ECO 2020 Tutorial 3

5 ① WTS: $\forall s' > s$, $\beta' > \beta$

$$f(s', \beta) \geq f(s, \beta) \rightarrow f(s', \beta') \geq f(s, \beta')$$

$$(u_1(w - s') - u_1(w - s)) + \beta(u_2(s') - u_2(s)) \geq 0$$

$$\longrightarrow \quad + \beta'(u_2(s') - u_2(s)) \geq 0$$

② Suppose $s^*(\beta) > s^*(\beta')$

$$f(s^*(\beta), \beta) > f(s^*(\beta), \beta')$$

$$\rightarrow f(s^*(\beta), \beta') > f(s^*(\beta'), \beta)$$

③ $$f(s, \beta) = u_1(w - s_1) + \sum_{t=2}^{T} \beta^{t-1} u_t(s_{t-1} - s_t)$$

$$= u_1(w - s_1) + V(s_1, \beta)$$

incomplete see solution

$s' > s$: $f(s', \beta) \geq f(s, \beta)$

$$V(s', \beta) - V(s, \beta) \geq u_1(w - s_1) - u_1(w - s_1')$$

$$\shortparallel$$

$\beta' > \beta$: $$\sum_{t=2}^{T} \beta^{t-1}(u_t(s_{t-1}' - s_t') - u_t(s_{t-1} - s_t)) \leq$$

$$\sum_{t=2}^{T} \beta'^{\,t-1}(u_t(s_t' - s_t) - u_t(s_{t-1} - s))$$

$$\geq u_1(w - s_1) - u_1(w - s_1)$$

$$\Rightarrow f(s', \beta') \geq f(s, \beta')$$

② 1) 3: $3 \times 40 + 1 \times 50 = 170$

95: $2 \times 55 + 2 \times 40 = 190$

$2 \times 40 + 2 \times 50 = 180$

2) $\max \ f(z) \cdot p \quad s.t. \quad w_1 z_1 + w_2 z_2 \le c$

$\mathcal{L} = f(z)p + \lambda \left(c - w_1 z_1 - w_2 z_2 \right)$

Envolope Theorom.

$\Rightarrow \frac{z_1}{z_2} = \frac{\alpha w_2}{(1-\alpha) w_1} \longrightarrow w_1 z_1 + w_2 z_2 = c$

$$\begin{cases} z_1 = \frac{\alpha c}{w_1} \\ z_2 = \frac{(1-\alpha) c}{w_2} \end{cases}$$

③ (1) $f : R \times T \to R$ satisfies ID

WTS $f(\cdot, t)$ SCE

$\forall x' > x, \ t' > t$

ID: $f(x', t') - f(x, t') \geq f(x', t) - f(x, t)$

If $\underbrace{f(x', t) - f(x, t)}_{>0} \geq 0 \xRightarrow{ID} \underbrace{f(x', t') - f(x, t')}_{>0} \geq 0$

(2) f, g have ID

$\forall x' > x, \ t' > t \quad f(x', t') - f(x, t') \geq f(x', t) - f(x, t)$

$g(x', t') - g(x, t') \geq g(x', t) - g(x, t)$

$$\left(f(x', t') + g(x', t') \right) - \left(f(x, t') + g(x, t') \right) \geq$$

$$\left(f(x', t) + g(x', t) \right) - \left(f(x, t) + g(x, t) \right)$$

(3) $g(x,t) = f(x)$

$g(x', t') - g(x', t) = f(x') - f(x') = 0$

$g(x, t') - g(x, t) = f(x) - f(x) = 0$

(4) $f(x,t) = a x^{\alpha} t^{\beta}$ $\quad a, \alpha, \beta > 0$

$f(x', t') - f(x', t) = a x'^{\alpha} t'^{\beta} - a x'^{\alpha} t^{\beta}$

$= a x'^{\alpha} (t'^{\beta} - t^{\beta})$

$f(x, t') - f(x, t) = a x^{\alpha} t'^{\beta} - a x^{\alpha} t^{\beta}$

$= a x^{\alpha} (t'^{\beta} - t^{\beta})$

$a x'^{\alpha} > \alpha x^{\alpha} \Rightarrow a x'^{\alpha} (t'^{\beta} - t^{\beta}) > a x^{\alpha} (t'^{\beta} - t^{\beta})$

④ (1) SCC $\quad x' > x \quad t' > t$

If $h(f(x', t)) \geq h(f(x,t))$

$\xRightarrow{h \text{ strictly } \uparrow} \quad f(x', t) \overset{>}{\geq} f(x,t)$

$\xRightarrow{\text{SCC of } f} \quad f(x', t') \overset{>}{\geq} f(x, t')$

$\xRightarrow{h \uparrow} \quad h(f(x', t')) \overset{>}{\geq} h(f(x, t'))$ V

(2) $f = xt$ $\quad$ ID

$h: R \to R \quad h(y) = \arctan(y)$

$x' = 2, \quad x = 1$

$h \circ f(x', t) - h \circ f(x, t) = \arctan 2t - \arctan t$

$$\frac{2}{1+4t^2} - \frac{1}{1+t^2} = \frac{(1-2t^2)}{\bigcirc}$$

①

$$\pi(w) = \max_{z} \underbrace{f(z) - w \cdot z}_{V(z,w)}$$

Apply Envolope Thm