Assignment 1 for Advanced Macroeconomics III (Fall 2009)

SUFE

Condiser a variation of Lucas's (1988, JME) model. Time is continuous in the economy. The representative household has the following utility function:

$$\int_0^\infty u(c(t))e^{-\rho t}dt, \text{ where } u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \ \sigma \neq 1, \text{ and } \sigma, \rho > 0.$$

The process of human capital accumulation by an individual agent is:

$$\dot{h}(t) = h(t)\delta\left[1 - l(t)\right], \ \delta > 0,$$

where $(1-l)$ is the time input in the accumulaiton. The produciton function of final goods is

$$F(k, lh, h_a) = k^\alpha (lh)^{1-\alpha} h_a^\gamma, \ \gamma > 0,$$

where h_a is the human capital stock per capita, which is taken as given by every individual agent, although $h_a = h$ in equilibrium. Assume that physical capital does *not* depreciate.

The government in this economy subsidizes human capital accumulation at a rate ϕ. That is, each household receives a subsidy $\phi w \dot{h}$ at each t, where w is the wage rate for raw labour. The subsidy is financed by a lump-sum tax S. Restrict $0 < \phi < 1/\delta$.

1. Formulate a representative household's optimization problem and derive the conditions for optimal choices.
2. A competitive firm's optimization problem yields $F_1 = r$ and $F_2 = w$. Use these conditions, the conditions in (1) and market clearing conditions to derive a dynamic system for (c, k, l, h). (There should not be any other variables in this system.)
3. Let $\frac{\dot{c}}{c} = \kappa$, and $\frac{\dot{h}}{h} = \nu$. Prove that $\frac{\dot{k}}{k} = \kappa$, and $\kappa = (1 + \frac{\gamma}{1-\alpha})\nu$ along the balanced growth path.
4. Solve for the long-run growth rate of consumption.
5. Assume $0 < l < 1$ along the balanced growth path. Compute the long-run level of intertemporal utility, and show that it is finite. (Note that σ could be greater than or less than 1.)
6. Does long-run utility necessarily increase with ϕ?
7. Formulate the social planner's problem in this economy and derive the socially optimal growth rate in the long run (denoted κ^o).
8. Show that, if $\phi = 0$, then the long-run growth rate in the decentralized economy is lower than the socially optimal one whenever the equilibrium satisfies $l > 0$. Briefly explain why this result holds.