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NATURAL CONVECTION FLOW ALONG A VERTICAL FLAT PLATE WITH CONDUCTION AND HEAT GENERATION EFFECT

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ABSTRACT
The heat generation effect on natural convection flow along and conduction inside a vertical flat plate is investigated. The developed governing equations with the associated boundary conditions for this analysis are transferred to dimensionless forms using a local non-similar transformation. The transformed nonlinear equations of the non-dimensional equations are then solved using the implicit finite difference method with Keller box-scheme. Numerical results are found for different values of the heat generation parameter and Prandtl number. The overall investigation of the velocity, temperature, skin friction and heat transfer rate represented graphically.

Keywords: Heat generation, Natural convection, Vertical flat plate, Conduction, Finite difference method

1. INTRODUCTION
The interaction between the conduction inside and the bouyancy forced flow of fluid along a solid surface is termed as conjugate heat transfer (CHT) process. In practical systems, such as heat exchangers, the convection in the surrounding fluid influences significantly the conduction in a tube wall. Accordingly, the conduction in the solid body and the convection in the fluid should be determined simultaneously. The CHT problems have been studied by several research groups [1, 2, 3] with the help of mathematical models on simple heat exchanger geometries. Gdalevich and Fertman [4] and Miyamoto et al [5] reviewed the early theoretical and experimental works of the CHT problems for a viscous fluid. Miyamoto observed that a mixed-problem study of the natural convection has to be performed for an acurate analysis of the thermo-fluid dynamic (TFD) field if the convective heat transfer depends strongly on the thermal boundary conditions. Pozzi and Lupo [6] investigated the entire TFD field resulting from the coupling of natural convection along and conduction inside a heated flat plate by means of two expansions, regular series and asymptotic expansions. Moreover, Vynnycky and Kimura [7] studied the two dimensional conjugate free convection for a vertical plate of finite extent adjacent to a semi-infinite porous medium using finite difference techniques. Pop et al [8] extended the analysis of Vynnycky for the mixed convection flow.

The CHT problems associated with the heat generating plate washed by laminar forced convection flow were studied by Karvinen [9], Sparrow and Chyu [10] and Garg and Velusami [11] using an approximate method. Moreover, analytical and numerical solutions were performed for the CHT problem associated with the forced convection flow over a conducting slab sited in an aligned uniform stream by Vynnycky et al [12].

In the present article, the natural convection flow along a vertical flat plate considering the conduction and heat generation effects is studied. The governing boundary layer equations are transformed into a non dimensional form and the resulting non linear partial differential equations are solved numerically using the implicit finite difference method together with the Keller box technique [13, 14]. The temperature distributions, velocity profiles, skin friction coefficients and the heat transfer rates are presented graphically.

2. MATHEMATICAL ANALYSIS
A time independent natural convection flow of a viscous incompressible fluid along a vertical flat plate of length l and thickness b (Figure 1) is considered. A greater temperature T_b than the ambient temperature T_∞ is maintained constant at the outer surface of the plate.

The governing equations of such flow under the usual boundary layer and the Boussinesq approximations in the presence of heat generation can be written as

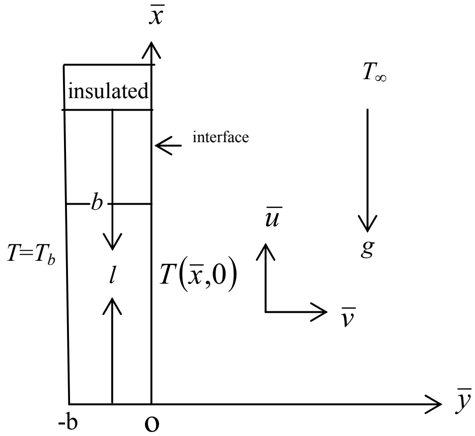

Fig 1: Physical model and coordinate system.

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta\left(T_f - T_\infty\right) \qquad (2)$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa_f}{\rho c_p}\frac{\partial^2 T_f}{\partial \bar{y}^2} + \frac{Q_0}{\rho c_p}\left(T_f - T_\infty\right) \qquad (3)$$

The term $\frac{Q_0}{\rho c_p}\left(T_f - T_\infty\right)$, Q_0 being a constant, represents the amount of generated or absorbed heat from per unit volume. Heat is generated or absorbed from the source term according as Q_0 is positive or negative.

The physical situation of the system suggests the following boundary conditions [15, 16, 17, 18]

$$\left.\begin{aligned} &\bar{u}=0, \bar{v}=0 \\ &T_f = T(\bar{x},0),\ \frac{\partial T_f}{\partial \bar{y}} = \frac{k_s}{b k_f}\left(T_f - T_b\right) \end{aligned}\right\} \text{on } \bar{y}=0,\ \bar{x}>0 \qquad (4a)$$

$$\bar{u} \to 0, T_f \to T_\infty \text{ as } \bar{y} \to \infty,\quad \bar{x} > 0 \qquad (4b)$$

The governing equations and the boundary conditions (Equation 1-4) can be made dimensionless by using the following dimensionless quantities:

$$x = \frac{\bar{x}}{l},\ y = \frac{\bar{y}}{l}Gr^{\frac{1}{4}},\ u = \frac{\bar{u}l}{\nu}Gr^{-\frac{1}{2}}$$
$$v = \frac{\bar{v}l}{\nu}Gr^{-\frac{1}{4}},\quad \frac{T - T_\infty}{T_b - T_\infty} = \theta, \qquad (5)$$
$$Gr = g\beta l^3\left(T_b - T_\infty\right)/\nu^2$$

where l is the length of the plate, Gr is the Grashof number and θ is the dimensionless temperature. The non-dimensional momentum and the energy equations can now be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \qquad (7)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Q\theta \qquad (8)$$

where $Q = \left(Q_0 l^2\right)/\left(\mu c_p Gr^{1/2}\right)$ is the dimensionless heat generation parameter and $\Pr = \left(\mu c_p\right)/\kappa_f$ is the Prandtl number .

The boundary conditions in dimensionless forms are obtained as:

$$u = v = 0,\ \theta - 1 = p\frac{\partial \theta}{\partial y} \quad \text{on } y = 0,\ x > 0 \qquad (9a)$$

$$u \to 0,\ \theta \to 0 \quad \text{as } y \to \infty,\ x > 0 \qquad (9b)$$

where $p = \left(k_f / k_s\right)\left(b/l\right)Gr^{1/4}$ is the conjugate conduction parameter.

The described problem is governed by the coupling parameter p. In actual fact, magnitude of O (p) depends on b/l and κ_f/κ_s , $Gr^{1/4}$ being the order of unity. Since l is small, the term b/l becomes greater than one. For air, κ_f/κ_s attains very small values if the plate is highly conductive and reaches the order of 0.1 for materials such as glass. Therefore in different cases p is different but not always a small number. In the present investigation we have considered p = 1.

To solve the equations (7) and (8) subject to the boundary conditions (9), the following transformations are introduced :

$$\psi = x^{4/5}\left(1+x\right)^{-1/20} f\left(\eta, x\right),$$
$$\eta = y x^{-1/5}\left(1+x\right)^{-1/20}, \qquad (10)$$
$$\theta = x^{1/5}\left(1+x\right)^{-1/5} h\left(\eta, x\right)$$

here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity and $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$ and $h(\eta, x)$ is the dimensionless temperature.

Substituting (10) into equations (7) and (8) we get the following transformed non-dimensional equations.

$$f''' + \frac{16+15x}{20(1+x)}ff'' - \frac{6+5x}{10(1+x)}f'^2 + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \qquad (11)$$

$$\frac{1}{P_r}h'' + \frac{16+15x}{20(1+x)}fh' - \frac{1}{5(1+x)}f'h$$
$$+ Q\,x^{2/5}(1+x)^{1/10}h = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \qquad (12)$$

In the above equations the primes denote partial differentiation with respect to η.

The boundary conditions (9) then take the following form:

$$f(x,0) = f'(x,0) = 0,$$
$$h'(x,0) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20}h(x,0 \qquad (13)$$
$$f'(x,\infty) \to 0,\, h'(x,\infty) \to 0$$

3. METHOD OF SOLUTION

To get the solutions of the parabolic differential equations (11) and (12) along with the boundary condition (13), we shall employ a most practical and accurate solution technique, known as implicit finite difference method together with Keller- box elimination technique.

4. RESULTS AND DISCUSSION

The main objective of the present work is to analyze the flow of the fluid and the heat transfer processes due to the conjugate heat transfer for a vertical flat plate. The values of the Prandtl number are considered 4.24, 1.74, 1.0 and 0.73 for the simulation that correspond to sulfur dioxide, water, steam and air, respectively. Detailed numerical results of the velocity, temperature, rate of heat transfer and skin friction coefficient for different values of the heat generation parameter and the Prandtl number are presented graphically.

The temperature and the velocity fields obtained from the solutions of the Equation 11 and 12 are depicted in Figure 2 to Figure 5. Figure 2 and Figure 3 illustrate the temperature distributions and the velocity profiles for different values of heat generation parameter with *Pr* = 0.73. It can be seen from Figure 2 and Figure 3 that the temperature profiles and the velocity profiles increase due to the increase of heat generation parameter. The increased value of the heat generation parameter means that more heat is produced and eventually increases the fluid motion. Moreover, the maximum values of the temperature are 0.8542, 0.8710, 0.8841 and 0.8932 for *Q* = 0.01, 0.05, 0.08 and 0.10, respectively and each of which occurs at the surface. It can be seen that the temperature increase by 4.366% as *Q* increases from 0.01 to 0.10. On the other hand, the maximum values of the velocity are 0.4895, 0.5026, 0.5127 and 0.5295 for *Q* = 0.01, 0.05, 0.08 and 0.10, respectively and each of which occurs at η = 1.3693. It is observed that the velocity increase by 5.775% when *Q* increases from 0.01 to 0.10. Temperature variation at the interface is also observed due to the conduction within the wall.

In Figure 4 and Figure 5, different values of Prandtl number with *Q* = 0.01 are considered for the velocity and temperature distributions. It can be seen from Figure 4 that the temperature profile decreases with the increasing *Pr*. The overall temperature profiles shift downwards with the increasing *Pr.* The physical fact that the thermal boundary layer thickness decreases with increasing *Pr* supports the result. Furthermore, the maximum values of the temperature are 0.8542, 0.8307, 0.7892 and 0.7240 for *Pr* = 0.73, 1.00, 1.74 and 4.24, respectively and each of which occurs at the surface. It can be observed that the temperature decrease by 15.242% as *Pr* increases from 0.73 to 4.24. From Figure 5, it is seen that the velocity within the fluid decreases when the value of Prandtl number, *Pr*, increases. The peak velocity decreases as well as its position moves toward the interface with the increasing *Pr*. Moreover, the maximum velocities are 0.4895, 0.4403, 0.3609 and 0.2541, respectively which occur at η = 1.3693 for the first and second maximum values and at η = 1.3025 for the third maximum value and at η = 1.1752 for the last maximum value. It is observed that the velocity decreases by 48.089% as *Pr* increases from 0.73 to 4.24.

The variation of the local skin friction coefficient C_{fx} and local rate of heat transfer N_{ux} with *Pr* = 0.73 for different values of *Q* at different positions are illustrated in Figure 6 and Figure 7, respectively. The heat generation accelerates the fluid flow, as mentioned earlier, and increases the shear stress at the wall. The increased skin friction coefficients with the increasing *Q* represent this phenomenon as illustrated in Figure 6. Moreover, the heat transfer rate depends on the gradient of temperature. As the gradient decreases with the increasing *Q* [Figure 2], the heat transfer rate also decreases as revealed in Figure7.

Skin friction coefficients and heat transfer rates at different positions along the plate surface for the variation of *Pr* with *Q* = 0.01 are illustrated in Figure 8 and Figure 9, respectively. From Figure 8, it is seen that the skin friction co-efficient decreases with the increasing *Pr*. Moreover, the skin friction increases along the plate for a particular value of *Pr*. The opposite situation is observed from Figure 9 for the rate of heat transfer.

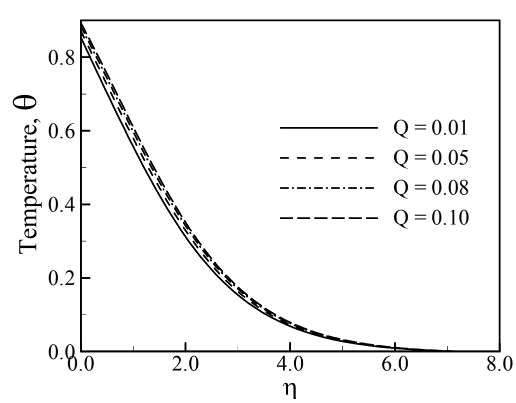

Fig 2: Variation of temperature profiles for different values of *Q* with *Pr* = 0.73

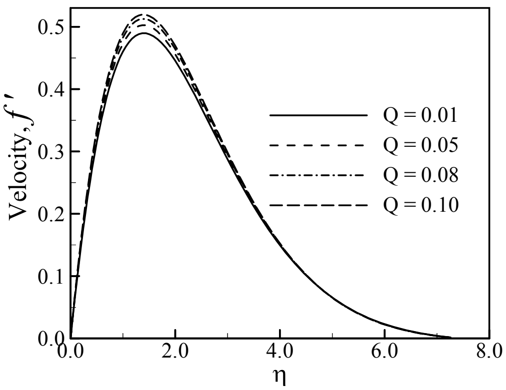

Fig 3: Variation of velocity profiles for different values of Q with Pr = 0.73

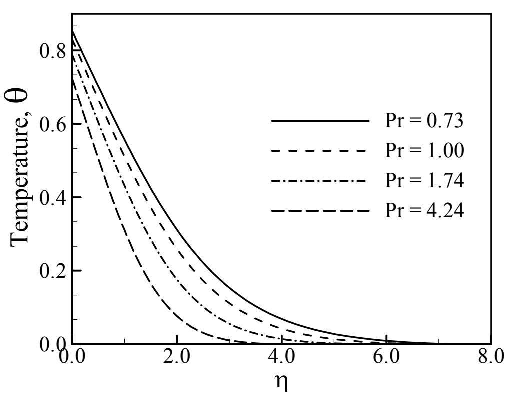

Fig 4: Variation of temperature profiles for different values of Pr with Q = 0.01

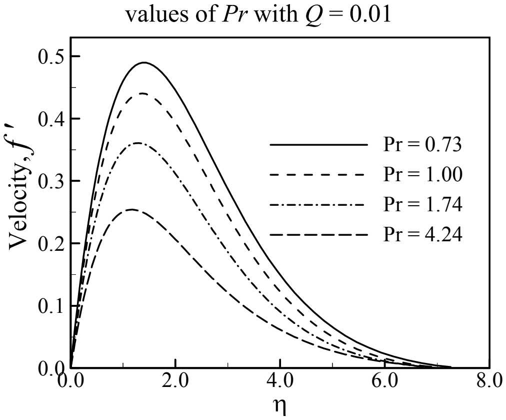

Fig 5: Variation of velocity profiles for different values of Pr with Q = 0.01

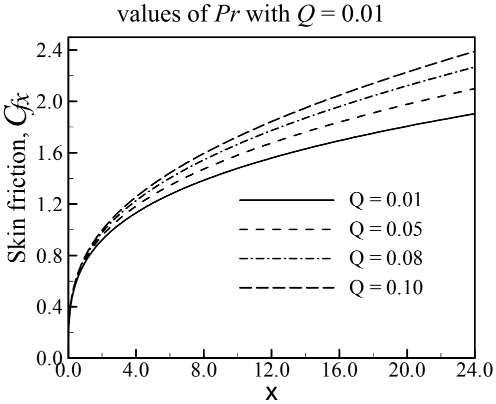

Fig 6: Variation of skin friction coefficients for different values of Q with Pr = 0.73

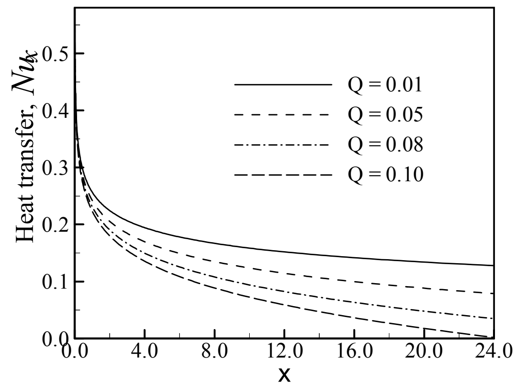

Fig 7: Variation of rate of heat transfer for different values of Q with Pr =0.73

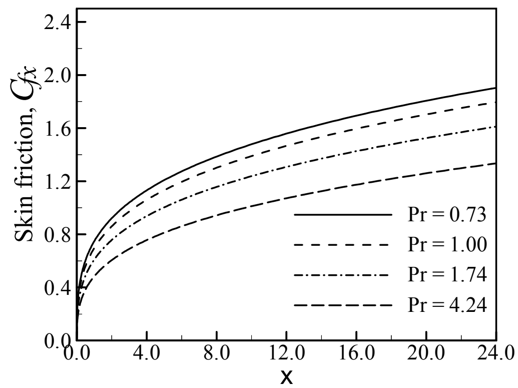

Fig 8: Variation of skin friction coefficients for different values of Pr with Q = 0.01

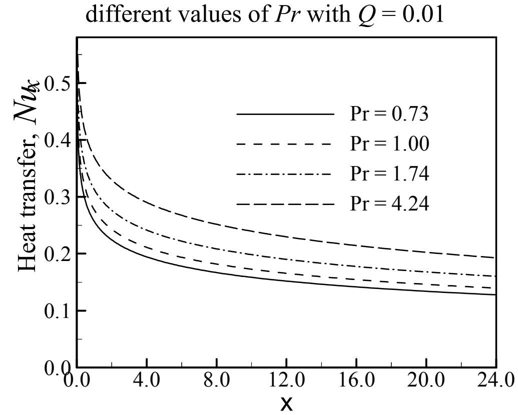

Fig 9: Variation of rate of heat transfer for different values of Pr with Q = 0.01

5. CONCLUSIONS

A steady, two-dimensional, laminar natural convection flow is analyzed considering conduction and heat generation effects. The transformed partial differential equations together with the boundary conditions are solved numerically by implicit finite difference method. The effects of the heat generation parameter and Prandtl number are studied on the fluid flow and at the solid-fluid interface. The velocity of the fluid and the skin friction at the interface increase with the increasing heat generation parameter while they decrease with the increasing Prandtl number. The temperature of the fluid increases with the increasing heat generation parameter and the decreasing Prandtl number. Furthermore, the rate of heat transfer decreases with the increasing heat generation parameter and the decreasing Prandtl number.

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7. NOMENCLATURE

Symbol	Meaning	Unit
b	Plate thickness	(cm)
C_p	Specific heat	(J/KgK)
f	Dimensionless stream function	
g	Acceleration due to gravity	(cm/s^2)
l	Length of the plate	(cm)
T_f	Temperature of the flow fluid	(K)
T_b	Temperature at outside of the plate	(K)
T_∞	Temperature of the ambient fluid	(K)
$\bar{u},\bar{v}$	Velocity components	(cm/s)
u,v	Dimensionless velocity components	
$\bar{x},\bar{y}$	Cartesian coordinates	(cm)
x,y	Dimensionless Cartesian coordinates	
β	Co-efficient of thermal expansion	
ψ	Dimensionless stream function	
η	Dimensionless similarity variable	
ρ	Density of the fluid inside the boundary layer	(kg/m^3)
ν	Kinematic viscosity	(m^2/s)
μ	Viscosity of the fluid	(N.s/m^2)
θ	Dimensionless temperature	
κ_f	Thermal conductivity of the ambient fluid	(kW/mK)
κ_s	Thermal conductivity of the ambient solid	(kW/mK)