

Business Cycles and Labor-Market Search

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The quantitative implications of labor-market search for economic fluctuations are evaluated in the context of a real-business-cycle model. Incorporating labor-market search into the model is found to improve its empirical performance along several dimensions. In particular, hours now fluctuate substantially more than wages and the contemporaneous correlation between hours and productivity falls. In addition, the model replicates the observation that output growth displays positive autocorrelation at short horizons. Overall, the empirical results suggest that the labor-market-search environment embodies a quantitatively important propagation mechanism. (JEL E32, J40)

In this paper, I evaluate the quantitative properties of a real-business-cycle (RBC) model in which the level of employment is determined using a search framework for the labor market instead of the standard Walrasian mechanism. This quantitative exercise is motivated by the appearance of a recent theoretical literature that organizes its thinking on aggregate labor-market dynamics and business-cycle activity around models based on search-theoretic principles. Some of this literature is concerned with explaining important business-cycle facts that RBC models are not designed to address. For example, Edmund S. Phelps et al. (1970) and Christopher A. Pissarides (1985, 1987) emphasize the role of labor-market search in generating empirical

regularities like the Beveridge curve and the Phillips curve. Other segments of this literature, for example, Randall Wright (1986) and Peter Howitt (1988), demonstrate how labor-market-search considerations may help resolve some of the well-known problems that RBC models have in explaining key features of the labor market. However, empirical investigations regarding the quantitative implications of search environments in general-equilibrium settings are rare; the purpose of the present paper is to help fill this void.¹

The search environment studied in this paper is based on the framework developed and studied extensively by Pissarides (1990). In this model, exit rates from unemployment and vacancy are determined by the search and recruiting decisions of workers and firms. These decisions serve as complementary inputs into an aggregate matching function. Changes in the expected returns to search, owing perhaps to changes in labor productivity or some structural disturbance, induce equilibrium responses in search and recruiting activities, the effects of which are propagated through time via changes in the stock of employment.

A search equilibrium can be consistent with a number of business-cycle facts that are accounted for with difficulty in standard theory. A partial list includes the following character-

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¹ Some closely related work includes Dale T. Mortensen (1990) and Monika Merz (1992).

istics: (i) the persistence and variability in unemployment; (ii) the large cyclical movements in job availability; (iii) the negative correlation between vacancies and unemployment; (iv) the large cyclical movements in the aggregate labor input in conjunction with relatively small movements in the real wage; and (v) the asymmetric dynamic correlation between hours and labor productivity. The question pursued here is whether or not and to what extent these qualitative implications match up quantitatively with observation.

A related question concerns the empirical relevance of the propagation mechanism that is embodied in this search environment; one contribution of the current paper is to quantify the degree of internal propagation induced by labor-market-search considerations vis-à-vis a benchmark RBC model. The exercise undertaken here is especially relevant in light of recent empirical results reported by Timothy Cogley and James M. Nason (1992). These authors demonstrate that many popular RBC environments embody quantitatively insignificant internal propagation mechanisms: in many cases, predicted output dynamics are virtually identical to assumed impulse dynamics. As a consequence, these models fail to account for the observed positive correlation in output growth, as documented by Charles R. Nelson and Charles I. Plosser (1982). The only exception reported by Cogley and Nason is the labor-hoarding model of Craig Burnside et al. (1993). Since costly search introduces a labor-hoarding motive for firms, there is some reason to believe that similar success may be enjoyed by a business-cycle model that incorporates search in the labor market. The extent to which this belief is justified will be examined below.

In order to study the implications of labor-market search for aggregate fluctuations, the search framework described above is integrated into an otherwise standard business-cycle model of the type introduced by Finn E. Kydland and Edward C. Prescott (1982), and John B. Long and Plosser (1983). Thus, apart from the labor market, where aggregate employment is determined by the matching process, all other markets operate as Walrasian auctions. The quantitative properties of the model are evaluated according to the

empirical methodology adopted in the RBC literature (Prescott, 1986). Specifically, an artificial economy is parametrized, calibrated, and the equilibrium is computed numerically. The equilibrium decision rules are then used to simulate time paths for the economic variables of interest; the statistical properties of these simulated time series are then compared to the statistical properties of the corresponding data generated by the postwar U.S. economy.

The empirical results can be summarized as follows. Overall, the search model studied here accounts for the observed pattern of aggregate economic activity reasonably well. In particular, the model generates persistent unemployment and a negatively-sloped Beveridge curve, similar to that generated by the U.S. economy. Also, the model is consistent with the observation that most of the variability in the aggregate-labor input is accounted for by cyclical adjustments in employment rather than hours worked per employee. The model's most notable shortcoming is in accounting for the observed volatility in job availability; the model predicts cyclical movements in vacancies that are substantially smaller than those displayed by the U.S. economy.

Incorporating labor-market search into a standard RBC model leads to a considerable improvement along three key dimensions. First, the model becomes consistent with the observation that hours fluctuate much more than wages. Second, the model implies a lower contemporaneous correlation between hours and productivity, with productivity displaying a slight lead. Finally, the equilibrium output dynamics are substantially different than the assumed impulse dynamics; the model is able to replicate the observed dynamic pattern of output growth.

The paper is organized as follows. In Section I, the model is described and the equilibrium is characterized. In Section II, the model is parameterized and calibrated. Section III reports the simulation results and provides an evaluation of the model. Section IV considers briefly the quantitative implications of introducing exogenous "structural" disturbances in the model economy. Section V provides some concluding remarks.

I. The Model

The model embeds a labor-market-search framework into an otherwise standard RBC environment. Assume that there are households distributed uniformly on the unit interval and that they have the usual preferences for consumption and leisure. In the model, households face the standard consumption-saving problem, but face altogether different opportunities for exchanging labor services. In particular, individuals either have a job opportunity or not, and job opportunities come and go at random, depending to some extent on individual search effort, the availability of jobs, and plain luck. Having a job opportunity means being matched with a firm, or to some position within the firm, and having the opportunity to negotiate a labor contract stipulating the terms by which labor services are exchanged for wages. Firms also face a standard wealth-maximization problem, except that, because finding new workers takes time and effort, firms view their existing workforce as a capital asset. Given constant returns in the production technology, it may be assumed without loss that each firm comprises a single job; in what follows, the terms *firm* and *job* will be used interchangeably.

A. The Search Process

In order to produce output, each job requires a worker. Let n_t denote the number of jobs that are matched with a worker at the beginning of period t ; hence, n_t is the measure of current period employment and $1 - n_t$ is the measure of nonemployed workers currently available for work.² Job-worker pairs are assumed to separate at the exogenous rate $0 < \sigma < 1$; so that the stock of active jobs (employment) is subject to continual depletion. Replenishing this stock takes time and consumes resources.

² The distinction between *unemployed* and *not in the labor force* is ignored here. Note that, as an empirical matter, the flows into employment from each of these pools are roughly of the same magnitude (Olivier J. Blanchard and Peter A. Diamond, 1990).

A firm interested in filling an available job must undertake recruiting and screening activities, which are necessary for finding a suitable employee. Let v_t denote the total number of new jobs made available by firms during period t , each of which incurs a flow cost equal to $\kappa > 0$, measured in units of physical output. Aggregate-recruiting intensity is assumed to be proportional to v_t , the number of job vacancies. In the version of the model studied here, workers are assumed to search passively. Letting e denote search effort per worker seeking employment, aggregate search effort by workers is given by $(1 - n_t)e$. Following Pissarides (1990), the rate at which new job matches form is governed by an aggregate-matching technology, $M(v_t, (1 - n_t)e)$, so that employment evolves according to the following dynamic equation:

$$(1) \quad n_{t+1} = (1 - \sigma)n_t + M(v_t, (1 - n_t)e).$$

Hence, a job vacancy can at best become productive only after a period of time has elapsed. This delay may be interpreted as reflecting the time-consuming nature of search, together with a period of training which is typically necessary for new employees. The matching technology is assumed to be a nondecreasing, concave function of aggregate search and recruiting effort and is assumed to display constant returns to scale.³ The uncoordinated nature of the search process is captured by the property that $M(v_t, (1 - n_t)e) \leq \min\{v_t, (1 - n_t)\}$. That is, during any given time interval, it will generally be the case that at least some searchers fail to contact an appropriate partner.

B. The Social Welfare Problem

The typical household has preferences represented by a utility function of the following form:

³ The assumption of constant returns to scale is supported empirically by Pissarides (1986) and Blanchard and Diamond (1989).

$$(2) \quad E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \phi(t)H(1 - x_t)]$$

where c_t denotes consumption, x_t denotes the fraction of time spent in nonleisure activities and $0 < \beta < 1$ is a discount factor. The functions U and H are increasing and concave. The value of the parameter $\phi(t) > 0$ depends on a household's employment status: $\phi(t)$ is equal to ϕ_1 if the household is employed and is equal to ϕ_2 if the household is nonemployed. This parameter may be interpreted as reflecting differences in the efficiency of a household's home production technology across different states of employment opportunities. No a priori restriction is placed on the relative magnitudes of ϕ_1 and ϕ_2 ; their values will be identified in the calibration procedure described below.⁴

Output is produced according to a standard neoclassical production technology, $y_t = F(k_t, n_t l_t; z_t)$, where k_t is the aggregate capital input; l_t is average hours worked by those employed; and z_t is a parameter reflecting the current state of technology, which evolves stochastically according to the transition function $G(z', z) = \Pr[z_{t+1} \leq z' | z_t = z]$. The capital stock depreciates at rate $0 < \delta < 1$, so that the economy-wide resource constraint is then given by

$$(3) \quad c_t + k_{t+1} + \kappa v_t = y_t + (1 - \delta)k_t.$$

The social welfare problem involves choosing a contingency plan $\{c_t, l_t, k_{t+1}, n_{t+1}, v_t\}_{t=0}^{\infty}$ in order to maximize the expression in (2) subject to the resource constraint in (3), the law of motion for employment in (1), the law of motion for the productivity shock, G , and an initial condition, (k_0, n_0, z_0) .

Exploiting the recursive structure of the problem, one may equivalently reformulate it in terms of a dynamic program. Let $\mathbf{s} \equiv (k, n, z)$ denote the current period capital stock, employment rate, and productivity level, respectively; refer to this vector as the *state* of the economic system. Let $W(\mathbf{s}_0)$ denote the maximum value of (2), given an ar-

bitrary initial condition (\mathbf{s}_0) , which is obtained by solving the welfare problem stated above. Let E_G denote the expectations operator associated with the transition function G and let primed variables denote "next period" values. The value function W satisfies the following Bellman equation:⁵

$$(4) \quad W(\mathbf{s}) = \max_{c, l, k', n', v} \{ U(c) + n\phi_1 H(1 - l) \\ + (1 - n)\phi_2 H(1 - e) \\ + \beta E_G W(\mathbf{s}') \}$$

where the maximization is subject to the constraints:

$$(5) \quad F(k, nl; z) + (1 - \delta)k \\ - k' - \kappa v - c \geq 0$$

$$(6) \quad (1 - \sigma)n + M(v, (1 - n)e) - n' \geq 0.$$

Letting (λ, μ) denote the multipliers associated with constraint (5) and (6), respectively, one can express the first-order conditions (assuming an interior solution) as follows:⁶

$$(7) \quad U_1(c) - \lambda = 0$$

$$(8) \quad -\phi_1 H_1(1 - l) + \lambda F_2(k, nl; z) = 0$$

$$(9) \quad \beta E_G W_1(\mathbf{s}') - \lambda = 0$$

$$(10) \quad \beta E_G W_2(\mathbf{s}') - \mu = 0$$

$$(11) \quad -\lambda \kappa + \mu M_1(v, (1 - n)e) = 0$$

⁵ Writing the equation in this manner anticipates that the planner will allocate consumption independently of employment status, a result that follows from the assumed separability of consumption and leisure in the utility function. Existence of a unique W satisfying (4) can be demonstrated easily using standard contraction mapping arguments, e.g., see Nancy Stokey and Robert E. Lucas (1989).

⁶ In what follows, the notation f_i will denote the derivative of the function f with respect to its i th argument.

⁴ Allowing this parameter to vary across employment status facilitates the calibration of the model to the data.

in addition to the constraints (5) and (6) holding with equality. From the envelope theorem, one derives

$$(12) \quad W_1(s) = \lambda [F_1(k, nl; z) + 1 - \delta]$$

$$(13) \quad W_2(s) = \phi_1 H(1 - l) - \phi_2 H(1 - e) \\ + \lambda F_2(k, nl; z)l \\ + \mu [1 - \sigma - (1 - \alpha)p(v, n)]$$

where $p(v, n) \equiv M(v, (1 - n)e)/(1 - n)$ is the fraction of nonemployed households that are allocated to new jobs. The parameter α is defined to be the elasticity of job matches with respect to the vacancy input, i.e., $\alpha = (vM_1)/M$. Given a Cobb-Douglas specification for the matching technology, α will take on a value between 0 and 1. Likewise, $(1 - \alpha) = ((1 - n)eM_2)/M$ is the elasticity of job vacancies with respect to the search input of households.

W_1 is the increase in utility that results from a small increase in the capital stock. W_2 can be interpreted as the surplus that results from the marginal job match. Increasing employment by one unit affects utility in three ways. The first two terms in (13) capture the difference in the utility of leisure that is enjoyed by a household when it is employed rather than unemployed. The third term is the value of the added output that is forthcoming from an additional worker and the fourth term captures the value of the "stock effect" on future employment. Note that this last term will be positive, since σ is calibrated below to be less than α . Hence, an increase in employment persists into the future, an effect which is valued at the margin by μ , where μ is equal to the expected marginal benefit of an additional future worker, i.e., see condition (10).

Combining the information above with conditions (7)–(11), the unknown policy functions (c, k', n', l, v, μ) are characterized by (P1)–(P6).

$$(P1) \quad U_1(c) = \beta E_G [F_1(k', n'l'; z') \\ + 1 - \delta] U_1(c')$$

$$(P2) \quad \phi_1 H_1(1 - l) = F_2(k, nl; z) U_1(c)$$

$$(P3) \quad \kappa v U_1(c) = \mu \alpha M(v, (1 - n)e)$$

$$(P4) \quad \mu = \beta E_G \{ \phi_1 H(1 - l') - \phi_2 H(1 - e) \\ + U_1(c') F_2(k', n'l'; z') l' \\ + \mu' [1 - \sigma - (1 - \alpha)p(v', n')] \}$$

$$(P5) \quad c + k' + \kappa v = F(k, nl; z) \\ + (1 - \delta)k$$

$$(P6) \quad n' = (1 - \sigma)n + M(v, (1 - n)e).$$

Conditions (P1) and (P2) have well-known interpretations: the former governs the intertemporal pattern of consumption and the latter determines the intratemporal allocation of consumption and leisure.⁷ As mentioned earlier, μ represents the expected discounted value of the marginal job match: condition (P4) ensures that the representative household is indifferent between small changes in the level of employment across any two time periods. Condition (P3) requires that the marginal recruiting cost is equal to its expected future return. Conditions (P5) and (P6) are simply restatements of the resource constraint and the law of motion for employment.

C. A Decentralized Search Economy

This section demonstrates how the Pareto optimal allocation characterized above might be implemented as a stationary equilibrium of a decentralized search economy.⁸ One benefit of this exercise is that it will result in an equilibrium wage equation, the implications of which may be compared to aggregate wage data. The conditions necessary for implementation are essentially twofold: (i) perfect insurance markets; and (ii) efficiency in the outcome of wage-employment contracts. These two conditions will be assumed to hold.

⁷ Note that, for $n \equiv 1$, these two conditions (together with the resource constraint) determine the optimal allocation for a standard RBC model.

⁸ This paper does not investigate the possible existence of other types of equilibria.

Firms are ultimately owned by individuals in the household sector. Assume that all firms discount expected future values according to

$$(14) \quad \Delta(\mathbf{s}', \mathbf{s}) \equiv \frac{\beta U_1(c(\mathbf{s}'))}{U_1(c(\mathbf{s}))}$$

where $c(\mathbf{s})$ is the equilibrium consumption in state \mathbf{s} . The use of this discount factor effectively weights current and future profits in terms of the values attached to these payoffs by households (i.e., state-contingent marginal utilities). In light of the ownership structure of the economy, the specification in (14) is reasonable.

Firms—or, more precisely, the jobs within a firm—exist in three possible states: active, vacant, and dormant. An active job is one that is matched with a worker and is currently producing output; let $J(\mathbf{s})$ denote the capital value of an active job when the state of the economy is \mathbf{s} . A vacant job is one that is seeking a suitable employee; let $Q(\mathbf{s})$ denote the capital value of a vacancy. A dormant job refers to a job that is neither matched with a worker nor looking for one. Assume that there is a large supply of such jobs each of which has zero capital worth.

A firm may choose to convert a dormant job into a vacant job within any period by undertaking the recruiting expense κ . Under the assumption that vacancies are matched with equal probability, the probability of making a successful transition is given by the ratio of the number of matches to the number of vacancies, (M/v) . Anticipating that $J > Q \geq 0$ in the equilibrium, the capital value of a vacant job must then satisfy the following recursive relationship:

$$(15) \quad Q(\mathbf{s}) = -\kappa + E_G \Delta(\mathbf{s}', \mathbf{s}) \times \left[\left(\frac{M}{v} \right) J(\mathbf{s}') + \left(1 - \left(\frac{M}{v} \right) \right) Q(\mathbf{s}') \right].$$

Condition (15) states that the capital value of a vacant job is equal to its current profit flow plus an expected, discounted, future capital value: with probability (M/v) , the firm is successful in recruiting a suitable employee and consequently attains the value J ; with proba-

bility $(1 - M/v)$, the firm is unsuccessful in its recruiting efforts thereby retaining the capital value Q ; these capital values are then converted into present value terms with the application of Δ .

As long as $Q(\mathbf{s}) > 0$, firms will convert dormant jobs into vacant positions. In this case, however, as the number of vacancies grows large, the probability that any one vacancy finds a suitable employee falls, owing to the added congestion in the search market. A lower probability of success reduces the attractiveness of recruiting activities, thereby reducing the value of Q . In equilibrium, free-entry ensures that $Q(\mathbf{s}) = 0$ in every state of the world. In other words, $v(\mathbf{s})$ adjusts to the point where firms are just indifferent between creating a vacant position or leaving it dormant, i.e., condition (15) reduces to

$$(16) \quad \kappa v = M(v, (1 - n)e) E_G \Delta(\mathbf{s}', \mathbf{s}) J(\mathbf{s}').$$

Consider next the determination of J , the capital value of an active job. This value will be determined by the wage-employment contract negotiated with the worker together with the firm's choice of capital input, k . The labor contract negotiated in state \mathbf{s} , denoted $(w(\mathbf{s}), l(\mathbf{s}))$, is assumed to be efficient. The equilibrium labor input, $l(\mathbf{s})$, must therefore satisfy condition (P2). The negotiated wage rate, $w(\mathbf{s})$, will depend on the nature of the share rule described below; in order for the share rule to implement the optimal allocation, it will have to induce equilibrium values for $v(\mathbf{s})$ and $\mu(\mathbf{s})$ that satisfy (P3) and (P4).

Recall from condition (13) that the utility value of the marginal job match in the optimal allocation is given by the value $W_2(\mathbf{s})$; this value, measured in units of goods, is given by $W_2(\mathbf{s})/U_1(c(\mathbf{s}))$. Let $0 < \xi < 1$ denote the firm's share of the value of a job that forms in the search equilibrium. Assuming for the moment that there exists a share parameter that implements the optimal allocation, the capital value of an active job is then given by

$$(17) \quad J(\mathbf{s}) = \xi W_2(\mathbf{s})/U_1(c(\mathbf{s}))$$

where $W_2(\mathbf{s})$ is given by expression (13) and where μ in expression (13) is characterized by condition (P4).

Having determined J in terms of μ , one may now rewrite condition (16), which determines equilibrium vacancies, as follows. From condition (10), one has $\mu = \beta E_G W_2(s')$; combining this with (17), one may equivalently write

$$(18) \quad \mu = \xi^{-1} \beta E_G J(s') U_1(c(s')).$$

Condition (18) may now be combined with condition (16) to yield

$$(19) \quad \kappa v U_1(c) = \mu \xi M(v, (1-n)e).$$

On comparing condition (19) with condition (P3), it is clear that in order that the equilibrium level of vacancies correspond to the socially-optimal level, it must be the case that $\xi = \alpha$. That is, the share rule agreed upon by workers and firms must be such that the share of match value accruing to firms must correspond to the elasticity of the matching technology with respect to recruiting effort.⁹

It will now be demonstrated how the conditions above may be used to derive a wage equation. An active job requires capital, which is rented on a competitive market at rate $r(s)$. Given the labor contract $(w(s), l(s))$, the decision problem of an active firm is simply

$$(20) \quad \pi(s) = \max_{\tilde{k}} \left\{ F(\tilde{k}, l(s); z) - r(s)\tilde{k} - w(s)l(s) \right\}$$

where $\pi(s)$ denotes the profit flow of an active job. In equilibrium, the aggregate demand for capital services must equal aggregate supply, i.e., $n\tilde{k} = k$; the equilibrium rental rate for capital will be given by the marginal product of capital, i.e., $r(s) = F_1(k, nl(s); z)$. Using the fact that $F = F_1\tilde{k} + F_2l$, together with the market-clearing condition, $\tilde{k} = k/n$, the equilibrium profit flow can be written as

$$(21) \quad \pi(s) = F_2\left(\frac{k}{n}, l(s); z\right)l(s) - w(s)l(s).$$

⁹ This result is similar to the one derived by Arthur J. Hosios (1990) and Pissarides (1990).

With $Q = 0$, the capital value of an active job must satisfy the following Bellman equation:

$$(22) \quad J(s) = \pi(s) + (1 - \sigma)E_G \Delta(s', s)J(s').$$

The interpretation of equation (22) is analogous to the interpretation of equation (15). Equation (22) may be thought of as determining, for any given wage rule, w , the equilibrium capital value of an active job, J . Alternatively, with J determined by condition (17), equation (22) may be used to recover the wage rule implied by the share parameter that implements the optimal allocation:

$$(23) \quad w(s) = F_2(k, nl(s); z) - [J(s) - (1 - \sigma)E_G \Delta(s', s)J(s')] \frac{1}{l(s)}.$$

Substituting for J from equation (17) (invoking $\xi = \alpha$) and utilizing equations (10) and (13) transforms the wage equation in (23) to

$$(24) \quad w(s) = (1 - \alpha)F_2(k, nl(s); z) + \alpha \times \left[\frac{b(s) + p(v(s), n)(1 - \alpha)\mu(s)}{U_1(c(s))} \right] \frac{1}{l(s)}$$

where $b(s) \equiv [\phi_2 H(1 - e) - \phi_1 H(1 - l(s))]$ is the gain in the utility of leisure to the household should it choose to search rather than work. The wage equation above is similar to the one derived by Merz (1992). In particular, a worker's wage bill turns out to be a weighted average of the worker's contribution to output and the worker's outside opportunity, with the weight $(1 - \alpha)$ having the interpretation of the worker's relative bargaining power in the wage negotiation process. The value of the outside opportunity is displayed in the square brackets in equation (24); it comprises two terms. Should the household choose to leave the bargaining process, it may enjoy the net gain b (described earlier) and re-engage in the search process, the value of which is $p(1 - \alpha)\mu$; i.e., with probability p the household contacts a different job and acquires the share

$(1 - \alpha)$ of the value of the match surplus, μ .¹⁰ The important development here is that the wage rate prevailing in the search equilibrium above does not equal (although it is related to) the marginal product of labor. As a result, average labor productivity and the real wage may potentially behave quite differently, as they apparently do in the data.¹¹

The equilibrium analysis turns next to the behavior of households. Households are assumed to begin time in an *ex ante* identical state. In particular, even though n_0 is known prior to trade, the identity of those belonging to the set of employed households is not known at this time. The random matchings and separations that occur in the labor market induce different employment histories among households and consequently can lead to heterogeneous wealth positions as well. However, given perfect insurance markets, risk-averse households will insure themselves fully against the income fluctuation attributable to idiosyncratic labor-market transitions; labor income will turn out not to depend upon the household's employment history. In equilibrium, labor income (net of insurance premiums) will be equal to the average wage bill, $nw(s)l(s)$, for all households. A detailed description of one market structure that implements the full-insurance result is provided in Appendix A.

Given full insurance, the equilibrium consumption and investment rules $(c(s), k'(s))$ for each household must satisfy

$$(25) \quad U_1(c(s)) = \beta E_G[1 + r(s') - \delta] \\ \times U_1(c(s'))$$

together with the budget constraint,

$$(26) \quad c(s) + k'(s) = [1 + r(s) - \delta]k \\ + nw(s)l(s) + d(s)$$

¹⁰ Dividing by the term U_1 then converts these values into "real" terms.

¹¹ Paul Gomme and Jeremy Greenwood (1995) develop an RBC model with heterogeneous agents in which the link between wages and productivity is broken owing to insurance considerations.

where $d(s) = n\pi(s) - \kappa v(s)$ is the aggregate dividend payment (net of recruiting costs). Combining (25) and (26) with the known pricing relationship $r(s)$ and the definition of profits in equation (20) results in two equations that correspond to condition (P1) and (P5). Hence, the search equilibrium described above is characterized by a set of functions $(c, n', l, k', v, \mu, w)$ satisfying conditions (P1) – (P6) and equation (24).

II. Model Parametrization and Calibration

Functional forms are required for U, H, F, M and G . The specifications used here are as follows:

$$U(c) = \log(c) \\ H(1 - x) = (1 - \eta)^{-1}(1 - x)^{1 - \eta} \\ F(k, nl; z) = \exp(z)\zeta k^\theta (nl)^{1 - \theta} \\ M(v, (1 - n)e) \\ = \min\{v, 1 - n, \chi v^\alpha ((1 - n)e)^{1 - \alpha}\}$$

where $\chi, \zeta > 0, 0 \leq \theta, \alpha \leq 1$ and $\eta \neq 1$. The productivity shock is assumed to be governed by the following stochastic process: $z' = \rho z + \tilde{\varepsilon}$, where $0 < \rho < 1$ and $\tilde{\varepsilon}$ is an independently and identically distributed random variable. In particular, assume that $\tilde{\varepsilon} \in \{-\varepsilon, \varepsilon\}$, $\varepsilon > 0$, and $\text{prob}(\varepsilon) = \text{prob}(-\varepsilon) = \frac{1}{2}$.

In order to compute the model's equilibrium, values must be assigned to the following list of parameters:

Preferences: $\beta, \eta, \phi_1, \phi_2$
Production technology: $\zeta, \theta, \delta, \rho, \varepsilon$
Search technology: $\chi, \alpha, \sigma, \kappa$.

The parameter values above are chosen according to the calibration procedure described by Kydland and Prescott (1994). In particular, parameter values are chosen to be consistent with the restrictions imposed by the theory on secular observations together with what is known from the cross-sectional data.¹²

When the variance of the shock process is set to 0, the model converges to a steady state.

¹² In this manner, parameter values are chosen independently of the phenomena to be explained (i.e., the higher-frequency movements in the data).

Let “starred” variables denote the steady-state values of the model’s endogenous variables. In the calibration procedure, these steady-state values are thought of as corresponding to the long-run averages of their counterparts in the data. The restrictions imposed by the theory on these long-run averages are the steady-state analogs to equations (P1)–(P6).¹³

$$(P1') \quad 1 = \beta[\theta\zeta(k^*)^{\theta-1}(n^*l^*)^{1-\theta} + 1 - \delta]$$

$$(P2') \quad \phi_1(1 - l^*)^{-\eta} \\ = (1 - \theta)\zeta(k^*)^\theta(n^*l^*)^{-\theta}(1/c^*)$$

$$(P3') \quad \kappa v^*/c^* = \mu^*\alpha\chi(v^*)^\alpha \\ ((1 - n^*)e)^{1-\alpha}$$

$$(P4') \quad \mu^* = \beta \left\{ \frac{\phi_1}{1 - \eta} + (1 - l^*)^{1-\eta} \right. \\ \left. - \frac{\phi_2}{1 - \eta} (1 - e)^{1-\eta}(1 - \theta) \right. \\ \left. \times \zeta(k^*)^\theta(n^*l^*)^{-\theta}(1/c^*) \right. \\ \left. + \mu^*[1 - \sigma - (1 - \alpha)\chi(v^*)^\alpha \right. \\ \left. \times ((1 - n^*)e)^{1-\alpha}/(1 - n^*) \right\}$$

$$(P5') \quad c^* + \delta k^* + \kappa v^* = \zeta(k^*)^\theta(n^*l^*)^{1-\theta}$$

$$(P6') \quad \sigma n^* = \chi(v^*)^\alpha((1 - n^*)e)^{1-\alpha}$$

To begin, the discount factor is set to $\beta = 0.99$, which is consistent with a steady-state real interest rate of 1 percent (per quarter). Following Prescott (1986), capital is assumed to depreciate at the rate of 10 percent per year; hence let $\delta = 0.025$. The parameter θ corresponds to capital’s share of output. Most RBC studies set $\theta = 0.36$; this value is used here as well, although, it should be pointed out that in the search economy, $(1 - \theta)$ no longer corresponds exactly to labor’s share of output.

¹³ Note: condition (P6') assumes that $\chi v^\alpha((1 - n)e)^{1-\alpha} < \min\{v, (1 - n)\}$.

With the technology parameter ζ chosen to normalize the steady-state level of output to unity, labor’s share can be computed from equation (23) to be

$$(27) \quad w^*n^*l^* = (1 - \theta) \\ - [1 - (1 - \sigma)\beta](\psi)/(\beta\sigma)$$

where $\psi = \kappa v^*$ is the ratio of recruiting expenditures to output.¹⁴ Unfortunately, there is little direct evidence pertaining to the magnitude of aggregate expenditures on search activity in the economy. In all likelihood, however, these costs are relatively small; with this in mind, set $\psi = 0.01$.¹⁵ In this case, the expenditure share of consumption implied by condition (P5') is $c^* \approx 0.74$, which is close to what is observed. Because ψ is a small number, it in fact turns out that labor’s share in (27) is approximately equal to $(1 - \theta)$.

The average employment ratio over the sample period is $n^* = 0.57$; the average fraction of discretionary time spent working is computed to be $l^* = 0.33$. Combining this information with (P1') results in a ratio of capital to (quarterly) output of about $k^* \approx 10$, which is consistent with the value reported by Prescott (1986).

The quarterly rate of transition from employment to nonemployment is set to $\sigma = 0.15$, which is based on the information reported in Kim B. Clark (1990) and Mortensen (1990).¹⁶ No hard evidence exists pertaining to the average fraction of time that nonemployed households spend searching; a reasonable guess is that it is probably no more than half the time an employed household spends

¹⁴ This derivation uses the following facts: $J^* = (\alpha\mu^*c^*)/\beta$ using equations (10) and (17), and $\mu^* = \psi/(c^*\sigma\alpha n^*)$ using (P3') and (P6').

¹⁵ As it turns out, none of the results reported below are very sensitive to the exact value of ψ ; for example, setting $\psi = 0.0001$ leaves the quantitative properties of the search model virtually unchanged.

¹⁶ Clark (1990 p. 191) reports monthly transition probabilities for the year 1974. According to this study, the probabilities of making transitions from employment to unemployment and employment to nonparticipation are 0.020 and 0.033, respectively, which yields a monthly transition probability of employment to nonemployment equal to 0.053.

working, i.e., $e = (1/2)l^*$.¹⁷ Let $m^* \equiv M(v^*, (1 - n^*)e)$. In a steady state, the flow of job destruction will equal the flow of job creation and condition (P6') reduces to $\sigma n^* = m^*$, which may be written alternatively as

$$(28) \quad \sigma n^* = q^* v^*$$

where $q^* \equiv m^*/v^*$ is the probability that a vacant position becomes a productive job by the end of a three-month period. A problem arises with respect to the appropriate way to set v^* , which in the model corresponds to the steady-state stock of available jobs (as a ratio of the working-age population). Our measure of vacancies is based on a count of help-wanted advertisements posted in various newspapers in cities across the country and likely underestimates the aggregate level of job availability.¹⁸ With $\sigma = 0.15$ and $n^* = 0.57$, setting v^* equal to the average stock of help-wanted advertisements (0.02) appears to be out of the question as this implies $q^* > 1$. Thus, the approach here will be to use evidence pertaining to the duration of vacancies in order to pin down a value for q^* ; the corresponding value for v^* will then be inferred from (28). The transition probability is set to $q^* = 0.90$, which is consistent with an average vacancy duration of about 45 days (Jan C. van Ours and Geert Ridder, 1992).¹⁹ Condition

¹⁷ The quantitative properties of the model are not very sensitive across a wide range of values for e between 0 and l^* .

¹⁸ For example, in a study on the use of help-wanted advertising in the cities of San Francisco and Salt Lake City in 1972, it was reported that "... in both cities a relatively small percentage [15–24] of employers hired workers through want ad advertising. These tended to be large firms concentrated in selected industries" (John Walsh et al., 1975 p. 22).

¹⁹ That is, q^* is approximately equal to $1 - (44/45)^{90}$. It should be noted that the average duration for vacancies is commonly reported to be under one month (e.g., Blanchard and Diamond, 1989 p. 21), which would imply $q^* \approx 1$. However, as pointed out by van Ours and Ridder (1992), a distinction should be made between the time a help-wanted advertisement is removed and the time it actually takes to fill a vacant position. These authors report that while 75 percent of all vacancies are filled by applicants who arrive in the first two weeks, it takes on average 45 days to select a suitable employee from the pool of applicants.

(28) then estimates average stock of available jobs to be $v^* = \sigma n^*/q^* = 0.095$, which is about five times larger than the average stock of help-wanted advertisements.²⁰ The vacancy cost parameter is then set to $\kappa = \psi/v^* = 0.105$.

The parameter α is the elasticity of the matching rate with respect to aggregate recruiting intensity. Blanchard and Diamond (1989) estimate this parameter to be $\alpha = 0.60$ for the United States. The technology parameter on the matching function, χ , is then determined by $m^* = \chi(v^*)^\alpha((1 - n^*)e)^{1-\alpha}$. Condition (P3') may then be used to calculate μ^* .

For the specification of preferences used in this paper, the individual labor supply elasticity is equal to

$$\gamma = \eta^{-1} \left(\frac{1}{l^*} - 1 \right).$$

Thomas MaCurdy (1981) provides estimates of γ for males that range from 0.10 to almost 0.5. Arguing that this elasticity is likely higher for females, Greenwood et al. (1988) suggest that $\gamma = 1.7$ is reasonable. For present purposes, an intermediate value is chosen: $\gamma = 1.0$, which implies setting $\eta = 2.0$. Given this information, (P2') may now be solved for $\phi_1 = 2.08$ and (P4') may be solved for $\phi_2 = 1.37$.²¹ The parameters remaining are those that describe the stochastic process for the technology shocks. Estimates for these parameters are provided by Prescott (1986), i.e., $\rho = 0.95$ and $\varepsilon = 0.007$.

With the parameter values so determined, the equilibrium may now be computed numerically. The computation procedure uses an algorithm suggested by Wilbur Coleman II (1990). Essentially, the procedure is to find function evaluations that satisfy the system (P1)–(P6) at discrete points on a state

²⁰ While the reported count of help-wanted advertisements likely underestimates the level of job availability, it is reasonable to assume that the behavior of actual job availability is approximately proportional to the number of help-wanted ads.

²¹ Hence, the calibration exercise here suggests that an employed household values any given amount of leisure time by a greater amount than a nonemployed household.

space.²² The true solution functions are then approximated by piecewise linear interpolations of these function evaluations.²³

III. Model Evaluation

Tables 1–3 below report statistics summarizing the cyclical properties of the U.S. and model economies. The economic variables of interest include real output (and its components) together with a variety of labor-market entities. Data for the U.S. economy is quarterly, for the 1953.1–1990.3 sample period.²⁴ All variables were first deflated by the 16+ population and transformed by taking logarithms. The data series were then detrended using the Hodrick-Prescott filter (as described in Prescott [1986]); statistics describing the cyclical component of these series were computed using the filtered data. A corresponding set of statistics, generated by the different model economies, was constructed by using the equilibrium decision rules to simulate time-series 5000 periods in length. All simulated series were transformed in a manner analogous to the transformation undertaken on the data.

Table 1 provides statistics describing the cyclical behavior of the U.S. economy, a standard RBC economy, and the search economy developed above. For each economy, three columns of statistics are reported. Column (1) provides a measure of volatility (the standard deviation of the variable relative to the standard deviation of output); column (2) provides a measure of comovement (the contemporaneous correlation of the variable with output); and column (3) provides a measure of the variable's phase shift relative to output.²⁵ The search economy is evaluated here

on its ability to replicate these key business-cycle facts; the value that is added by introducing labor-market search can to some extent be ascertained by comparing its performance vis-à-vis the RBC model, which is nested within the search model considered here.

The standard deviation of real per-capita output around trend is 1.58 percent for the U.S. economy, 1.22 percent for the RBC economy, and 1.45 percent for the search economy. Hence, the first thing to note is how the presence of trading frictions in the search economy amplify the volatility of the business cycle: the standard deviation of output increases by almost 20 percent. The RBC economy does well in mimicking the behavior of consumption and investment spending; labor-market search is seen to have little impact on the behavior of these components.

In the data, the aggregate labor input varies almost as much as output, is procyclical, and lags the cycle by a quarter. Most of the variance in hours is accounted for by adjustments in employment rather than hours per worker; the extensive margin is seen to fluctuate twice as much as the intensive margin (see also Gary Hansen, 1985). As well, note that while employment lags the cycle by one quarter, hours per worker varies contemporaneously with output. The search economy replicates these features of the data remarkably well. The only deficiency appears to be with respect to generating a sufficient amount of variability in the labor input: the search economy can account for only two thirds of the variance in hours. Nevertheless, the search model delivers a substantial improvement in this regard relative to the predictions of the standard RBC economy.

The aggregate wage bill in the U.S. economy varies almost as much as output, is procyclical, and lags the cycle. On the other hand, labor's share of output is countercyclical and leads the cycle by a full three quarters. The standard RBC economy cannot account for this last feature, given that labor's share of income is constant in this environment. Note, however, that the search economy is able to replicate the countercyclical behavior of labor's share and its tendency to lead the cycle. Unfortunately, the model underestimates the extent of the lead and also fails to capture the lagging behavior of the wage bill.

²² Here, we use a $5^3 = 125$ point grid distributed evenly over the state space.

²³ See Gomme and Greenwood (1995) for details describing the implementation of this algorithm.

²⁴ Refer to Appendix B for information regarding the source and construction of the data series used here.

²⁵ In column (3), an entry of, for example, "+1" indicates that the variable *lags* the cycle by one quarter; an entry of "-2" indicates that the variable *leads* the cycle by two quarters; and an entry of "0" means that the variable is most strongly correlated with contemporaneous output.

TABLE 1—CYCLICAL PROPERTIES: U.S. ECONOMY AND MODEL ECONOMIES

Variable (x)	U.S. economy $\sigma(y) = 1.58$			RBC economy $\sigma(y) = 1.22$			Search economy $\sigma(y) = 1.45$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Consumption	0.56	0.74	0	0.34	0.90	0	0.32	0.91	0
Investment	3.14	0.90	0	3.05	0.99	0	2.98	0.99	0
Total hours	0.93	0.78	+1	0.36	0.98	0	0.59	0.96	0
Employment	0.67	0.73	+1	0.00	0.00	0	0.51	0.82	+1
Hours/worker	0.34	0.66	0	0.36	0.98	0	0.22	0.66	0
Wage bill	0.97	0.76	+1	1.00	1.00	0	0.94	1.00	0
Labor's share	0.68	-0.38	-3	0.00	0.00	0	0.10	-0.62	-1
Productivity	0.64	0.43	-2	0.64	0.99	0	0.46	0.94	0
Real wage	0.44	0.04	-4	0.64	0.99	0	0.39	0.95	0

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between x and y . Column (3) is the phase shift in x relative to y : $-j$ or $+j$ corresponds to a lead or lag of j quarters.

The last two variables in Table 1 are Productivity and the Real wage; these refer to the average product of labor and the ratio of the wage bill to total hours, respectively. In the U.S. economy, the real wage is smoother than labor productivity and both of these variables display considerably less movement than hours over the cycle. As well, the real wage displays virtually no relationship with contemporaneous output, although the relationship is somewhat stronger at a one-year lead (-0.27). Labor productivity also leads the cycle, but unlike the real wage, this relationship is positive (0.56). In the standard RBC model, the assumed market structure is such that the real wage is proportional to the average product of labor; this model will not be able to account for any differences that might exist between the behavior of wages and productivity. The ratio of the volatility of hours to productivity is about 1.45 in the data. It is well known that, given any reasonable value for the elasticity of labor supply, the standard RBC model dramatically underestimates the magnitude of this ratio; here, it is predicted to be 0.56.²⁶ The search model, on the other hand,

predicts a value for this ratio of about 1.28, which is reasonably close to observation. Finally, note that the search model overestimates the degree to which productivity and the real wage are correlated with output (although the correlation falls somewhat, relative to the RBC model). In light of the single disturbance afflicting this model economy, this last result is not too disappointing; adding the government spending shocks considered by Merz (1993) would presumably improve matters along this dimension. What is more disappointing is the failure of the search model to mimic the leading behavior observed in productivity and wages: this failure may be linked to the nature of the model's propagation mechanism.

Table 2 provides a closer look at the relationship between hours, labor productivity, and the real wage. In the U.S. economy, labor productivity and real wages behave in an interesting and different manner.²⁷ In particular, observe that hours are positively correlated with past productivity, but negatively correlated with future productivity. Interestingly, this pattern appears to be reversed for real

²⁶ The indivisible-labor modification proposed by Richard Rogerson (1988) and Hansen (1985) leads to a considerable improvement in this dimension.

²⁷ Note: it is well known that the measure for the real wage used here may suffer from an aggregation bias resulting from cyclical changes in the skill composition of the workforce. See, for example, Eswar Prasad (1993) and Gary Solon et al. (1994).

TABLE 2—CROSS CORRELATIONS OF HOURS WITH PRODUCTIVITY AND THE REAL WAGE

Variable (x)	$x(t - 4)$	$x(t - 3)$	$x(t - 2)$	$x(t - 1)$	$x(t)$	$x(t + 1)$	$x(t + 2)$	$x(t + 3)$	$x(t + 4)$
U.S. economy:									
Productivity	0.47	0.43	0.31	0.10	-0.22	-0.25	-0.27	-0.31	-0.36
Real wage	-0.25	-0.10	-0.01	0.01	-0.13	0.10	0.19	0.18	0.09
RBC economy:									
Real wage	-0.10	0.09	0.33	0.61	0.95	0.77	0.59	0.42	0.25
Search economy:									
Productivity	0.14	0.38	0.63	0.82	0.81	0.56	0.40	0.28	0.18
Real wage	0.05	0.30	0.57	0.80	0.84	0.66	0.52	0.40	0.29

wage. A priori, there are reasons for believing that the search economy studied here may account for such behavior. It is possible, for example, that the asymmetric lead-lag relationship between hours and productivity reflects certain delays in responding to various impulses: the search environment provides a natural rationale for the existence of such delays. Secondly, the direct link between productivity and wages is broken owing to bargaining considerations: these two variables may behave quite differently in the search equilibrium.

Alas, the actual results are somewhat disappointing, although there are some bright spots. To begin, the search model does in fact generate a faint hint of asymmetry in the dynamic correlation between hours and productivity: productivity leads hours and the contemporaneous correlation drops to 0.81, compared to the 0.95 predicted by the RBC model. Again, it is reasonable to suppose that the quantitative magnitudes at all leads and lags would be much more in accord with the data if some type of aggregate demand shock were included in the model.²⁸ On the other hand, the predicted dynamic correlation between hours and the real wage appears to be completely wrong. In particular, the equilibrium real wage behaves “too much” like labor

productivity.²⁹ Reflecting upon the form of the wage rule in (24), perhaps this result should have been anticipated; it is likely that the value of a worker’s outside opportunity adjusts in the same direction as productivity in response to a technology shock.

Table 3 displays statistics summarizing the cyclical relationship between vacancies and unemployment (actually, nonemployment), which is sometimes referred to as the Beveridge curve. The first thing to note is the high degree of persistence exhibited by unemployment in the U.S. economy; in fact, the search economy performs very well in mimicking the dynamic pattern of persistence. Secondly, observe that, in the data, job availability displays a strong tendency to move counter to unemployment over the cycle and appears to lead unemployment by a full quarter. The search economy is able to match these patterns reasonably well, although it appears to overestimate the lead in vacancies over unemployment. One dimension in which the search model fails dramatically is its ability to replicate the observed volatility of job availability: the percent standard deviation in vacancies (not reported in Table 3) is over nine times that of output for the U.S. economy. The search model studied here can account for only about one third of this variance.

²⁸ Examples include the preference shocks considered by Valerie Bencivenga (1992) or the government spending shocks considered by Lawrence J. Christiano and Martin Eichenbaum (1992).

²⁹ At least, this is true for the wage rule that implements the Pareto-optimal allocation. It is conceivable that some other equilibrium wage rule, based on the outcome of an explicitly-formulated noncooperative-bargaining game, may possibly yield very different wage behavior. This issue is not pursued here.

TABLE 3—CROSS CORRELATIONS OF UNEMPLOYMENT WITH UNEMPLOYMENT AND VACANCIES

Variable (x)	$x(t-4)$	$x(t-3)$	$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$	$x(t+4)$
U.S. economy:									
Unemployment	0.23	0.46	0.69	0.89	1.00	0.89	0.69	0.46	0.23
Vacancies	-0.39	-0.62	-0.82	-0.92	-0.89	-0.72	-0.47	-0.21	0.02
Search economy:									
Unemployment	0.20	0.41	0.65	0.87	1.00	0.87	0.65	0.41	0.20
Vacancies	-0.51	-0.65	-0.73	-0.65	-0.19	0.05	0.17	0.24	0.27

As a final step toward assessing the quantitative importance of the propagation mechanism embedded in the search economy, I pursue a line of enquiry originating with the recent contribution of Cogley and Nason (1992). These authors begin by reviewing two stylized facts concerning output dynamics that were first pointed out by Nelson and Plosser (1982):

- (F1) There is a unit root or near unit root in real output.
- (F2) Output growth is positively correlated over short horizons and negatively correlated over longer horizons.

The question that Cogley and Nason ask is: can RBC models replicate the Nelson and Plosser facts? They demonstrate that any model with persistent impulse dynamics can replicate (F1), but that to replicate (F2), a model must also be able to propagate shocks over time. Hence, the ability to replicate the second stylized fact can be thought of as a test concerning the quantitative importance of a model's propagation mechanism. In examining the autocorrelation functions (ACF's) for output of eight popular RBC models, their principal results can be summarized as follows:

- (R1) For most RBC models, output dynamics are essentially the same as impulse dynamics.
- (R2) Only the labor-hoarding model of Burnside et al. (1993) is able to match the sample ACF for output.

The first result highlights the fact that most RBC environments are incapable of generat-

ing quantitatively-important propagation mechanisms. Because labor-market search induces a labor-hoarding motive for firms, the second result suggests that the search economy studied here may enjoy success similar to the model of Burnside et al.

Consider first Figure 1. This figure plots the ACF for output (GNP) growth and total-factor-productivity (TFP) growth; the top panel does so for the standard RBC model, while the bottom panel does so for the search economy studied above. The top panel confirms the first result reported by Cogley and Nason, i.e., output dynamics for this model are virtually indistinguishable from the assumed impulse dynamics. On the other hand, the bottom panel reveals that the search economy generates output dynamics that are substantially different than impulse dynamics; this model embodies a significant propagation mechanism. Furthermore, as is revealed by Figure 2, the model's output dynamics are broadly consistent with the pattern generated by the U.S. economy, i.e., output growth is positively correlated at short horizons and negatively correlated at longer horizons.

IV. Structural Disturbances

This section of the paper considers briefly the quantitative implications of introducing exogenous "structural" disturbances in the search economy studied above. The analysis here is motivated largely by suggestions put forth by David Lilien (1982) as well as Steve Davis and John Haltiwanger (1992) regarding the macroeconomic implications of structural disturbances that impinge directly on an economy's allocative mechanism (as distinct from disturbances that affect production technologies).

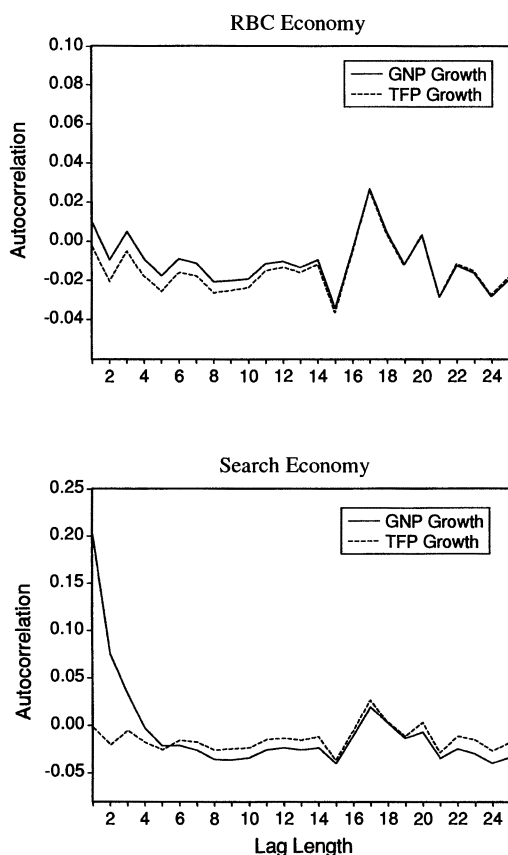


FIGURE 1. AUTOCORRELATION FUNCTIONS:
GNP AND TFP GROWTH

In the search model studied here, the efficiency of the economy's allocative mechanism is captured by the technological properties of the aggregate matching function. Changes in this function can be thought of as reflecting, at least in part, the technological ramifications of various types of structural disturbances.

Consider, for example, the following generalization of the matching technology:

$$(30) \quad m_t = \chi_t v_t^\alpha ((1 - n_t)e)^{1-\alpha}$$

where χ_t varies over time according to some exogenous stochastic process. According to Lilien (1982), random shifts in the sectoral composition of aggregate demand and supply can have adverse consequences in an economy

where resources are not instantaneously mobile across sectors. In the present context, a sectoral shift would likely require a temporary increase in the number of matches that must form across sectors as opposed to within sectors. If matching across sectors is relatively more difficult, then a sectoral shock could manifest itself as a decrease in the technical efficiency with which matches are formed at the aggregate level. Such an interpretation is also consistent with the view offered Katherine Abraham and Lawrence Katz (1986). James Medoff (1983) also describes how "spurts in labor market imbalances" might cause shifts in the Beveridge curve. Finally, a decrease in χ_t may also plausibly capture the effects of what Davis and Haltiwanger (1990 p. 3) call "changes in the intensity of shifts in employment opportunities across establishments." In what follows below, the parameter χ_t will be referred to as an "allocative" shock.

Given observations on $\{m_t, v_t, n_t, e_t\}$ and an estimate for α , one may proceed as in Prescott (1986) to back out the residual χ_t . Unfortunately, the search-effort variable, e_t , is unobservable: as a working hypothesis, it is simply assumed here that $e_t = e$. Hence, some care must be taken in interpreting movements in χ_t as reflecting structural disturbances; in particular, it is conceivable that much of the measured movement in this efficiency parameter is caused by unobserved changes in the search intensity of workers. Matters are further complicated by the fact that there does not appear to be any direct measure of aggregate hires, m_t , available. However, there is a new-hires series available for the U.S. manufacturing sector for the period 1958:1–1981:12.³⁰ In what follows, χ_t is computed under the assumption that the aggregate hiring rate is proportional to the hiring rate in the U.S. manufacturing sector.

Letting $a_t = \ln(\chi_t)$, the joint process governing $\{z_t, a_t\}$ is assumed to take the form

³⁰ The series label is NHR and is described as: New hires per 100 employees, manufacturing, U.S., seasonally adjusted. Source: Employment and Earnings, Bureau of Labor Statistics.

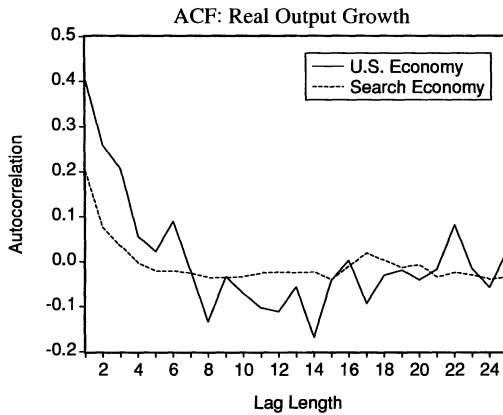


FIGURE 2. ACF REAL OUTPUT GROWTH

$$z_{t+1} = \rho_1 + \rho_{11}z_t + \rho_{12}a_t + \varepsilon_{t+1}$$

$$a_{t+1} = \rho_2 + \rho_{21}z_t + \rho_{22}a_t + u_{t+1}$$

This system was estimated using data from the 1958.1–1981.4 time period.³¹ The initial estimation yielded 2-stage-least-squares parameter estimates for ρ_{12} and ρ_{21} with t ratios around -1 ; restricting these values to 0 had little effect on the remaining parameter estimates. The point estimates of interest (for the restricted version) are: $\rho_{11} = 0.96$ and $\rho_{22} = 0.85$.³² In addition, the standard deviations for the innovations $\{\varepsilon, u\}$ were 0.008 and 0.070, respectively, with a correlation coefficient of 0.48. Hence, both the technology shock and the allocative shock are highly persistent. The standard deviation of the innovations to the allocative shock is estimated to be almost ten times the size of the corresponding innovations to the technological shock process. Given the crude measurements available for vacancies and hires, much of this variation could simply be the result of measurement error. The innovations are also estimated to be positively correlated. There are at least two interpretations for this correlation. One possibil-

ity is that it reflects the behavior of search intensity in response to a positive productivity shock. The other possibility is that more reallocation is required (χ_t falls) during recessions and less (χ_t rises) during booms. This latter interpretation is consistent with the popular idea that recessions tend to be periods of “cleaning up.”

The dynamic properties of the model are now examined by recomputing the equilibrium with the joint process for $\{z_t, a_t\}$ embedded in the model. Assume that the innovation to each shock lies in a two point set: $\tilde{\varepsilon} \in \{-\varepsilon, \varepsilon\}$ and $\tilde{u} \in \{-u, u\}$, $\varepsilon, u > 0$ with

$$\text{Prob}(\varepsilon, u) = \text{Prob}(-\varepsilon, -u) = (1/2)\omega$$

$$\text{Prob}(\varepsilon, -u) = \text{Prob}(-\varepsilon, u)$$

$$= (1/2)(1 - \omega)$$

and where $0 \leq \omega \leq 1$. Under this specification, each innovation has a standard deviation equal to (ε, u) , respectively, and the correlation between the two innovations is equal to $(2\omega - 1)$. Hence, set $\varepsilon = 0.008$; $u = 0.070$ and $\omega = 0.74$.

Table 4 reports some of the implications for labor-market variables from incorporating allocative shocks into the search model. The three columns report the behavior of the search economy under three different specifications for the standard deviation in the innovation to the allocative disturbance: $u = 0.070$, $u = 0.035$, and $u = 0.010$. The results of this experiment reveal that the search model now performs considerably worse along several margins with the inclusion of the allocative disturbances. In model M1, the behavior of aggregate hours appears to be closer to observation, but the components of the labor input now behave quite differently relative to the actual economy and the benchmark search model. In particular, employment now fluctuates more than total hours, and hours per worker behaves countercyclically. Reducing the standard deviation of the allocative shock (models M2 and M3) mitigates this counterfactual behavior, but what remains true for each specification is that employment and hours per worker are predicted to covary negatively; in the data, these two variables are positively correlated.

³¹ The Solow residual was computed using the data described in Appendix B. The capital-stock series used here is from Andreas Hornstein and Jack Praschnik (1994).

³² The t ratios were 71.77 and 15.91, respectively.

TABLE 4—CYCLICAL PROPERTIES: SEARCH ECONOMIES WITH STRUCTURAL DISTURBANCES

Variable (x)	M1: $u = 0.070$ $\sigma(y) = 1.93$			M2: $u = 0.035$ $\sigma(y) = 1.78$			M3: $u = 0.010$ $\sigma(y) = 1.67$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Total hours	0.85	0.92	0	0.71	0.93	0	0.63	0.91	0
Employment	1.05	0.82	+1	0.79	0.82	+1	0.63	0.95	+1
Hours/worker	0.29	-0.33	+1	0.22	0.22	+2	0.20	0.41	-1
Productivity	0.40	0.49	0	0.42	0.79	0	0.44	0.92	0
Real wage	0.34	0.73	0	0.38	0.88	0	0.39	0.93	0

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between x and y . Column (3) is the phase shift in x relative to y : $-j$ or $+j$ corresponds to a lead or lag of j quarters.

Evidently, when the model economy is subjected to (say) a negative allocative disturbance, the rate of job creation is affected adversely, leading to a subsequent decline in employment. *Ceteris paribus*, the decline in employment leads to an increase in the productivity of labor (as the capital released by firms is absorbed by remaining firms): exiting firms would like to hire more workers, but this is costly and takes time. In the meantime, firms attempt to compensate for the decline in employment by working the existing workforce harder, hence employment and hours per worker tend to move in opposite directions.³³ Of course, to the extent that an adverse allocative disturbance is accompanied by an adverse technology shock, the incentive to increase hours per worker is reduced and may even be reversed if the technology shock is large enough relative to the allocative shock. In the parametrizations above, however, the effects of the allocative disturbances appear to swamp these technological considerations.

V. Summary and Conclusions

Labor-market search has been held by many theorists as a promising framework from which to study and to better understand the nature of aggregate fluctuations. The purpose of this paper has been to evaluate the

empirical content of a business-cycle theory based on labor-market-search considerations. The quantitative results indicate that, when labor-market search is incorporated into a standard RBC model, the empirical performance of the model improves along several dimensions. In particular, the search model becomes consistent with the observation that the labor input fluctuates substantially more than the real wage and that most of the variability in aggregate hours is accounted for by cyclical adjustments in employment rather than hours worked per employee. Also, the search model correctly predicts the countercyclical movement in labor's share and replicates the feature that the real wage is "stickier" than labor productivity over the cycle. Furthermore, the search model implies a lower contemporaneous correlation between productivity and hours, with productivity displaying a slight lead. A reasonable conjecture is that with the inclusion of some aggregate demand disturbance, the search model would also be able to account for the asymmetric pattern in the dynamic correlation between hours and productivity. Finally, in contrast to the RBC model (and several of its variants), the search model embodies a quantitatively important propagation mechanism in the sense that it replicates the observed dynamic pattern of output growth.

The search model also accounts reasonably well for a number of business-cycle facts that RBC models were not designed to address. In particular, the model was able to replicate the observed pattern of persistence in unemploy-

³³ The same type of behavior is present in this search economy when it is subjected to a stochastic separation rate; the reason is similar to that above.

ment and the Beveridge curve relationship reasonably well. The model's most notable shortcoming in this regard was in terms of accounting for the observed volatility in job availability; the model predicted cyclical movements in vacancies that were substantially smaller than those displayed by the U.S. economy. The other disappointing feature of the search model was its inability to capture the dynamic pattern of real wage movements vis-à-vis movements in labor productivity. It would be interesting to examine exactly what modifications to the search environment can possibly reconcile this discrepancy.

The search model studied here obviously leaves out some important features of actual labor markets. It is in this light that the results reported above may be viewed as encouraging. An extension worthy of consideration would involve endogenizing the rate of job destruction. Work along this line is currently being pursued by a number of authors, in particular, see Mortensen (1993) and Merz (1993). Empirical investigations regarding the quantitative implications of search environments in general-equilibrium settings are still at an early stage. Whether the search paradigm will prove itself as a useful econometric tool remains to be seen.

APPENDIX A

This appendix outlines in greater detail a simple institutional structure that implements the full-insurance result described in Section I of the paper. Suppose that there is a distinction between job and worker flows in the model. In particular, job flows are governed by the matching/separation process described in the paper, while worker flows are determined exogenously by a game of "musical chairs." That is, at the beginning of any period, n represents the number of jobs that can actively produce output during the period. Imagine that the entire workforce is "shuffled" randomly across this given set of jobs at the beginning of each period, before any trading occurs. Hence, as far as the representative household is concerned, the probability of employment in any period is given by $0 < n < 1$.

Imagine that the insurance market operates in the following way. At the beginning of each

period, households may choose to purchase y units of insurance at a price $q(s)$ per unit, where y is the quantity of the consumption good that is delivered to the household contingent on nonemployment during the period. In each period, the representative household is assumed to choose a level of insurance (y), consumption (c_j), investment (i_j) contingent upon the household's employment status j , where $j = 1$ denotes employment and $j = 2$ denotes nonemployment.³⁴ The employment-contingent period budget constraints faced by the representative household are

$$(A1) \quad c_1 + i_1 + q(s)y \leq w(s)l(s) + r(s)x + d(s)$$

$$(A2) \quad c_2 + i_2 + q(s)y \leq y + r(s)x + d(s)$$

where x denotes the household's capital holdings and $d(s)$ represents the profits earned by firms and redistributed to households in the form of a dividend payment.³⁵ Also, the evolution of the household's capital stock must obey:

$$(A3) \quad x'_j = (1 - \delta)x + i_j \quad \text{for } j = 1, 2.$$

From the viewpoint of an individual household, the relevant state is given by the vector (x, s) . Let $V(x, s)$ denote the maximum utility attainable by the representative household given that it begins the period in state (x, s) and behaves optimally. Then, under mild restrictions, there is a concave (in its first argument) value function V satisfying:

$$(A4) \quad V(x, s) = \max_{c_j, i_j, y} \{ n[U(c_1) + \phi_1 H(1 - l(s)) + \beta E_G V(x'_1, s')] \}$$

³⁴ Recall that $l(s)$ is determined separately by the bargaining process.

³⁵ That is, each household is assumed to hold a single share in a fully diversified portfolio of claims on the profits earned by firms. The market for these shares is suppressed since, given the representative-agent framework, these shares will not be traded in equilibrium.

$$+ (1 - n)[U(c_2) + \phi_2 H(1 - e) + \beta E_G V(x'_2, s')] \}$$

where the maximization is subject to the constraints (A1) and (A2). The solution to this dynamic programming problem will be a set of policy functions (c_j, i_j, x'_j, y) , which prescribe the household's optimal action given any particular state of the world, and a value function, V , which returns the maximum utility of such an action.

Substituting the constraints (A1) and (A2) into the objective function (A3), the maximization may be undertaken with respect to the choice of x'_j , and y . The first-order necessary conditions are given by

$$(A5) \quad U'(c_j) = \beta E_G V_1(x'_j, s') \quad j = 1, 2$$

$$(A6) \quad nq(s)U'(c_1) = (1 - n)(1 - q(s))U'(c_2).$$

The conditions (A5)–(A6) may be interpreted as follows. A household contemplating a one-unit increase in its future capital holdings must forego one unit of current consumption: the loss of utility is given by U' and the (discounted) expected gain in utility by $\beta E V_1$. Condition (A5) states that the consumption-savings choice will be made in a manner that balances these two margins. Condition (A6) determines the level of insurance chosen by the household. Buying one more unit of insurance ends up costing q (in units of consumption) if the household is employed and $(1 - q)$ if nonemployed. Condition (A6) equates the expected marginal cost of insurance, $nqU'(c_1)$, to its expected marginal benefit, $(1 - n)(1 - q)U'(c_2)$.

The expected profits of a representative insurance company are given by: $q(s)y - (1 - n)y$ (revenue minus expected payout). Competition in the insurance market implies that the equilibrium price adjusts to eliminate profits; the price a household must pay for insurance will equal the probability that the household collects on the insurance:

$$(A7) \quad q(s) = (1 - n).$$

Inserting (A7) into (A6) yields $U'(c_1) = U'(c_2)$, which, given the strict concavity of U implies that $c_1 = c_2$. That is, the household insures itself in a manner that guarantees an equal level of consumption across possible states of employment.³⁶ This being the case, then the concavity of V together with condition (A4) implies $x'_1 = x'_2$, which in turn implies (using condition (A2)) that $i_1 = i_2$. Because the budget constraints will, in equilibrium, hold with equality, the conditions above combined with (A1) imply that $y = w(s)l(s)$. In other words, households will choose to insure themselves fully.

The notation used to distinguish employment status may be eliminated; the solution to the household's problem may be characterized in terms of two functions, (f, g) satisfying

$$(A8) \quad U'(f(x, s)) = \beta E_G(1 + r(s') - \delta)$$

$$\times U'(f(g(x, s), s'))$$

$$(A9) \quad f(x, s) + g(x, s) = (1 + r(s) - \delta)x$$

$$+ w(s)nl(s) + d(s)$$

where the right-hand side of condition (A8) is derived from an application of the envelope theorem on equation (A3). Condition (A8) is the standard Euler equation that governs the intertemporal allocation of consumption and condition (A10) is the budget constraint. In equilibrium, households will have identical capital holdings, i.e., $x = k$. Furthermore, in equilibrium, $c = f(k, s)$ and $k' = g(k, s)$; substituting these relations into (A8) and (A9) results in the pair of equations (25) and (26) which were used in the text to characterize the behavior of the household.

APPENDIX B

The data used in this study are real aggregate data of the United States for the sample period 1954:1–1990:3; the source is Citicorp's citibase data bank.

³⁶ This result is an artifact of our assumption regarding the separability of consumption and leisure in the utility function.

A. Raw Data Series

- [1] GCN82: Personal consumption expenditures on nondurables (1982 dollars).
- [2] GCS82: Personal consumption expenditures on services (1982 dollars).
- [3] GCD82: Personal consumption expenditures on durables (1982 dollars).
- [4] GGE82: Government purchases of goods and services (1982 dollars).
- [5] GIF82: Gross private domestic investment, fixed (1982 dollars).
- [6] LHEM: Total employment (seasonally adjusted).
- [7] LHCH: Average hours of work per week (household data, seasonally adjusted).
- [8] LHUR: Unemployment rate, all workers 16+ (seasonally adjusted).
- [9] LHELX: Employment ratio (help-wanted ads divided by the number of unemployed workers).
- [10] LHPAR: Labor force participation rate, total 16+.
- [11] PO16: Noninstitutional population, total 16+.
- [12] GAP: Compensation of employees, all industries.
- [13] GDY: Implicit price deflator, national income.

B. Constructed Data

Consumption = $([1] + [2] + [4]) \div [11]$.

Investment = $([3] + [5]) \div [11]$.

Output = consumption + investment.

Hours = $[6] \times [7] \div [11]$.

Employment = $[6] \div [11]$.

Hours per worker = $[7]$.

Productivity = output \div hours.

Vacancies = $[9] \times [8] \times [10]$.

Wage bill = $[12] \div [13] \div [11]$.

Labor's share = wage bill \div output.

Real wage = wage bill \div hours.

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