ECO 2020 Tutorial 4

④ g is twice differentiable need to show ID

take the element (x, y', z) and (x', y, z)

$$x \leq x', \quad y \leq y'$$

by supermodularity,

$$f(x, y', z) + f(x', y, z) \leq f(x', y', z) + f(x, y, z)$$

$$f(x', y, z) - f(x, y, z) \leq f(x', y', z) - f(x, y', z)$$

$$f(x', y) - f(x, y) \leq f(x', y') - f(x, y')$$

and so f has ID $\Rightarrow f_{xy} \geq 0$

Conversely, $f_{xy} \geq 0$

Suppose $x \leq x', \ y \leq y', \ z \leq z'$

by ID $f(x', y, z) - f(x, y, z) \leq f(x', y', z') - f(x, y', z')$

$$\Rightarrow f(x', y, z) + f(x, y', z') \leq f(x', y', z) + f(x, y, z)$$

$$\Rightarrow f(x', y, z) + f(x, y', z') \leq f(x \vee x', y \vee y', z \vee z') + f(x \wedge x', y \wedge y', z \wedge z')$$

and so f is supermolar.

NEED additional proof of $f_{xy} \geq 0$ iff ID

modify the proof in 2D.

(9) a) 1 state only: $S = \{*\}$

$Z = \{a, b, c\}$

$f \sim g \sim \alpha f + (1-\alpha) g \qquad \alpha \in [0,1]$

$$u(\alpha f + (1-\alpha) g) = (\alpha p_a^f + (1-\alpha) p_a^g)\, u(a)$$
$$+ (\alpha p_b^f + (1-\alpha) p_b^g)\, u(b)$$
$$+ (\alpha p_c^f + (1-\alpha) p_c^g)\, u(c)$$

$\Rightarrow u(f) = p_a^f \cdot u(a) + p_b^f u(b) + p_c^f u(c)$

$U(\alpha f + (1-\alpha) g) = \alpha U(f) + (1-\alpha) U(g)$

$\Rightarrow$ since $f \sim g$, then

$= U(f) = U(g)$

b) Betweenness Axiom

$\forall L, L'$ and $\lambda \in (0,1)$ if $L \sim L'$, then

$\lambda L + (1-\lambda) L' \sim L$

(a) $L \succsim L'$ take any lottery $L^* \in \Delta Z$ by independence

$\lambda L + (1-\lambda) L^* \succsim \lambda L' + (1-\lambda) L^*$

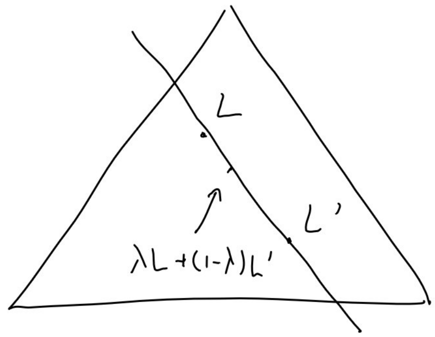
take $L^* = L' \Rightarrow \lambda L + (1-\lambda) L' \succsim L'$

$L \sim L' \Rightarrow L \succsim L'$ and $L \precsim L'$

$\lambda L + (1-\lambda) L' \succsim L'$ and $\lambda L + (1-\lambda) L' \precsim L'$

$\Rightarrow \lambda L + (1-\lambda) L' \sim L' \Rightarrow$ betweenness axiom holds.

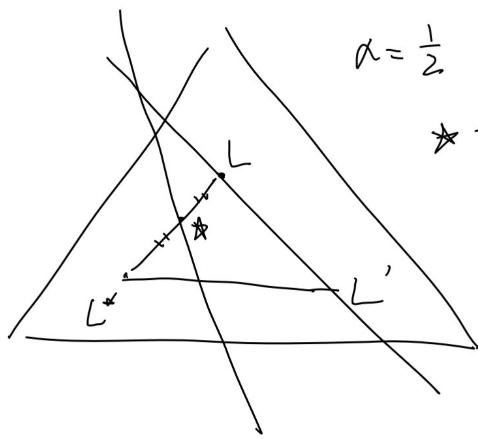
(b)



if L, L' and $\lambda L + (1-\lambda) L'$ are equally preferred

$\Rightarrow$ same indifference curve (straight line)

(c)



$\alpha = \frac{1}{2}$

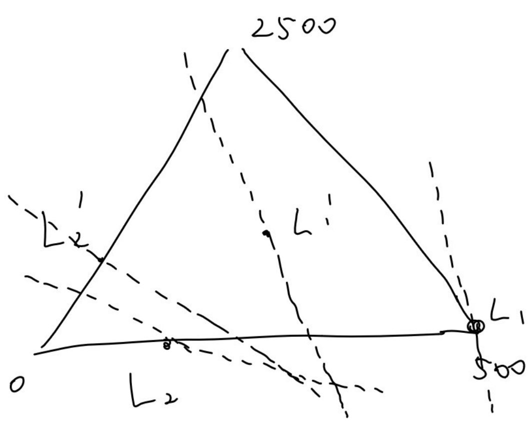
$☆ = \frac{1}{2} L + \frac{1}{2} L^*$

☆☆ not $\frac{1}{2}, \frac{1}{2}$

$\lambda L + (1-\lambda) L^* \gtrsim L'$

$\lambda L + (1-\lambda) L^* \lesssim L'$

(d)



$L_1 > L_1'$

$L_2' > L_2$

⑤ $E[\pi(\underbrace{a, q}_{X}, \underbrace{A, c}_{T})] = p(a)\, q A + q P(q) - qc - q$

WTS π is supermolar on $X \times T$ let $t = -c$

1) $h(a, q, A) = p(a)\, q A$

WLOG take $x = (a, q, A)$ $a < a'$

$y = (a', q', A)$ $q > q'$

$h(x \vee y') + h(x \wedge y) - h(x) - h(y) \geq 0$

$p(a')\, q A + p(a)\, q' A - p(a)\, q A - p(a')\, q' A \geq 0$

$\underbrace{(p(a') A - p(a) A)}_{\geq 0} \underbrace{(q - q')}_{> 0} \geq 0$ since $A > 0$

$\therefore$ h is supermodular

(2) $-qc = qt$

$h(q, t) = qt$ take $q < q'$, $t > t'$

$h(x \vee y) + h(x \wedge y) - h(x) - h(y) \geq 0$

$q' t + q t' - qt - q' t' \geq 0$

$\underbrace{(q' - q)}_{> 0} \underbrace{(t - t')}_{> 0} \geq 0$ $\Rightarrow$ qt is supermodular

π is S. and $(a^*, q^*)(A, t)$ is monotonically increasing

(decreasing in c)

③ a) $(X, \leq_X)$

$f: X \to \mathbb{R}$ $\nearrow$ ← increasing

$v: \mathbb{R} \to \mathbb{R}$ convex $\nearrow$

$v \circ f \to SM$

$$\underbrace{f(x)}_{a} + \underbrace{f(y)}_{b} \leq \underbrace{f(x \wedge y)}_{c} + \underbrace{f(x \vee y)}_{d}$$

$$v(a) + v(b) \leq v(c) + v(d)$$

1) $f \nearrow$ $\quad a \geq c$

$b \leq d$

f SM $\quad a + b \leq c + d$

$$\alpha_x = \frac{d-a}{d-c} \Rightarrow 1 - \alpha_x = \frac{a-c}{d-c}$$

$$\alpha_y = \frac{d-b}{d-c} \Rightarrow 1 - \alpha_y = \frac{b-c}{d-c}$$

$$v(a) \leq \alpha_x v(c) + (1-\alpha_x) v(d)$$

$$+ \quad v(b) \leq \alpha_y v(c) + (1-\alpha_y) v(d)$$

$$v(a) + v(b) \leq \left(\frac{d-a}{d-c} + \frac{d-b}{d-c}\right) v(c) + \left(\frac{a-c}{d-c} + \frac{b-c}{d-c}\right) v(d)$$

$$\leq \frac{2d - (a+b)^{+c-c}}{d-c} v(c) + \frac{(a+b) - 2c^{+d-d}}{d-c} v(d)$$

$$\leq v(c) + v(d) + \underbrace{\frac{(c+d) - (a+b)}{b-c}}_{>0} \underbrace{(v(c) - v(d))}_{<0}$$

$$\leq v(c) + v(d)$$

b) f g

$$(\alpha f + (1-\alpha) g)(x) + (\alpha f + (1-\alpha) g)(y)$$

$$= \underbrace{\alpha f(x)} + \underbrace{(1-\alpha) g(x)} + \underbrace{\alpha f(y)} + \underbrace{(1-\alpha) g(y)}$$

$$\hookrightarrow \alpha (f(x) + f(y)) + (1-\alpha)(g(x) + g(y))$$

$$\leq \alpha (f(x \wedge y) + f(x \vee y)) + (1-\alpha)(g(x \wedge y) + g(x \vee y))$$

c) $X \subseteq \mathbb{R}$ $(X, \leq)$

$X = (a, b)$

$f : X \to \mathbb{R}$

take $c, d \in (a, b)$ s.t $c \leq d \Rightarrow c \wedge d = c$
$c \vee d = d$

$$f(c \wedge d) = f(c)$$
$$f(c \vee d) = f(d)$$

$$f(c) + f(d) = f(c) + f(d)$$

⑧ u_s represents $\precsim$. Show (v_s) preserve $\precsim$

$$v_s = a u_s + b_s \quad (a > 0,\ b_s \in \mathbb{R})$$

$$U(f) = \sum_{s,z} f_s(z)\, u_s(z) \qquad V(f) = \sum_{s,z} f_s(z)\, v_s(z)$$

For any $f, g \in X$, $f \succsim g$

$$\overset{\text{def}^{n}}{\Longleftrightarrow} \quad \sum_{s,z} f_s(z)\, u_s(z) \geq \sum_{s,z} g_s(z)\, u_s(z)$$

$$\Longleftrightarrow \quad a \sum_{s,z} f_s(z) u_s(z) + \sum_{s,z} f_s(z)\, b_s$$

$$\geq a \sum_{s,z} g_s(z) u_s(z) + \sum_{s,z} g_s(z)\, b_s$$

① $a > 0$

② $\forall\, h \in X$

$$\sum_{s,z} h_s(z) b_s = \sum_s b_s \sum_z h_s(z)$$

$$\Longleftrightarrow \quad \sum_{s,z} f_s(z) \big(\overbrace{a\, u_s(z) + b_s}^{v_s(z)} \big)$$

$$\geq \sum_{s,z} g_s(z) \big(\underbrace{a u_s(z) + b_s}_{v_s(z)} \big)$$

$$\Longleftrightarrow \quad V(f) \geq V(g)$$

(10)

+10   −10

Buy   Sell

\$5.65   \$5.63

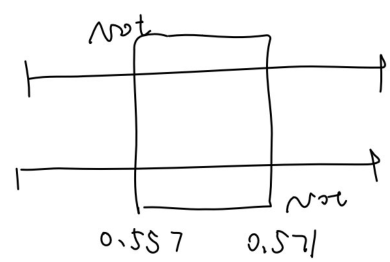
$$10 = u(0) \geq p(u(10 - 5.65)) + (1-p)u(-5.65)$$

$$\geq \sqrt{94.35} + p(\sqrt{104.35} + \sqrt{\ })$$

$$p \in P_B = 0.571$$

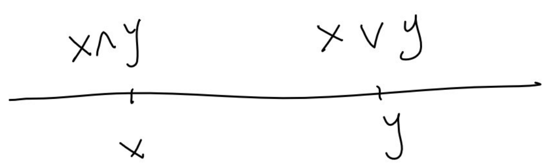
$$u(0) \geq p(u(-10 + 5.65)) + (1-p)u(5.63)$$

$$\geq 0.557$$

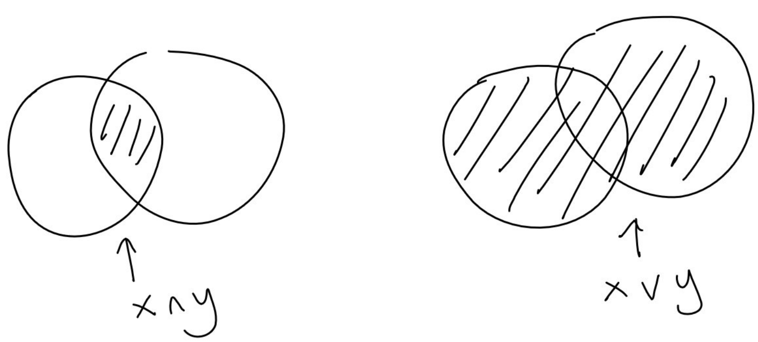


(2)

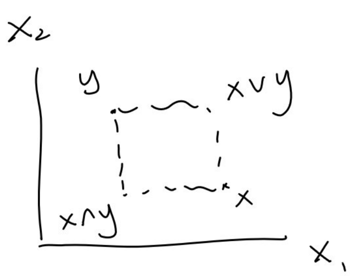
a)



b)



c)



d) $A = \{(0,1), (1,0)\}$

e). Same

⑦ $U(f) = \sum_{s,z} \mu_s(z) f_s(z)$

Independence : Suppose $f \lesssim g, h$

$$U(\alpha f + (1-\alpha) h) = \sum_{s,z} \mu_s(z)(\alpha f + (1-\alpha)h)_s(z)$$

$$= \alpha \sum_{s,z} \mu_s(z) f_s(z) + (1-\alpha) \sum_{s,z} \mu_s(z) h_s(z)$$

$$= \alpha U(f) + (1-\alpha) U(h)$$

$$\leq \alpha U(g) + (1-\alpha) U(h)$$

Axiom 1: $V \to \mathbb{R}$. $\mathbb{R}$ complete and transitive

Axiom 2:

⑥ $u(f_1, f_2) = \min(f_1, f_2)$

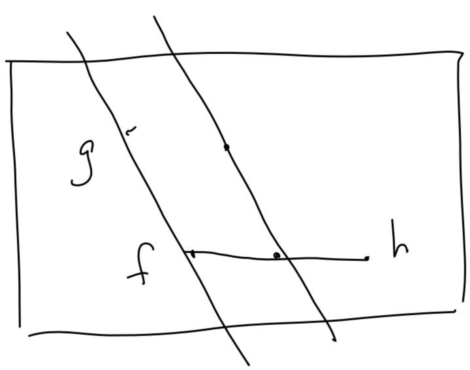
$f = (1, 0)$, $g = (0, 1)$

$h = \frac{1}{3} f + \frac{2}{3} g = (\frac{1}{3}, \frac{2}{3})$

$u(h) > u(f) = u(g)$

$\alpha = \frac{1}{2}$

b)



Ind $L \sim L'$

$\alpha L + (1-\alpha) L' \sim L'$

Non-linear

$\exists\, \alpha' \in (0,1)$ s.t. $\alpha L + (1-\alpha) L' \gtrsim L'$