Assignment 4 for Advanced Macroeconomics III

(Fall 2009)

SUFE

Question 1. Consider a variation of Kiyotaki and Moore (1997, JPE). In a discrete-time economy with a continuum of agents who use land to produce fruits. Everyone lives for ever and has the same preferences

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \ln c_t \right), \tag{1}$$

where c_t is consumption of fruits at date t. At each date, some agents are productive and the others are unproductive. The production technology is

$$y_t = \alpha k_{t-1} \tag{2}$$

and

$$y_t = \gamma k_{t-1} \tag{3}$$

for productive and unproductive agents, respectively, $\alpha > \gamma > 1/\beta$. Assuming that the initial ratio of productive agents to unproductive agents is $n : 1$. Each agent shifts stochastically between productive and unproductive states according to a Markov process. Specifically, each agent who is productive in this period may become unproductive in the next period with probability δ, and each unproductive agent may become productive with probability $n\delta$. The shifts of the productivity of individuals are exogenous, and are independent across agents and over time. So the ratio of productive agents to unproductive agents is constant over time. The productivity of each producer between dates t and $t+1$ is known to the public at date t. Assume $\delta + n\delta < 1$. Assume the production technology is specific to each producer and each producer is free to walk away from the production. The after dividend value of land could be used as collateral for borrowing. The total land is normalized to 1.

Please complete the following parts.

(1) Define and characterize the equilibrium without credit constraint (i.e. no collateral is required for borrowing). How does the aggregate output depend on the distribution of wealth between productive and unproductive agents? Solve the steady state equilibrium.

(2) Suppose there is a negative productivity shock, that is, α and γ both decrease by Δ. What is the impulse response of aggregate output? Will this response be persistent?

(3) Now characterize the equilibrium with collateral requirement, that is, debt b_t (actually $r_{t+1}b_t$) is restricted by the future value of land, $q_{t+1}k_t$. Moreover, we assume that there

is some deadweight loss if the collateral is transferred to the lender. Specifically, a lender gets only $\theta q_{t+1}k_t$, $0 < \theta < 1$ (assume k_t is not changed, but the lender pays some fee $1 - \theta$ such that he gets only θq_{t+1} for one unit of land). Assume that n is small and δ is large such that the productive agents cannot hold all the land given the collateral requirement. In the neighborhood of the steady state, show that the equilibrium user cost $u_t = \frac{\gamma}{r_{t+1}}$. Solve the steady state equilibrium and show the condition for unproductive agents to produce (a condition of θ related to n and other parameters).

(4) Show that with collateral constraint, the aggregate shock on productivity is propagated persistently.