right) & \text{if} \quad x > v^+ \\ J_1(x) - J_0(x) \leq \left(x - \frac{1 - H_1(x)}{h_1(x)} \right) - \left(x - \frac{1 - H_0(x)}{h_0(x)} \right) & \text{if} \quad x < v^+ \end{array}.$$

Now suppose the valuations of informed bidder i and uninformed bidder j are such that the values of resulting virtual surplus function are tied in the Myerson's optimal auction with exogenous information, so that bidder i and j will win the object with equal probability. With endogenous information, however, bidder i wins if their valuations are above v^+ and bidder j wins if their valuations are below v^+. That is, compared to the case with exogenous information, informed bidder i is treated more favorably if bidders' valuations are above v^+, and uninformed bidder j is treated more favorably otherwise. In both cases, endogenous information acquisition reduces price discrimination against "strong" bidders.■

Appendix B: Sufficient Conditions for the First-Order Approach

This Appendix provides several sets of sufficient conditions under which the first-order approach is valid. Recall that bidder i chooses α_i to maximize his payoff given other bidders choose $\alpha_j\,(j \neq i)$. Bidder i's payoff under mechanism $\{q_i(v), t_i(v)\}$ is

$$\pi_i(\alpha_i) = \mathbb{E}_{v_{-i}} \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} [1 - H(v_i, \alpha_i)]\, q_i(v_i, v_{-i})\, dv_i - C(\alpha_i).$$

Here, we explicitly write the support of v_i as $\left[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}\right]$, which could vary with respect to the choice of α_i.

We differentiate $\pi_i(\alpha_i)$ with respect to α_i to obtain

$$\frac{\partial \pi_i}{\partial \alpha_i} = -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} H_{\alpha_i}(v_i, \alpha_i)\, Q_i(v_i)\, dv_i - Q_i\left(\underline{\omega}_{\alpha_i}\right) \frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i} - C'(\alpha_i)$$

and differentiate it twice to obtain

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial \alpha_i^2} &= -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i - H_{\alpha_i}(\overline{\omega}_{\alpha_i}, \alpha_i)\, Q_i(\overline{\omega}_{\alpha_i}) \frac{\partial \overline{\omega}_{\alpha_i}}{\partial \alpha_i} \\ &\quad + H_{\alpha_i}\left(\underline{\omega}_{\alpha_i}, \alpha_i\right) Q_i\left(\underline{\omega}_{\alpha_i}\right) \frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i} - Q_i'\left(\underline{\omega}_{\alpha_i}\right) \left(\frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i}\right)^2 - Q_i\left(\underline{\omega}_{\alpha_i}\right) \frac{\partial^2 \underline{\omega}_{\alpha_i}}{\partial \alpha_i^2} - C''(\alpha_i) \end{aligned}$$

By our rotation order assumption, $\underline{\omega}_{\alpha_i}$ is decreasing in α_i, $\overline{\omega}_{\alpha_i}$ is increasing in α_i, and $H_{\alpha_i}\left(\underline{\omega}_{\alpha_i}, \alpha_i\right) \geq 0$. Therefore,

$$\frac{\partial^2 \pi_i}{\partial \alpha_i^2} \leq -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i - H_{\alpha_i}(\overline{\omega}_{\alpha_i}, \alpha_i)\, Q_i(\overline{\omega}_{\alpha_i}) \frac{\partial \overline{\omega}_{\alpha_i}}{\partial \alpha_i} - Q_i\left(\underline{\omega}_{\alpha_i}\right) \frac{\partial^2 \underline{\omega}_{\alpha_i}}{\partial \alpha_i^2} - C''(\alpha_i) \tag{24}$$

The first-order approach is valid if $\partial^2 \pi_i / \partial \alpha_i^2 < 0$, which holds as long as the cost function is sufficiently convex.[24]

[24] Persico (2000) makes such an assumption in his example of information acquisition.

If $C''(\alpha_i) > 0$ and the support $\left[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}\right]$ is invariant with respect to α_i, then a sufficient condition for $\frac{\partial^2 \pi_i}{\partial \alpha_i^2} < 0$ is

$$\frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} \geq 0 \text{ for all } v_i. \tag{25}$$

That is, the distribution of the posterior estimates is convex in the bidder's information choice. This condition is analogous to the CDFC (convexity of the distribution function condition) in the principal-agent literature, which requires that the distribution function of output be convex in the action the agent takes (Mirrlees (1999), and Rogerson (1985)).[25]

For the two leading information structures, we provide sufficient conditions for the first-order approach to be valid.

Proposition 6 (Sufficient Conditions for First-Order Approach)

1. *For the truth-or-noise technology: if* $C''(\alpha_i)\alpha_i \geq f(\overline{\omega})(\overline{\omega} - \mu)^2$ *for all* α_i*, the first-order approach is valid when either (1)* $F(x)$ *is convex, or (2)* $F(x) = x^b\,(b > 0)$ *with support* $[0, 1]$.

2. *For the Gaussian specification, the first-order approach is valid if, for all* α_i*,*

$$\sqrt{\beta^3 / \left[\alpha_i^3 (\alpha_i + \beta)^5\right]} < 2\sqrt{2\pi} C''(\alpha_i).$$

For the truth-or-noise technology, the condition, $C''(\alpha_i)\alpha_i \geq f(\overline{\omega})(\overline{\omega} - \mu)^2$, is to ensure that the relative gain from information acquisition is not too high so that bidders will not pursue extreme information choice $\overline{\alpha}$. Condition (2) indicates that the convexity of F is not necessary for the first-order approach. For the Gaussian specification, the sufficient condition for first-order approach is satisfied when $\underline{\alpha}$ is large relative to β, which ensures that information acquisition is profitable.

Proof: For the truth-or-noise technology, it follows from (24) that

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial \alpha_i^2} &\leq -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i - H_{\alpha_i}(\overline{\omega}_{\alpha_i}, \alpha_i) Q_i(\overline{\omega}_{\alpha_i}) \frac{\partial \overline{\omega}_{\alpha_i}}{\partial \alpha_i} - Q_i(\underline{\omega}_{\alpha_i}) \frac{\partial^2 \underline{\omega}_{\alpha_i}}{\partial \alpha_i^2} - C''(\alpha_i) \\
&= -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i + f(\overline{\omega}) \frac{(\overline{\omega} - \mu)^2}{\alpha_i} Q_i(\overline{\omega}_{\alpha_i}) - C''(\alpha_i) \\
&\leq -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i + f(\overline{\omega}) \frac{(\overline{\omega} - \mu)^2}{\alpha_i} - C''(\alpha_i) \\
&\leq -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i, \alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i.
\end{aligned}$$

The equality follows from equation (5) and the fact that

$$\partial \overline{\omega}_{\alpha_i} / \partial \alpha_i = \overline{\omega} - \mu, \quad \text{and } \partial^2 \underline{\omega}_{\alpha_i} / \partial \alpha_i^2 = 0.$$

[25] See also Jewitt (1988).

The last inequality follows from the assumption that $C''(\alpha_i)\alpha_i \geq f(\overline{\omega})(\overline{\omega}-\mu)^2$.

Note that

$$\begin{aligned}
& -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H(v_i,\alpha_i)}{\partial \alpha_i^2} Q_i(v_i)\, dv_i \\
= & -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \left\{ f'\left(\frac{v_i-(1-\alpha_i)\mu}{\alpha_i}\right)\frac{(\mu-v_i)^2}{\alpha_i^4} - f\left(\frac{v_i-(1-\alpha_i)\mu}{\alpha_i}\right)\frac{2(\mu-v_i)}{\alpha_i^3}\right\} Q_i(v_i)\, dv_i \\
= & -\int_{\underline{\omega}}^{\overline{\omega}} \left\{ f'(s_i)\frac{(s_i-\mu)^2}{\alpha_i} + f(s_i)\frac{2(s_i-\mu)}{\alpha_i}\right\} Q_i(\alpha_i s_i+(1-\alpha_i)\mu)\, ds_i.
\end{aligned}$$

If $F(\cdot)$ is convex, then

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial \alpha_i^2} &< -\frac{2}{\alpha_i}\int_{\underline{\omega}}^{\overline{\omega}} (s_i-\mu) f(s_i) Q_i(\alpha_i s_i+(1-\alpha_i)\mu)\, ds_i \\
&< -\frac{2}{\alpha_i}\int_{\underline{\omega}}^{\mu} (s_i-\mu) f(s_i) Q_i(\mu)\, ds_i - \frac{2}{\alpha_i}\int_{\mu}^{\overline{\omega}} (s_i-\mu) f(s_i) Q_i(\mu)\, ds_i \\
&= -\frac{2}{\alpha_i} Q_i(\mu)\int_{\underline{\omega}}^{\overline{\omega}} (s_i-\mu) f(s_i)\, ds_i \\
&= 0.
\end{aligned}$$

If $F(x)=x^b$ $(0<b\leq 1)$ with support $[0,1]$, then

$$\begin{aligned}
\frac{\partial^2 \pi_i}{\partial \alpha_i^2} &< -\int_{\underline{\omega}}^{\overline{\omega}} \left\{ f'(s_i)\frac{(s_i-\mu)^2}{\alpha_i} + f(s_i)\frac{2(s_i-\mu)}{\alpha_i}\right\} Q_i(\alpha_i s_i+(1-\alpha_i)\mu)\, ds_i \\
&= -\frac{1}{\alpha_i}\int_0^1 [(b+1)s+(1-b)\mu]\, bs^{b-2}(s-\mu) Q_i(\alpha_i s+(1-\alpha_i)\mu)\, ds \\
&< -\frac{1}{\alpha_i} Q_i(\mu)\int_0^1 ((b+1)s+(1-b)\mu)\, bs^{b-2}(s-\mu)\, ds \\
&= -\frac{1}{\alpha_i} Q_i(\mu)(b+1)\int_0^1 bs^{b-1}(s-\mu)\, ds - \frac{1}{\alpha_i} Q_i(\mu)(1-b)\mu b\int_0^1 s^{b-2}(s-\mu)\, ds \\
&= -\frac{1}{\alpha_i} Q_i(\mu)\frac{b}{(1+b)^2} \\
&< 0.
\end{aligned}$$

For the Gaussian specification,

$$\frac{\partial^2 \pi_i}{\partial \alpha_i^2} = -\mathbb{E}_{v_{-i}}\int_{-\infty}^{\infty} \frac{\partial^2 H(v_i,\alpha_i)}{\partial \alpha_i^2} q_i(v_i,v_{-i})\, dv_i - C''(\alpha_i).$$

With some algebra, we can obtain

$$\frac{\partial^2 H(v_i,\alpha_i)}{\partial \alpha_i^2} = \frac{4\alpha_i+3\beta}{2\alpha_i(\alpha_i+\beta)}\frac{(v_i-\mu)}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}}\exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right)\left(1-\frac{\alpha_i+\beta}{4\alpha_i+3\beta}\frac{\beta^2}{\alpha_i}(v_i-\mu)^2\right).$$

Thus, we can write the second derivative as

$$\frac{\partial^2 \pi_i}{\partial \alpha_i^2} = \left(\begin{array}{c} -\int_{-\infty}^{\infty} \frac{4\alpha_i+3\beta}{2\alpha_i(\alpha_i+\beta)} \frac{(v_i-\mu)}{2\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i(\alpha_i+\beta)}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) Q_i(v_i)\, dv_i \\ +\int_{-\infty}^{\infty} \frac{\beta^2}{4\alpha_i^2} \frac{(v_i-\mu)^3}{\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) Q_i(v_i)\, dv_i \end{array} \right) - C''(\alpha_i)$$

$$= \left(\begin{array}{c} -\frac{4\alpha_i+3\beta}{2\alpha_i(\alpha_i+\beta)} \int_{-\infty}^{\infty} \left(-H_{\alpha_i}(v_i,\alpha_i)\right) Q_i(v_i)\, dv_i \\ +\int_{-\infty}^{\infty} \frac{\beta^2}{4\alpha_i^2} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \frac{\beta}{\alpha_i(\alpha_i+\beta)} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) Q_i(v_i)\, dv_i \end{array} \right) - C''(\alpha_i).$$

Given that the information structure is rotation-ordered, bidders always prefer a higher α_i, which implies

$$\int_{-\infty}^{\infty} \left(-H_{\alpha_i}(v_i,\alpha_i)\right) Q_i(v_i)\, dv_i > 0.$$

Thus, a sufficient condition for the second-order condition is

$$\int_{-\infty}^{\infty} \frac{\beta^2}{4\alpha_i^2} \frac{\beta}{\alpha_i(\alpha_i+\beta)} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) Q_i(v_i)\, dv_i < C''(\alpha_i) \Leftrightarrow$$

$$\frac{\beta^3}{4\alpha_i^3(\alpha_i+\beta)} \int_{-\infty}^{\infty} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) Q_i(v_i)\, dv_i < C''(\alpha_i).$$

A sufficient condition for the above inequality is,

$$\frac{\beta^3}{4\alpha_i^3(\alpha_i+\beta)} \int_{\mu}^{\infty} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) dv_i < C''(\alpha_i) \Leftrightarrow$$

$$\frac{1}{2\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)^5}} < C''(\alpha_i),$$

which is satisfied if β/α_i is small.■

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