Welfare of Price Discrimination and Market Segmentation in Duopoly^{*}

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May 9, 2022

Abstract

We apply information design approach to studying market segmentation and third-degree price discrimination in a duopoly market with captive and contested consumers. A market segmentation divides the market into segments that contain different proportions of captive and contested consumers. Firm-optimal segmentation divides the market into two segments and in each segment only one firm has captive consumers. In contrast to the existing literature with exogenous segmentation, price discrimination under firm-optimal segmentation unambiguously reduces consumer surplus for all market configurations. Consumer-optimal segmentation divides the market into a "maximal symmetric" segment and the remainder, and yields the lowest producer surplus among all segmentations.

^{*}We are grateful to Dirk Bergemann, Tibor Heumann, Sridhar Moorthy, Kai Hao Yang, Jidong Zhou, and seminar participants at CUHK, HKU, HKUST, PUC-Chile, UTS, Zhejiang University, and Econometric Society China Meeting 2021 for helpful comments and suggestions. We thank Pegah Rahmani and Uluc Sengil for excellent research assistance. Shi thanks the Social Sciences and Humanities Research Council of Canada for financial support. Xianwen Shi: Department of Economics, University of Toronto, xianwen.shi@utoronto.ca. Jun Zhang: Economics Discipline Group, School of Business, University of Technology Sydney, zhangjunqueens@gmail.com.

1 Introduction

Third-degree price discrimination is ubiquitous and is probably the most common form of price discrimination. Almost all firms with some market power would attempt to increase profit by charging different prices for consumers in different submarkets (or market segments). To engage in third-degree price discrimination, a firm must decide how to divide consumers into different groups and what price to charge for each consumer group.

Following the seminal work of Pigou (1920) and Robinson (1933), most of the literature on third-degree price discrimination takes the segmentation of consumers into different groups as exogenously given and finds that welfare consequences of price discrimination are generally ambiguous. For example, Schmalensee (1981) and Varian (1985) show that the effect of monopolistic price discrimination on social welfare, relative to uniform pricing, depends on whether the overall output increases.¹ In a symmetric duopoly model, Holmes (1989) shows that the effects of price discrimination on output and profit depend on cross-price elasticities and concavities of demand functions in the two submarkets.²

The choice of how to divide the market, however, is clearly a very important consideration for firms (and data brokers) who can choose what kind of consumer data to collect, keep and process, and for regulators who can limit the nature and extent of consumer data to be collected, traded and used. Before the era of big data, consumers were segmented into different submarkets by easily observable characteristics such as ages and locations. With the advance of information technology and social media, the amount of consumer data available for firms to differentiate consumers grows exponentially and the number of ways for firms to segment the market is enormous. Social media platforms build user profiles by gathering data from mobile apps, e.g., what messages they post, read, comment, and forward and what products they search and buy. These user profiles can be used to feed machine learning algorithms to classify users into different consumer groups. The digital footprints of consumers, together with traditional offline consumer data, allow firms to perform increasingly fine and intricate market segmentations.

In this paper, we follow Bergemann, Brooks and Morris (2015) (BBM hereafter) to

¹See also Aguirre, Cowan and Vickers (2010) and Cowan (2012). Aguirre, Cowan and Vickers (2010) show that the effect of price discrimination in general depends on the relative curvature of the direct or inverse demand functions in the two submarkets. Cowan (2012) shows that consumer surplus may rise with discrimination if the ratio of pass-through to the elasticity at the uniform price is higher in the high-elasticity submarket.

²See also Corts (1998) who shows that if firms disagree over which submarkets are strong or weak, then price discrimination may lower profit and increase consumer surplus.

formulate the problem of third-degree price discrimination as a problem of information design in which the designer first chooses how to divide the market and then firms choose what price to charge in each submarket. We take an agnostic view and consider all possible segmentations. The only restriction we impose on segmentations is that they must be public in the sense that the designer must reveal the same segmentation to both firms. Public disclosure is robust to communications among firms, and it also satisfies the requirement for transparency and non-favoritism. Most importantly, it allows for a more direct comparison of our results to the classical literature of price discrimination where firms share the same exogenous market segmentation.

We consider two possible objectives for the designer: producer surplus maximization and consumer surplus maximization. The first objective is relevant if the designer is an industry association who collects consumer information and wants to maximize member firms' aggregate welfare or if the designer is a regulator who would like to understand how data brokers and third-party platforms may control market competition through information provision in the product markets. The second objective is relevant if the designer is a regulator or a consumer association who would like to advocate consumer welfare.

In our baseline model, two firms produce a homogeneous product and compete in prices. Each firm has their own captive consumers who can only buy from the firm they are captive to.³ There are also contested consumers who are loyal to neither firms and will buy from the firm that offers the lower price. All consumers have the same downward-sloping demand. This model framework is developed by Varian (1980) and Narasimhan (1988) for the case of unit demand, and later generalized by Armstrong and Vickers (2019) to the case of downward-sloping demand.⁴

A market segmentation divides the market into segments that contain different proportions of captive and contested consumers. We characterize the *unique* firm-optimal segmentation and the *unique* consumer-optimal segmentation among all possible segmentations. Both segmentations take simple forms. To succinctly describe them, let $(\gamma_1, 1-\gamma_1-\gamma_2, \gamma_2)$ with $\gamma_1 \geq \gamma_2$ denote a prior market where γ_i is the share of consumers captive to firm *i* and $1 - \gamma_1 - \gamma_2$ is the share of contested consumers. Let $\ell = \gamma_1 + \gamma_2$ denote the total share of captive consumers. The firm-optimal segmentation divides the market into two nested submarket $(\ell, 1 - \ell, 0)$ and submarket $(0, 1 - \ell, \ell)$ with size γ_1/ℓ and γ_2/ℓ , respectively.⁵ In contrast, the consumer-optimal segmentation divides

 $^{^{3}}$ For example, consumers may become captive to a brand either because they are loyal to the brand or because they have made brand-specific investments and hence it is costly for them to switch.

⁴This model has been a working horse in the marketing literature for studying promotional strategies. See for example, Chen, Narasimhan and Zhang (2001) and references therein.

⁵This form of market segmentation is first noted by Armstrong and Vickers (2019). They observe

the market into a "maximal symmetric" submarket of $(\gamma_2/(1-\gamma_1+\gamma_2), \gamma_2/(1-\gamma_1+\gamma_2))$ with size $1-\gamma_1+\gamma_2$ and the remainder of (1,0,0) with size $(\gamma_1-\gamma_2)$.

To further illustrate, consider the following toy example. Imagine that there are two firms competing for 36 consumers in the market. Each consumer is a member of an association and owns an association email account. In the figure below, we represent each consumer by a colored email address — different colors represent different consumer types. The association collects members' information and learns perfectly that 16 of them are captive to firm 1 (yellow), 8 of them are captive to firm 2 (green), and the rest are contested (red).



The association divides the 36 email addresses into several email lists, publicly discloses the size and relative compositions of consumer types in each email list without revealing the color of each consumer, and controls the two firms' access to the email lists. For each email list, each firm sends via the association one and only one price offer. The simple segmentation in the left panel—dividing contested consumers proportional to captive consumers—is firm optimal, while the segmentation in the right panel—one maximal symmetric submarket plus the reminder—is consumer optimal.⁶

In sharp contrast to the existing literature on price discrimination with exogenous submarkets where the effect of price discrimination on consumer surplus is generally ambiguous, we show that the firm-optimal segmentation always reduces consumer surplus compared to uniform pricing (i.e., no segmentation) for all prior markets. Nev-

that this segmentation arises if two regional monopolists are allowed to serve each other's customer bases, consumers differ in their switching costs, and firms engage in price discrimination by geographical regions. It is shown to be firm-optimal in the case of unit demand by Albrecht (2020) and Bergemann, Brooks and Morris (2020).

⁶If the association compiles different email lists for different firms, then it becomes an example of private segmentation.

ertheless, the firm-optimal segmentation may not minimize consumer surplus. The consumer-optimal segmentation, however, also minimizes producer surplus. A key insight is that a more symmetric market fosters stronger competition between firms. Market segments $(\ell, 1 - \ell, 0)$ and $(0, 1 - \ell, \ell)$ in the firm-optimal segmentation feature the maximal level of asymmetry for a fixed total share of captive consumers (ℓ) while the maximal symmetric segment in the consumer-optimal segmentation minimizes such asymmetry.

Our analysis directly builds on Armstrong and Vickers (2019) who show that, if firms are sufficiently symmetric, consumers are better off under uniform pricing than under price discrimination with all possible public segmentations. Although their result inspires our construction of the consumer-optimal segmentation, it does not imply that a symmetric segment must be part of the consumer-optimal segmentation.

Methodologically, we follow the seminal work of BBM to formulate the segmentation problem as an information design problem. Instead of applying the standard concavification technique in the information design literature, we take a different approach.⁷ We first identify the forms of market segments that can possibly be part of the optimal segmentation and then reformulate the information design problem as a problem of choosing the distributions of these segments. Our two-step solution procedure, more elementary and intuitive in our setup, can easily establish uniqueness as we solve the optimal segmentation. The uniqueness property is important for our welfare analysis, because, for example, different firm-optimal segmentations may have different welfare implications for consumers.

In a monopoly setting with unit demand, BBM show that any surplus division (or equivalently any point in the surplus triangle) can be attained by some market segmentation. The analysis of BBM has been applied to a wide range of monopoly applications, such as multiproduct monopoly (Ichihashi (2020), Haghpanah and Siegel (2021), Hidir and Vellodi (2021)), lemons market with interdependent values (Kartik and Zhong (2019)), and revenue-maximizing data brokers (Yang (2022)).⁸

There have been several attempts to extend the analysis of BBM at least partially to the oligopoly setting. One strategy is to identify a possible welfare target and then examine how to attain it. In an oligopoly model with unit demand, Elliott et al. (2021) provide a necessary and sufficient condition under which a firm-optimal segmentation extracts the full surplus and characterizes a consumer-optimal segmentation which induces an efficient allocation and delivers to each firm its minimax profit. If no obvious

 $^{^{7}}$ See Bergemann and Morris (2019) and Kamenica (2019) for surveys of standard solution techniques and recent developments in this literature.

⁸See also Ali, Lewis and Vasserman (2019) for an analysis of how consumer information control can affect consumer welfare by influencing the learning of and the competition between firms.

welfare target is available, however, it is necessary to characterize all possible equilibria in the baseline pricing model to identify the target. As observed by Armstrong and Vickers (2019), even for duopoly pricing models, "[e]xcept in symmetric and other special cases ... the form of the equilibrium is not known." Hence, a stylized baseline model is often necessary for tractability. Albrecht (2020), Bergemann, Brooks and Morris (2020), and Bergemann, Brooks and Morris (2021) use the unit demand version of Armstrong and Vickers (2019) as their baseline model, and identify the firm-optimal segmentations among all possible public and private segmentations.⁹ Our analysis of downward-sloping demand is complementary to theirs. The setting of downwardsloping demand is better suited for our purpose of comparison since the literature of price discrimination has shown that elasticities and curvatures of demand are crucial in evaluating the welfare consequences of price discrimination.

All the above papers take consumer demand as given and study how to design information structures to influence learning by firms. One can also consider the design of information structures to affect consumer learning. Roesler and Szentes (2017) consider a monopoly model with privately informed consumers and derive consumeroptimal information structures. Armstrong and Zhou (2022) extend their analysis to a duopoly setting and characterize firm-optimal and consumer-optimal information structures. Assuming that firms rather than the designer choose information structures, Ivanov (2013) and Boleslavsky, Hwang and Kim (2019) derive equilibrium information structures in games where firms compete in both pricing and advertising.

2 The Model

Our baseline model is taken from Armstrong and Vickers (2019). There are two firms who can produce a homogeneous product at zero cost and compete for consumers in prices. There are three types of consumers: consumers who are captive to (and hence can only buy from) firm 1, consumers who are captive to firm 2, and contested consumers who will buy from the firm that charges a lower price. Let γ_1 and γ_2 denote the share of consumers captive to firm 1 and firm 2, respectively, and the share of contested consumers is then $1 - \gamma_1 - \gamma_2$. Without loss of generality, we assume that $\gamma_2 \leq \gamma_1$.

Consumers have quasilinear preferences and their demand D(p) is downward sloping and continuously differentiable. If a consumer buys from a firm who charges price p, this consumer will buy D(p) units of the product, yielding a profit of $\pi(p) \equiv pD(p)$

⁹With unit demand, perfectly revealing consumer information to both firms is consumer-optimal, but it is not optimal for downward-sloping demand.

to the firm. As in Armstrong and Vickers (2019), we impose the following assumption:

Assumption 1 The elasticity of demand $\eta(p) \equiv -pD'(p)/D(p)$ is strictly increasing.

Under Assumption 1, $\pi(p)$ is single-peaked and hence is strictly increasing for all $p \in [0, p^*]$ where p^* is the revenue-maximizing price $p^* = \arg \max \pi(p)$. Moreover, consumer surplus $V(\pi)$ as a function of profit π is strictly decreasing and strictly concave in $[0, \pi^*]$, where $\pi^* \equiv p^*D(p^*)$ is the maximal profit. To rule out triviality, we assume that $\pi^* > 0$ and $V(\pi^*) > 0$.

The overall duopoly market, referred to as the prior market, can be segmented into different submarkets or market segments which may have different relative shares of capped and contested consumers. We will use the terms of "submarket" and "market segment" interchangeably. In a market segment $(q_1, 1 - q_1 - q_2, q_2)$, q_1 and q_2 are the fraction of consumers captive to firm 1 and firm 2, respectively, and $(1 - q_1 - q_2)$ is the fraction of contested consumers. To simplify notation, we write a market segment $(q_1, 1 - q_1 - q_2, q_2)$ as (q_1, q_2) and a prior market $(\gamma_1, 1 - \gamma_1 - \gamma_2, \gamma_2)$ as (γ_1, γ_2) . The set of possible market segments is

$$\mathcal{M} = \{(q_1, q_2) \in [0, 1]^2 : 0 \le q_1 + q_2 \le 1\}.$$

A market segmentation can be represented as a probability distribution $m(q_1, q_2) \in \Delta \mathcal{M}$ of different segments such that, for $i = 1, 2, {}^{10}$

$$\gamma_{i} = \sum_{(q_{1},q_{2})\in\mathcal{M}} m(q_{1},q_{2}) q_{i}$$

We assume that, once a market segmentation is chosen, it is publicly observable to both firms. That is, we restrict attention to public segmentations. This assumption allows for a more direct comparison of our results to the classical literature of price discrimination where firms observe the same exogenous market segmentation. It is appropriate if information or signals on which the market segmentation is based are shared or publicly observable. See Section 4 for further remarks on this assumption.

Given a market segmentation m, firms decide what prices to charge for each submarket (q_1, q_2) in the support of m to maximize their profit. The producer surplus

¹⁰As argued by Kamenica and Gentzkow (2011), the required number of segments is no larger than the minimum of the number of states (three in our paper) and the number of actions (continuous). We thus need at most three segments.

under segmentation m is

$$PS(m) = \sum_{(q_1,q_2)\in\mathcal{M}} m(q_1,q_2) \left[\pi_1(q_1,q_2) + \pi_2(q_1,q_2)\right],$$

where $\pi_1(q_1, q_2)$ and $\pi_2(q_1, q_2)$ denote the profit in market segment (q_1, q_2) for firm 1 and firm 2, respectively. The total consumer surplus under segmentation m is

$$CS(m) = \sum_{(q_1,q_2)\in\mathcal{M}} m(q_1,q_2) C(q_1,q_2)$$

where $C(q_1, q_2)$ denotes the consumer surplus in market segment (q_1, q_2) .

A market segmentation is *firm-optimal* if it maximizes producer surplus among all possible market segmentations. A market segmentation is *consumer-optimal* if it maximizes consumer surplus among all possible market segmentations.

It is easy to see that if a prior market (γ_1, γ_2) does not contain any contested consumers (i.e., $\gamma_1 + \gamma_2 = 1$), both firms will offer the maximal profit π^* for every market segment. All market segmentations yield the same payoffs for firms and consumers. Therefore, from now on, we assume that $\gamma_1 + \gamma_2 < 1$.

Remark 1 Our downward sloping demand model can nest a unit demand specification with a random taste shock. Let i = 0, 1, 2, denote the types of consumers who are contested, captive to firm 1, and captive to firm 2, respectively. Suppose that each consumer has a unit demand and that a type i consumer's valuation for product j is

$$v_{ij} = \theta_{ij} + \varepsilon_i$$

where θ_{ij} is the normalized mean utility of type i consumers for firm j's product (j = 1, 2), and ε_i is type i's taste shock which is randomly drawn from a common distribution Φ with density ϕ . The normalized utility θ_{ij} takes the value of 0 if i = 0 or i = j and the value of $-\infty$ otherwise, so captive consumers (i = 1, 2) will only buy from their favorite firms. Furthermore, the taste shock ε_i is common across products, so contested consumers (i = 0) will buy from the firm that offers the lower price. The taste shock is realized upon receiving product offers. If a type 1 consumer is offered product 1 at price p, this consumer will buy if $\varepsilon_i \ge p$, which happens with probability $1 - \Phi(p)$. If we define the "demand function" as $D(p) = 1 - \Phi(p)$, then Assumption 1 is equivalent to the requirement that $p\phi(p)/[1 - \Phi(p)]$ is strictly increasing in p.

3 Firm- and Consumer-Optimal Segmentations

We first characterize the unique equilibrium for a generic market segment (q_1, q_2) . The equilibrium characterization is then used to find the firm-optimal segmentation and the consumer-optimal segmentation.

3.1 Preliminaries

Fix a market segment (q_1, q_2) with $q_2 \leq q_1$. As demonstrated in Armstrong and Vickers (2019), it is more convenient to view firms as choosing the per-customer profit π rather than the price p they ask from their customers, and consumers choose the firm who offers the lowest profit among the firms they can buy from. The following equilibrium characterization is standard and is taken from Narasimhan (1988) and Armstrong and Vickers (2019). We omit its proof.

Lemma 1 In the unique equilibrium for market segment (q_1, q_2) with $q_1 \ge q_2$, both firm 1 and firm 2 play mixed strategies on a common support $[\underline{\pi}, \pi^*]$ where the minimum profit $\underline{\pi} = q_1 \pi^* / (1 - q_2)$. Firm 1 chooses per-consumer profit according to distribution

$$F_{1}(\pi) = \frac{1 - q_{1}}{1 - q_{1} - q_{2}} \left(1 - \frac{\pi}{\pi}\right)$$

with an atom of size $(q_1 - q_2) / (1 - q_2)$ at $\pi = \pi^*$, and firm 2 chooses per-consumer profit according to distribution

$$F_{2}(\pi) = \frac{1 - q_{2}}{1 - q_{1} - q_{2}} \left(1 - \frac{\pi}{\pi}\right)$$

with no atom. The equilibrium profits are $\pi_1 = q_1 \pi^*$ and $\pi_2 = (1 - q_1) q_1 \pi^* / (1 - q_2)$.

In this game, the unique Nash equilibrium is in mixed strategy. Firms randomize on a common support, and the firm with more captive consumer has a mass point at the bottom. Note that when $q_1 + q_2 = 1$, the above equilibrium is not well-defined, but it converges to a situation where each firm operates as a monopoly and plays a pure strategy π^* . It follows from Lemma 1 that the equilibrium producer surplus obtained in market segment (q_1, q_2) is

$$P(q_1, q_2) = \pi_1(q_1, q_2) + \pi_2(q_1, q_2) = \frac{(2 - q_1 - q_2)q_1}{1 - q_2}\pi^*.$$
 (1)

Let $G(\pi; q_1, q_2)$ denote the equilibrium probability that a consumer in market segment (q_1, q_2) is offered a minimum profit weakly lower than π . Since firm *i*'s profit offer is

considered only by consumers captive to firm i and contested consumers, we have

$$G(\pi; q_1, q_2) = (1 - q_2) F_1(\pi) + (1 - q_1) F_2(\pi) - (1 - q_1 - q_2) F_1(\pi) F_2(\pi)$$

= $\frac{(1 - q_1) (1 - q_2)}{1 - q_1 - q_2} \left(1 - \frac{q_1^2}{(1 - q_2)^2} \left(\frac{\pi^*}{\pi} \right)^2 \right)$ (2)

with an atom of size $q_1(q_1 - q_2)/(1 - q_2)$ at $\pi = \pi^*$. Therefore, the equilibrium consumer surplus is

$$C(q_1, q_2) = \int_{\underline{\pi}}^{\pi^*} V(\pi) \, dG(\pi; q_1, q_2)$$

= $\frac{q_1(q_1 - q_2)}{1 - q_2} V(\pi^*) + \frac{2q_1^2(1 - q_1)}{(1 - q_2)(1 - q_1 - q_2)} (\pi^*)^2 \int_{\frac{q_1}{1 - q_2} \pi^*}^{\pi^*} \frac{V(\pi)}{\pi^3} d\pi.$ (3)

We now introduce three forms of market segments that will play an important role in our characterization of firm- and consumer-optimal segmentations. We follow Armstrong and Vickers (2019) and call a market segment *nested* if either $q_1 = 0$ or $q_2 = 0$. A market segment (q_1, q_2) is symmetric if $q_1 = q_2$. A market segment (q_1, q_2) is perfect if it contains only one type of consumers (i.e., either $q_1 = 1$, or $q_2 = 1$, or $q_1 + q_2 = 0$).

The following lemma identifies some important properties of the nested segment and the symmetric segment. It will be repeatedly used for our later characterization.

Lemma 2 Producer surplus is strictly concave in q for a nested segment, (q, 0) or (0, q). Producer surplus is linear in q and consumer surplus is strictly concave in q for a symmetric segment (q, q).

Proof. For a nested segment, producer surplus $P(q, 0) = (2 - q)q\pi^*$, which is strictly concave in q. For a symmetric segment, producer surplus is $2q\pi^*$, which is linear in q. Consumer surplus is given by

$$C(q,q) = \frac{2q^2}{1-2q} (\pi^*)^2 \int_{\frac{q}{1-q}\pi^*}^{\pi^*} \frac{V(\pi)}{\pi^3} d\pi.$$

The argument for the strict concavity of C(q, q) is first sketched out in Armstrong and Vickers (2019). Here, we provide a formal proof. To show it is strictly concave in q, we need to show that, for any $\lambda \in (0, 1)$, any $0 < q_L < q_H < 1/2$ and $q = \lambda q_L + (1 - \lambda) q_H$,

$$C(q,q) > \lambda C(q_L,q_L) + (1-\lambda) C(q_H,q_H).$$

In other words, if we segment a symmetric prior market (q, q) into two symmetric submarkets, (q_L, q_L) with size λ and (q_H, q_H) with size $(1 - \lambda)$, then consumer surplus must strictly decrease. Since $V(\pi)$ is strictly concave, it is sufficient to show that the distribution of the minimum of the profits in the two submarkets is a mean-preserving spread of the minimum profit distribution in the single prior market.

Let $G(\pi; q) \equiv G(\pi; q, q)$ denote the probability that a consumer in market segment (q, q) is offered a minimum profit weakly lower than π . Then we must have

$$G(\pi;q) = \frac{(1-q)^2}{1-2q} - \frac{q^2}{1-2q} \left(\frac{\pi^*}{\pi}\right)^2.$$

 $G(\pi; q)$ is strictly concave in q for all $\pi < \pi^*$ because

$$\frac{\partial^2 G\left(\pi;q\right)}{\partial q^2} = -\frac{2}{(1-2q)^3} \left(\left(\frac{\pi^*}{\pi}\right)^2 - 1 \right) < 0.$$

Consider $q_L < q_H$, $\lambda \in (0, 1)$ and $q = \lambda q_L + (1 - \lambda) q_H$. Then for all $\pi \in \left[\frac{q_H}{1 - q_H} \pi^*, \pi^*\right]$, $\overline{G}(\pi; q) \equiv \lambda G(\pi; q_L) + (1 - \lambda) G(\pi; q_H) < G(\pi; q)$.

Since $q > q_L$, the support of $\overline{G}(\pi; q)$ contains the support of $G(\pi; q)$. Furthermore, for $\pi \in \left[\frac{q}{1-q}\pi^*, \frac{q_H}{1-q_H}\pi^*\right]$,

$$\frac{G'\left(\pi;q\right)}{\overline{G}'\left(\pi;q\right)} = \frac{1}{\lambda} \frac{q^2}{1-2q} \left(\frac{q_L^2}{1-2q_L}\right)^{-1} > 1$$

because function $f(x) = x^2/(1-2x)$ is strictly increasing and $q > q_L$. It follows that $G(\pi;q)$ crosses $\overline{G}(\pi;q)$ only once and from below. Finally, the two submarkets yield the same producer surplus of $2q\pi^*$ as the prior single symmetric market (q,q). Therefore, $\overline{G}(\pi;q)$ is a mean-preserving spread of $G(\pi;q)$. The strict concavity of $V(\pi)$ then implies that

$$C(q,q) > \lambda C(q_L,q_L) + (1-\lambda) C(q_H,q_H)$$

That is, C(q,q) is strictly concave in q.

The strict concavity of producer surplus for nested segments implies that a merger of two different nested segments always strictly increases producer surplus. The strict concavity of consumer surplus and linearity of producer surplus for symmetric segments imply that a merger of two different symmetric segments always strictly increases consumer surplus, while leaving producer surplus unchanged.

3.2 Firm-Optimal Segmentation

Fix a prior market (γ_1, γ_2) with $\gamma_1 \geq \gamma_2$. To characterize the (unique) firm-optimal segmentation, we proceed in two steps. First, we argue that every market segment that is part of a firm-optimal segmentation must be a nested segment. Second, we reformulate the designer's problem and show that a firm-optimal segmentation either contains one nested segment (if the prior market is nested) or two nested segments which take the form of $(\gamma_1 + \gamma_2, 0)$ and $(0, \gamma_1 + \gamma_2)$.

Lemma 3 Every submarket in a firm-optimal segmentation is a nested segment.

Proof. We first show that a submarket (q_1, q_2) with $q_1 > 0$, $q_2 > 0$ and $q_1 + q_2 < 1$ cannot be part of a firm-optimal segmentation. If it is, we can further decompose it into two submarkets as follows:

$$(q_1, q_2) = \frac{q_1}{q_1 + q_2} \left(q_1 + q_2, 0 \right) + \frac{q_2}{q_1 + q_2} \left(0, q_1 + q_2 \right).$$

The decomposition yields a strictly higher producer surplus because

$$\frac{q_1}{q_1+q_2} P\left(q_1+q_2,0\right) + \frac{q_2}{q_1+q_2} P\left(0,q_1+q_2\right) - P\left(q_1,q_2\right)$$
$$= \frac{q_2}{1-q_2} \left(1-q_1-q_2\right) \left(2-q_1-q_2\right) \pi^* > 0.$$

A contradiction to the optimality. It remains to show that a submarket (q, 1 - q) with $q \in (0, 1)$ cannot be part of a firm-optimal segmentation. Suppose by contradiction that a firm-optimal segmentation includes such a submarket. We can decompose this submarket into q(1,0) + (1-q)(0,1) and maintain the same producer surplus. Since $\gamma_1 + \gamma_2 < 1$, a firm-optimal segmentation must involve at least one nested segment with strictly positive amount of shoppers. Without loss of generality, suppose this nested segment takes the form of (x, 1 - x, 0) with x < 1. Since both segments (1, 0, 0) and (x, 1 - x, 0) are nested segments, by Lemma 2, a merger of the two yields a strictly higher producer surplus. Again a contradiction to the optimality.

Given Lemma 3, it is natural to consider the following segmentation for a prior market (γ_1, γ_2) :

$$(\gamma_1,\gamma_2) = \frac{\gamma_1}{\gamma_1 + \gamma_2} \left(\gamma_1 + \gamma_2, 0\right) + \frac{\gamma_2}{\gamma_1 + \gamma_2} \left(0, \gamma_1 + \gamma_2\right).$$

Since all submarkets are nested segments, the above segmentation will be referred to as a "nested segmentation."

Proposition 1 The nested segmentation uniquely maximizes producer surplus among all possible market segmentations.

Proof. Lemmas 2 and 3 imply that a firm-optimal segmentation can contain at most two nested segments and if there are two nested segments they must take the form of (q, 0) and (0, q') with $q, q' \in [0, 1]$. Therefore, the designer's problem of finding all firm-optimal segmentations can be simplified as

$$\max_{\left(q,q',m_{q},m_{q'}\right)} m_{q} P\left(q,0\right) + m_{q'} P\left(0,q'\right)$$

subject to $\gamma_1 = m_q q$, $\gamma_2 = m_{q'} q'$ and $m_q + m_{q'} = 1$. By substituting the three constraints and expressions for P(q, 0) and P(0, q'), we can rewrite the objective as a function of q as

$$\gamma_1(2-q)\pi^* + \gamma_2\left(2 - \frac{\gamma_2}{1 - \gamma_1/q}\right)\pi^*$$

which is strictly concave for all $q \ge \gamma_1$ and has a unique maximizer of $q = \gamma_1 + \gamma_2$. Therefore, the unique firm-optimal segmentation must take the form of a nested segmentation.

To get intuition, consider all market segments (q_1, q_2) with $q_1 \ge q_2$ that have the same total share of captive consumers, that is, $q_1 + q_2 = \ell$ for some constant $\ell \in (0, 1)$. Then the larger q_1 is, the more asymmetric the segment becomes. As q_1 increases (keeping ℓ fixed), the incentive of firm 1 to attract contested consumers with low profit offers decreases, the equilibrium profit distribution shifts up. Formally, by (1), the total profit of segment (q_1, q_2) is

$$P(q_1, q_2) = \frac{(2-\ell) q_1}{1-\ell+q_1} \pi^*,$$

which is increasing in q_1 . Nested segments such as $(\ell, 0)$ and $(0, \ell)$ has the maximal asymmetry and hence maximize profit while symmetric segments minimizes profit.

Two remarks are in order on the connection to and the difference from the analysis of unit demand (without taste shock). First, in the case of unit demand, Albrecht (2020) and Bergemann, Brooks and Morris (2020) have shown that the same nested segmentation is also firm-optimal. It is not surprising that their results extend to downward sloping demand, because, by viewing firms as competing in profits rather than in prices, the analysis of equilibrium profits is identical to the one with unit demand.

Second, although producer surplus varies in the same way with respect to the choice of market segmentation in both demand settings, the analysis of consumer welfare and social welfare differs in the two settings. In the case of unit demand, consumers are always served in every possible segmentation and hence the total social welfare is constant across segmentations, which implies that what is best for firms must be worst for consumers and vice versa. In the case of downward-sloping demand, however, consumers are risk averse with respect to variation in profit and total welfare varies across different segmentations. As a result, the analysis of the effects of price discrimination on consumer welfare and social welfare is subtler and more intricate here.

Our main contribution in this section is to show that the firm-optimal segmentation always makes consumers worse off relative to uniform pricing without segmentation, in sharp contrast to the existing literature on price discrimination.

Proposition 2 The unique firm-optimal segmentation yields a lower consumer surplus than uniform pricing for any prior market.

Proof. Note that consumer surplus under the firm-optimal segmentation is $C(\gamma_1 + \gamma_2, 0)$, so we need to show $C(\gamma_1, \gamma_2) \ge C(\gamma_1 + \gamma_2, 0)$. Define $\ell = \gamma_1 + \gamma_2$ and rewrite

$$C\left(\gamma_{1},\gamma_{2}\right)=C\left(\gamma_{1},\ell-\gamma_{1}\right).$$

We prove that $C(\gamma_1, \ell - \gamma_1)$ is decreasing in γ_1 for fixed ℓ . That is, for a fixed total share of captive consumers, consumer surplus decreases as the distribution of captive consumers becomes more uneven between the two firms. We can use (3) to write

$$C\left(\gamma_{1}, \ell - \gamma_{1}\right) = \frac{\gamma_{1}\left(2\gamma_{1} - \ell\right)}{1 - \ell + \gamma_{1}}V\left(\pi^{*}\right) + \frac{2\gamma_{1}^{2}\left(1 - \gamma_{1}\right)\left(\pi^{*}\right)^{2}}{\left(1 - \ell\right)\left(1 - \ell + \gamma_{1}\right)}\int_{\underline{\pi}}^{\pi^{*}}\frac{V\left(\pi\right)}{\pi^{3}}d\pi,$$

where the minimum profit $\underline{\pi}$ is a function of γ_1 and ℓ :

$$\underline{\pi} = \frac{\gamma_1}{1 - \ell + \gamma_1} \pi^*.$$

We take the total derivative with respect to γ_1 :

$$\begin{aligned} \frac{dC\left(\gamma_{1},\ell-\gamma_{1}\right)}{d\gamma_{1}} &= \frac{2\gamma_{1}^{2}+\left(4\gamma_{1}-\ell\right)\left(1-\ell\right)}{\left(1-\ell+\gamma_{1}\right)^{2}}V\left(\pi^{*}\right) - \frac{2\left(1-\gamma_{1}\right)}{\gamma_{1}}V\left(\underline{\pi}\right) \\ &+ \frac{2\left(2\gamma_{1}-3\gamma_{1}^{2}\right)\left(1-\ell+\gamma_{1}\right)-2\gamma_{1}^{2}\left(1-\gamma_{1}\right)}{\left(1-\ell+\gamma_{1}\right)^{2}\left(1-\ell\right)}\left(\pi^{*}\right)^{2}\int_{\underline{\pi}}^{\pi^{*}}\frac{V\left(\pi\right)}{\pi^{3}}d\pi. \end{aligned}$$

Since $V(\pi)$ is decreasing in π , we have $V(\pi) \leq V(\underline{\pi})$ for all $\pi \in [\underline{\pi}, \pi^*]$. It follows that

$$(\pi^*)^2 \int_{\underline{\pi}}^{\pi^*} \frac{V(\pi)}{\pi^3} d\pi \le V(\underline{\pi}) (\pi^*)^2 \int_{\underline{\pi}}^{\pi^*} \frac{1}{\pi^3} d\pi = V(\underline{\pi}) \frac{(1-\ell)(2\gamma_1 - \ell + 1)}{2\gamma_1^2}.$$

Noting that the coefficient for the integral is positive and $V(\pi^*) \leq V(\underline{\pi})$, we obtain that

$$\frac{dC\left(\gamma_{1},\ell-\gamma_{1}\right)}{d\gamma_{1}} \leq 0 \cdot V\left(\underline{\pi}\right) = 0.$$

Therefore, $C(\gamma_1, \gamma_2) \ge C(\gamma_1 + \gamma_2, 0)$.

The intuition for $C(\gamma_1, \gamma_2) \geq C(\gamma_1 + \gamma_2, 0)$ is as follows. Consider a market (γ_1, γ_2) with $\gamma_1 > \gamma_2$. Suppose we increase γ_1 but keep $\ell = \gamma_1 + \gamma_2$ unchanged. There are two effects associated with an increase in γ_1 . First, as we argue earlier, when the market becomes more asymmetric, the equilibrium profit distribution shifts up, which tends to lower consumer surplus. Second, as γ_1 increases, the support of profit distribution shrinks and the variability of profit may go down, which tends to benefit consumers who are risk averse in offered profit π . It turns out the first effect always dominates since the second effect is of higher order compared with the first one.

If for a nested segment $(\ell, 0)$ consumer surplus $C(\ell, 0)$ is convex in ℓ , then any further division of segment $(\ell, 0)$ can only increase consumer surplus. In this case, the firm-optimal segmentation also minimizes consumer surplus. But $C(\ell, 0)$ is not necessarily convex in ℓ . To see this, note that

$$C(\ell, 0) = \ell^2 V(\pi^*) + 2\ell^2 (\pi^*)^2 \int_{\ell\pi^*}^{\pi^*} \frac{V(\pi)}{\pi^3} d\pi$$

and

$$\frac{\partial^2 C\left(\ell,0\right)}{\partial \ell^2} = 2V\left(\pi^*\right) + 4\left(\pi^*\right)^2 \int_{\ell\pi^*}^{\pi^*} \frac{V\left(\pi\right)}{\pi^3} d\pi - \frac{2}{\ell^2} V\left(\ell\pi^*\right) - \frac{2\pi^*}{\ell} V'\left(\ell\pi^*\right) d\pi$$

It is easy to see that $C(\ell, 0)$ is convex in ℓ as $\ell \to 1$, and is concave as $\ell \to 0$ if $V'(\pi)$ is bounded. Therefore, if the share of captive consumers is large $(\ell \to 1)$, then the nested segmentation is also consumer-surplus minimizing. If the share of captive consumers is small $(\ell \to 0)$, however, there are alternative segmentations that generate lower consumer surplus than the nested segmentation.

Proposition 2 has an antitrust implication. The ambiguous welfare impact of price discrimination found in the existing literature does not provide a clear justification for antitrust authorities to intervene. In contrast, Proposition 2 suggests that, if data brokers or third-party platforms are allowed to freely choose information structures for the product markets through public information provision, consumers are likely worse off compared to the case when no information is provided. To protect consumers, it may be necessary for antitrust authorities to step in, for example, by initiating a ban on price discrimination or on selling personal information.

3.3 Consumer-Optimal Segmentation

As for the firm-optimal segmentation, we follow a two-step procedure to find the unique consumer-optimal segmentation. We first show that every segment in a consumeroptimal segmentation must be either symmetric or perfect, and then we use this observation to simplify and solve the information design problem.

Armstrong and Vickers (2019) show that, if the prior market is symmetric, uniform pricing generates a higher consumer surplus than price discrimination for any market segmentation. This insight is partially captured in Lemma 2. It hints (but does not imply) that a consumer-optimal segmentation may involve symmetric market segments.

Lemma 4 Every submarket in a consumer-optimal segmentation must be either symmetric or perfect.

Proof. Suppose by contradiction that a consumer-optimal segmentation contains a segment that is neither symmetric nor perfect. Let (q_1, q_2) denote this segment. Then we must have $q_1, q_2 \in [0, 1)$ and $q_1 \neq q_2$. Without loss we assume $q_1 > q_2$. We first rule out the case with $q_1 + q_2 < 1$. Consider the following ε -segmentation:

$$(1-\varepsilon)\left(\frac{q_1-\varepsilon}{1-\varepsilon},\frac{q_2}{1-\varepsilon}\right)+\varepsilon(1,0)$$

where $\varepsilon \in (0, q_1 - q_2]$. Consumer surplus under the ε -segmentation is

$$C^{\varepsilon}(q_{1},q_{2}) = (1-\varepsilon)C\left(\frac{q_{1}-\varepsilon}{1-\varepsilon},\frac{q_{2}}{1-\varepsilon}\right) + \varepsilon V(\pi^{*})$$

$$= \left(\frac{q_{1}-\varepsilon}{1-\varepsilon-q_{2}}\left(q_{1}-\varepsilon-q_{2}\right) + \varepsilon\right)V(\pi^{*}) + \frac{2\left(1-q_{1}\right)}{1-q_{1}-q_{2}}\frac{\left(q_{1}-\varepsilon\right)^{2}\left(\pi^{*}\right)^{2}}{1-\varepsilon-q_{2}}\int_{\frac{q_{1}-\varepsilon}{1-\varepsilon-q_{2}}\pi^{*}}^{\pi^{*}}\frac{V(\pi)}{\pi^{3}}d\pi$$

With some algebra, we can show that

$$\frac{\partial C^{\varepsilon}(q_{1},q_{2})}{\partial \varepsilon} = \frac{(1-q_{1}-q_{2})(1-q_{1})}{(1-\varepsilon-q_{2})^{2}}V(\pi^{*}) + \frac{2(1-q_{1})}{q_{1}-\varepsilon}V\left(\frac{q_{1}-\varepsilon}{1-\varepsilon-q_{2}}\pi^{*}\right) - \frac{2(1-q_{1})(q_{1}-\varepsilon)(2-q_{1}-2q_{2}-\varepsilon)}{(1-q_{1}-q_{2})(1-\varepsilon-q_{2})^{2}}(\pi^{*})^{2}\int_{\frac{q_{1}-\varepsilon}{1-\varepsilon-q_{2}}\pi^{*}}^{\pi^{*}}\frac{V(\pi)}{\pi^{3}}d\pi$$

Therefore,

$$= \frac{\frac{\partial C^{\varepsilon}(q_{1},q_{2})}{\partial \varepsilon}|_{\varepsilon=0}}{\left(1-q_{1}-q_{2}\right)\left(1-q_{1}\right)}V(\pi^{*}) + \frac{2\left(1-q_{1}\right)}{q_{1}}V\left(\frac{q_{1}}{1-q_{2}}\pi^{*}\right) \\ -\frac{2q_{1}\left(1-q_{1}\right)\left(2-q_{1}-2q_{2}\right)}{\left(1-q_{2}\right)^{2}\left(1-q_{1}-q_{2}\right)}\left(\pi^{*}\right)^{2}\int_{\frac{q_{1}}{1-q_{2}}\pi^{*}}^{\pi^{*}}\frac{V(\pi)}{\pi^{3}}d\pi$$

$$\geq \frac{2\left(1-q_{1}\right)}{q_{1}}V\left(\frac{q_{1}}{1-q_{2}}\pi^{*}\right) - \frac{2q_{1}\left(1-q_{1}\right)\left(2-q_{1}-2q_{2}\right)}{\left(1-q_{2}\right)^{2}\left(1-q_{1}-q_{2}\right)}\left(\pi^{*}\right)^{2}\int_{\frac{q_{1}}{1-q_{2}}\pi^{*}}^{\pi^{*}}\frac{V\left(\frac{q_{1}}{1-q_{2}}\pi^{*}\right)}{\pi^{3}}d\pi$$

where the inequality follows because the term involving $V(\pi^*)$ is non-negative and $V\left(\frac{q_1}{1-q_2}\pi^*\right) \geq V(\pi)$ for all $\pi \in \left[\frac{q_1}{1-q_2}\pi^*, \pi^*\right]$. Note that

$$\int_{\frac{q_1}{1-q_2}\pi^*}^{\pi^*} \frac{(\pi^*)^2}{\pi^3} d\pi = \frac{1-q_1^2+q_2^2-2q_2}{2q_1^2}$$

Hence, after collecting coefficients for $V\left(\frac{q_1}{1-q_2}\pi^*\right)$, we deduce that

$$\frac{\partial C^{\varepsilon}(q_1, q_2)}{\partial \varepsilon}|_{\varepsilon=0} \ge \frac{(1-q_1)(1-q_1-q_2)}{(1-q_2)^2} V\left(\frac{q_1}{1-q_2}\pi^*\right) > 0$$

Therefore, the ε -segmentation strictly increases consumer surplus, a contradiction to the optimality.

It remains to show that a submarket (q, 1 - q) with $q \in (0, 1)$ cannot be part of a consumer-optimal segmentation. Suppose by contradiction that a consumeroptimal segmentation includes such a submarket. This submarket yields a consumer surplus $V(\pi^*)$. If $q \ge 1/2$, we can decompose this submarket into $2(1-q)(\frac{1}{2},\frac{1}{2}) + (2q-1)(1,0)$ and maintain the same consumer surplus $V(\pi^*)$. Since $\gamma_1 + \gamma_2 < 1$, a consumer-optimal segmentation must involve at least one symmetric segment with a strictly positive fraction of contested consumers. But by Lemma 2, a merger of this symmetric segment with $(\frac{1}{2}, \frac{1}{2})$ yields a strictly higher consumer surplus. Again a contradiction to the optimality. The case of q < 1/2 is similar.

In applying the ε -segmentation to an asymmetric segment (q_1, q_2) with $q_1 > q_2$, we separate out ε -fraction of firm 1's captive consumers to form a new perfect segment and make the remaining segment more symmetric. Consumers in the new segment face monopoly pricing from firm 1 and hence are worse off, while consumers in the remaining segment are better off because, as the segment becomes more symmetric, competition between firms is heightened. It turns out that the gain of these consumers always outweighs the loss of consumers in the perfect segment, so one can successively apply the ε -segmentation to improve consumer welfare until the asymmetric segment becomes symmetric.

Now consider the following market segmentation for a prior market (γ_1, γ_2) :

$$(\gamma_1, \gamma_2) = (1 - \gamma_1 + \gamma_2) \left(\frac{\gamma_2}{1 - \gamma_1 + \gamma_2}, \frac{\gamma_2}{1 - \gamma_1 + \gamma_2} \right) + (\gamma_1 - \gamma_2) (1, 0),$$

which contains a maximal symmetric segment and a perfect segment. Since the symmetric segment levels the playing field for the two firms, we will refer to the above segmentation as a "field-levelling segmentation."

Proposition 3 The field-levelling segmentation uniquely maximizes consumer surplus among all possible market segmentations.

Proof. By Lemma 2, a consumer-optimal segmentation can contains only one symmetric segment. Next, we argue that it cannot contain two perfect segments, (0, 1) and (1, 0), simultaneously. To see this, suppose by contradiction, it contains (0, 1) and (1, 0) with size x and y, respectively. Suppose that $x \ge y$ (the case of x < y is analogous). We can replace x(1, 0) + y(0, 1) with $(x - y)(1, 0) + 2y(\frac{1}{2}, \frac{1}{2})$ and merge $(\frac{1}{2}, \frac{1}{2})$ and the symmetric segment in the consumer-optimal segmentation to strictly increase consumer surplus according to Lemma 2 since they are two different symmetric segments. Finally, since $\gamma_1 \ge \gamma_2$ by assumption, the consumer-optimal segmentation can contain only the perfect segment of the form (1, 0). Hence, the consumer-optimal segmentation must be a field-levelling segmentation.

Intuitively, the mean of firm profits is minimized in a symmetric segment. Moreover, any further division of a symmetric segment can only increase the variability of profit distribution and hence lower consumer surplus. As a result, the symmetric segment must be maximal in the consumer-optimal segmentation.

If the segmentation is chosen optimally for the consumers, what would be its payoff implication for firms and the society overall? The following proposition answers the question.

Proposition 4 The field-levelling segmentation minimizes producer surplus. If $V(\pi^*) \ge \pi^*$, it also maximizes social surplus.

Proof. It is easy to verify that, under the field-levelling segmentation, the two firms receive profit $\gamma_1 \pi^*$ and $\gamma_2 \pi^*$, respectively. These are the profits they can guarantee

themselves in any market segmentation by uniformly charging π^* across all market segments. Hence, the field-levelling segmentation minimizes producer surplus.

For social welfare, we first use the ε -segmentation to show that, if $V(\pi^*) \geq \pi^*$, every segment in a social welfare maximizing segmentation must be either symmetric or perfect. Suppose not and consider any segment (q_1, q_2) with $q_1, q_2 \in [0, 1)$ and $q_1 \neq q_2$. The ε -segmentation

$$(1-\varepsilon)\left(\frac{q_1-\varepsilon}{1-\varepsilon},\frac{q_2}{1-\varepsilon}\right)+\varepsilon(1,0)$$

yields a producer surplus of

$$P^{\varepsilon}(q_1, q_2) = \frac{q_1(2 - q_1 - q_2) - \varepsilon}{1 - q_2 - \varepsilon} \pi^*$$

with

$$\frac{\partial P^{\varepsilon}(q_1, q_2)}{\partial \varepsilon}|_{\varepsilon=0} = -(1 - q_1) \frac{1 - q_1 - q_2}{(1 - q_2)^2} \pi^*.$$

It follows that

$$\begin{aligned} \frac{\partial S^{\varepsilon}(q_{1},q_{2})}{\partial \varepsilon}|_{\varepsilon=0} &= \frac{\partial P^{\varepsilon}(q_{1},q_{2})}{\partial \varepsilon}|_{\varepsilon=0} + \frac{\partial C^{\varepsilon}(q_{1},q_{2})}{\partial \varepsilon}|_{\varepsilon=0} \\ &\geq -(1-q_{1})\frac{1-q_{1}-q_{2}}{(1-q_{2})^{2}}\pi^{*} + \frac{(1-q_{1})(1-q_{1}-q_{2})}{(1-q_{2})^{2}}V\left(\frac{q_{1}}{1-q_{2}}\pi^{*}\right) \\ &= \frac{(1-q_{1})(1-q_{1}-q_{2})}{(1-q_{2})^{2}}\left(V\left(\frac{q_{1}}{1-q_{2}}\pi^{*}\right) - \pi^{*}\right) \\ &> 0. \end{aligned}$$

The first inequality follows from the bound for $\frac{\partial C^{\varepsilon}(q_1,q_2)}{\partial \varepsilon}|_{\varepsilon=0}$ obtained in the proof of Lemma 4, and the last inequality follows from the fact that $V\left(\frac{\gamma_1}{1-\gamma_2}\pi^*\right) > V(\pi^*) \ge \pi^*$. By Lemma 2, producer surplus is linear in q and consumer surplus is strictly concave in q for a symmetric segment (q, q). Hence, social surplus S(q, q) is also strictly concave in q. An argument similar to the one in the proof of Proposition 3 can be used to establish that the field-levelling segmentation maximizes social surplus.

The field-levelling segmentation minimizes producer surplus because producer surplus is minimized in a symmetric segment. For social welfare, when $V(\pi^*) \ge \pi^*$, consumer surplus carries a sufficiently large weight in the composition of social surplus. The gains of consumers from the process of gradually levelling the playing field outweigh the losses of firms. Since consumer surplus is strictly concave and producer surplus is linear for symmetric segment, social surplus is also strictly concave. As a result, the symmetric segment in the socially optimal segmentation must be maximal.

It is worth pointing out that although the perfect segmentation

$$(\gamma_1, \gamma_2) = \gamma_1 (1, 0) + (1 - \gamma_1 - \gamma_2) (0, 0) + \gamma_2 (0, 1)$$

induces perfect price competition in the segment of contested consumers it does not yield the maximal consumer surplus. It generates the same expected producer surplus of $(\gamma_1 + \gamma_2) \pi^*$ but its associated profit distribution is more volatile than the one under the field-levelling segmentation. In the case of unit demand, both the perfect segmentation and the field-levelling segmentation simultaneously maximize consumer surplus and minimize producer surplus. Hence, in some sense, our field-levelling segmentation maximizes consumer surplus more robustly.

4 Concluding Remarks

This paper has characterized the firm-optimal and the consumer-optimal public market segmentation in a specialized duopoly model. We have also showed that the firmoptimal segmentation yields a lower consumer surplus than no segmentation, and that the consumer-optimal segmentation also minimizes producer surplus among all segmentations.

A natural next step is to characterize the set of payoff vectors that are attainable by all market segmentations, as BBM do in their monopoly setting. If consumers in our model have unit demand as in BBM, then the set of attainable payoffs takes a simple form. With offered profits replaced by offered prices, the unique equilibrium for every market segment is the same as the one characterized in Lemma 1. All consumers are served in any market segment, so full surplus is always realized. This fact, together with risk-neutrality associated with unit demand, implies that the interests of firms and consumers are diametrically opposed. It follows that the nested segmentation attains the point of maximal producer surplus and minimal consumer surplus and that the field-levelling segmentation attains the point of minimal producer surplus and maximal consumer surplus. The set of attainable payoffs is then the line segment connecting these two points. With general downward-sloping demand, however, the task of characterizing the full payoff set is difficult.

Throughout of the paper, we restrict attention to market segmentations that are based on signals publicly observable to both firms. In many applications, firms often do not share the same information about consumers. Allowing for multiple firms, Bergemann, Brooks and Morris (2021) identify an upper bound for the equilibrium distribution of prices and for any symmetric prior they construct a private segmentation that attains this upper bound. One can show, however, if the prior market is strongly asymmetric (e.g., with either $\gamma_1 = 0$ or $\gamma_2 = 0$), their upper bound is not attained by their constructed private segmentation. It remains an open question about the form of firm-optimal private segmentation for an asymmetric prior. For the downward-sloping demand setting here, we conjecture that our field-levelling segmentation remains consumer-optimal among all (private and public) segmentations, but we are unable to prove it.

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