Assignment 3 for Advanced Macroeconomics III

(Fall 2009)

SUFE

Question 1. Introduce capital accumulation into the asset pricing model of Lucas (1978, Econometrica) as follows. There is a unit measure of firms, each of which is specialized in collecting fruit from a particular tree. The quantity of fruit produced by a tree in a period depends on the amount of capital the firm uses. That is, output is $y_t = z_t f(k_t)$, where k_t the level of capital, and z_t is productivity which obeys a two-state Markovian process. The two possible realizations of z are z_1 and z_2, with $\infty > z_1 > z_2 > 0$. The transition probabilities are

$$prob\{z_{+1} = z_j | z = z_i\} = \begin{cases} \lambda & \text{if } z_j = z_i \\ 1 - \lambda & \text{if } z_j \neq z_i \end{cases}.$$

Assume $0.5 < \lambda < 1$.

The function f has the following properties: $f(0) = 0$, $f' > 0$, $f'' < 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. The firm chooses investment it so that the capital stock next period is $k_{t+1} = k_t + i_t$. The firm gives all residual earnings after investment to the shareholders as dividends; that is, $d_t = y_t - i_t$.

The firms are owned by a unit measure of households who have the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \ \beta \in (0, 1).$$

Here, the utility function $u(c)$ has the standard properties: $u' > 0, u'' < 0, u'(0) = \infty$, and $u'(\infty) = 0$. In each period t, the household chooses consumption, c_t, and the number of shares of the firm to hold, s_{t+1}. In contrast to a firm, a household does not have a storage technology and, hence, dividends that are not consumed will perish.

The shares of a firm are traded competitively in the asset market. Each firm issues one share. Let p_t be the ex-dividend price of a share measured in terms of consumption goods (fruit). Denote the aggregate state in period t as $A_t = (K_t, z_t)$, where K is the capital stock per firm. Individual households and firms take A as given. Use the notation $E_A(x)$ to denote the expectation of any variable x conditional on A. Denote the variables in the next period by adding the subscripts $+1$ to the variables.

Assume that the price p and dividends d can be expressed as $p = \psi(A)$ and $d = D(A)$.

1. Let the state variables for a household be (s, A) and the value function be $v(s, A)$. Formulate the household's optimization problem as a dynamic programming problem. Specify explicitly the state variables, the choices, and the information which expectations are conditional on. Derive the optimality conditions.

2. Assume that consumption per household is $C = C(A)$. Denote $M(A, A_{+1}) \equiv \beta u'(C(A_{+1}))u'(C(A))$. Let a firm's state variables be (k, A) and its value function be

$J(k, A)$. Assume that each firm discounts next period's value with the factor $M(A, A_{+1})$. Formulate a firm's optimization problem as a dynamic programming problem. Specify explicitly the state variables, the choices, and the information which expectations are conditional on. Derive the optimality conditions.

3. Focus on a symmetric equilibrium where all households have the same level of consumption and all firms have the same capital stock. Prove that the assumptions $p = \psi(A)$, $d = D(A)$, and $C = C(A)$ are consistent with a symmetric equilibrium (i.e. to prove that p, d, and C are all only a function of A).

4. For this part only, assume that the utility function is linear. Prove that the firm's optimal choice of k_{+1} in state A can be written as $k_{+1} = \phi(z)$. Also, prove that $\phi(z_1) > \phi(z_2)$.

5. Return to the general specification of the model. Does this model explain the observed equity premium better than the model where the capital stock is fixed? Justify.

Question 2. Consider a special case of the Diamond and Dybvig model (JPE 1983). A large number of agents live for 3 periods. Preference is isoelastic and the discounting factor is one:

$$\theta \frac{c_1^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{(c_1+c_2)^{1-\sigma}}{1-\sigma},$$

where the taste shock $\theta = \begin{cases} 1 \text{ if type 1} & \text{with prob. } t \\ 0 \text{ if type 2} & \text{with prob. } 1-t \end{cases}$. Each agent is endowed with 1 unit of goods in time period 0. The good can be privately stored. The goods could also be invested in a project that matures in time period 2. If a project matures, it gives R units of goods; if the project is interrupted, it gives 1 unit of goods. Assume $R > 1$. Each agent learns his type in time period 1. In time period 0, an agent can deposit his money in a bank. If an agent withdraws in time period 1, the bank pays r_1, and if an agent withdraws in time period 2, the bank pays r_2.

1. Sovle for the demand deposit contract (r_1, r_2).
2. Show that if $\sigma > 1$, as assumed in the paper, $r_1 > 1$ and $r_2 < R$.
3. What would happen if $\sigma < 1$? Would bank exist? If so, could bank runs happen?