Environmental Policy with Heterogeneous Plants

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Abstract

We show that accounting for plant heterogeneity is important for the evaluation of environmental policies. We develop a general equilibrium model in which monopolistic competitive plants differ in productivity, produce differentiated goods and optimally choose a discrete emission-reduction technology. Calibrated to the Canadian data, the model shows two main results. First, the aggregate costs of using an emission tax to reduce 20% of current emissions are twice as large as what would result with homogenous plants due to the selection of an abatement technology. Second, an emission standard outperforms an emission tax since the tax causes price distortions.

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Reducing Greenhouse Gas (GHG) emissions has become a public and political concern. The purpose of this paper is to analyze how plant heterogeneity may affect the impact of emission-reduction policies on aggregate output and welfare. The paper develops a general equilibrium model with heterogeneous plants that differ in productivity and that produce differentiated goods. Unlike the standard macroeconomic models with heterogeneous plants, plants in this model generate emissions which can be reduced at a cost. Since emissions cause disutility to consumers but not to the plants that generate them, the plants will internalize the externality caused by their production only if they are forced to do so. This externality thus creates a role for government policy.

The emission-reduction policies most often discussed include an emission standard, an emission tax, an emission quota and a tradable emission permit. An emission standard requires that the plant control its emission-intensity, usually measured by the emission-output ratio. This is a command and control instrument. Historically, this has been the instrument of choice for policy makers because it is direct and it encourages plants to invest in cleaner production and abatement technologies. Economists, however, prefer a price instrument such as an emission tax. An emission tax is thought to be efficient since it equalizes the marginal costs of reducing emissions across plants.

An emission quota restricts the amount of emissions that a single plant (or a group of plants) can generate. This policy instrument is effective if the goal is to reach an emission target. However, it may cause inefficiency if a plant that has a higher marginal cost of reducing emissions is assigned a lower quota, or vice versa. A solution to this problem is to allow the quota to be tradable. A tradable emission permit allows a plant with a marginal cost of reducing emissions that is higher than the price of a permit to buy the permit, instead of going through the costly process of reducing its emissions. So, eventually, this market-oriented instrument equalizes the marginal costs of reducing emissions across plants, just like an emission tax does. In practice, the United States has used a tradable emission permit to reduce SO_2 emissions, while some European countries have used an emission tax.

The traditional assessment on the efficiency of environmental policies has not been based on a model that places heterogeneous plants in an economic system where plants' decisions on emissions reduction can affect not only other plants but also consumers. It is recognized in the literature that when the government imposes an emission-reduction policy, heterogeneous plants react to it differently. But, what are the aggregate effects of an emission-reduction policy when plants react differently? Will the price instruments always outperform a command and control instrument?

The answer to the latter question is negative when there is a wedge between how much consumers value the goods and the marginal cost of producing them, i.e., when plants make profits. When an emission tax is imposed, the marginal costs of producing the goods (which now includes the cost of paying for generating the emissions) increase. In response, plants increase the prices of their goods in order to transfer part of those costs to consumers. The wedge between the price and the marginal cost, i.e. the profit margin, increases at the expense of consumers. This is probably one reason why policy makers are often reluctant to use an emission tax. On the other hand, if a command and control instrument is used, a plant's main concern is which abatement technology to adopt in order to satisfy the standard, rather than changing their marketing behavior. Equalizing across plants the marginal costs of reducing emissions is not enough to guarantee efficiency. Since the heterogeneous plants have different ability to transfer their emission taxes and emission-reduction costs to consumers and to compete with other plants for market shares, the social marginal costs are not equalized even if the marginal costs across plants are equalized.

The answer to the question about the aggregate effects of emission-reduction policies with heterogeneous plants is non-trivial and it is quantitatively analyzed in this paper. If the plants change the prices of their goods, they change their market shares and the proportions of the goods that consumers consume. Depending on the heterogeneity, this may reduce the welfare of consumers when there is a wedge between the price and the marginal cost of the goods. Since the plants differ in their productivity, the aggregate productivity is also affected by changes in the allocation of resources across plants. More directly and probably more importantly, plants with different levels of productivity may have to choose different abatement technologies and, hence, incur different abatement costs. This dispersion of costs may also increase the aggregage abatement costs. These heterogeneity-associated channels through which an emission-reduction policy affects the aggregate economy are missing in the literature; accountting for them could be important for the evaluation of emission-reduction policies.

In the model, monopolistic competitive plants produce differentiated goods and generate emissions in the so called "dirty sector" in contrast with a "clean sector" in which production generates no emissions. The dirty sector represents the proportion of the economy that is directly affected by environmental policies. The dirty plants differ in productivity and their emissions are proportional to their input. The higher the productivity, the smaller the amount of resources required to produce one unit of goods. Since those resources, including energy, are the source of the emissions, the emission intensity, as measured by emission-output ratio, is lower in higher productivity plants.

Driven by the enforcement of emission-reduction policies, plants may actively invest to reduce their emissions. Plants can modify their production processes to improve production cleanliness and/or improve their abatement processes to reduce emissions. To incorporate the plants' active emission-reduction behavior, the model assumes that a plant can engage in lowering the emission intensity from production by incurring a fixed investment. For expositional simplicity, this type of behavior is called investment in an advanced abatement technology. The model predicts that the lower the productivity, the more likely the plant will use the less efficient abatement technology and will incur a higher average abatement cost. The implied productivity-emission-abatement relationship is consistent with empirical observations.[1]

The model is calibrated to the Canadian data. The main quantitative results in the steady state analysis are two. First, the paper compares two policy instruments used to reduce emissions: an ad valorem emission tax and an emission standard on emission-output ratio. The paper shows that the emission standard could be more efficient than the emission tax, while in the literature an emission tax is considered more efficient. The reason is as follows.

First, the emission tax causes such a severe price distortion that the consumers cannot consume their desired combination of goods. In the presence of monopoly power, there is

[1] The Environmental Accounts and Statistics Division of Statistics Canada (2004, 2005) reported that there exists a large variation in abatement expenditures and choices of abatement technologies across plants, both within an industry and across industries. There are also some empirical studies that find a negative relationship between productivity and emission-output ratio (Shadbegian and Gray 2003) and between productivity and average abatement costs (Gray and Shadbegian 1995).

a wedge between the prices and the marginal costs of producing goods. The emission tax increases the marginal costs of producing goods. In response, plants increase their prices of goods by an even higher proportion. As a result, they transfer part of their costs of reducing emissions to consumers by increasing their profit margin. Moreover, the wedge between the price and the marginal cost is not identical for all the plants, so an emission tax also causes distortions in the allocation of resources to different dirty plants. Second, the average productivity of plants in the dirty sector is higher when the emission standard, as opposed to the emission tax, is imposed. This is because the low productivity plants, which have the highest emission-output ratio, are disproportionately punished when the emission-output ratio is targeted by an emission standard. This leads to a larger proportion of goods being produced by high productivity plants when the emission standard is imposed (as opposed to the emission tax). Third, there will be deadweight losses caused by plant exit (without loss of generality, we assume that there is some irreversible sunk investment). Since the low productivity plants which also have little profits are affected the most by the emission standard, some of them may exit. In general, there is a higher exit rate when the emission standard is imposed. Since the first and the second effects favor the emission standard, but the third effect favors the emission tax, it will be a quantitative question whether the emission tax is more efficient.

The second main result is that accounting for plant heterogeneity is important for the evaluation of environmental policies. This result is obtained by comparing a model with heterogeneous plants with a model with homogeneous plants. The aggregate GDP loss caused by an emission tax that reduces current emissions by 20% in the model with plant heterogeneity is twice as large as that predicted by the model without plant heterogeneity. The higher costs are mainly due to the following three reasons. (a) In the benchmark model with plant heterogeneity, only about 28% of plants adopt the advanced abatement technology, while the other plants use inefficient methods to reduce their emissions. In the economy with no productivity dispersion, all the plants utilize the advanced abatement technology. So the average cost of reducing emissions is higher with productivity dispersion. (b) In the model with plant heterogeneity, the combination of goods changes when the plants have different ability to transfer their costs to consumers. This distortion reduces welfare. (c) A large proportion of the cost of an emission tax comes from plant exit. Eliminating plant exit, the cost of reducing current emissions by 20% declines by 40%, from 1.9% to 1.1% of GDP.

This paper also provides a tractable framework for studying the transition of the economy from one steady state to another after an emission tax is imposed. Imposing the steady state welfare-maximizing tax rate, it takes over 80 years to reach half of the optimal steady state level. Imposing this emission tax reduces the present value of the expected GDP by 2.58% in the model where the exit is eliminated, much higher than the loss of 1.92% of GDP from the steady state analysis. After imposing the emission tax, the emission levels and the dirty goods output drop immediately below the levels in the long run steady state. This overshooting of dirty goods and emissions in the early stage makes the dynamic cost of the emission policy higher than in the long run. The cost is even higher if the plants can exit the industry. Taking into account productivity dispersion and exit, the cost of reducing emissions is much higher than the estimate in the Stern Review (2007), which is 2% of GDP.

This is not the first paper that emphasizes how productivity dispersion across plants influences the effects of government policies. Hopenhayn and Rogerson (1993) find that government policies that make it costly for firms to adjust their employment levels have sizable impacts on employment and aggregate productivity in a model with productivity dispersion among firms. Melitz (2003) and Bernard, Eaton and Kortum (2003) show that productivity dispersion is important in explaining plant export activities. Restuccia and Rogerson (2008) show that policy distortions can cause misallocation of resources across heterogeneous plants, leading to cross-country inequality. From the technical aspect of constructing the model with productivity dispersion, the paper by Ghironi and Melitz (2005) is most closely related to the current paper.

This paper is also related to the large literature on the computable general equilibrium (CGE) models that have been developed over the past few decades. In the quantitative assessments of the impact of climate policies, for example, Nordhaus (1993) has integrated a macroeconomic model of the global economy with a climate system to address the science of climate change, in what is known as the Dynamical Integrated Models of Climate and the Economy (DICE). The Global Trade Analysis Project (GTAP) at Purdue University has developed a multiregional, multisectoral computable general equilibrium model to study the impact of climate change agreements on international trade. Contributing to this literature, the current paper emphasizes the importance of plant heterogeneity.

The rest of this paper proceeds as follows: Section I describes the economic environment; Section II characterizes the optimal choices; Section III provides aggregation and the equilibrium, and analyzes the effects of environmental policies; Section IV parameterizes the model; Section V conducts numerical experiments; Section VI concludes. Finally, appendices provide data description and lengthy algebra.

1 The Economic Environment

1.1 The Households

The economy is populated by a unit measure of identical households. The representative household is infinitely lived and has preferences over streams of consumption goods and pollutant stocks (pollution) at each date. The expected discounted life time utility is

$$U_0 = \max_{\{m_t, q_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\alpha) g\left(D_t\right) m_t^{\rho} + \alpha {q_t}^{\rho}\right]^{1/\rho}. \tag{1}$$

Here, m_t is the consumption of clean goods, q_t is the consumption of an aggregate of dirty goods, and D_t is the level of the pollutant stock. The subjective discount factor is $\beta \in (0,1)$. Restrict $-1 < \rho < 0$, so that the clean goods and the dirty aggregate are poor substitutes. The elasticity of substitution between m_t and q_t is $\frac{1}{1-\rho}$, and $0 < \alpha < 1$ parameterizes the relative importance of q_t. Let $\bar{D}$ be a threshold level of the pollutant stock, above which the pollution causes disutility. Let $g\left(D_t\right) = 1$ for all $D_t \leq \bar{D}$, and $g\left(D_t\right) = \left(\frac{D_t}{\bar{D}}\right)^{\Psi}$, where $\Psi > 0$, for all $D_t > \bar{D}$. This specification implies that pollution generates externalities, that is $\frac{\partial U_0}{\partial D_t} < 0$, which will be referred to later. This specification also implies that the marginal utility of clean goods relative to dirty goods increases as pollution increases.

Every household is endowed with l units of resource per period. This resource is the only input required for production. The empirical counterpart of this resource is a Cobb-Douglas combination of capital, labor and energy. The household supplies l inelastically. The production of dirty goods generates emissions. The total amount of emissions generated in period t is denoted as E_t. Emissions accumulate according to

$$D_t = (1-\delta_1)D_{t-1} + E_t, \tag{2}$$

where $\delta_1 \in (0,1)$ is a natural decay factor for the pollutant stock. Let D_{t-1} be the pollutant stock at the beginning of the period. Different dirty goods appear in the utility function through the following aggregator:

$$q_t = \left[\int_{i\in\Omega_t} q_{i,t}^{\frac{(\sigma-1)}{\sigma}} di\right]^{\frac{\sigma}{(\sigma-1)}}. \tag{3}$$

Here, $\sigma > 1$ is the elasticity of substitution across dirty goods, and Ω_t is the set of dirty goods available at period t. Let $\Omega = \cup_t \Omega_t$ be the entire set of dirty goods available over time, where Ω is assumed to be a continuum.

1.2 The Producers

Potential plants can choose to enter either the clean sector (with no entry cost) or the dirty sector (with a sunk entry cost $f_{e,t}$ in units of resource). The plants enter the dirty sector if the present value of the expected profit stream can cover this entry cost. If a plant chooses to enter the dirty sector at time t, the plant can start producing only at time $t+1$. This setup introduces a one-period time-to-build lag in the model. After entry, the plant draws a productivity level. As shown later, the plant's profit is an increasing function of the level of productivity. Because every plant needs to pay a positive fixed cost in order to produce, some low productivity plants do not produce and exit immediately after entry. The producing plants keep their productivity levels until they are hit by an exogenous exit-inducing shock with probability μ. This exit-inducing shock is independent of the plants' productivity levels, so $G(x)$, truncated at a threshold level, $x_{e,t}$, above which plants produce, also represents the productivity distribution of all producing plants.[2]

[2] In order to simplify the analysis, we will only look at the equilibria in which the tax rate or the standard stays the same in every period. A change in pollution regulation can be seen as a permanent shock which can move one stationary equilibrium to another.

1.2.1 The Clean Sector

The clean goods M_t are produced by a linear production technology: $M_t = XL_{m,t}$, where X represents the level of productivity and $L_{m,t}$ is the quantity of the resources used to produce goods M_t in period t. The competitive feature of this market ensures that the factor price equals the level of productivity, $w_t = X$.

1.2.2 The Dirty Sector

Potential plants are identical before they enter the dirty sector. Upon entry, each plant draws a productivity level x from a common distribution $G(x)$ with support on $[x_{\min}, \infty)$. This productivity level remains constant for the plant thereafter. Thus, a plant with productivity x is referred to as plant x. Plants are monopolistically competitive, with each producing a variety of goods. So the goods produced by plant x can also be indexed by x. Under the specification of the aggregator in (3), the elasticity of the demand for each good is σ. Hence, the optimal pricing strategy is to set the price as a constant markup, $\sigma/(\sigma - 1)$, over the marginal cost.

In order to produce, a plant needs to pay a fixed cost $f > 0$ in units of resource. For simplicity, assume that this fixed cost is the same for all the plants in all the periods. Hence, some plants never produce if they draw a low level of productivity after entry. Producing the dirty goods generates air emissions. The production technology of a plant that draws productivity x and its emission generation function are

$$q_t^s(x) = xL_{g,t}(x), \text{ and } e_t(x) = b_I L_{g,t}(x), \tag{4}$$

where $L_{g,t}(x)$ is the variable input required for producing goods x. The parameter $b_{II} > 0$ captures the intensity of emissions generated from production. b_{II} is usually refered to as an emission factor.

In practice, a plant can become cleaner by modifying its production or/and abatement process, which usually requires an investment, or simply by adding on some pollution treatement equipment to reduce its emissions. Accordingly, in the model, we allow for two ways of reducing emissions. One is to reduce b_{II} to b_I by investing in a fixed amount. We use I

to refer to type I plants that invest in new abatement technology, and II to refer to type II plants that do not invest. The other way to reduce emissions is to use some variable abatement input $L_{a,t}(x)$. For tractability, and without loss of generality, we normalize $L_{a,t}(x)$ by the amount of resource used in production, $L_{g,t}(x)$, and define a cleanliness index as $z_{j,t}(x) = 1+\frac{L_{a,t}(x)}{L_{g,t}(x)}$, for $j = I, II$. An increase in $L_{a,t}(x)$ will reduce emissions according to the new emission generation function

$$e_t(x) = z_{j,t}(x)^{1-h} b_j L_{g,t}(x), \tag{5}$$

where $h \geq 1$ is a constant.

1.2.3 The Government's Policies

As clear from (1), pollution is an externality that is not internalized in a laissez-faire economy. This creates role for government policy. The government monitors and regulates emissions on behalf of consumers. Two instruments are available for that: one is an ad valorem tax on emissions, the other is a uniform standard on the emission-output ratio. When the tax instrument is applied, the tax revenue is returned to consumers as a lump sum transfer. The government's budget constraint is

$$T_t = \tau_t E_t, \tag{6}$$

where T_t is a lump sum transfer to consumers, and τ_t is the tax rate. When a standard s_t on the emission-output ratio is set, all the plants are required to satisfy $\frac{e_t(x)}{q_t^s(x)} \leq s_t$.

2 The Optimal Choices

2.1 The Households' Problem

Let $p_{i,t}$ denote the price of dirty good $i \in \Omega_t$, and P_t denote the price of the aggregate of the dirty goods at t. The demand for good i is

$$q_{i,t} = q_t \left(\frac{P_t}{p_{i,t}} \right)^{-\sigma}, \tag{7}$$

which is solved from the following problem:

$$\max_{q_{i,t}} q_t \qquad\qquad s.t. \int_{i\in\Omega_t} p_{i,t} q_{t,t} di = P_t q_t$$

given P_t and q_t. The relative price is $P_t = \left[\int_{i\in\Omega_t} p_{i,t}{}^{(1-\sigma)} di\right]^{\frac{1}{(1-\sigma)}}$, derived from (7).

The representative household enters period t with an endowment of the resource l and mutual fund share holdings A_t, which finance the continuing operation of all pre-existing dirty plants and all new entrants in the dirty sector during period t. During period t, a mass N_t of dirty plants is in operation and pays dividend, and a mass $N_{e,t}$ of new dirty plants enters. The average value of the operating dirty plants is denoted as $\tilde{v}_t$ and the average value of the new dirty plants is denoted as $\tilde{v}_{e,t}$. $\tilde{v}_t$ and $\tilde{v}_{e,t}$ will be defined later. In each period the mutual fund pays a total profit that is equal to the total profit of all the dirty plants producing in that period. The average profit of a share is denoted as $\tilde{\pi}_t$. Let T_t be the lump sum transfer from the government. The period budget constraint of the representative household (in units of the clean goods) is

$$P_t q_t + m_t + (\tilde{v}_t N_t + \tilde{v}_{e,t} N_{e,t}) A_{t+1} = w_t l + (\tilde{v}_t + \tilde{\pi}_t) N_t A_t + T_t, \tag{8}$$

where w_t is the resource price denominated in clean goods.

The mass of dirty plants evolves according to

$$N_{t+1} = (1-\mu) N_t + (1-\mu)(1-G(x_{e,t})) N_{e,t}. \tag{9}$$

The plants with productivity levels lower than $x_{e,t}$ exit immediately after entry, so only a proportion $1-G(x_{e,t})$ of new entrants will stay in operation. A proportion μ of the remaining plants will be hit by the exogenous exit shock at the end of period t, and so only a proportion $1-\mu$ of them continues to the next period.

Given the budget constraint (8), the household chooses m_t and q_t to maximize expected intertemporal utility (1). The first order conditions imply that the marginal utility of the clean goods relative to the dirty goods increases in the aggregate level of pollution, that is

$$\frac{U_m}{U_Q} = \frac{1-\alpha}{\alpha} \left(\frac{m_t}{q_t} \right)^{\rho-1} \left(\frac{D_t}{\bar{D}} \right)^{\Psi} = \frac{1}{P_t}. \tag{10}$$

As a result, as pollution stock increases, consumers will continously reduce their consumption of dirty goods. The Euler equation for the share is

$$\tilde{v}_t = E_t \left[R_{t+1} (\tilde{v}_{t+1} + \tilde{\pi}_{t+1}) \right], \tag{11}$$

where $R_s = [\beta(1-\mu)]^{s-t} \left(\frac{\eta_s}{\eta_t} \right)^{\frac{1}{\rho}-1} \frac{g(D_s)}{g(D_t)} (\frac{m_s}{m_t})^{\rho-1} \frac{N_{t+1}}{N_t}$ is the stochastic subjective discount factor, for $s > t$, and $\eta_t = (1-\alpha) g(D_t) m_t^{\rho} + \alpha {q_t}^{\rho}$.

2.2 The Producers' Problem

2.2.1 Plants' Optimal Decisions under an ad valorem Emission Tax

Given the resource price w_t and the tax rate τ_t, the plant x chooses abatement technology and variable abatement input, and sets price according to a constant markup over variable cost. It is easy to show that this pricing strategy is optimal for the plant. The producers do not choose the quantity of output, which is determined by equilibrium demand given the prices.

If an emission tax is imposed, a plant can choose to put some resouces to reduce emissions in order to save some tax payment. The objective of the plant is to minimize its total cost, consisting of $w_t(L_{a,t}(x) + L_{g,t}(x))$ and $\tau_t e_t(x)$. The problem is equivalent to choosing the optimal cleanliness index $z_{j,t}(x)$ to minimize the total cost given the production input $L_{g,t}$:

$$\min_{z_{j,t}(x)} \left[w_t z_{j,t}(x) + \tau_t b_j z_{j,t}(x)^{1-h} \right] L_{g,t}(x), \text{ for } j = I, II.$$

The cleanliness index is

$$z_{j,t} = \begin{cases} [\tau_t b_j (h-1)/w_t]^{(1/h)} & \text{if } \tau_t b_j (h-1)/w_t > 1 \\ 1 & \text{otherwise} \end{cases}$$

Note that this amount is identical for all type j plants. Note that, in both cases, if the tax rate is so low that $\frac{\tau_t}{w_t}(h-1) \leq \frac{1}{b_j}$, the plants find that incurring variable abatement costs to reduce emissions is not worthwhile.

The profit maximizing plants will set the prices according to a fixed markup over variable costs, including the cost of producing goods, the cost of reducing emissions and the tax payment. That is

$$p_{j,t}(x) = \phi_j x^{-1} ,$$

for $j = I, II$. ϕ_j is defined as follows:

$$\phi_j = \begin{cases} \sigma/(\sigma-1)w_t h/(h-1)z_{j,t} & \text{if } z_{j,t} > 1 \\ \sigma/(\sigma-1)(w_t + \tau_t b_j) & \text{otherwise} \end{cases}$$

A plant can invest in a new abatement technology by a fixed amount $w_t f_{a,t}$ in every period, reducing b_{II} to b_I. By incurring this fixed investment, the plants can save some variable abatement input or/and emission tax. To make this decision, the plant compares the profit stream in each case. The profits are

$$\begin{cases} \pi_{I,t}(x) = \frac{P_t^\sigma Q_t}{\sigma}\left[\frac{\phi_I}{x}\right]^{1-\sigma} - w_t f_{a,t} - w_t f \\ \pi_{II,t}(x) = \frac{P_t^\sigma Q_t}{\sigma}\left[\frac{\phi_{II}}{x}\right]^{1-\sigma} - w_t f \end{cases}$$

The comparison gives that a plant will adopt the new abatement technology if and only if its productivity is above a threshold level. This threshold is given by

$$x_{a,t} = \left\{ \frac{w_t f_{a,t}\sigma}{P_t^\sigma Q_t \left[\phi_I^{(1-\sigma)} - \phi_{II}^{(1-\sigma)}\right]} \right\}^{\frac{1}{\sigma-1}} ,$$

where $Q_t = \left\{\int [q_t^s(x)]^{\frac{(\sigma-1)}{\sigma}} G(dx)\right\}^{\frac{\sigma}{(\sigma-1)}}$, and the dirty aggregate supplied, Y_Q, equals $N_t Q_t$. If $x \geq x_{a,t}$, the plant adopts the new abatement technology. Such plants are type I plants. The remaining plants are type II plants.

The emission level is

$$e_{j,t}(x) = z_{j,t}^{1-h} b_j \phi_j^{-\sigma} x^{\sigma-1} P_t^\sigma Q_t, \text{ for } j = I, II.$$

A notable feature is that the elasticity of substitution among dirty goods, σ, influences the dispersion of emissions across plants. The higher σ is, the easier the goods can be substituted by others, and the larger the dispersion of emissions is.

2.2.2 Plants' Optimal Decisions under an Emission Standard

Given the resource price w_t and the standard s_t, the plants choose whether to adopt the new abatement technology, make variable abatement choice, z_t, and set prices according to the constant markup over variable costs. Only the case $s_t < b_{II}$ is considered here, because the other case $s_t \geq b_{II}$ is trivial.

A plant's choices on production and abatement depend on the plant's productivity. Recall that $e_{j,t}(x) = z_{j,t}(x)^{1-h} b_j \frac{q_t^s(x)}{x}$, so the standard requires that $\frac{z_{j,t}(x)^{1-h} b_j}{x} \leq s_t$. Note that if $z_{j,t}(x) = 1$ (i.e., if there are no variable abatement costs) the emission-output ratio is negatively related to the productivity level. Thus, the plants can be classified into five groups according to their choices of abatement methods, with the nice feature that they are sorted by their productivity levels.[3]

(1) Type 1 plants, $x \geq x_{1,t}$. These plants do not abate emissions. They have high productivity and low emission-output ratio. They satisfy the standard without any abatement given $x_{1,t} = \frac{b_{II}}{s_t}$.

(2) Type 2 plants, $x_{1,t} > x \geq x_{2,t}$. These plants incur only variable abatement costs. They have slightly higher emission-output ratio than the type 1 plants. Since they need to reduce only a minor amount of emissions to satisfy the standard, they do not invest in the abatement technology, but simply use some variable input to reduce emissions. The threshold value of type 2 plants, $x_{2,t}$, is obtained when

$$\frac{P_t^\sigma Q_t}{\sigma} \left[\frac{\sigma}{\sigma - 1} z_{II,t}(x) \frac{w_t}{x} \right]^{1-\sigma} - \frac{P_t^\sigma Q_t}{\sigma} \left(\frac{\sigma}{\sigma - 1} \frac{w_t}{x} \right)^{1-\sigma} - w_t f_{a,t} = 0, \quad (12)$$

where $z_{II,t}(x) = \left(\frac{b_{II}}{s_t x} \right)^{\frac{1}{h-1}}$. The left hand side of equation (12) is the difference between the profit if the plant uses only variable abatement input and the profit if the plant invests in

[3] For illustrative purposes, we assume the proportion of exiting plants to be very small here, such that only a few of the type 5 plants exit.

the new technology but does not use variable inputs.

(3) Type 3 plants, $x_{2,t} > x \geq x_{3,t}$. These plants invest in the new abatement technology only. These plants have even higher emission-output ratio, so they generate emissions on a large scale. Therefore, they find it cheaper to reduce emissions by investing in the new abatement technology, which brings the emission-output ratio below the standard. Thus, no variable abatement costs are incurred. The threshold value $x_{3,t}$ is obtained when $x_{3,t} = \frac{b_I}{s_t}$.

(4) Type 4 plants, $x_{3,t} > x \geq x_{4,t}$. These plants adopt the new technology and use variable input to reduce emissions. The threshold value $x_{4,t}$ is obtained when

$$\frac{P_t^\sigma Q_t}{\sigma}\left[\frac{\sigma}{\sigma-1}z_{I,t}(x)\frac{w_t}{x}\right]^{1-\sigma} - w_t f_{a,t} - \frac{P_t^\sigma Q_t}{\sigma}\left[\frac{\sigma}{\sigma-1}z_{II,t}(x)\frac{w_t}{x}\right]^{1-\sigma} = 0, \tag{13}$$

where $z_{I,t}(x) = \left(\frac{b_I}{s_t x}\right)^{\frac{1}{h-1}}$. The left hand side of (13) is the difference between the profit when the plant invests in the new technology and also uses some variable input and the profit when the plant only uses variable inputs to reduce emissions. The threshold value is

$$x_{a,t} = x_{4,t} = \left\{\frac{w_t f_{a,t}\sigma}{P_t^\sigma Q_t\left[\chi_I^{(1-\sigma)} - \chi_{II}^{(1-\sigma)}\right]}\right\}^{\frac{h-1}{h(\sigma-1)}},$$

where $\chi_j = \frac{\sigma}{\sigma-1}\left(\frac{b_j}{s_t}\right)^{\frac{1}{h-1}} w_t$.

(5) Type 5 plants, $x_{a,t} > x \geq x_{e,t}$. These plants incur only the variable abatement costs. They have the lowest productivity, so both the production and the emission-generation scales are so small that investing in a new abatement technology becomes too expensive.

For reasonable parameters all these 5 types of producers exist. For some extreme values of policy and technology parameters, some types may not exist. For example, for a stringent pollution policy it is possible that all the type 5 plants go out of business and even some type 4 or type 3 plants exit.

The profits of plants using different technologies are depicted in figure 1. Line 1 depicts the profit of plants if they do not reduce emissions. Line 2 is the profit of plants with only variable abatement costs. Line 3 is the profit of plants with only fixed investment in abatement technology. Line 4 is the profit of plants with both fixed and variable abatement costs. Given a level of productivity, a plant will choose the method of abatement that gives the highest profit.

3 Aggregation and Equilibrium

3.1 Plant Average and Aggregation

Following Melitz (2003) and some empirical evidence this paper assumes that productivity x obeys Pareto distribution with a lower bound $x_{\min}$ and shape parameter $k > \sigma - 1$.[4] That is $G(x) = 1 - (x_{\min}/x)^k$, where k governs the dispersion of productivity. As k increases, productivity dispersion decreases, and the levels of the plants' productivity become increasingly concentrated toward their lower bound $x_{\min}$.

3.1.1 Under the Emission Tax

According to the above analysis, the proportion of producing plants that use the new abatement technology is $n_{I,t} = \frac{1-G(x_{a,t})}{1-G(x_{e,t})}$, and the proportion of producing plants that do not adopt the new abatement technology is $n_{II,t} = \frac{G(x_{a,t})-G(x_{e,t})}{1-G(x_{e,t})}$.

Let us define two special "average" productivity levels, an average $\tilde{x}_{I,t}$ for all type I producing plants, and an average $\tilde{x}_{II,t}$ for all type II producing plants:

$$\tilde{x}_{I,t} = \left[\frac{1}{1-G(x_{a,t})}\int_{x_{a,t}}^{\infty} x^{\sigma-1}dG(x)\right]^{\frac{1}{\sigma-1}} = \upsilon x_{a,t},$$

and

$$\tilde{x}_{II,t} = \left[\frac{1}{G(x_{a,t})-G(x_{e,t})}\int_{x_{e,t}}^{x_{a,t}} x^{\sigma-1}dG(x)\right]^{\frac{1}{\sigma-1}} = \upsilon x_{e,t}\left[\frac{1-\vartheta^{k+1-\sigma}}{1-\vartheta^{k}}\right]^{\frac{1}{\sigma-1}},$$

where $\upsilon = \left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}$ and $\vartheta = \frac{x_{e,t}}{x_{a,t}}$. Note that the integration requires $k+1-\sigma > 0$ for $\sigma > 1$.

[4] See Axtell (2001) for empirical evidence that the Pareto distribution approximates the observed distribution of firm sizes.

It is easy to show that $\tilde{x}_{I,t}$ and $\tilde{x}_{II,t}$ completely summarize the information in the distribution of productivity levels $G(x)$ relevant to all aggregate variables. Thus, this economy is isomorphic, in terms of all aggregate outcomes, to one where $N_{j,t}$ plants with productivity $\tilde{x}_{j,t}$ are type j. Accordingly, $\tilde{p}_{j,t} \equiv p\ (\tilde{x}_{j,t})$ represents the average price of type j plants. The price of the dirty aggregate is written as $P_t = \left[n_{I,t}(\tilde{p}_{I,t})^{1-\sigma} + n_{II,t}(\tilde{p}_{II,t})^{1-\sigma}\right]^{1/(1-\sigma)}$. Similarly, denote $\tilde{\pi}_{j,t} \equiv \pi\ (\tilde{x}_{j,t})$ as the average profit of type j plants. The average profit of all dirty plants is $\tilde{\pi}_t = n_{I,t}\tilde{\pi}_{I,t} + n_{II,t}\tilde{\pi}_{II,t}$. It is easy to show that

$$\begin{cases} \tilde{\pi}_{I,t} = \frac{P_t^{\sigma} Q_t}{\sigma} \left[\frac{\phi_I}{\tilde{x}_{I,t}}\right]^{1-\sigma} - w_t f_{a,t} - w_t f \\ \tilde{\pi}_{II,t} = \frac{P_t^{\sigma} Q_t}{\sigma} \left[\frac{\phi}{\tilde{x}_{II,t}}\right]^{1-\sigma} - w_t f \end{cases}$$

The aggregate emission level is $E_t = N_{I,t} e\ (\tilde{x}_{I,t}) + N_{II,t} e\ (\tilde{x}_{II,t})$.

3.1.2 Under the Emission Standard

Under the emission standard the aggregation of variables is similar to that under the emission tax. The algebra is provided in Appendix II.

3.2 The Value of Plants

The prospective entrants are forward looking and correctly anticipate their future average profits $\tilde{\pi}_t$ in every period. The discounted present value of an entrant is given by $\tilde{v}_{e,t} = [1 - G(x_{e,t})] E_t \left(\sum_{s=t+1}^{\infty} R_s \tilde{\pi}_s\right)$. Plants discount future profits using the household's subjective discount factor, R_s (to be defined in the next subsection). Entry occurs until the average plant value is equalized with the entry cost, leading to the free entry condition $\tilde{v}_{e,t} = w_t f_{e,t}$. This condition holds so long as the mass $N_{e,t}$ of entrants is positive.

After drawing a productivity level x, a plant exits if its present value of profit stream $v_{x,t}$ is negative. Plant x has the value $v_{x,t} = E_t \left(\sum_{s=t+1}^{\infty} R_s \pi_{x,s}\right)$, given its anticipated profit $\pi_{x,s}$ in period s, for all s. Let x_e be the productivity level such that $v_{x_e,t} = 0$. Hence, plants exit after entry if $x < x_e$. The average value of the incumbent plants is $\tilde{v}_t = E_t \left(\sum_{s=t+1}^{\infty} R_s \tilde{\pi}_s\right)$.

3.3 A Steady State Equilibrium

In equilibrium, all the goods markets, resource markets and share markets clear. That is

$$
\begin{aligned}
M_t &= m_t, &(14)\\
NQ_t &= q_t, &(15)\\
L_{m,t} + N_{n,t} f_{e,t} + N_t f + N_{a,t} f_{a,t} + N \int L_t(x) dG(x) &= l, &(16)\\
\text{and } A_t &= 1. &(17)
\end{aligned}
$$

The steady state equilibrium is defined as follows.[5] For all t, and for $x \in [x_{\min}, \infty)$,

Definition 1 *An allocation is comprised of quantities of* $(m_t,\ q_{i,t},\ A_t)$ *for consumers,* $(L_{m,t},$ $M_t)$ *for producers in the clean sector, and* $(L_{g,t}(x),\ L_{a,t}(x),\ q_t^s(x),\ e_t(x),\ F_{a,t})$ *for producers in the dirty sector, where* $F_{a,t} = f_{a,t}$ *if the plants invest in the new abatement technology, and* $F_{a,t} = 0$ *otherwise;*

Definition 2 *A price system is comprised of* $(w_t,\ P_t,\ p_t(x))$*;*

Definition 3 *A government policy is comprised of* s_t *for the standard or* $(\tau_t,\ T_t)$ *for the tax;*

Definition 4 *A steady state equilibrium is a time-invariant allocation, a time-invariant price system, a law of motion of the aggregate level of pollution with pollution level constant over time, i.e.* $\delta_1 D_{t-1} = E_t$*, and a time-invariant government policy such that (a) given the government policy, the law of motion of the aggregate level of pollution, and the price of resource* w_t *and the relative price* P_t*, the prices* $p_t(x)$ *and the quantities* $(L_{g,t}(x),\ L_{a,t}(x),$ $q_t^s(x),\ e_t(x),\ F_{a,t})$ *solve the plant's problem in the dirty sector; (b) given the price system, the government policy, and the law of motion of the aggregate level of pollution, the allocation solves both the consumer's problem and the plant's problem in the clean sector; (c) given the allocation, the price system, and the law of motion of the aggregate level of pollution, the government policy satisfies the budget constraint (6); (d) market clearing conditions from (14) to (17) are satisfied; (e) the free entry condition holds; (f) the distributions for plants'*

[5] It is not difficult to show that there is a unique steady state equilibrium given a pollution policy.

size, emissions, profit, and value are stationary; and (g) there is consistency between the individual plants' behavior and aggregate variables.

4 Calibration

We calibrate the model with commonly used empirical evidence found in the literature whenever it is possible. The parameters specific to the paper, mostly related to emissions, are calibrated to Canadian output, emissions, and abatement expenditure data between 1990 and 2006. During the period 2000-2006, some regulations on Greenhouse Gas (GHG) emissions in the near future were anticipated after Canada signed the Kyoto Protocol. Some agreements on reducing GHG emissions between the government and some industries and polluting plants were signed.[6] As a result, some plants adopted new systems or equipment to reduce GHG emissions, although no explicit regulation on reducing GHG was announced.

Table 1 lists the values of parameters that are taken to fit the empirical evidence commonly used in the literature. According to Dunne et. al. (1989), the average failure rate of plants in U.S. manufacturing during any five years is 0.391. Hence, the annual failure rate implied by their study is 0.08. This value is used as the exogenous exit rate μ. Again from Dunne et. al. (1989), the annual new entrants rate $\frac{N_n}{N}$ is approximately 0.095. A stationary distribution of plants requires that

$$\frac{N_n}{N} = \frac{\mu}{(1-\mu)(1-G(x_e))}. \tag{18}$$

Solving equation (18)) gives that $G(x_e)$ equals 0.08, where x_e is the threshold value of productivity above which plants produce. The value of $G(x_e)$ will be used later to identify the fixed cost of production.

[6] Some industries and provinces have signed agreements with the government. For example, by April 2005 all major companies of Canada's automobile industry had signed an agreement with the government to voluntarily reduce their greenhouse gas emissions and thus help Canada meet its commitments under the Kyoto climate protocol. The pact focuses on immediate action to achieve reductions in greenhouse gas emissions. In June 2005, the Government of Canada and the Air Transport Association of Canada signed an agreement to reduce the growth of greenhouse gas emissions in Canada's aviation sector. In December 2006, the government of Ontario announced Bill 179, an act for the reduction of greenhouse gas emissions in Ontario.

Table 1 Parameters identified to conventional targets

	Parameter	Value	Comments
Time preference	β	0.96	Real interest rate 4% per year
Exit shock	μ	0.08	Dunne et. al. (1989)
Emission decay rate	δ_1	0.008	Kolstad (1996)
Threshold of emission stock	$\bar{\text{D}}$	32.0	1965 pollution stock
Initial level of emission stock	D_{-1}	32.1	1990 pollution stock

The annual decay rate of GHG emissions is 0.008 per decade found in the literature (Kolstad 1996). The threshold value $\bar{D}$ is taken as the 1965's stock level of GHG (1965 is usually taken as a reference point; see Nordhaus 1993 and Kolstad 1996). $\bar{D}$ is set to equal 32 gigaton CO_2 equivalent.[7] Given $\bar{D}$, D_{-1} is calculated as 32.1 gigaton at the beginning of 1990.

The preference parameters depend on the definition of the dirty sector and the clean sector. The definition of the dirty sector is provided in appendix I. The clean to dirty goods sales ratio $\frac{Y_m}{Y_Q}$, the pollutant stock D_t, and the relative price P_t during 1990 and 2006 are used to calibrate α, ρ, and Ψ. The relative price of dirty goods to clean goods is constructed (see appendix I for detail). In order to identify α, ρ, and Ψ, rewrite equation (10) from the households' problem as

$$\ln \frac{Y_m}{Y_Q} = \frac{1}{1-\rho} \ln \left(\frac{1-\alpha}{\alpha} \right) + \frac{\Psi}{1-\rho} \ln \left(\frac{D_t}{\bar{D}} \right) + \frac{\rho}{1-\rho} \ln P_t. \tag{19}$$

Given the data on P_t, D_t, and $Y_{m,t}/Y_{Q,t}$, the preference parameters can be estimated by using equation (19).

Equation (19) predicts that $Y_{m,t}/Y_{Q,t}$ decreases in the relative price P_t since the dirty goods and the clean goods are complements, $-1 < \rho < 0$. The coefficient ρ determines the magnitude of this effect. The higher the absolute value of ρ, the lower the substitutability, and the larger the effect of the price change on $Y_{m,t}/Y_{Q,t}$. Equation (19) also predicts that an increasing pollution level leads to consumers demanding more clean goods, which leads

[7] The literature (e.g. Nordhaus 1993 and Kolstad 1996) uses 667 gigaton as the stock of GHG for U.S. in 1965. Since Canada emits roughly 10 percent of what US emits and this paper cuts off the emissions other than industrial emissions (about 52% percent of total GHG), 4.8% of 667 is used as $\vec{D}$.

to a higher ratio $Y_{m,t}/Y_{Q,t}$. Ψ influences the impact of D_t on $Y_{m,t}/Y_{Q,t}$. A higher Ψ implies a higher disutility from pollution for consumers. Ψ should be higher than $|\rho|$. If not, D_t would have a positive effect on utility. Also, assume Ψ to be less than $|\rho| * 1.5$ to avoid unreasonably large disutility from pollution. The value of the exogenous share of dirty goods α in the model should not be very different from the dirty goods share $Y_{Q,t}/Y$ in the data, 0.38. Given these restrictions, parameters α, ρ and Ψ are estimated by searching the estimates that minimize the divergence between the model and the data. $\alpha = 0.36$, $\rho = -0.4$, and $\Psi = 0.45$ give the best fit. The model simulated $\ln \frac{Y_{m,t}}{Y_{Q,t}}$ and the data are depicted in figure 2.

In order to calibrate the abatement technology and policy parameters, three episodes from 1990 to 2006 are defined. In the first episode (1990-1994) there was no emission-reduction effort reported. During the second episode (1995-1999), the Kyoto Protocol was signed and Canada committed to reduce GHG emissions to a level 6% below the 1990 level between 2008 - 2012. Accordingly, during the third episode (2000-2006) some agreements between industries and government were signed that focus on immediate reduction of emissions. Some plants started using new abatement technologies during the period 2002 and 2006. So the parameters characterizing the basic economy without emission-reduction are calibrated using data in the first period. The abatement technology related parameters are calibrated using the data in the third episode, specifically, the data on the expenditures on investments in abatement technology, the impact of these investments, and the average investment rate. Since in the model plants will adopt new abatement technology only if there is some enforcement to reduce emissions, it is assumed that there is an identical emission standard in the third episode that generates the emission-reduction activities reported by Environment Canada.

(1) The economy in the first episode is taken as a reference economy. This episode is used to identify the parameters that describe the basic economic structure in which no plants use the advanced abatement technology. The calibrated parameter values listed in table 2.

Table 2 Parameters identified in episode without emission-reduction

	Parameter	Value	Targets or constraints (1990-94)
Clean sector productivity	X	1	Normalization
Minimum productivity	x_{min}	0.706	$\int_{x_{min}}^{\infty} x dG(x) = 1$
Emission factor	b_{II}	2.92	Emission sales ratio E/Y_Q, 2.15
Substitution among dirty goods	σ	3.8	$b_{II}=\frac{\sigma}{\sigma-1}\frac{E}{Y_Q}$
Fixed production costs	f	0.0066	Entering rate $\frac{N_n}{N} = 0.095$
Fixed entry costs (Y_Q/N)	f_e	1.5	Free entry condition
Productivity dispersion	k	3.4	Dirty goods sales share, 0.39
Resource	l_0	0.35	Emission level, 0.299 gigaton

The average productivity in the clean sector X and the prior average productivity in the dirty sector are also normalized to 1. Other parameters are identified by simulating the model to match the moments in the first period. The exit and entry rates and equilibrium condition mentioned above implied that $G(x_e) = 0.08$, which in turn implies that $f = 0.0475\frac{Y_Q}{wN}$ (0.0066 in the numerical model), where Y_Q/N is the average revenue of the dirty plants. f_e is calculated from the free entry condition given the average profits of dirty plants $\frac{1}{\sigma}Y_Q/N - wf$. The relationship $b_{II} = \frac{\sigma}{\sigma-1}\frac{E}{Y_Q}$ from integrating emissions across plants and the moment of the emission-sales ratio $\frac{E}{Y_Q} = 2.15$ in the data imply that $\sigma = 3.8$ and $b_{II} = 2.92$ kilo-ton CO_2 emissions equivalent per million dollars. The endowment in this period l_0 is set to be 0.35 trillion dollars in order to equate the level of emissions generated in the model to the average level of emissions in the data.

(2) In the third episode, the emission-sales ratio in the dirty sector declines dramatically, $\frac{E}{Y_Q} = 1.96$. In the dirty sector, 24% of dirty plants reported using new system or equipment to reduce Greenhouse Gas emissions. This is, by assumption, because of enforcing an emission standard s. To achieve the emission-sales ratio in this episode, s has to be $0.74b_{II}$. The plants that adopted new abatement technology also reported the impact of using the new abatement technology. According to the reported impact of the technology in 2002, b_I is set to be $0.8b_{II}$.[8]

[8] Among the plants reporting adoption of a new abatement technology, 13% reported significant reduction of GHG emissions, 44% reported medium reduction, and 44% reported small reduction. I interpret significant effect as 40%, medium effect as 24%, and small effect as 10%. This leads to an effect of 20% on average. See appendix I table A2.

The fixed cost of adopting the new abatement technology, $f_{a,t}$, is approximately 1.3% of the dirty goods sales, $0.013Y_Q/N$, according to the fact that the capital expenditure on abatement was 0.31% of the dirty goods sales. Using the percentage of plants that invested in the new abatement technologies reported during the third episode we find h to be 5.8. This implies that a one percent increase in operating abatement expenditure reduces emissions by 4.8%. For a plant with an average productivity level it would be more efficient to use the new abatement technology than to use the variable inputs to reduce emissions. The same expenditure could reduce emissions by 20% if the plant used the new abatement technology. Finally, the endowment l is set to be 0.492 trillion dollars in order to equate the level of emissions generated in the model to the average level of emissions in the data.

Among all the parameters the productivity dispersion parameter k and the substitution parameter σ are crucial for the distributional effects and aggregate effects of an environmental policy. The quantitative experiment is carried out with different values of k. The robustness tests on σ are provided at the end of the next section.

5 Quantitative Experiment

5.1 Compare Emission Tax and Emission Standard

An emission standard and an emission tax generate different incentives for plants to reduce their emissions. If the emission-output ratio is targeted, the low productivity plants, which have highest emission-output ratio, will reduce more emissions, while the higher productivity plants will produce more goods. The average productivity in the dirty sector will be higher if the emission standard rather than the emission tax is imposed.

An emission standard and an emission tax also increase the relative price of dirty goods to different extents. The emission standard is a quantity instrument. It increases the amount of resources required to produce one unit of dirty goods, by adding the resources used to reduce emissions and satisfy the standards. Therefore, it indirectly increases the price of products to cover the additional variable abatement costs. Recall that the price is set according to a fixed mark-up over the variable costs. In contrast, the emission tax is a price instrument.

As plants incorporate the emission tax into their production plan, the price of dirty goods increases directly. When the emission tax is imposed there are some additional tax costs for the producers of dirty goods besides the variable abatement costs induced by reducing emissions. As a result, imposing the emission tax increases the price of dirty goods by a larger percentage.

Since an emission standard and an emission tax cause the plants to respond differently, the relative prices of goods across plants and the relative marginal costs of producing goods across plants are also different under the two different policies. Consequently, the wedge between the price — how consumers value the goods — and the marginal cost of producing the goods is affected differently. As a result, the two policies cause different degrees of price distortion. When the emission tax is imposed, the plants increase their price and thus transfer more costs to consumers. This price distortion causes more welfare to be lost. The following experiment compares the effects of the emission tax and the emission standard in terms of aggregate outputs and relative prices for an equal amount of emission-reduction ($E = 0.3680$ gigaton). Key variables are listed in table 3.

Table 3 Compare models under tax and standard

	Tax $\tau = \$3.33/per\ ton$	Standard $s = 0.74b_{II}$
m	0.3185	0.3183
q	0.1857	0.1863
p	1.0169	1.0110
N_a/N (%)	0.82	24.41
$G(x_e)$ (%)	8.55	12.08
L_x	0.1379	0.1388
Average productivity	0.7427	0.7450
Value of utility function	0.2018	0.2020

As shown in table 3, the results of imposing the emission standard, as opposed to the emission tax, are a higher average productivity of the dirty sector and a lower price of dirty goods. As a result, the emission tax has a stronger demand shift from dirty to clean goods than the emission standard. So if the emission standard is imposed, the quantities of resources allocated to the dirty sector are higher and the value of the utility function is also slightly higher. This result differs from the existing literature. The existing literature, since it does not consider that plants have different levels of productivity and different abilities to transfer their costs to consumers, finds that the emission tax is most efficient.

5.1.1 Effects of Productivity Dispersion — Imposing Emission Tax

This subsection studies the economic and environmental performance of an emission tax in an economy without productivity dispersion relative to economies with different degrees of productivity dispersion. In order to have a fair comparison, a model without productivity dispersion is constructed in such a way that the outputs of both the clean goods (0.3185) and the dirty goods (0.1857) are the same as in the model with productivity dispersion specified above. In the two models, all the parameters are the same with the exception of the productivity parameters, which are listed in table 4. The same method is used to construct two models with different degrees of productivity dispersion: $k = 4$ and $k = 3.2$. Recall that the smaller the value of k, the higher the degree of productivity dispersion. Table 5 also shows that the economy with a higher degree of productivity dispersion generates less emissions in the production of the same amount of outputs.

Table 4 Productivity dispersion parameters and emissions - $\tau = 3.33$ $\$/ton$

	$k = \infty$ (no dispersion)	$k = 4$	$k = 3.4$ (benchmark)	k = 3.2
E ($gigaton$)	0.3952	0.3721	0.3680	0.3618
X	0.9856	0.997	1	1.002
Mean of x	1.3720	1.184	1	0.882

5.1.2 The Estimates of Emissions

The reduction of emissions, when an emission tax is imposed, depends on the degree of productivity dispersion. Figure 3 depicts the proportion of emissions estimated by the models with different degrees of productivity dispersion after imposing ad valorem emission taxes. As seen from the figure 3, starting from the initial level of emissions (normalized to 1) and increasing the emission tax, the proportion of emissions reduced in the economy without dispersion is less than that in the economy with dispersion up until a threshold value of the tax rate; after that threshold value, the opposite is the case. In the simulated economies, the threshold value of the tax rate is around 33$ per ton of emissions. When the tax rate is lower, the model without dispersion underestimates the reduction of emissions. When the tax rate is above the threshold value, the model without dispersion overestimates the reduction of emissions.

Why does the degree of productivity dispersion affect the estimates of emissions when an emission tax is imposed? As solved in section II, whether a plant chooses to adopt the new abatement technology or not is contingent on its productivity. In the economy without dispersion, all the plants have an identical level of productivity. When the tax rate is low, no plants adopt the new abatement technology. As the emission tax increases, the prices of dirty goods increase and less quantities are demanded. The emissions decline slightly as a consequence of the reduction of dirty-goods production. When the tax rate is high enough, all the plants invest in the abatement technology. This conversion of abatement methods generates a sharp drop of emissions once the threshold value of tax rate is reached (see figure 3). In the economy with productivity dispersion, when the tax rate is low, the very high productivity plants invest in the new abatement technology. As the tax rate increases, plants with lower levels of productivity start investing in the abatement technology. As a result, the reduction of emissions is smooth.

Figure 3 also compares the reduction of emissions in economies with different degrees of productivity dispersion. Starting from the initial levels of emissions, the economy with a higher degree of productivity dispersion reduces a slightly larger proportion of emissions under a moderate tax rate. As the tax rate increases, the economy with a higher degree of productivity dispersion reduces less emissions. The reason is shown in table 5: the higher the degree of productivity dispersion, the smaller the investing rate in the abatement technology under the same tax rate.

Table 5 Reduction of emissions $\tau = 88.05$ (\$/ton)

	k = 3.2	k = 3.4	k = 4	No dispersion
Emission reduction (%)	−15.34	−17.08	−20.00	−24.50
Variable abatement costs	0.00079	0.00088	0.00079	0.00003
Investing rate Na/N (%)	20.45	28.30	48.14	100

5.1.3 The Effects of Targeting Emissions

The welfare consequences of environmental policies in an economy with heterogeneous plants and in the economy with homogeneous plants may be significantly different. Tables 6 and 7 show the effects of reducing the initial levels of emissions by 3%, 20%, and 25%. The costs of reducing emissions are not linear. The economy with productivity dispersion could bear more or less costs, depending on the proportion of emissions to be reduced. If the goal is to reduce 3% of emissions, the economy with productivity dispersion bears less costs. In the economy without productivity dispersion, no plants adopt the new abatement technology, so the reduction of emissions comes completely from the reduction of dirty goods production. The plants also transfer a larger proportion of the costs to consumers. The price of the dirty goods increases by a much higher proportion than that in the economy with productivity dispersion. In the economy without productivity dispersion, this distortion induced by the emission tax leads to a greater loss of GDP (or consumption).

Table 6 Effects of targetting emissions $k = 3.4$

Reduction of emissions	Initial level	-3%	-20%	-25%
Tax rate ($/ton)	3.33	10.59	92.96	100.75
Consumption $\triangle\%$[9]	–	-0.14	-1.95	-2.44
Welfare (%)	100	102.03	114.32	118.83
M	0.3185	0.3184	0.3201	0.3174
Q	0.1857	0.1851	0.1748	0.1752
Dirty sector output / input	1.3692	1.3696	1.3733	1.3745
P $	1.0169	1.0354	1.2320	1.2490
Aggregate GDP $\triangle\%$[10]	–	-0.14	-1.85	-2.31
$G(x_e)$	0.0855	0.0864	0.0948	0.0954
Investing rate Na/N (%)	0.82	3.26	27.84	28.12
Exit rate (%)	8.07	8.29	9.32	9.33

If the goal is to reduce 25% of emissions, however, the economy with productivity dispersion bears more costs. Reducing emissions to 25% below the current level is approximately the requirement in the Kyoto Protocol —– Canada needs to reduce Greenhouse Gas emissions by 6% below the 1990 level. In an economy with heterogeneous plants, there are two difficulties in curbing GHG emissions: (1) a large percentage of low productivity plants will not adopt the new abatement technology, and (2) some low productivity plants will go out of business if the tax is imposed, leading to more waste of sunk entry costs. When the tax rate increases to 100 $ per ton, the percentage of plants that adopt the new abatement technology is just 28%. The dirty sector reduces emissions using less efficient methods. Reducing emissions by 25% costs the economy with heterogeneous plants an additional 0.87% of GDP compared to the economy with homogeneous plants. In the economy with heterogeneous plants, as the tax rate increases, the entry and exit rate — and, therefore, the total entry costs — also increase.

[9] It is the welfare cost measured by the percentage of consumption that has to be increased to achieve the same welfare level as in the initial state, keeping the emissions level at 0.3680.

[10] The real GDP is calculated using the price in the initial state.

Table 7 Effects of targetting emissions - no dispersion

Reduction of emissions	Initial level	-3%	-20%	-25%
Tax rate ($/ton)	3.33	29.80	39.90	90.37
Consumption $\triangle\%$	–	-0.26	-0.90	-1.67
Welfare (%)	100	101.93	115.65	120.07
M	0.3185	0.3227	0.3140	0.3177
Q	0.1857	0.1801	0.1857	0.1781
Dirty sector output / input	1.3705	1.3705	1.3705	1.3705
P $	0.9907	1.0610	1.0671	1.1837
Aggregate GDP $\triangle\%$	–	-0.26	-0.90	-1.65
Investing rate Na/N (%)	0	0	100	100
Exit rate (%)	8	8	8	8

Tables 8 shows the decomposition of costs in the economy with and without productivity dispersion, respectively. In the model with productivity dispersion, the variable abatement costs increase dramatically in order to reduce 25% of emissions, while the investment expenditure in abatement technology is limited by the proportion of high productivity plants that choose to invest in the new abatement technology. In the model without productivity dispersion, all the plants invest in the new abatement technology and reduce emissions more efficiently, so they save some variable abatement inputs.

In an economy with productivity dispersion, a uniform emission tax has two side effects that go in opposite directions. On the one hand, it leads to a resource reallocation from low productivity plants (including exiting plants) to high productivity plants. This increases the overall productivity of the producing plants. On the other hand, the uniform emission tax also increases the aggregate sunk entry cost, since some low productivity plants exit the industry and the turnover of plants increases. In the model, the second effect tends to dominate, leading to a decline in the quantity of dirty goods. For example, when the target is to reduce 25% of emissions, table 8 shows that the increased entry costs account for a large proportion of the losses of dirty goods.

Table 8 Decomposition of costs

	k = 3.4			no dispersion		
Emission reduction	Initial level	−3%	−25%	Initial level	−3%	−25%
L_m trillion $	0.3185	0.3184	0.3174	0.3231	0.3274	0.3223
L_g trillion $	0.1379	0.1374	0.1297	0.1354	0.1313	0.1299
$L_a$trillion $	0	0	0.0038	0	0	0.0006
Investment	0.0000	0.0001	0.0008	0	0	0.0028
Entry cost	0.0269	0.0274	0.0316	0.0242	0.0253	0.0279
Fixed cost	0.0087	0.0087	0.0088	0.0093	0.0080	0.0085
Total expenditure	0.4920	0.4920	0.4920	0.4920	0.4920	0.4920

5.1.4 The Sensitivity of Substitution

The effects of an emission tax may depend on how easily goods can be substituted by others. This subsection studies how different values of the substitution parameter σ can affect the effects of an emission tax. The prior distribution of productivity is adjusted so the shares of dirty goods sales remain unchanged when varying σ. Since the model has a restriction that $1 + k - \sigma > 0$, the value of σ cannot be too large. In the model a value for σ larger than 4 makes the goods too easy to be substituted and makes it difficult to match the productivity dispersion with $k = 3.4$. Table 9 shows that the more easily the goods can be substituted by others (with a higher value of σ), the larger the costs of reducing the same proportion of emissions. If goods are more easily substitutable, a larger proportion of goods is produced in the high productivity plants, which is equivalent to having a higher degree of productivity dispersion. As a result, the low productivity plants are at a disadvantage, leading to a lower investment rate and a higher GDP loss. However, the magnitude of the GDP loss does not vary much.

Table 9 Sensitivity of substitution

	$\sigma = 3.6$	$\sigma = 3.8$	$\sigma = 3.9$
Prior average productivity	1.096	1	0.9480
Tax rate ($/ton)	90.80	92.96	94.33
M	0.3199	0.3201	0.3202
Q	0.1756	0.1748	0.1747
Dirty sector output / input	1. 3680	1. 3733	1. 3154
Aggregate GDP $\triangle$%	-1.75	-1.85	-1.89
Investing rate Na/N (%)	36.78	27.84	23.93

5.2 The Effects of Exit

To better understand the sources of the different welfare consequences of an emission tax in the economies with and without productivity dispersion, this subsection studies the models without exit. To construct a model without exit, we re-calibrate some parameters. In order to exclude exit, assume that there is no fixed cost in production and there is no death of plants. The profit margin is increased by these adjustments. The free entry condition still holds, although there is no entry in the equilibrium. As the profit margin increases, the entry cost implied by the free entry condition also increases. The value of f_e rises to $6.316Y_Q/N$. Since the profit margin goes up, the average productivity in the dirty sector has to be lower in order to keep the dirty goods sales share at 0.39 without adjusting the value of productivity dispersion parameter k. The value of $x_{\min}$ becomes 0.6354. Other parameters are unchanged. Finally, to keep the emissions at the level of 0.3680 gigatons, the amount of resources l is adjusted to 0.4447. After eliminating the exit, the costs of reducing emissions decrease as shown in table 10. For instance, in the reduction of 20% of emissions, the GDP loss is reduced from 1.85% to 1.13%. This indicates that the exit cost accounts for a large proportion of the emission-reduction costs.

Table 10 Effects of targeting emissions - without exit

Reduction of emissions	Initial level	-3%	-20%	-25%
Tax rate ($/ton)	3.33	10.59	93.92	101.92
Consumption $\triangle\%$	–	-0.04	-1.21	-1.54
M	0.3068	0.3071	0.3111	0.3088
Q	0.1628	0.1623	0.1542	0.1547
Aggregate GDP $\triangle\%$	–	-0.04	-1.13	-1.49
Investing rate Na/N %	0.83	3.38	30.10	30.10

5.3 Transition after a Policy Change

This subsection studies the transition of an economy after an emission tax is imposed. The emission tax is seen as a permanent policy shock. The experiment here is to impose a steady-state welfare-maximizing tax rate on the current steady state and let the economy evolve to the optimal steady state. The steady-state welfare-maximizing tax rates are calculated and reported in tables 11. For simplicity, this section uses the models without exit.

Table 11 Steady-state welfare-maximizing tax rate

	k = 3.4		no dispersion	
	Initial level	Optimal level	Initial level	Optimal level
Tax rate ($/ton)	3.33	112.1	3.33	109.9
Emissions (gigaton)	0.3680	0.2560	0.3940	0.2560
Consumption $\triangle\%$	–	-2.00	–	-1.96
Welfare (%)	100	125.00	100	131.34
M	0.3068	0.3061	0.3068	0.3061
Q	0.1628	0.1552	0.1628	0.1552
Aggregate GDP $\triangle\%$	–	-1.92	–	-1.84

Starting from the current state of the economy, impose the steady state welfare-maximizing tax rate and let the economy evolve to the optimal steady state. As shown in figure 4, it takes over 80 years for the economy to reach half of the welfare-maximizing steady state level. The steady state welfare-maximizing tax rate overshoots the dirty sector, making its output fall below its future steady state level. As the emission stock declines over time, the dirty sector recovers itself, but the clean sector declines over time. After imposing the steady state optimal tax rates, the emissions fall by 33% below the existing level right after the policy shock, which is below the level in the future steady state. Compared to the current economy without further actions, imposing this steady state welfare-maximizing tax rate costs 2.58% of GDP in the economy with productivity dispersion. This dynamic cost is higher than the static cost — 1.92% of GDP. The cost should be even higher if plant exit were considered.

6 Conclusion

This paper evaluates emission-reduction policies in an economy with heterogeneous plants. It calls attention to the different reactions of plants to environmental policies and the resulting efficiency problem. Calibrated to Canadian data, the model predicts that the cost of reducing 20% of current emissions through an emission tax is much higher than that estimated by a model with homogeneous plants. This is due to a selection effect; that is, high productivity plants choose to invest in advanced abatement technology, while low productivity plants do not. For the latter, since both their production and their emissions are small, the benefit of such an investment is also relatively small. These low productivity plants, instead, use less efficient methods to reduce their emissions, or simply pay more tax. As a consequence, the average abatement cost is high. Furthermore, this paper shows that a large proportion of the cost of reducing emissions arises from policy-driven plant exit.

The model also compares an emission tax and an emission standard and finds the surprising result that the standard outperforms the emission tax. The main reason is that the emission tax increases to a large extent the wedge between the price of goods and the marginal cost of producing them (including the added costs of paying emission tax and reducing emissions), resulting in a transfer of part of the tax costs to consumers. This result is based on the assumption that plants are heterogeneous and can set the prices of their goods. This method can be applied to cross-country studies. The relative advantage of the two policies may depend on the degree of plant heterogeneity in different countries.

Finally, the paper provides a tractable framework to study the transition of the economy after an emission tax is imposed. To reduce about 30% of emissions would cost 2.58% of GDP in the economy with productivity dispersion but without plant exit. The cost should be even higher if exit were considered. As a result, the costs estimated from this paper are much higher than that in the Stern Review, 2% of GDP.

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A Appendix I: Data Description

1. Define the dirty sector and the clean sector. According to the Environment Canada, there are 16 industries whose abatement costs per employee are more than 1,000$. The emissions from these 16 industries account for about 90% of all industrial emissions. These 13 industries are defined as the empirical counterpart of the dirty sector in the model. These 16 industries and their NAICS codes are as follows: Forestry and Logging (113000), Oil and Gas Extraction (211000), Mining (212000), Electric Power Generation, Transmission and Distribution (221110), Natural Gas Distribution (221200), Food Manufacturing (311000), Beverage and Tobacco Products (312000), Wood Products (321000), Pulp, Paper, and Paperboard Mills (322100), Petroleum and Coal Products (324000), Chemicals (325000), Non-Metallic Mineral Products (327000), Primary Metals (331000), Fabricated Metal Products (332000), Transportation Equipment (336000), and Pipeline Transportation (486000).

2. The aggregate emission data in Canada from 1990 to 2006 come from the Greenhouse Gas Emissions National Inventory Report (NIR) by the Environment Canada. The GDP data come from the Statistics Canada, CANSIM II. The industrial level emission and GDP data come from the Canadian Industrial End-use Energy Data and Analysis Centre and CANSIM II. The emissions from these 16 industries account for about 50% of the total emissions in Canada. Since this paper focuses only on industrial emissions, the emissions from transportation, agriculture, residence, and other sources are excluded. The aggregate GDP in this paper is therefore the GDP from the 16 dirty industries plus a half of the total GDP of the sectors that do not generate emissions. It is approximately 50% of the aggregate GDP in Canada. The series of GDP of the clean sector and the dirty sector used in estimating the parameters in the relationship between GDP ratio and price ratio in equation (19) are nominal GDP adjusted by the price indices constructed below.

3. Construct the relative price. The relative price is the ratio between the price of the dirty aggregate and the price of the clean goods. The dirty goods price is constructed as a GDP-weighted average of 12 dirty goods: Electric Power Generation, Petroleum and Coal Products, Fabricated Metal Products, Food Manufacturing, Beverage and Tobacco Products, Wood Products, Pulp, Paper, and Paperboard Mills, Primary Metals, Fabricated Metal Products, Gasoline, Chemical and Chemical Products, and Transportation Equipment. The clean goods price is constructed as a weighted average of 3 clean goods: new houses, electrical and communication products, and farm product, with weights of $54\%, 30\%$ and 16%, respectively. The relative price at the initial date, i.e. 1990, is normalized to 100.

Table A1 Price indices for clean and dirty goods

Year	1990	91	92	93	94	95	96	97	98	99
Clean	100	94	93	95	97	98	98	97	98	98
Dirty	100	98	99	100	104	112	111	111	104	108

Year	2000	01	02	03	04	05	06
Clean	99	102	106	108	113	114	122
Dirty	129	130	125	128	150	162	181

4. The total operating and capital expenditures on environmental processes and technologies

to reduce greenhouse gas emissions by industry are reported by the Environment Accounts and Statistics Division of the Statistics Canada (2004).

Table A2 Adoption of new abatement technologies to reduce GHG emissions by industry

		Impact on emissions		
Industries	Introduced new tech.	Small	Medium	Large
Logging	11	71	29	0
Oil and Gas Extraction	65	31	57	12
Mining	18	70	30	0
Electric Power Generation	29	45	23	32
Natural Gas Distribution	58	0	71	29
Food Manufacturing	10	59	41	0
Beverage and Tobacco Prod.	16	60	40	0
Wood Products	14	50	36	14
Paper Manufacturing	35	40	36	24
Petroleum and Coal Products	39	62	38	0
Chemicals	18	55	33	13
Non-Metallic Mineral Prod.	18	46	31	23
Primary Metals	21	30	51	19
Fabricated Metal Products	18	43	50	7
Transportation Equipment	23	59	32	9
Pipeline Transportation	71	17	80	3
Average	24	44	44	13

These data are the empirical counterpart of the variable costs of emission abatement and the investment in new abatement technology. The Environment Accounts and Statistics Division has also reported the adoption of new or significantly improved systems or equipment to reduce GHG emissions by industry during 2000-2002. These data, provided in table A2, are used to calibrate the parameters affecting the relative importance of using new abatement technology. Respondents who answered Yes to the adoption of new or significantly improved systems or equipment were asked to rank the impact on greenhouse gas emission reductions as being small, medium or large.

B Appendix II: Aggregation under Emission Standard

Given the threshold values that determine the types of plants, the mass of each type of plants can be calculated. Denote the mass of type j plants as $N_{j,t}$, the proportion of type j plants as $n_{j,t}$, and the mass of producing plants as N_t. Thus,

$$n_{1,t} = \frac{N_{1,t}}{N_t} = \left(\frac{x_{\min}}{x_{1,t}}\right)^k,$$

$$n_{j,t} = \frac{N_{j,t}}{N_t} = \left(\frac{x_{\min}}{x_{j,t}}\right)^k - \left(\frac{x_{\min}}{x_{j-1,t}}\right)^k,$$

for $j = 2,\ 3,\ 4$, and

$$n_{5,t} = \frac{N_{5,t}}{N_t} = 1 - \left(\frac{x_{e,t}}{x_{a,t}}\right)^k.$$

Define the special "average" productivity levels for the 5 types of plants. The average productivity of the type 1 plants is

$$\tilde{x}_{1.t} = \upsilon x_{1,t},$$

recall $\upsilon = (\frac{k}{k+1-\sigma})^{\frac{1}{\sigma-1}}$. The average productivity of the type 2 plants is

$$\begin{aligned}\tilde{x}_{2,t} &= \left[\frac{1}{G(x_1) - G(x_2)} \int_{x_2}^{x_1} x^{\frac{h(\sigma-1)}{h-1}} dG(x)\right]^{\frac{h-1}{h(\sigma-1)}} \\ &= \varpi x_{2,t} \left[\frac{1 - (\frac{x_{2,t}}{x_{1,t}})^{k - \frac{h(\sigma-1)}{h-1}}}{1 - (\frac{x_{2,t}}{x_{1,t}})^k}\right]^{\frac{h-1}{h(\sigma-1)}},\end{aligned}$$

where $\varpi = \left[\frac{k}{k - \frac{h(\sigma-1)}{h-1}}\right]^{\frac{h-1}{h(\sigma-1)}}$. Note that when the variable abatement cost is positive, the average productivity is influenced by h. The average level of productivity of the type 3 plants is

$$\tilde{x}_{3,t} = \upsilon x_{3,t} \left[\frac{1 - (\frac{x_{3,t}}{x_{2,t}})^{k+1-\sigma}}{1 - (\frac{x_{3,t}}{x_{2,t}})^k}\right]^{\frac{1}{\sigma-1}}.$$

The average productivity level of the type 4 plants is

$$\tilde{x}_{4,t} = \varpi x_{a,t} \left[\frac{1 - \left(\frac{x_{a,t}}{x_{3,t}} \right)^{k - \frac{h(\sigma-1)}{h-1}}}{1 - \left(\frac{x_{a,t}}{x_{3,t}} \right)^{k}} \right]^{\frac{h-1}{h(\sigma-1)}} .$$

The average productivity of the type 5 plants is

$$\tilde{x}_{5,t} = \varpi x_{e,t} \left[\frac{1 - \vartheta^{k - \frac{h(\sigma-1)}{h-1}}}{1 - \vartheta^{k}} \right]^{\frac{h-1}{h(\sigma-1)}} .$$

The average price ($\tilde{p}_{j,t}$) and quantity ($\tilde{q}^{s}_{j,t}$) of type j plants can be calculated according to the average productivity. The aggregate price of the dirty goods is defined as

$$P_t = \left[\sum_{j=1}^{5} n_{j,t} (\tilde{p}_{j,t})^{1-\sigma} \right]^{1/(1-\sigma)} .$$

The average profit is defined as $\tilde{\pi}_t = \sum_{j=1}^{5} n_{j,t} \tilde{\pi}_{j,t}$, where $\tilde{\pi}_{j,t}$ is the average profit of type j plants. The average emissions of type j plants is $\tilde{e}_{j,t} = e\left(\tilde{x}_{j,t}\right)$, and the aggregate emission level is $E_t = \sum_{j=1}^{5} N_{j,t} \tilde{e}_{j,t}$.

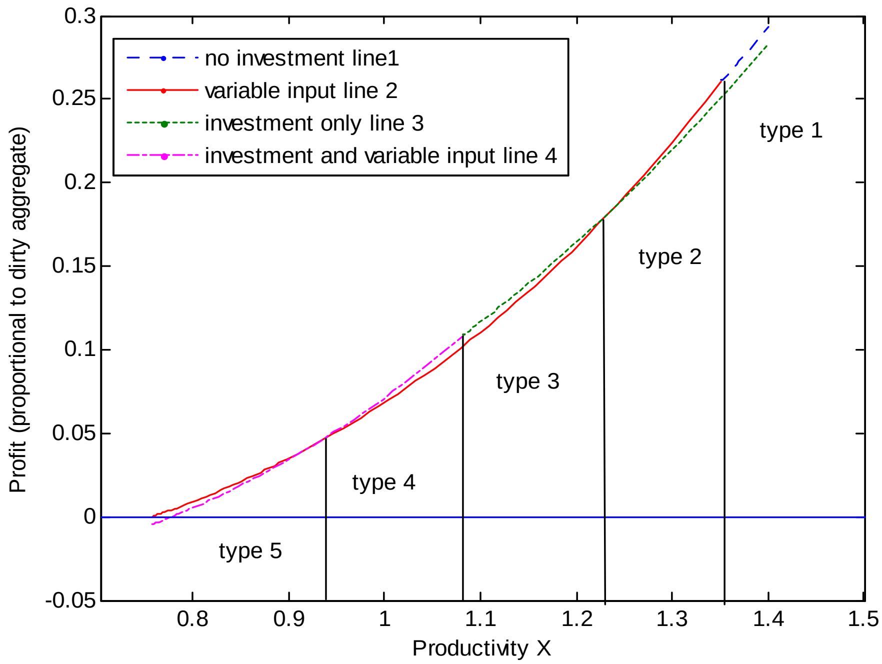

Figure 1: The choice of abatement methods w.r.t. x.

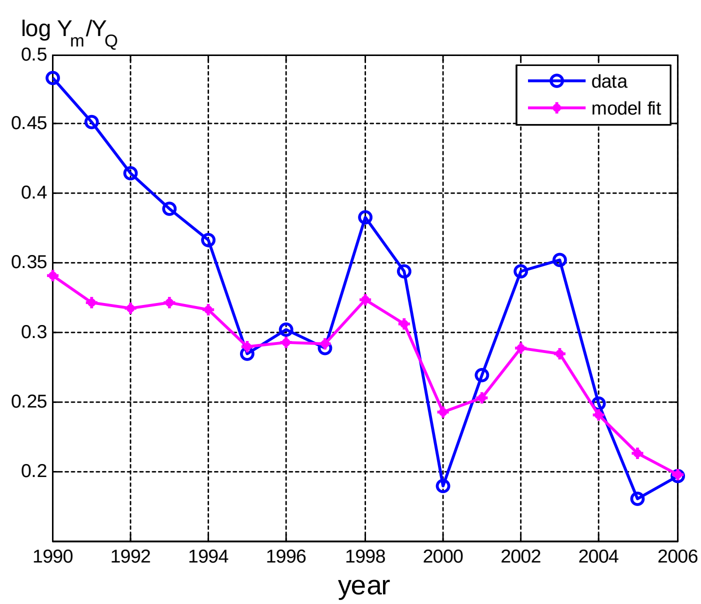

Figure 2: Model's fit

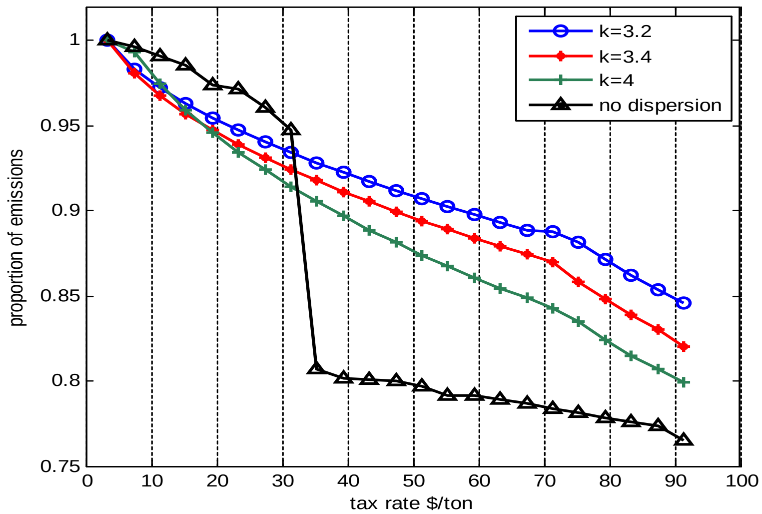

Figure 3: Tax rates and emission reduction

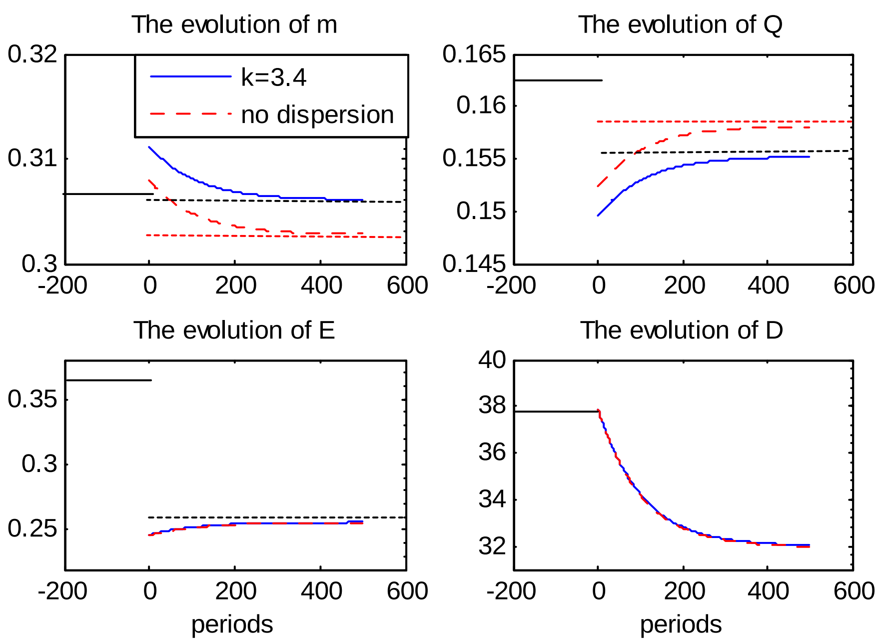

Figure 4: Transition of variables under steady state welfare-maximizing tax rate