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Design, Analysis, and Fabrication of a Differential Pump Driven by Whitworth Mechanisms

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Abstract—*A new differential pump is presented in this paper. A pump of this type is mainly composed of a motor, two trains of gearings, two speed transforming mechanisms of the same type and a pump body. The pump body is composed of two impellers and a pump case. The operating principle is: the motor indirectly drives two speed transforming mechanisms, which in turn drive their corresponding impellers separately but cooperatively, so that the volumes of the four compartments separated by the two impellers vary periodically. Consequently liquid can be drawn in and discharged through two inlets and two outlets of the pump case, respectively. The function of the speed transforming mechanism is to transform uniform rotation into nonuniform rotation. Whitworth mechanisms are used as the speed transforming mechanisms. The operation principle is presented, the kinematics is analyzed, and the performance indexes are discussed. Finally, the experimental result is described.*

Keywords: differential pumps, nonuniform rotation, Whitworth mechanisms, gears

I. Introduction

Several types of positive displacement pumps are extensively used in various industries for their advantage of large displacement. Nevertheless, some common drawbacks of these pumps such as small flow and volume ratios hinder their applications in some fields. Therefore, multistage centrifugal pumps are usually utilized in such applications requiring high pressure and large flow capacity as water flooding to oil field strata.

Differential pumps are a new type of pump firstly presented by Ming Chen [2], [3], [4], [5]. A pump of this type is composed of a motor, two trains of gearings, two speed transforming mechanisms of the same type, two impellers and a pump case. Two speed transforming mechanisms are used to indirectly drive two impellers to rotate with periodical nonuniform velocities, which makes the volumes of the compartments of the pump body changes and thus liquid flow in and out. The pumps are characterized by the simple structure, small volume, small mass, high efficiency, large flow and volume ratio, and can be used in occasions requiring high pressure and large flow, such as water flooding to oil field strata, water supply to high buildings, and distant conveyance of industrial liquid raw materials. The study of the differential pump is interesting from both practical and theoretical viewpoints.

This paper presents a new driving system for the differential pump. The Whitworth mechanism [1] is utilized to produce the nonuniform velocities. Compared with the preexisting differential pump driven by noncircular gears, this pump has the advantage of easy fabrication since Whitworth mechanisms are much more easily fabricated than noncircular gears.

II. Operating Principle

Fig. 1 shows the driving system of the pump. The motor (1) drives the gear (2) to rotate, which meshes with the gear (3) and gear (14) and force them to rotate in the same direction. The crank (4) of one Whitworth mechanism is fixed to the gear(3), and the crank (13) of the other Whitworth mechanism is fixed to the gear (14). Thereby the both cranks rotate in the same direction, and they respectively force the guide-bar (6) and guide-bar (11) to rotate with periodical nonuniform velocities through the slider (5) and slider (12). Besides, the crank (4) and crank (13) are fixed with the same phase. The guide-bar (6) and guide-bar (11) are fixed to gear (7) and gear (10) respectively, and force them to rotate in the same direction with nonuniform velocities. The gear (7) and gear (10) mesh with gear (8) and gear (9) respectively which are coaxially fixed, and force them to rotate in the same direction with nonuniform velocities. The gear ratio of gear (8) (or the gear (9)) to the gear (7) (or the gear (10)) is 2, i.e., $z_8/z_7 = z_9/z_{10} = 2$. As shown in Fig. 1, e denotes the distance between the rotating axes of the cranks and their corresponding guide-bars. And l denotes the kinematic length of the cranks.

Fig. 2 shows the pump body. The impeller (3) and impeller (4) fixed coaxially inside the hull (5) are fixed to gear (8) and gear (9) in Fig. 1, respectively, and they rotate with nonuniform angular velocities. The impeller (3) and impeller (4) have two symmetrically distributed vanes respectively. The neighboring vanes and the hull form four compartments denoted by I, II, III, and IV. When the pump is working, the four compartments enlarge and decrease periodically and hence the liquid is drawn in and discharged through the inlets and outlets, respectively.

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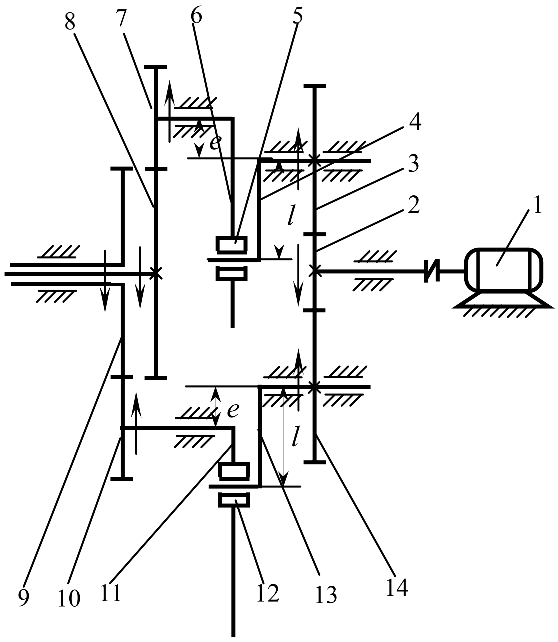
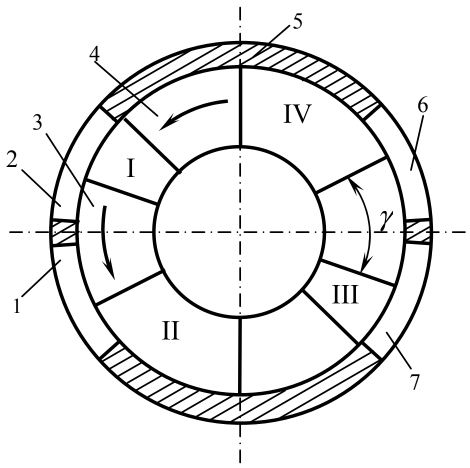


1. motors; 2. 3. 7. 8. 9. 10. 14. gears; 4. 13. cranks; 5. 12. sliders; 6. 11. guide-bars.

Fig. 1. Driving system of the pump.



1. 6. inlets; 2. 7. outlets; 3. 4. impellers; 5. hull.

Fig. 2. Pump body.

III. Kinematic Analysis of the Driving System

From Fig. 2, it is easily known that the cranks of the Whitworth mechanisms have the same rotating direction and angular velocity, which is expressed by

$$\omega_4 = \frac{z_2}{z_3}\,\omega_1 \tag{1}$$

$$\omega_{13} = \frac{z_2}{z_{14}}\,\omega_1 \tag{2}$$

where ω_1 denotes the angular velocity of the motor, and z_2, z_3 and z_{14} respectively denote the teeth numbers of the gear (2), gear (3), and gear (14).

From the velocity relationship between the crank and guide-bar of a Whitworth mechanism, the angular velocities of the guide-bar (6) and guide-bar (11) are respectively given by

$$\omega_6 = \frac{1 - c\cos\varphi}{1 - 2c\cos\varphi + c^2}\,\omega_4 \tag{3}$$

$$\omega_{11} = \frac{1 + c\cos\varphi}{1 + 2c\cos\varphi + c^2}\,\omega_{13} \tag{4}$$

where φ denotes the rotating angle of the cranks, and c represents a length ratio e/l.

The angular velocities of the gear (8) and gear (9) can be expressed as

$$\omega_8 = \frac{z_7}{z_8}\,\omega_6 \tag{5}$$

$$\omega_9 = \frac{z_{10}}{z_9}\,\omega_{11}. \tag{6}$$

The impeller (3) and impeller (4) respectively have the same angular velocities with the gear (8) and gear (9), i.e.,

$$\omega_{i3} = \omega_8 \tag{7}$$

$$\omega_{i4} = \omega_9 \tag{8}$$

where ω_{i3} and ω_{i4} respectively denote the angular velocity of the impeller (3) and impeller (4).

Taking Eq. 1, Eq. 3 and Eq. 5 into Eq. 7, and taking Eq. 2, Eq. 4 and Eq. 6 into Eq. 8 give

$$\omega_{i3} = \frac{z_2}{z_3}\,\frac{z_7}{z_8}\,\frac{1 - c\cos\varphi}{1 - 2c\cos\varphi + c^2}\,\omega_1 \tag{9}$$

$$\omega_{i4} = \frac{z_2}{z_{14}}\,\frac{z_{10}}{z_9}\,\frac{1 + c\cos\varphi}{1 + 2c\cos\varphi + c^2}\,\omega_1. \tag{10}$$

Because

$$\frac{z_8}{z_7} = \frac{z_{10}}{z_9} = \frac{1}{2} \tag{11}$$

and it is set that

$$\frac{z_2}{z_3} = \frac{z_2}{z_{14}} = u \tag{12}$$

hence, the angular velocity of the impellers can be written as

$$\omega_{i3} = \frac{u}{2}\,\frac{1 - c\cos\varphi}{1 - 2c\cos\varphi + c^2}\,\omega_1 \tag{13}$$

$$\omega_{i4} = \frac{u}{2}\,\frac{1 + c\cos\varphi}{1 + 2c\cos\varphi + c^2}\,\omega_1. \tag{14}$$

Fig. 3 shows the the variation of the angular velocities of the two impellers with the rotating angles of the cranks. It can be seen that the impellers rotate with periodical nonuniform velocities. Hence the volumes of the four compartments formed by the neighboring vanes vary periodically, which result in the flowing in and out of the liquid.

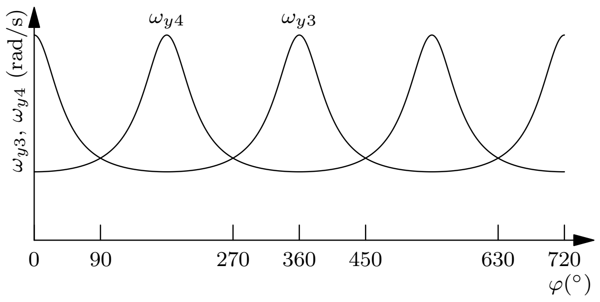


Fig. 3. Angular velocity of the impellers.

IV. Calculation of the Vane Angle

The shape of the vanes on the impellers is sectorial, as shown in Fig. 4. The vane angle refers to the radius angle formed by two sides of a vane.

When the angular velocities of the two impellers are equal, the two impellers close. From Fig. 3, when the cranks rotate for 90°, 270°, 450° or 630°, the two impellers have the equal angular velocity, and therefore the vanes on them close. Fig. 4(a) shows the closing state of the impellers when the cranks rotate for 90° or 450°. And Fig. 4(b) shows the closing state of the impellers when the cranks rotate for 270° or 630°.

When the cranks rotate from 0° to 270°, the impellers rotate from the closing state shown in Fig. 4(a) to the closing state shown in Fig. 4(b). Thereby the rotating angle of the impeller (4) is equal to the vane angle denoted by γ, i.e.,

$$\gamma = \int_{t_1}^{t_2} \omega_{i4}\, \mathrm{dt}. \tag{15}$$

Since the angular velocity of the cranks is

$$\omega_4 = \frac{\mathrm{d}\varphi}{\mathrm{dt}} = \frac{z_2}{z_{14}}\,\omega_1 = u\omega \tag{16}$$

we have

$$\mathrm{dt} = \frac{\mathrm{d}\varphi}{\mathrm{u}\omega_1}. \tag{17}$$

Thereby Eq. 15 can be written as

$$\gamma = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\omega_{i4}}{u\omega_1}\, \mathrm{d}\varphi. \tag{18}$$

By taking Eq. 14 into Eq. 19, we have

$$\gamma = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}\, \frac{1 + c\cos\varphi}{1 + 2c\cos\varphi + c^2}\, \mathrm{d}\varphi. \tag{19}$$

Fig. 5 shows that the vane angle γ varies with the ratio c whose value spans from 0 to 1. When $c = 1$, the guide-bars have the same angular velocity as the cranks. Thereby the impeller (3) and impeller (4) have the same angular velocity all the time, and the pump cannot work. When $c > 1$, the aforementioned Whitworth mechanism is no longer a real

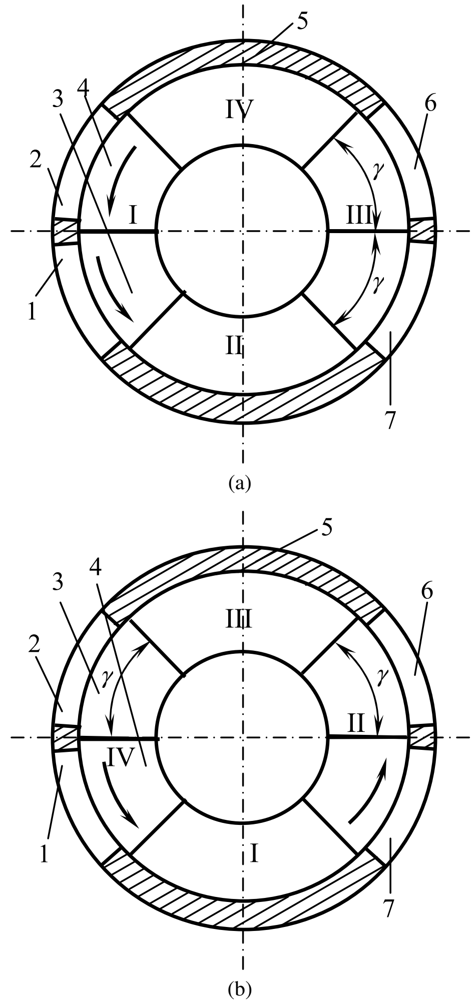


Fig. 4. Closing state of the vanes.

Whitworth mechanism (actually, it becomes the oscillating guide-bar mechanism), and hence can not be used as part of the driving system of the pump.

It can also be seen that the vane angle becomes smaller when c increases. When $c = 0$, the vane angle attains the largest ($\gamma_{max} = 90°$), all the volume in the hull is occupied by the four vanes on the two impellers, and hence the pump cannot work. When $c = 1$, the vane angle become the smallest ($\gamma_{min} = 45°$). However, the corresponding mechanisms do not exist. From the above discussion concerning the vane angle, the feasible value of the vane angle should span from 45°to 90°.

V. Calculation of the Displacement and Instantaneous Flow of the Pump

A. Calculating the Displacement

When the two impellers open and close once, the liquid flows in and out once. See Fig. 6, the corresponding volume can be written as

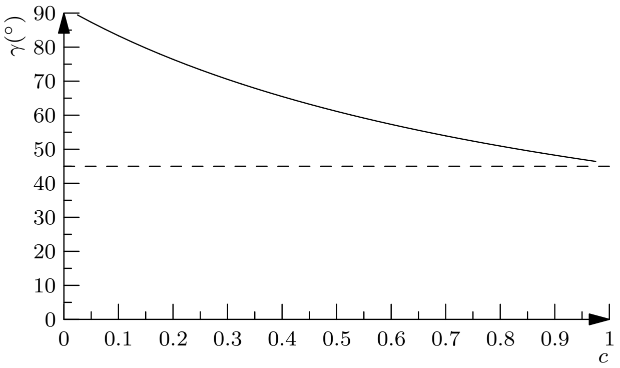


Fig. 5. Vane angle.

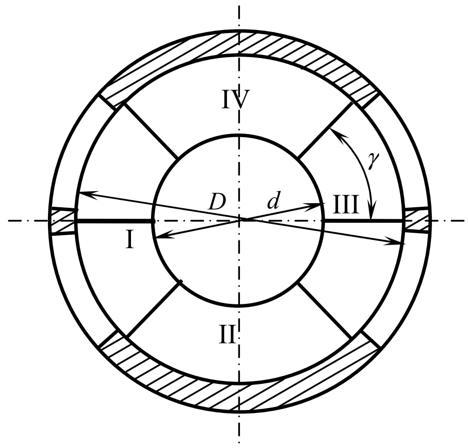


Fig. 6. Calculating the displacement.

$$Q = \pi \frac{D^2 - d^2}{4} \frac{2\pi - 4\gamma}{2\pi} h \tag{20}$$

where D denotes the external diameter of the vane, d denotes the internal diameter of the vane, and h is the dimension of the vanes along the axis of the impellers.

When the impellers rotate once, the liquid flows in and out for four times. Thereby the displacement of the pump is

$$\begin{aligned} Q &= 4\pi \frac{D^2 - d^2}{4} \frac{2\pi - 4\gamma}{2\pi} h \\ &= (D^2 - d^2)(\pi - 2\gamma)h. \end{aligned} \tag{21}$$

From Eq. 21, it is readily seen that the displacement Q increases as the vane angle decreases. Thereby the displacement Q increases as c increases, as illustrated by the curve in Fig. 7.

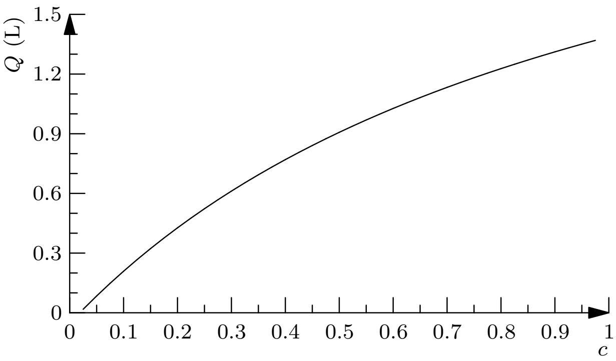


Fig. 7. Displacement.

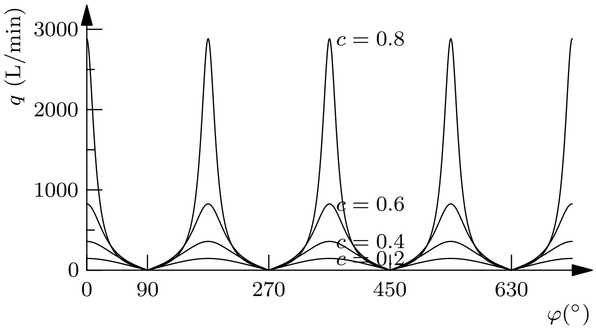


Fig. 8. Flow.

B. Calculating the Instantaneous Flow

In Fig. 8, the instantaneous flow is denoted by q, the flow during $\mathrm{d}t$ is

$$\begin{aligned} q\mathrm{d}t &= 2\pi \frac{D^2 - d^2}{4} \left|\frac{\mathrm{d}\varphi_{i4} - \mathrm{d}\varphi_{i3}}{2\pi}\right| h \\ &= \frac{D^2 - d^2}{4} \left|\mathrm{d}\varphi_{i4} - \mathrm{d}\varphi_{i3}\right| h. \end{aligned} \tag{22}$$

Thereby, we have

$$\begin{aligned} q &= \frac{D^2 - d^2}{4} \left|\frac{\mathrm{d}\varphi_{i4} - \mathrm{d}\varphi_{i3}}{\mathrm{d}t}\right| h \\ &= \frac{D^2 - d^2}{4} \left|\omega_{i4} - \omega_{i3}\right| h. \end{aligned} \tag{23}$$

Taking Eq. 13 and Eq. 14 into Eq. 23 gives

$$q = \frac{u}{2} \frac{D^2 - d^2}{4} h \left|\frac{(2 - c^2)c\cos\varphi}{(1 + c^2)^2 - 4c^2\cos^2\varphi}\right| \omega_1. \tag{24}$$

As it is shown in Fig. 8, the four curves respectively correspond to $c = 0.8$, $c = 0.6$, $c = 0.4$ and $c = 0.2$. It is known that the flow pulsation increases as c increases. Thereby, the value of c should not be too large. And it is proper for c to take values between 0.3 and 0.5.

VI. Calculation of the Driving Torque

The liquid pressure at the inlets is denoted by p. And the liquid pressure at the outlets is assumed to be zero. At the state shown in Fig. 9(a), the compartment I and compartment III are connected to the inlet (2) and outlet (7), respectively. The torques on the impeller (3) and impeller (4) can be derived as

$$T_{y3} = \frac{D^2 - d^2}{2}\, hp \tag{25}$$

$$T_{y4} = -\frac{D^2 - d^2}{2}\, hp. \tag{26}$$

At the state shown in Fig. 9(b), the compartment II and compartment IV are connected to the outlet (2) and outlet (7), respectively. The torques on the impeller (3) and impeller (4) are

$$T_{y3} = -\frac{D^2 - d^2}{2}\, hp \tag{27}$$

$$T_{y4} = \frac{D^2 - d^2}{2}\, hp. \tag{28}$$

The torques are illustrated in Fig. 10. From Fig. 1, it is known that the torque on the guide-bar (6) (or the gear (7)) is

$$T_6 = \frac{z_7}{z_8}\, T_{y3} = \frac{1}{2}\, T_{y3}. \tag{29}$$

The torque on the guide-bar (11) (or the gear (10)) is

$$T_{11} = \frac{z_{10}}{z_9}\, T_{y4} = \frac{1}{2}\, T_{y4}. \tag{30}$$

The torque on the crank (4) (or the gear (3)) is

$$T_4 = \frac{1 - c\cos\varphi}{1 - 2c\cos\varphi + c^2}\, T_6 \tag{31}$$

The torque on crank (13) (or the gear (14)) is

$$T_{13} = \frac{1 + c\cos\varphi}{1 + 2c\cos\varphi + c^2}\, T_{11} \tag{32}$$

Fig. 11 shows that the torques on the cranks varies with the rotating angles of the cranks, and that the torques fluctuate periodically. As c increases, the fluctuation amplitude increases. When the torque is positive, its direction is opposite to the rotation direction of the cranks. And when the torque is negative, its direction is the same to the rotation direction of the cranks.

It is known from Fig. 1 that the torque on the gear (2) (or the input shaft) is

$$T_2 = T_4\, \frac{z_2}{z_3} + T_{13}\, \frac{z_2}{z_{14}} = (T_4 + T_{13})\, u. \tag{33}$$

The torque on the input shaft (or gear (2)) T_2 is shown in Fig. 12. Although the torque fluctuation still exists, it is largely diminished compared with the torque fluctuation on the cranks. This merit makes it very convenient for the designers to choose the motor.

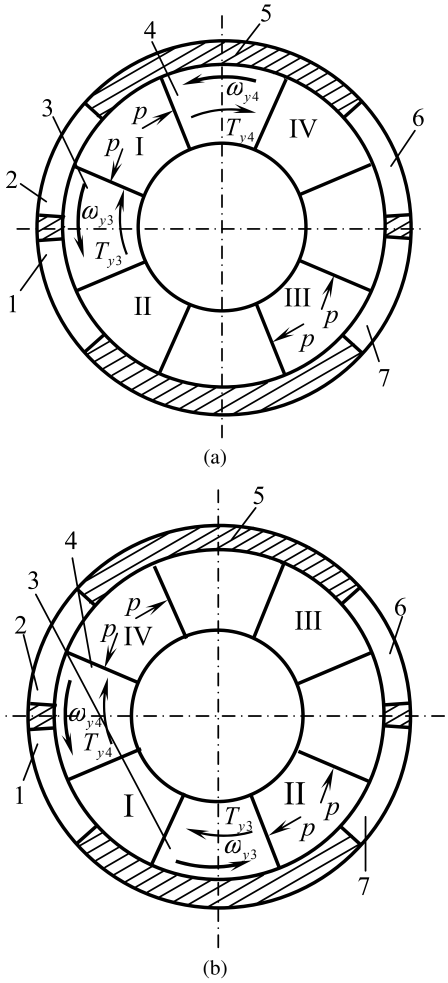


Fig. 9. Force diagram of the vanes.

VII. Experiment

The fabricated pump is shown in Fig. 13. The frequency converter is utilized to adjust the velocity of the motor. When the velocity is increased, the noise becomes louder due to the mechanism gaps. The maximum oil pressure is 8MPa. When the oil pressure becomes larger, the oil leakage from the case to the velocity-transforming case becomes more serious.

VIII. Conclusions

A new driving system of differential pumps has been presented. More specifically, the Whitworth mechanism can be used as an alternative to noncircular gears to produce the nonuniform velocity. The Whitworth mechanism has the advantage of easy fabrication over noncircular gears. The experiment showed that the oil pressure produced by the

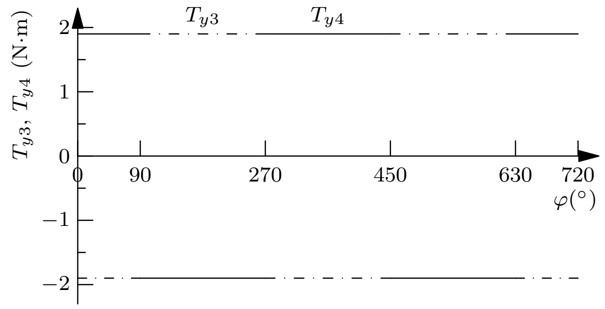


Fig. 10. Torques on the impellers.

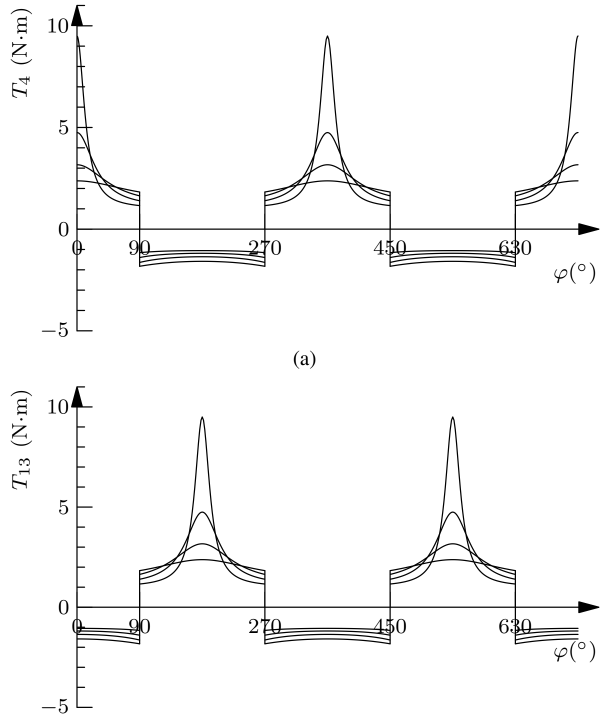


Fig. 11. Torques on the cranks.

pump is fairly high. The phenomenon of oil leakage may be solved by redesigning the detailed structures between the pump case and the velocity transforming case, and using high performance rubber seal rings.

For the purpose of optimizing the mechanism parameters, the dynamics analysis is in progress.

Appendix

The parameters of the pump used in this paper is shown in TABLE I.

TABLE I. Parameters

D	d	h	u	ω_1	p
250mm	200mm	40mm	$\frac{1}{3}$	$1458 \times \frac{\pi}{30}$	8MPa

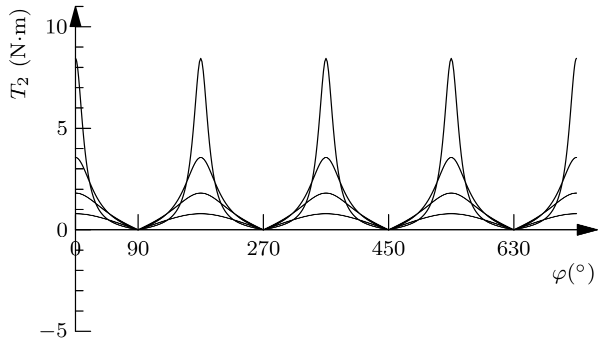


Fig. 12. Torques on the input shaft.

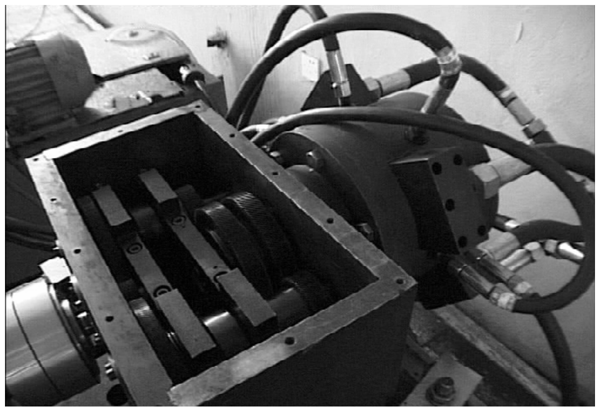


Fig. 13. Fabricated pump.

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