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Estimation of Direction of Arrival (DOA) Using Real-Time Array Signal Processing

Md. Shahedul Amin, Ahmed-Ur-Rahman, Saabah-Bin-Mahbub,
Khawza I. Ahmed[1] and Zahidur Rahim Chowdhury[1]

Department of Electrical and Electronic Engineering, Islamic University of Technology (IUT)
Board Bazar, Gazipur-1704, Bangladesh.
[1]Department of Electrical and Electronic Engineering, United International University (UIU)
Sat Masjid Road, Dhaka, Bangladesh.
E-mail: shahedul_amin2000@yahoo.com, romel_iut@yahoo.com, saabah.mahbub@gmail.com, khawza@eee.uiu.ac.bd and zahidur@eee.uiu.ac.bd

Abstract- Array Signal Processing (ASP) is a relatively new technique in Digital Signal Processing (DSP) with many potential applications in communication and speech processing. Direction of arrival (DOA) can be estimated using different techniques evolved with ASP. Spectral-based algorithm and subspace-based methods are implemented using two widely used softwares, MATLAB™ and National Instrument's LabVIEW™, to demonstrate the feasibility of introducing the topics in course curriculum of graduate or undergraduate program. It is observed that subspace method provides superior performance in resolving closely spaced sources. The blocks developed using LabVIEW™ can be used for processing signals obtained from data acquisition card in real time.

I. Introduction

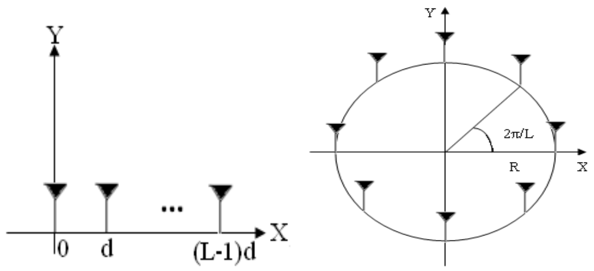
Digital Signal Processing is one of the fastest growing sectors of Electrical Engineering and many applications, available in our day to day life, have been developed using this technology. Array signal processing (ASP) is one of the techniques of DSP which has many potential applications [1]. Sensor ASP has emerged as an active area of research and is centered on the ability to analyze data collected at several sensors [1]. Such topics are not taught in undergraduate level in the most of the universities. This paper addresses method to introduce the topic and ASPs practical implications in undergraduate level. The application developed in this article may also be introduced in the laboratory as experiments in Digital Signal Processing Laboratory. The students can perform the simulation both in MATLAB and National Instruments LabVIEW. National Instrument's LabVIEW offers graphical interface for simulation with a number of advantages like – direct implementation of design from this software in DSP kit, capturing real time data using data acquisition card and easy manipulation of captured or stored data or parameters. The most common applications of array signal processing involve detecting location of acoustic signals [2] which is the focus of this paper. The sensors in this case are microphones and arrangement of microphone positions is significant. We have considered a linear array to collect signals from relatively low frequency sounds (0 to 8 kHz) coming out from a specific direction.

II. Related Terms

This section briefly introduces the relevant terms associated with array signal processing.

A. Array Signal Processing

Array signal processing is a part of signal processing that uses sensors organized in patterns or arrays to detect signals and to determine information about the signals [2]. Arrays can be arranged in a line or a circle as shown in Figure 1. Uniform linear array (ULA) where L numbers of sensors are spaced linearly with equal distance d is shown in Figure 1(a). Figure 1(b) demonstrates the uniform circular array (UCA) where L numbers of sensors are spaced circularly with equal amount of angle $2\pi / L$.



(a) (b)

Fig. 1 Array arrangements (a) Uniform linear array, (b) Uniform circular array

B. Spatial Frequency Transform

Analogous to the Discrete Fourier Transform (DFT) [2], Spatial Frequency Transform is the sampled and windowed spatial equivalent that is used to filter signal in space. The information in the space domain or wave number domain is directly related to the angle the signal is coming from relative to the ULA.

C. Spatial Aliasing

It is well known fact from the sampling theorem that aliasing occurs in the frequency domain if the signal is not sampled at high enough rate (the minimum rate is Nyquest sampling rate given by the twice of the bandwidth of the signal). We have the same sort of considerations to take into account when we analyze the spectrum of the spatial frequency as well. The Nyquest equivalent of the sampling rate to avoid spatial aliasing implies that the distance between the sensors d should be less than or equal to the half of the minimum wavelength [3], i.e., $d \le \lambda_{\min}/2$ where $\lambda_{\min}$ is the minimum wavelength corresponding to the maximum frequency $f_{\max}$. This is due to the fact that the velocity of sound, $v = f\lambda$ is fixed in a medium and thus, when the frequency is maximum, the wavelength is minimum.

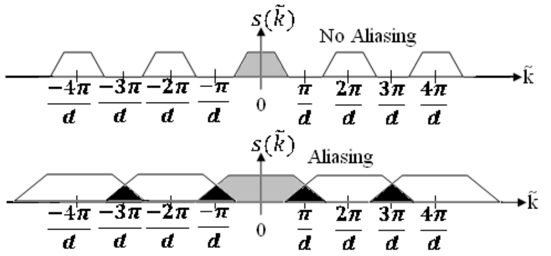


Fig. 2 Visualization of Spatial Aliasing

In Figure 2, $\tilde{k}$ is the space domain or wavenumber whereas $s(\tilde{k})$ is the spectrum of the space domain sampled signal. In the top of Figure 2 Nyquest Sampling rate is maintained and as a result there is no overlap of the spectra of the sampled signals but in the bottom of Figure 2 aliasing occurs as Nyquest criterion is not maintained.

D. Beamforming

Beamforming is the process of combining sounds or electromagnetic signals that come from only one particular direction and impinges different sensors at the receiver. Due to the coherent combining after the appropriate phase compensation at each sensor the resultant signal provides higher strength. Thus, the resultant gain of the sensor would look like a large dumbbell shaped lobe aimed in the direction of interest [3]. This important concept is used in different communication, voice and sonar applications [3].

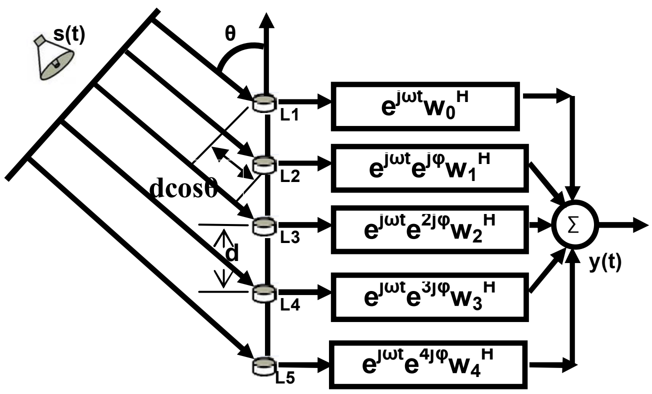
E. Direction of Arrival (DOA)

It is a process of finding the exact location of the source from where the sound is coming. There are three ways of finding the Direction of Arrival (DOA) [3]:

- Spectral-based algorithm
  1. Conventional beamformer
  2. Capon's beamformer
- Subspace-based methods
  1. Multiple signal classification (MUSIC) algorithm
  2. Extension to MUSIC algorithm
- Parametric Methods
  1. Deterministic Maximum Likelihood Method
  2. Stochastic Maximum Likelihood Method

III. System Model for Estimating DOA

It is assumed that the array is linear with L sensors and sound source is located at $\theta°$ away from the axis of the array as shown in Figure 3. A useful property of the ULA is the delay from one sensor to the next is uniform across the array because of their equidistant spacing. Planar sinusoidal sound waves are considered to avoid complexity. Trigonometry reveals that the additional distance the incident signal travels between sensors is $d\cos\theta$. Thus, the time delay between consecutive sensors is given by, $\tau = d\cos\theta / c$.



Velocity of sound, v = 330.7 m/s, f_{max} = 1600 Hz, L=5

Distance between two sensors, d = 9.9 cm, Φ = -ωτ

Fig. 3 Application of uniform Linear Array

Let's say, the highest narrowband frequency we are interested is $f_{\max}$. To avoid spatial aliasing, we would like to limit phase differences between spatially sampled signals to π or less because phase differences above π causes incorrect time delays to be seen between received signals and we have, $2\pi\tau f_{\max} \le \pi$ [4]. Substituting for τ, we get, $d \le c/2f_{\max}\cos\theta$. Since, worst delay occurs for $\theta = 0°$, we obtain the fundamentally important condition to avoid spatial aliasing, $d \le \lambda_{\min}/2$. Referring to Figure 3 for an L-element ULA, the array output vector is obtained as [4]

$$\mathbf{x}(t) = \mathbf{a}(\theta)s(t), \qquad (1)$$

where $\mathbf{x}(t)$ is the array output, $s(t) = \exp(j\omega t)$ is the signal coming from the source and steering vector, $\mathbf{a}(\theta) = [1, \exp(j\phi), ..., \exp(j(L-1)\phi)]^T$ assuming the propagation delay between the source and the first sensor is normalized to unity and the phase delay between the sensors, $\phi = -\omega d\cos\theta / c$. A single signal at the DOA θ, thus results in a scalar multiple of the steering vector. If M signals impinge on an L-dimensional array from distinct DOAs $\theta_1, \theta_2, ..., \theta_M$, the output vector takes the form

$$\mathbf{x}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) s_m(t) \quad (2)$$

where $s_m(t)$ denotes the baseband signal waveforms from m-th source. The output equation can be put in a more compact form by defining a steering matrix and a vector of signal waveforms as [4]

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_M)] \quad (3)$$

$$s(t) = [s_1(t), ..., s_M(t)]^T \quad (4)$$

In the presence of an additive noise $\mathbf{n}(t)$, we now get the model commonly used in array processing

$$\mathbf{x}(t) = \mathbf{A}(\theta)s(t) + \mathbf{n}(t). \quad (5)$$

The methods to be presented all require $M < L$, which is therefore assumed throughout the paper.
To find DOA the idea is to "steer" the array in one direction at a time and measure the output power. The steering locations which result in maximum power yield the DOA estimates. The array response is steered by forming a linear combination of the sensor outputs [4]

$$y(t) = \sum_{l=1}^{L} w_l^* x_l(t) = \mathbf{w}^H \mathbf{x}(t) \quad (6)$$

where $\mathbf{w}$ is the weighting vector used for cancelling the phase delay between the sensors and $\mathbf{w}^H$ is the Hermitian of $\mathbf{w}$. N-samples of $y(t)$ are taken with time interval T between the samples and $t = kT$, where $k = 1,2,...,N$. The output power is measured by

$$P(\mathbf{w}) = \frac{1}{N}\sum_{t=1}^{N} |y(t)|^2 = \frac{1}{N}\sum_{t=1}^{N} \mathbf{w}^H \mathbf{x}(t)\mathbf{x}^H(t)\mathbf{w}$$
$$= \mathbf{w}^H \mathbf{R}\mathbf{w}, \quad (7)$$

where $\mathbf{R} := \frac{1}{N}\sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t)$. The steps for finding the DOA are shown graphically in Figure 4, which we have used in all the proposed experiments [5].

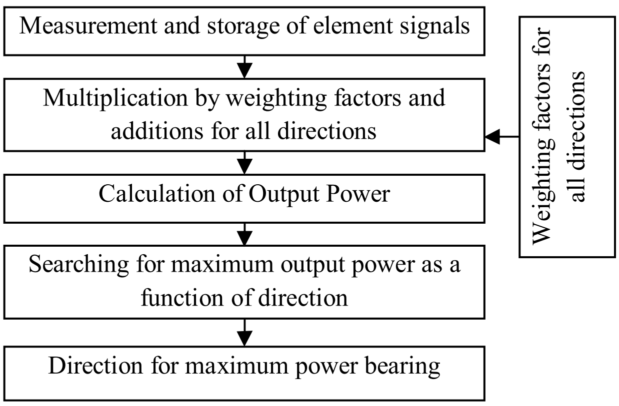


Fig. 4 Steps for location determination by DOA

IV. Experiments

In this section we would like to introduce the proposed experiments based on DOA. Here we have introduced the spectral-based methods and subspace-based methods of estimating DOA.

A. EXP. 1: Conventional Beamformer

Theory: The conventional beamformer is a natural extension of classical Fourier-based spectral analysis to sensor array data [5]. For an array of arbitrary geometry, this algorithm maximizes the power of the beamforming output for a given input signal. Let, we wish to maximize the output power from a certain direction θ. The problem of maximizing the output power is then formulated as [6],

$$\max E\{\mathbf{w}^H \mathbf{x}(t)\mathbf{x}^H(t)\mathbf{w}\} = \max\{\mathbf{w}^H E[\mathbf{x}(t)\mathbf{x}^H(t)]\mathbf{w}\}$$
$$= \max\{E|s(t)|^2 |\mathbf{w}^H \mathbf{a}(\theta)|^2 + \sigma^2 |\mathbf{w}|^2\} \quad (8)$$

where σ^2 is the noise covariance and the assumption of spatial white noise is used [6]. To obtain a non-trivial solution, the norm of $\mathbf{w}$ is constrained to $|\mathbf{w}| = 1$ when carrying out the above maximization. The resulting solution for $\mathbf{w}$ is then,

$$\mathbf{w}_{BF} = \frac{\mathbf{a}(\theta)}{\sqrt{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}} \quad (9)$$

Inserting the weighting vector from Equation 9 into Equation 7, the classical *spatial spectrum* is obtained [6],

$$P_{BF} = \frac{\mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)} \quad (10)$$

MATLAB simulation: We have assumed six sensors and two sound sources located at $45°$ and $135°$ from the axis of the array. The array consists of $L = 10$ sensors arranged in the form of ULA.

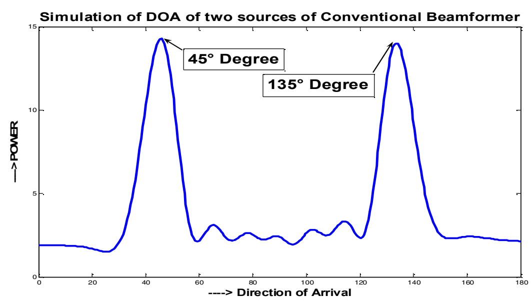


Fig. 5 DOA of two sources of Conventional Beamformer using MATLAB

Figure 5 shows the MATLAB simulation of power measurement in all the positions of space and maximum power is obtained at an angle of $45°$ and $135°$ from the axis of the array where the two sources were located. The power is measured in watts throughout the experiments.

LabVIEW Implementation:

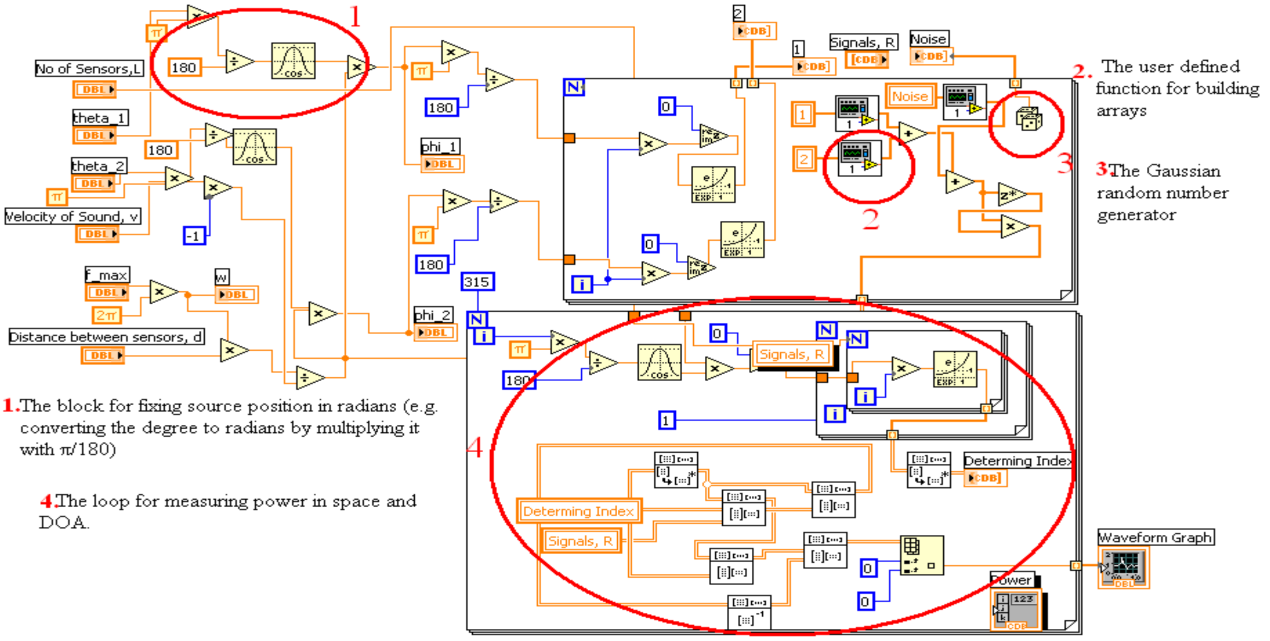


Fig. 6 Block Diagram of Conventional Beamformer in LabVIEW

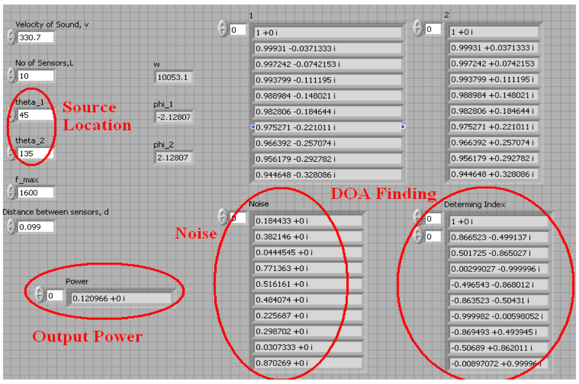


Fig. 7 Portion of Front Panel of Conventional Beamformer in LabVIEW

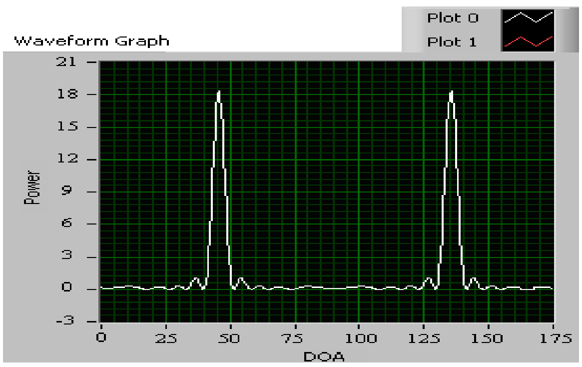


Fig. 8 Result from LabVIEW of Conventional Beamformer

LabVIEW implementation of Conventional Beamformer is shown in Figure 6 which describes graphical representation of the basic mathematical equations where some of the functional blocks are shown. Figure 7 shows some portion of the front panel of LabVIEW where important blocks are shown in the figure. The output for Conventional Beamformer in LabVIEW is shown in Figure 8 and the result is same that obtained from MATLAB as shown in Figure 5.

Limitation of Conventional Beamformer: The standard beamwidth for a ULA is $\varphi_B = 2\pi / L$, and sources whose electrical angles are closer than φ_B will not be resolved by the Conventional Beamformer, regardless of the available data quality [6].

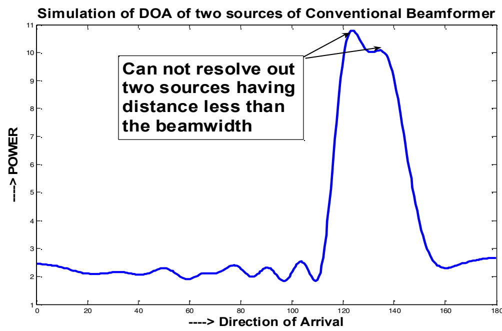


Fig. 9 DOA of two sources of having distance less than the beamwidth in Conventional Beamformer using MATLAB

A ULA of $L = 10$ sensors of half wavelength inter-element spacing has a beamwidth of $2\pi / 10 = 0.63$ radians, implying that sources need to be at least $12°$ apart in order to be separated by the beamformer. We have assumed the two sources located at $124.2°$ and 135° which is less than $12°$ and thus spatial aliasing takes place. Therefore, the sensors can not resolve out two sources as desired which is shown in Figure 9.

B. EXP. 2: Capon's Beamformer

Theory: In an attempt to alleviate the limitation of the conventional beamformer, such as its resolving power of two sources spaced closer than the beamwidth, proposed modifications is given by Capon which is also known as the Minimum Variance Distorsionless Response Filter [7]. The optimization problem is proposed as,

$$Min\ P(\mathbf{w}) \text{ subject to } \mathbf{w}^H \mathbf{a}(\theta) = 1 \tag{11}$$

where $P(\mathbf{w})$ is as defined in Equation 7. This beamformer attempts to minimize the power contributed by noise and any signals coming from other directions than θ, while maintaining a fixed gain in the "look direction θ" like as a sharp spatial bandpass filter. The optimal $\mathbf{w}$ can be found using the technique of Lagrange multipliers, resulting in [8]

$$\mathbf{w}_{CAP} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)} \tag{12}$$

Inserting the above weight into Equation 7 leads to the following "*spatial spectrum*" [8]

$$P_{CAP}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)} \tag{13}$$

MATLAB simulation: Here we consider two sources at 135° and 124.2° away from the axis of the array and we see that it resolves out them perfectly and its MATLAB Simulation is shown in Figure 10.

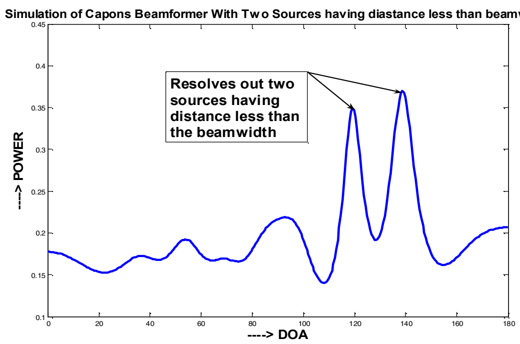


Fig. 10 Simulation of DOA for Capon's Beamformer with two sources having distance less than beamwidth

LabVIEW Implementation: Block diagram for Capon's beamformer is pretty similar to that of Conventional Beamformer except the power calculation loop as mentioned in Figure 6. It is shown in Figure 11. The results from LabVIEW of Capon matches with the results obtained from MATLAB simulation shown in Figure 10.

C. EXP. 3: MUSIC Algorithm

Theory: The main features of MUSIC are [8]:

- Its properties are directly related with the Eigen-structure of the covariance matrix.
- Unlike others, MUSIC was originally presented as a DOA estimator.

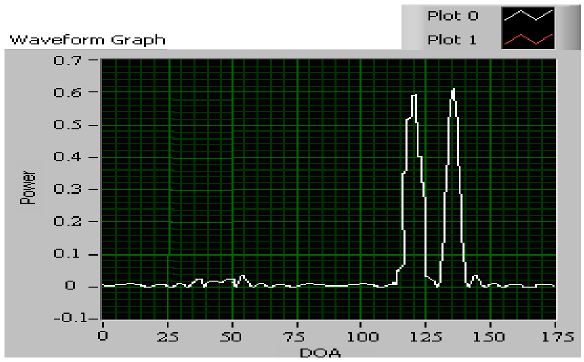


Fig. 11 Result from LabVIEW of Capon's Beamformer

- It is a frequency estimation technique.
- It reduces noise to a great extent.

The signal parameters which are of interest are spatial in nature and thus require the cross covariance information among the various sensors, i.e., the spatial covariance matrix given by [8]

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}E\{s(t)s^H(t)\}\mathbf{A}^H + E\{\mathbf{n}(t)\mathbf{n}^H(t)\} \tag{14}$$

with

$$E\{s(t)s^H(t)\} = \mathbf{P} \tag{15}$$

is the source covariance matrix and

$$E\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \sigma^2\mathbf{I} \tag{16}$$

is the noise covariance matrix. To allow for unique DOA estimates, the array is usually assumed to be unambiguous; that is, any collection of M steering vectors corresponding to distinct DOAs $\theta(k), k = 1, \ldots, M$ forms a linearly independent set of $\{\mathbf{a}(\theta(1)),\ldots,\mathbf{a}(\theta(M))\}$ and $\mathbf{P}$ has full rank [9]. In practice, an estimate $\mathbf{R}$ of the covariance matrix is obtained and its eigenvectors are separated into the signal and noise subspace as

$$\mathbf{R} = \mathbf{APA}^H + \sigma^2\mathbf{I} = \mathbf{U\Lambda U}^H \tag{17}$$

with $\mathbf{U}$ unitary and $\mathbf{\Lambda}$ = diag$\{\lambda_1, \lambda_2, \ldots, \lambda_L\}$, a diagonal matrix of real eigenvalues ordered such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L > 0$. It is observed that any vector orthogonal to $\mathbf{A}$ is an eigenvector of $\mathbf{R}$ with the eigenvalue σ^2 [9]. There are $L - M$ linearly independent such vectors. Since, the remaining eigenvalues are all larger than σ^2, we can partition the eigenvector pairs into noise eigenvector (corresponding to eigenvalues $\lambda_{M+1} = \ldots = \geq \lambda_L = \sigma^2$) and signal eigenvectors (corresponding to eigenvalues $\lambda_1 \geq \ldots \geq \lambda_M > \sigma^2$). Hence we can write [10]

$$\mathbf{R} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H , \qquad (18)$$

where $\mathbf{\Lambda}_n = \sigma^2 \mathbf{I}$. Since all noise eigenvectors are orthogonal to $\mathbf{A}$, the columns of $\mathbf{U}_s$ must span the range space of $\mathbf{A}$ whereas those of $\mathbf{U}_n$ span its orthogonal complement. The projection operators onto this signal and noise subspaces are defined as [10]

$$\mathbf{\Pi} = \mathbf{U}_s \mathbf{U}_s^H = \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \qquad (19)$$

$$\mathbf{\Pi}^{\perp} = \mathbf{U}_n \mathbf{U}_n^H = \mathbf{I} - \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \qquad (20)$$

Thus, MUSIC "Spatial Spectrum" is defined as

$$P_M(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{\Pi}^{\perp}\mathbf{a}(\theta)} \qquad (21)$$

MATLAB simulation: This experiment assumes two sources at *124.2°* and *135°* which was taken in the previous experiments. The output obtained from MATLAB Simulation of MUSIC is shown in Figure 12. From the figure it is apparent that through MUSIC sensors are able to detect sources having distance less than beamwidth.

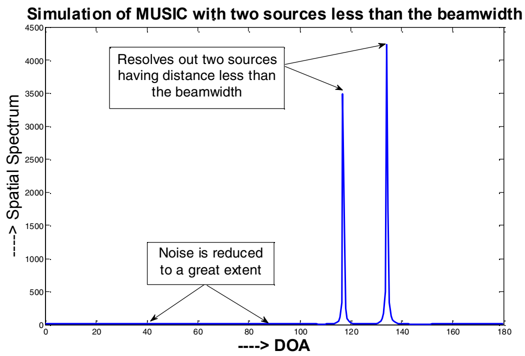


Fig. 12 Simulation of MUSIC

We get much more improved result here since the noise is reduced to a very good extent and also it works as a very sharp *spatial bandpass filter* [7].

LabVIEW Implementation: The block diagram is also similar with the Figure 6 except the power calculation loop due to the difference in power calculation method. Figure 13 shows the output from LabVIEW of MUSIC algorithm and no difference exists between Figure 12 and 13.

Results: From the above Three experiments it is apparent that MUSIC is the best algorithm for finding the DOA.

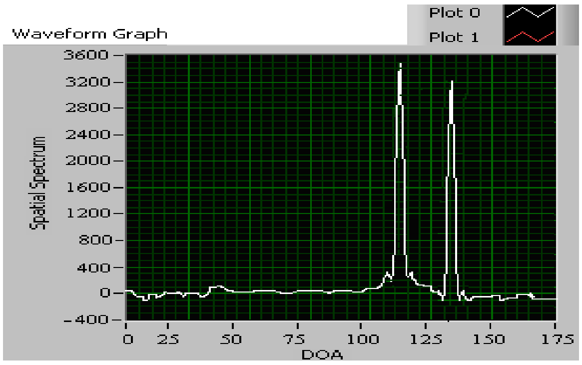


Fig. 13 Result of MUSIC in LabVIEW

V. Conclusion

In this paper, the theory of Array Signal Processing is introduced for voice signals to find the location of source. It can be inferred that estimates of an arbitrary location of signal source can be performed with moderate accuracy if the data collection time is sufficiently long or the SNR is adequately high, and the signal model is sufficiently accurate. A hardware implementations using LabVIEW data acquisition card that would use the blocks developed in LabVIEW in real time is in progress. Since the proposed set of experiments augments the practical implication of ASP in addition to the theoretical understanding, it will be beneficial for the undergraduate students if it is introduced in the Digital Signal Processing Laboratory experiments.

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