Lending Competition and Endogenous Credit Supply: A General Equilibrium Theory of Loan Sales*

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Abstract

We develop a general equilibrium theory of loan sales based on bank competition. We show how credit shortage can arise endogenously in response to an increase in bank competition for lending, and how such credit shortage can motivate loan sales as a means to financing risky yet potentially profitable projects that would otherwise be rationed. Our results highlight an environment in which lower lending standards and sub-prime loans, and higher aggregate risk, can arise endogenously due to an increase in bank competition, without misalignment of incentives, mis-perception of risks, or mis-pricing of assets. Our theory has a number of testable implications, which are all supported by empirical evidence. On a general level, by demonstrating the rudimentary role of lending competition in motivating off-balance-sheet activities, our theory provides a novel account for the fundamental shift, over the past two decades or so, in the lending practice of U.S. and European banks, from the traditional 'originate to hold' model of credit provision, towards the 'originate to distribute' approach for credit extension, or, the emergence of the 'shadow banking system'.

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1 Introduction

The past twenty years or so have observed a fundamental shift in the lending practice of U.S. banks, from the traditional 'originate to hold' model of credit provision, where banks used deposits to fund loans that they then kept on their balance sheets until maturity, towards the so-called 'originate to distribute' approach for credit extension, under which banks sell loans that they originate, either in whole or in part, to investors, rather than fund them with deposit liabilities.[1] A similar transformation has also occurred in the European banking system.[2]

The recent financial crisis and ensuing recession have raised concerns about the implications of these off-balance-sheet activities for the safety and soundness of the financial system. A main concern is that potential incentive misalignments in the application of the OTD model could lead to weakening of lending standards, which is widely accepted as a root cause for the recent crisis. This concern is shared by not only academic researchers, but also policymakers.[3] As a matter of fact, this is an issue singled out by the U.S. President's Working Group on Financial Markets in its efforts to identify the sources of the financial turmoil during the onset of the crisis, and discussed in length by the U.S. Federal Reserve Chairman Ben Bernanke in his speeches at the World Affairs Council of Greater Richmond's Virginia Global Ambassador Award Luncheon on April 10, 2008, and at the Conference Co-sponsored by the Center for Economic Policy Studies and the Bendheim Center for Finance at Princeton University on September 24, 2010. In the European Union, the Economic and Financial Affairs Council mandated the European Central Bank, in cooperation with the Banking Supervision Committee, to assess "...how the so-called 'originate and distribute' model ... has impacted on the incentive structures of credit markets, in a context characterised by a shift from the more traditional retail to interbank borrowing."

To address this issue, and to assess the broad implications of this recently developed shadow banking system, one needs to better understand the incentives of banks in using the OTD model. One strand of the literature has focused on loan sales as a risk transfer tool.[4] If banks sell loans mainly to transfer risk, the potential for misalignment of incentives among market participants

[1] The secondary market (where loans are sold after origination) for direct sales of individual loans grew from a mere \$8 billion in 1991, to \$154.8 billion in 2004, \$176 billion in 2005, \$238.6 billion in 2006, and further to \$342 billion in 2007. The syndicated loan market (where loans are sold at origination) rose from \$339 billion in 1988 to \$2.2 trillion in 2007. The growth of the market for securitization of pooled loans had also been spectacular in the years leading up to the financial crisis of 2007. These facts are documented by Lucas et al. (2006), Drucker and Puri (2009), Ahn (2010), and Bord and Santos (2012), among others. See, also, Duffie [1] and Loutskina and Strahan [2] for related facts.

[2] See, for example, ECB (2008).

[3] See, among others, Ashcraft and Schuermann (2008), ECB (2008), Mian and Sufi (2009), Keys, Mukherjee, Seru, and Vig [13], CRS Report for Congress (2010), Demyanyk and van Hemert (2011), and Bord and Santos (2010, 2012) for some comprehensive and in-depth discussions of this issue.

[4] See, among others, Allen and Carletti [10], Allen and Gale [11], and Wagner and March [12]).

would be a major weakness of the OTD model for the policymakers to address. Reflections on the recent financial crisis are mainly along this line of thinking, which also is a mission set out in the 2008 ECB Eurosystem Report, as well as in the plan for regulatory reform proposed in 2009 by the U.S. Committee on Capital Markets Regulation.

Recent empirical studies do not seem to suggest risk transfer as the main thrust for loan sales. The evidence provided by Keys, Mukherjee, Seru, and Vig [13], Greenlaw, Hatzius, Kashyap, and Shin [14], Gordon [15], and Berndt and Gupta [16] suggests that the application of the OTD model actually increases the risk faced by loan originators. More direct evidence based on U.S. bank holding company data from 2001 to 2007 is provided by Sarkisyan, Casu, Clare, and Thomas [17], who find that banks use loan sales mainly as a financing strategy, rather than a risk transfer tool. This conforms to the finding by Drucker and Puri (2009) based on four different data sources that banks sell loans mainly for the purpose of increasing their credit supply.[5]

To be consistent with these empirical findings, a theory about loan sales should also provide a coherent account for what might have propelled banks into a state of credit shortage so they felt the need to increase their credit supply in the first place. Such joint account shall also help predict how bank decisions and the financial system may respond to changing economic conditions and policy reforms looking forward. To meet this challenge, it is essential to endogenize credit supply in tandem with bank decisions of selling loans in a general equilibrium environment.

We provide such a joint account based on a general equilibrium theory of bank competition. We demonstrate how credit shortage can arise endogenously in response to an increase in bank competition for lending opportunities, and how such credit shortage can motivate loan sales as a means to financing an additional set of potentially profitable projects that would otherwise be rationed. As we show below, the mechanism of our model is supported by empirical evidence.

One crucial and relevant observation is that much of the past two decades that has experienced the aforementioned growth in loan sales has also witnessed increased bank competition for lending opportunities, as technological advance, and deregulation and globalization weakened geographic boundaries and encouraged interstate (U.S.) and cross-border (Europe) banking.[6] The increased competitive pressure has been widely perceived as reflecting a permanent shift in the loan market.[7]

[5]Consistent evidence can be found in the earlier study by Cebenoyan and Strahan (2004), which shows that the chief benefit of loan selling is greater bank credit availability but not lower bank risk. Corroborating evidence can also be found in Faulkender and Petersen (2006) and Sufi (2006).

[6]The technological and regulatory changes also weakened product boundaries and encouraged an "all-finance" practice in the lending business, and thus opened up other sources of interbank competition and competition from non-bank financial institutions and the capital market.

[7]See, for example, the Senior Loan Officer Opinion Survey on Bank Lending Practices (1997-2006), conducted quarterly by the Board of Governors of the U.S. Federal Reserve System. See, also, Boot and Schmeits [4], Hakenes and Schnabel [5], Ahn and Breton [6], and Ahn [7].

Our theory suggests that the observed surge in loan sales over the past twenty years could be an equilibrium response to this increase in lending competition. The prediction of our model is also consistent with the empirical evidence based on micro-level data that banks facing more intense competition are more likely to sell loans.[8]

A bank in our model plays the dual role, of channeling funds from households to entrepreneurs, who rely on external sources to finance their risky projects, and of screening and monitoring the risky projects. The lending practice generates proprietary information about the entrepreneurs, which is not observable by third parties.[9] This gives the bank a comparative advantage in originating loans, which is suggested by empirical evidence as a primary motivation for loan sales[10] A defining feature of the model is that, both sides of the bank's balance sheet are endogenously determined and affected by macroeconomic conditions.[11] The joint presence of the proprietary information and the endogenous credit supply of banks generates a tradeoff facing entrepreneurs, between interest rate on loans and probability of obtaining the loans.

An increase in competition for lending opportunities among banks in this environment lowers interest rate on loans and, thus, interest rate on deposits by households falls too. In consequence, returns to banks' assets fall relative to their costs of funds in the presence of a regulatory capital requirement, given that banks are less patient than households so equity is more costly than debt. This leads to a fall in deposits and a decline in on-balance-sheet supply of credit, and, therefore, a rationing of potentially profitable projects. This creates an incentive for the banks to use the OTD model to extend the size of their profitable investments, even if such innovations may be costly. The banks may even originate risky investments with negative expected returns and sell them to other investors, in order to fully explore their comparative advantages in loan origination to maximize their total profits.

In our model, loans that are originated and sold to other investors are sold at their par values, as is consistent with the empirical evidence presented in Drucker and Puri (2009), and a "skin in the game" constraint addresses potential adverse selection and moral hazard problems associated with this off-balance-sheet activity. Thus our results in this paper show how lower lending standards and sub-prime loans can arise from the application of the OTD model due to

[8] See, for example, DeYoung (2007) for a survey of the related literature.

[9] This assumption, as is commonly made in the banking literature (e.g., Rajan, 1992), is supported by empirical evidence (e.g., Lummer and McConnell, 1989). It captures the intuition that the screening process can be time consuming and thus may not be conducted in sequel by a competing bank before the entrepreneur misses the profit opportunity (e.g., Dell'Ariccia, 2000), and that an important part of the information acquired by the bank may be soft in nature and thus cannot be credibly communicated to outsiders (e.g., Parlour and Plantin, 2008).

[10] See, for example, Pavel and Phillis [21], and Demsetz [22].

[11] The idea that the effects of changes in banks' balance sheets on the supply of credit might be important for understanding the functioning of the shadow banking system is also emphasized by Shin [25].

increased bank competition, without misalignment of incentives, mis-perception of risks, or mis-pricing of assets. However, one should not take this as suggesting that these latter problems are not important issues concerning the off-balance-sheet activity. Clearly, they are. Rather, we view our results as highlighting a fundamental role of lending competition in motivating loan sales, under which weakening lending standards and rising aggregate risk are equilibrium responses of the economy to increased bank competition for lending opportunities.

As discussed above, the fact that the supply of credit responds endogenously to changes in the general economic conditions plays a central role in our analysis. In this spirit our paper is related to Shin [25]. This defining feature of our model helps generate a number of testable implications, which are verified by exiting empirical studies. This is in contrast to a few recent papers that use partial equilibrium models with exogenous supply of credit to study the role of bank competition in asset sales.[12]

Our central mechanism also differs from the traditional models of bank loan sales that appeal to evolution of capital requirement regulation as a motivation for this off-balance-sheet activity.[13] Whereas the bank capital requirement that is kept at a constant level in our model does make on-balance-sheet intermediation more costly, it does not create a large enough regulatory arbitrage opportunity for banks to engage in costly loan sales in the absence of lending competition. Instead, it is the increase in competition for lending, through creating an endogenous credit shortage, that triggers widespread loan sales. Therefore, our model may help explain why enormous growth in loan sales had already occurred even before the U.S. adopted the Basel II Accord in 2005.

This is related to another contribution of our paper. While some models have the feature that banks with more opportunities in originating loans are more likely to adopt the OTD practice, such loan originating opportunities are mostly treated as exogenous in the existing studies.[14] In this paper, in contrast, bank competition for lending serves as a mechanism to endogenize loan originating opportunities, and the endogeneity of credit supply acts as the nexus between the two. Therefore, our model may help explain why there was the tremendous growth in loan origination opportunities in the first place during the past two decades, as increased bank competition resulted in excess demand for credit.

As stated earlier, the predictions of our model are consistent with broad empirical evidence: the prediction that increased lending competition may result in a shortage of credit for funding risky investments that require careful screening and monitoring is supported by the empirical findings of Petersen and Rajan [9], Rajan and Zingales (1998), Cetorelli and Gambera (2001),

[12] See, for example, HS [5], AB [6], and Ahn [7].
[13] See, for example, Pennacchi [18], Duffee and Zhou [19], and Calomiris and Mason [20].
[14] See, for example, Gordon and Pennacchi [23], and Parlour and Plantin [24].

and Bonaccorsi and Dell'Ariccia (2004); that lower interest rates and growth in loan sales may arise in tandem is confirmed by the empirical study of Guner (2006);[15] and that increased bank competition may lead to a decline in lending standards conforms to the empirical facts reported in Dell'Arrica, Igan, and Laeven [3] as well as those documented in the U.S. Federal Reserve's Senior Loan Officer Opinion Survey on Bank Lending Practices (1997-2006).[16]

2 Environment

Time is infinite in the forward direction and is divided into discrete periods indexed by t, $t = 0, 1, 2, \ldots$. The economy consists of B (≥ 2) islands indexed by i, $i = 1, 2, \ldots B$. In each island, there is one bank. A bank may operate for multiple periods until it defaults. There are overlapping generations of two-period lived households (and an initial "old" generation in period zero). Each generation has a unit measure. A representative young household is endowed with one unit of labor, which is supplied inelastically to produce a perishable consumption good. The period t good can be consumed or used as input in projects owned by entrepreneurs in period t. There is a large number, N $(>> B)$, of entrepreneurs, each of whom is endowed with a one-period project in every period t, $t = 1, 2, \ldots$. However, the household does not have access to the projects directly. The entrepreneurs have no input to carry out their projects. A bank is required as the intermediary between a household and an entrepreneur.

Banks are assumed to have the expertise to screen and monitor entrepreneurs, since the later may falsely report their type if there is no screening and hide the output of its project if there is no monitoring. For simplicity, we assume that the banks can screen and monitor the entrepreneurs with zero cost, while the households cannot screen and monitor the entrepreneurs. We assume that the households have access to all the banks.

The consumption good is produced by a constant returns to scale technology using intermediate good and labor. Since the labor supply is fixed, we may write the production function in per young-household terms. For any period t, the production function of the consumption good y_t is $y_t = z_t f(m_{t-1})$, where m_{t-1} is the amount of intermediate good per young household (the production of m will be defined later) and z_t is an aggregate productivity shock. We take the

[15] As a corroborating evidence, for much of the past two decades that has witnessed the increased bank competition for lending and growth in loan sales, bank prime loan rate has also been low, when compared with its level in much of the two preceding decades. This is not to exclude other factors, such as loose monetary policy and global imbalances, that could also have contributed to generating the low interest rates in this more recent episode.

[16] Almost all surveyed domestic and foreign respondents (from investment and commercial banks, as well as other financial intermediaries) cited more aggressive competition from other banks or non-bank lenders as the most important reason for easing their lending standards and terms. As a result, more than 20 percent of the banks eased lending standards for Commercial and Industrial loans, and around 50 percent of the banks decreased spreads on loan pricing.

random variable z_t to be i.i.d. over time, to be distributed continuously over a finite positive support, and to have a mean equal to $\hat{z}$.

The consumption good in period t can be transformed into period $t+1$ intermediate good (without the use of labor) by means of an investment technology. This investment technology comes in discrete, indivisible units, called "projects". Each entrepreneur is endowed with one of these projects (and we assume that it is too costly to trade or transfer a project away from the original owner). A project takes exactly one unit of the consumption good as input. With less than 1 unit of the consumption good, nothing is produced, and the marginal product of increments of the consumption good to a project that already has its requisite quantity of input is zero.

Any project that is undertaken in period t produces a quantity of intermediate good available for use in period $t+1$. The amount of intermediate good produced by a given project is a discrete random variable with possible outcomes ξ_j, $j = 1, 2$. We focus on the case of only two outcomes: a good outcome $\xi_1 = 1$ with probability θ, and a bad outcome $\xi_2 = 0$ with probability $1-\theta$. The entrepreneur's type θ obeys an i.i.d distribution with a Cumulative Distribution Function (CDF) $G(\theta)$ and a Probability Density Function (PDF) $g(\theta)$ on the support of $\left[\underline{\theta}, \bar{\theta}\right]$ with $0 \leq \underline{\theta} < \bar{\theta} \leq 1$. The intermediate good cannot be consumed but it can be used in the production of the consumption good. The intermediate good is assumed to depreciate fully in one period.

Besides their traditional intermediation function as delegated monitors (Diamond, [36]), banks are also assumed to have a new function: originate-to-distribute (O&D). The banks' new business of loan-sales is to fully exploit their special expertise of analyzing the credit worthiness of borrowers. That is, the banks originate a larger pool of loans and resell some of their loans to other investors. In order to avoid banks' moral hazard problem of investing in bad projects and reselling them to other investors, the banks are often required to keep a proportion of their packed loans. This is usually called "skin in the game".

An authority (central bank) is assumed to regulate the banks' behavior. First, the central bank sets a capital requirement for loans. That is, for a certain amount of loan, k ($1 > k \geq 0$) proportion of it has to be financed by bank's capital, and only $1-k$ proportion of it could be financed by households' deposit. Second, if a bank sells its loans, the bank has to hold at least λ proportion of the loans.

In each period, given the central bank's regulation, the banks make an investment plan. According to their plan, they choose a quantity of bank capital K_i. Given K_i, the banks raise deposit in a competitive market. All the banks take deposit rate r as given, and the total volume of deposit that the bank i will get, $S_i(r)$, has to satisfy $S_i(r)/[K_i + S_i(r)] \leq 1 - k$. We assume that the depositors have full insurance, so the volume of deposits depends only on the interest rate, and it does not depend on any risk.

After raising the funds, the bank lends them to entrepreneurs. The procedure of applying for funds is as follows. (1) The banks post their loan contract conditions, which are observable to other banks and to all the entrepreneurs. (2) Given the posted contracts, every entrepreneur chooses a strategy (a probability profile) on which island to attend. This strategy is a public information. (3) Every entrepreneur visits an island to apply for funds. (4) The banks evaluate the risk of each project that comes to their own island and discover the quality of the project, θ. The quality of a project θ is a common information in the island where the project is evaluated, but it is sealed to other banks. (5) The banks decide which entrepreneurs to finance. (6) The entrepreneurs that have been financed produce the intermediate goods, while the ones that have no funding do nothing but stay in the island. Some of the entrepreneurs who have relatively good quality may have the chance to be invested later if the banks could raise more money by selling their loans. Note that, during the period, a bank can evaluate projects only once and only the evaluated projects can be invested. As a result, the entrepreneurs do not have incentive to move to other islands during the period.

After the banks have invested in their selected projects, they can sell a proportion of their loans to the young households. Let $\acute{\theta}$ denote the average success probability of the loan-sales in the market. The loan-sales will entail a return r^a with probability $\acute{\theta}$. Both the banks and the households will take the contract of loan-sales $(r^a, \acute{\theta})$ as given. The total volume of loan-sales from bank i is denoted by $S_i^a(r^a, \acute{\theta})$. The banks can use the funds from selling the loans to invest in new projects. All these invested projects produce the intermediate goods, which will be available for the next period.

3 Optimal Decisions

3.1 Households

A representative young household in period t supplies 1 unit of labor inelastically and earns wage income w_t. A representative old household owns the entrepreneurs who produce the intermediate goods and takes all their profit. The old household passes the ownership of the entrepreneurs to the next generation when it dies. The expected profit of entrepreneurs is denoted by $\tilde{\pi}_{e,t+1}$.

The utility function satisfies the usual assumptions and the discounting factor satisfies $0 < \beta < 1$. The representative young household's problem is to maximize the expected life time utility,

$\max_{\{c^y_t, c^o_{t+1}, S_{i,t}, S^a_{i,t}\}} U(c^y_t) + \beta EU\left(c^o_{t+1}\right)$, subject to the following budget constraints

$$c^y_t + \sum_{i=1}^{B} S_{i,t} + \sum_{i=1}^{B} S^a_{i,t} = w_t, \text{ and } c^o_{t+1} = r_t \sum_{i=1}^{B} S_{i,t} + \acute{\theta}_t r^a_t \sum_{i=1}^{B} S^a_{i,t} + \tilde{\pi}_{e,t+1}, \tag{1}$$

where c^y_t, c^o_{t+1}, $S_{i,t}$, $S^a_{i,t} \geq 0$. The saving decisions are made according to the deposit rate, r_t, and the interest rate of the securities, r^a_t, together with the average probability of getting the returns of the loan-sales, $\acute{\theta}_t$. We restrict our attention to the case where the deposit at bank i is non-decreasing in interest rate r_t, i.e., $S_{i,t}(r_t)$ weakly increases in r_t.

3.2 Entrepreneurs

The entrepreneurs make one period decision only, so we omit the subscript t. After observing the loan contract conditions, the entrepreneurs choose a strategy on which island to attend. The contract conditions are described by $\left\{\gamma_i,\ \vec{\theta}_i\left(n_i\right)\right\}_{i=1}^{B}$, where γ_i is the bank's lending rate, and $\vec{\theta}_i$ is a threshold value of project quality θ above which the project will be financed. As in Peters [28], for simplicity, we let γ_i not contingent on the number of visitors. However, the selection criterion $\vec{\theta}_i$ will depend on the number of visitors. The larger the number of visitors, the higher the $\vec{\theta}_i$, given the bank's fixed lending capacity. We also assume that the banks can commit their γ_i, so we do not consider the possible bargaining after the banks meet with the entrepreneurs as in Camera and Selcuk [35]. Also note that the bank's profit is contingent on the realization of the project, but it does not depend on the evaluated quality θ of a project. This contingence means that a bank i will get a positive profit from the project if and only if the project succeeds.

After an entrepreneur has arrived at its chosen island, it draws a success probability θ from the distribution $G(\theta)$. The information of θ is unknown to anybody. The bank needs to evaluate the project in order to discover the value of θ. For simplicity, we assume zero physical cost associated with evaluation. After the bank's evaluation, θ is discovered to both the bank and the entrepreneur, but it is still a sealed information for other banks. Since banks can evaluate projects only once in a period, we shut down the incentive for entrepreneurs to move to other islands during the same period.[17] According to the revealed θ, the bank decides whether or not to lend funds to the entrepreneur. All the entrepreneurs bear limited liability, i.e., an entrepreneur pays back to the bank at most the amount ξ_i if state i is realized. With limited liability, an entrepreneur always has a positive expected return if it invests in its project, so it will always be willing to borrow from the bank.

[17] This assumption of entrepreneurs being locked to an island during one period does not affect the general results, but it makes the model much simpler. This assumption leads to that banks cannot compete for clients in the loan-sales stage. This lack of competition in the loan-sales stage affects only the magnitude of selling loans, but not the motive for selling loans.

If an entrepreneur goes to island i, it faces a contract $\left\{\gamma_i,\ \vec{\theta}_i\left(n_i\right)\right\}$. An entrepreneur can expect to be financed with probability $p_i = 1 - G(\vec{\theta}_i\left(n_i\right))$. Let $\mathring{\theta}_i = \left[\int_{\vec{\theta}_i(n_i)}^{\bar{\theta}} \theta dG(\theta)\right] / p_i$ be the expected average probability of success, the expected profit of an entrepreneur that chooses island i is

$$\pi_{e,i} = p_i \mathring{\theta}_i \left(\hat{q} - \gamma_i\right), \tag{2}$$

where $\hat{q}$ is the expected price of the intermediate good in the next period.

The conditional contract value $\vec{\theta}_i\left(n_i\right)$ depends not only on n_i, but also implicitly on both the profitability of the projects and the available funding $D_i = K_i + S_i$ at bank i. First, if the expected price of the intermediate goods $\hat{q}$ is high, or/and the marginal cost of funding, denoted by η (to be defined later), is low, then the projects are more profitable for the bank. If the projects are more profitable in general, then a project with a lower θ might be worth investing. We define $\hat{\theta}_i$ as the lower bound of project quality, above which projects are profitable. Then $\hat{\theta}_i$ is determined by $\eta = \hat{\theta}_i \gamma_i$. To satisfy the profitability condition, we should have the selection criterion $\vec{\theta}_i \geq \hat{\theta}_i = \eta/\gamma_i$. Second, the conditional contract value $\vec{\theta}_i$ also depends on the potential number of projects attracted to island i, n_i, and the available funding at bank i, D_i. Given any D_i and n_i, there is a $\tilde{\theta}_i$ that satisfies $n_i x \left[1 - G(\tilde{\theta}_i)\right] = D_i$. If $\tilde{\theta}_i > \hat{\theta}_i$, then D_i is not sufficient to support all the profitable projects. As a consequence, bank i selects only the good projects that have $\theta \geq \tilde{\theta}_i$. The threshold value $\vec{\theta}_i$ is, therefore, determined by $\vec{\theta}_i = \max\left\{\hat{\theta}_i, \tilde{\theta}_i\right\}$.

An entrepreneur chooses an island according to $\max_i \left\{\pi_{e,i}\right\}$ across all i. Given $\pi_{e,i}$ determined in (2), an entrepreneur faces a trade-off between γ_i and $\vec{\theta}_i$: in an island with a lower γ_i, the probability of being invested, p_i, is lower. This is because the island i with a lower γ_i may attract a larger number of entrepreneurs (higher n_i) competing for the limited funding. Given this trade-off, the entrepreneurs' expected profits in all islands should be equal in an equilibrium. Otherwise, if an island offers lower expected profit than other islands, the entrepreneurs would choose not to come to this island.

3.3 Banks

We assume that banks are risk neutral and very impatient: the discount factor of a bank, $1/\rho$, is much smaller than β. A bank $i's$ expected utility is $u_{b,i} = E_0 \sum_{t=0}^{\infty} \rho^{-t} c_{b,i,t}$, where $c_{b,i,t}$ is the bank $i's$ consumption of the period t good. The impatience assumption is a simple way to motivate a high cost of acquiring bank capital: if there is no capital requirement, the bank would rather consume everything it has, and borrow from the household to invest in its available projects. If there is capital requirement, the bank maintain as low bank equity as required. We assume that every bank is endowed with a large amount of bank equity at period 0, such that the banks have enough consumption good to cover the capital requirement. In the future period, the banks can either save from their profit to maintain the bank equity or borrow from outside with a fixed cost ρ.[18]

With this linear utility function, a bank's objective is equivalent to maximizing the expected present value of its life time profits (or the Franchise value). The Franchise value of a bank i in period t is $V_{i,t} = \max \pi_{b,i,t} + \rho^{-1} E_t \left(V_{i,t+1} \right)$, where $\pi_{b,i,t}$ is the expected profit of bank i in period t.

Here we do not allow banks to strategically default on the deposits of households (moral hazard). "Strategic default" means that the default plan is made before the aggregate states and the individual states are realized. If a bank strategically defaults, it may earn excess profit in the event of default at the expenses of depositors. Invulnerable default, on the contrary, is due to bad state and the bank earns zero profit when it defaults. Whether or not to strategically default may depend crucially on the default regulation and the capital requirement rate k. If any strategic default (when banks earn positive profits) will be caught and severely punished, then there will be no strategic default. But if not all the strategic defaults will be caught, some speculators may take the chance to default strategically. Capital requirement may reduce the bank's incentive of strategic default.

In every period, a bank i makes decisions on its capital, deposit, loan contract, and loan sales, sequentially. We divide every period into four stages accordingly. Without confusion, we omit the subscript t below. In the first stage, the bank chooses an amount of capital, K_i, with the fixed opportunity cost ρ. The banks have to make rational expectation about the optimal decisions in the following stages, in order to decide how much K_i to hold. In the second stage, taking the market rate r as given, the bank i raises deposit S_i from the young households, with a constraint $k \left(S_i + K_i \right) \leq K_i$. In the third stage, the bank i posts the contract $\left\{ \gamma_i, \vec{\theta}_i(n_i) \right\}$ and lends the funds to entrepreneurs after entrepreneurs' types are discovered. Those entrepreneurs who receive funding have the top tier projects, which are projects with quality θ on the right tail of the distribution of θ. If the banks are still interested in some second tier projects, which are not invested through funding from bank capital (equity) and deposit (debt), they may look for "out of balance sheet" method to raise funds. In the fourth stage, the bank i decides how much of its loan should be sold. The banks use funding through selling loans to invest in the second

[18] In order to avoid the case in which bank industry's total capital (equity) is constrained by the total previous period profit in the bank sector, we assume that banks can aquire capital from outside with a fixed cost ρ. This assumption does not affect steady state analysis. If we take seriously the constraint of bank industry's total capital, there might be interesting business cycle dynamics from the bank industry. But that is out of the scope of the current paper.

Here we also implicitly assumed that the households would not buy bank equity to avoid losses from bank's bankraptcy.

tier projects, which we can also call "subprime" loans.

We first consider a case in which banks have not innovated loan-sales, and leave the loan-sales analysis to section 5. We solve the bank's problem by backward induction. In this case, there is only three stages of bank's decision. In the third stage, the bank i chooses γ_i, given that K_i and S_i have been determined. Let $\pi_{b,i}$ be the bank $i's$ expected profit and $n_i p_i$ be the total number of projects that the bank i finances, then,

$$\pi_{b,i} = \max_{\gamma_i} \left\{ n_i p_i \mathring{\theta}_i \gamma_i - \rho K_i - r S_i \right\}. \tag{3}$$

The banks will always ensure this expected profit non-negative. However, if n_i is a finite number, then it is possible that, ex post, a bank earns a negative profit. To avoid this problem of deposit risk, we assume full insurance in the banking sector. Moreover, the rate of this potential failure of a bank is small if capital requirement k is large.

Let $\Phi_i(\gamma_i)$ be the total loan the bank i would have if it posts γ_i, we have

$$\Phi_i(\gamma_i) = \min \left[n_i\left(\gamma_i\right) \left[1 - G\left(\hat{\theta}_i\left(n_i\left(\gamma_i\right)\right) \right) \right],\ D_i \right]. \tag{4}$$

The cut-off value of θ, $\hat{\theta}_i$, is determined by the bank's profit break-even condition, i..e $\eta = \hat{\theta}_i \gamma_i$. If the bank increases γ_i, more projects become worth investing for the bank, resulting in an decrease in $\hat{\theta}_i$, $\hat{\theta}_i'\left(\gamma_i\right) \leq 0$. The total number of attracted entrepreneurs, however, decreases in γ_i, $n_i'\left(\gamma_i\right) < 0$.

4 Symmetric equilibrium without loan-sales

We first shut down the technology for loan-sales and restrict our attention to a stationary symmetric strong Nash equilibrium where $K_i,\ S_i,\ D_i,\ \gamma_i,\ n_i,\ p_i$ and $\vec{\theta}_i$ are identical for all $i = 1, 2, ...B$, and all the entrepreneurs choose an identical mixed strategy on which banks to attend. As shown in Peters [28] and BSW [32], such a capacity-constrained Bertrand equilibrium always exists and it is unique.

The aggregate state variables in the economy are the total quantity of the intermediate good, m, and the aggregate productivity, z, at the beginning of each period. The wage rate and the price of the intermediate good are determined by these two state variables, that is, $q = zf'(m)$ and $w = y - qm$. Here we have assumed that the output production function features constant returns to scale.

We are going to compare an equilibrium with only one bank and a symmetric equilibrium with many banks. In the equilibrium with many banks, the banks compete with each other. In this capacity-constrained Bertrand competition with a larger number of entrepreneurs N, the number of banks B does not affect the equilibrium, as long as there are more than two banks and N/B is always very large. By comparing the two equilibria, we show that the competition across banks lowers the equilibrium lending rate and creates excess demand for funding.

4.1 One bank equilibrium

If there is only one bank, the bank can earn the highest possible profit by posting $\gamma = \hat{q}$. We assume that an entrepreneur always invests in its project as long as it receives funds, even if it earns zero profit. So $\gamma = \hat{q}$ means that the bank gets all the surplus from the invested projects and the entrepreneurs earn zero profits. In this case, the marginal project $\vec{\theta}$, above which all projects will be financed, satisfies $\vec{\theta} = \hat{\theta} = \eta/\hat{q}$, where $\eta = (1-k)\,r + k\rho$ is the marginal cost of one unit of funds.

In the equilibrium, the bank lends exactly what it has raised. The bank has no incentive to raise more funds than what is needed for its investment, since there is no benefit but cost from additional funding. In lemma 1 we can show that this statement is true for both an equilibrium with only one bank and a symmetric equilibrium with many banks.

Lemma 1 *In a symmetric equilibrium, all the banks raise an amount of funds such that* $\Phi_i(\gamma_i) = n_i(\gamma_i)\left[1 - G\left(\hat{\theta}_i(n_i(\gamma_i))\right)\right] = D_i$.

Proof. In a symmetric equilibrium, knowing that the total loan is $\Phi_i(\gamma_i)$ given by (4), a bank will not raise more funds than necessary for its planed investment since additional funding is costly. As a result, $D_i = S_i + K_i \leq n_i(\gamma_i)\left[1 - G\left(\hat{\theta}_i(n_i(\gamma_i))\right)\right]$. Next we show that it will not be optimal if $D_i < n_i(\gamma_i)\left[1 - G\left(\hat{\theta}(\gamma_i)\right)\right] x$. Since if so, some profitable projects would not be financed, then the foresighted banks would increase bank equity K_i and deposit S_i in the first place until $S_i + K_i = n_i(\gamma_i)\left[1 - G\left(\hat{\theta}_i(n_i(\gamma_i))\right)\right]$. Moreover, since bank equity is expensive, the bank will let K_i and S_i satisfy $K_i/(K_i + S_i) = k$. ■

According to lemma 1, $N\left[1 - G(\hat{\theta})\right] = D$. Given that $\hat{\theta} = \eta/\hat{q}$, $\eta = (1-k)\,r + k\rho$, and $D = S/(1-k)$, we have $N\{1 - G[((1-k)\,r + k\rho)/\hat{q}]\} = S/(1-k)$. The latter gives a fund-demand function $S^d = S^d(r)$. It is easy to show that $S^{d\prime}(r) < 0$. Together with the fund-supply function $S(r)$ from the consumer's problem, we can solve for a unique equilibrium interest rate given $S'(r) \geq 0$. So there is a unique equilibrium with $\gamma = \hat{q}$.

4.2 A symmetric equilibrium with many banks

If there are more than one bank, then banks cannot maintain a symmetric equilibrium with $\gamma_i = \hat{q}$, for all $i = 1, 2, ..., B$. If all the banks post $\gamma_i = \hat{q}$, the equilibrium outcome is equivalent to that with only a single bank. However, if all the banks post $\gamma_i = \hat{q}$, then a bank has an incentive to deviate from it. If a bank i decreases its γ_i a little bit, so that $\gamma_i < \hat{q}$ and all other banks still post $\gamma = \hat{q}$, then an entrepreneur can expect a positive profit from visiting bank i. As a consequence, all the entrepreneurs would be attracted to the deviating bank i. If all the entrepreneurs come to island i, the bank i can select better projects than before the deviation. Let $\vec{\theta}_i^d$ be the project selection criterion by the bank i, let $\vec{\theta}^*$ be the project selection criterion if all the banks post $\gamma = \hat{q}$, then $\vec{\theta}_i^d > \vec{\theta}^*$. The bank faces all N potential projects when it deviates from $\gamma_i = \hat{q}$, while it faces N/B if it posts $\gamma_i = \hat{q}$. The total funding D_i can now be used to support $\left[1 - G(\vec{\theta}_i^d)\right] N$ number of projects, that is, $\left[1 - G(\vec{\theta}_i^d)\right] N = D_i$, while before the deviation D_i can be used to support $\left[1 - G(\vec{\theta}^*)\right] N/B$ number of projects, that is, $\left[1 - G(\vec{\theta}^*)\right] N/B = D_i$. Since $B \geq 2$, we have $\vec{\theta}_i^d > \vec{\theta}^*$. Therefore, the average success probability of the invested projects is higher. Using the same funding D_i, now the bank i can invest in the same number of projects with a much higher average probability of success, so the bank i would deviate from posting $\gamma_i = \hat{q}$. As a result, it is not an equilibrium if all the banks post $\gamma = \hat{q}$. If there is a symmetric equilibrium with bank competition, then $\gamma_i < \hat{q}$.

The markets clearing conditions are apparent in the labor market, the intermediate good market, and the credit market. In the consumption good market, it should be $c_1^y + c_1^o + S_1 + c_{b,1} + K_1 = y_1 + b_1 + a_0$, where $y_1 = z_1 f(m_0)$, and $c_t^y + c_t^o + S_t + c_{b,t} + K_t + \rho b_{t-1} = y_t + b_t$, for $t = 2, 3, \ldots$. We assume that the initial intermediate good m_0 is owned by the old households. The bank's initial capital a_0 is large enough so that the bank does not need to borrow in order to satisfy the capital requirement in the first period, that is, $b_1 = 0$ and $a_0 \geq K_1$, and more specifically, $c_{b,1} + K_1 = a_0$. The demand for the consumption good consists of the total consumption by the young and the old households, c_t^y and c_t^o, respectively, the deposit of the young households, S_t, the total consumption by the banks, $c_{b,t}$, the total bank capital, K_t, and the debt repayment, ρb_{t-1}. The supply of the consumption good consists of the total output y_t, the total new debt of the banks from outside, b_t. All the variables in the above market clearing condition are aggregate variables, for example, $b_t = \int b_{i,t} di$.

Definition 2 *A symmetric equilibrium with bank competition is defined by sequences of quantities* $\left\{ \{n_{i,t},\ S_{i,t},\ K_{i,t},\ D_{i,t},\ b_{i,t}\}_{i=1}^{B},\ m_t,\ y_t,\ c_t^y,\ c_t^o \right\}_{t=1}^{\infty}$, *prices* $\{\gamma_t^c,\ r_t^c,\ q_t,\ w_t\}_{t=1}^{\infty}$, *an initial value of intermediate good* m_0, *an initial value of bank capital* a_0, *and a policy parameter* k *such that: (i) the representative young household maximizes its expected life-time utility subject to (1), taking as given the wage rate, the interest rates, and the expected profit from entrepreneurs; (ii) the representative old household consumes everything it gets from its income; (iii) taking as given* *the market deposit rate, the strategy of entrepreneurs and the strategy of other banks, the capital requirement rate k, and the expected price of the intermediate good $\hat{q}_{t+1}$, the banks choose their capital $K_{i,t}$, raise deposit $S_{i,t}$ from the young households, post a committed contract $\left(\gamma_t^c,\ \vec{\theta}_{i,t}\right)$ to maximize the life-time utility; (iv) an entrepreneur chooses a strategy on which islands to attend to maximize its expected profit; (v) the total consumption good is produced according to $y_t = z_t f(m_{t-1})$; (vi) all the markets clear; (vii) all the prices and quantities are identical across islands; and (viii) no banks deviate from the equilibrium.*

In a symmetric equilibrium, we have $n_i = N/B$, $\hat{\theta}_i^c = \eta/\gamma^c$ and $n_i\left[1 - G\left(\tilde{\theta}_i^c\right)\right] = D_i$. Without confusion we have dropped the subscript for t and $\hat{q}$ is the expected price of intermediate goods in the next period. If a symmetric competitive equilibrium with $\gamma^c \in (-\infty, \hat{q})$ exists, we have to ensure that no banks deviate from it. We prove that such a symmetric equilibrium exists and it is unique under certain conditions. This is the most important result in this paper and it is summarized in proposition 3.

Proposition 3 *There exists a stationary symmetric equilibrium with bank competition. This symmetric equilibrium is unique if the distribution of θ satisfies the condition that the term $\left[\int_{\vec{\theta}}^{\bar{\theta}} \theta dG(\theta)\right] / \left\{\vec{\theta}\left[1 - G(\vec{\theta})\right]\right\}$ weakly decreases in $\vec{\theta}$.*

Proof. We will focus on a symmetric strong Nash equilibrium in which all the banks post $\left\{\gamma^c, \vec{\theta}^c\right\}$ for every period, where $\gamma^c \in (-\infty, \hat{q})$. In order to prove the existence of such an equilibrium, we need to prove that there exists a pair of $\left\{\gamma^c, \vec{\theta}^c\right\}$ from which no banks will deviate by posting $\gamma_i^d \neq \gamma^c$. In order to find this equilibrium γ^c, we need to show that $\pi\left(\gamma_i^d\right) \leq \pi\left(\gamma^c\right)$ for any γ_i^d.

First we show that, in the third stage after K_i and S_i are determined, there is an equilibrium pair of $\left\{\gamma^c, \vec{\theta}^c\right\}$ such that if a bank i posts a contract $(\gamma_i^d, \vec{\theta}_i^d)$ with $\gamma_i^d < \gamma^c$, then $\pi\left(\gamma_i^d\right) \leq \pi\left(\gamma^c\right)$. If a bank posts γ_i^d, the corresponding selection rule of the threshold value of project quality $\vec{\theta}_i^d$ should satisfy $\vec{\theta}_i^d > \vec{\theta}_i^c$. This is because a larger number of projects, n_i^d, will be attracted by the new contract, i.e., $n_i^d > n_i^c$, such that the bank can select better projects given a larger pool available. As a consequence, the average quality of projects, $\mathring{\theta}_i^d = \int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta) / \left[1 - G(\vec{\theta}_i^d)\right]$, becomes higher. The bank's profit becomes $\pi_{b,i}^d = n_i^d p_i^d \mathring{\theta}_i^d \gamma_i^d - \eta D_i$. For an entrepreneur, the probability of being financed by bank i, $p_i^d = 1 - G(\vec{\theta}_i^d)$, becomes lower. Since the bank faces a better pool of projects, it uses up all of its funding to finance the projects, i.e., $n_i^d\left[1 - G(\vec{\theta}_i^d)\right] = D_i$.

When a bank varies its contract, it faces a trade-off between the lending rate γ_i^d, and the number of potential projects attracted, n_i^d. It is crucial to figure out how n_i^d moves in response to γ_i^d. Observing $(\gamma_i^d, \vec{\theta}_i^d)$, an entrepreneur will visit island i if its expected profit from borrowing at island i is higher than or equal to what it could get from other islands. If we consider an economy with a large number of banks and entrepreneurs, the last visitor (marginal visitor) will have the same profit as if it visited any other islands, that is

$$p_i^d \mathring{\theta}_i^d (\hat{q} - \gamma_i^d) = (\hat{q} - \gamma_i^d) \int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta) = \pi_e^c. \tag{5}$$

Equation (5) gives an indifference curve over the choices of $\left(\gamma_i^d,\ \vec{\theta}_i^d\right)$ for an entrepreneur. The expected profit of an entrepreneur in the initial symmetric equilibrium with γ^c is

$$\pi_e^c = \left[1 - G(\vec{\theta}_i^c)\right] \mathring{\theta}_i^c (\hat{q} - \gamma^c) = (\hat{q} - \gamma^c) \int_{\vec{\theta}_i^c}^{\bar{\theta}} \theta dG(\theta).$$

Differentiating equation (5) completely, we get

$$\frac{d\vec{\theta}_i^d}{d\gamma_i^d} = -\frac{\int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{(\hat{q} - \gamma_i^d)\vec{\theta}_i^d g(\vec{\theta}_i^d)}. \tag{6}$$

Given the indifference curve of the entrepreneurs, the expected profit of the deviating bank is

$$\pi_{b,i}^d \left(\gamma_i^d\right) = \frac{D_i}{1 - G(\vec{\theta}_i^d)} \frac{\gamma_i^d \pi_e^c}{(\hat{q} - \gamma_i^d)} - \eta D_i.$$

Here we have used $n_i^d = D_i / \left[1 - G(\vec{\theta}_i^d)\right]$. Define $\psi(\gamma_i^d) \equiv \frac{\partial \pi_{b,i}^d(\gamma_i^d)}{\partial \gamma_i^d}$, then

$$\psi(\gamma_i^d) = \frac{\pi_e^c D_i}{\left[1 - G(\vec{\theta}_i^d)\right] (\hat{q} - \gamma_i^d)^2} \left\{ \hat{q} - \gamma_i^d \frac{\int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{\vec{\theta}_i^d \left[1 - G(\vec{\theta}_i^d)\right]} \right\}.$$

Here we have used (6) from the entrepreneurs' indifference curve.

Notice that $\psi(\gamma_i^d)$ has the same sign as $\phi(\gamma_i^d)$, the later is defined as

$$\phi(\gamma_i^d) = \hat{q} - \gamma_i^d \frac{\int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{\vec{\theta}_i^d \left[1 - G(\vec{\theta}_i^d)\right]}. \tag{7}$$

Now we will show that $\psi(\gamma_i^d) = 0$ has a unique solution. First, $\phi(\gamma_i^d)$ is weakly decreasing in γ_i^d given the assumption that $\int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta) / \left\{\vec{\theta}_i^d \left[1 - G(\vec{\theta}_i^d)\right]\right\}$ is weakly decreasing in $\vec{\theta}_i^d$. Second, $\lim_{\gamma_i^d \to 0} \phi(\gamma_i^d) = \hat{q} > 0$. Third, $\lim_{\gamma_i^d \to \hat{q}} \phi(\gamma_i^d) = \lim_{\gamma_i^d \to \hat{q}} \hat{q} - \gamma_i^d \int_{\vec{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta) / \left\{\vec{\theta}_i^d \left[1 - G(\vec{\theta}_i^d)\right]\right\} < 0$. All the above three imply that there exists a unique γ_i^{d*} such that $\phi(\gamma_i^{d*}) = 0$. It will be easy to show that γ_i^{d*} is also the unique solution for $\psi(\gamma_i^{d*}) = 0$.

Now we show that no banks post γ_i^d such that $\gamma_i^d > \gamma_i^{d*}$. The proof is relatively simpler. If a bank i can post a contract $(\gamma_i^d, \vec{\theta}_i^d)$ and $\gamma_i^d > \gamma_i^{d*}$ to earn a higher expected profit, then other banks would do the same. However, all the banks posting $(\gamma_i^d, \vec{\theta}_i^d)$ with $\gamma_i^d > \gamma_i^{d*}$ is not an equilibrium; as shown above, banks will deviate from it until $\gamma_i^d = \gamma_i^{d*}$. This γ_i^{d*} defines the unique stationary symmetric equilibrium, i.e., $\gamma_i^{d*} = \gamma^c$. ■

We have shown that $\gamma < \hat{q}$ in the equilibrium with bank competition. Consequently, some projects that could be profitable if $\gamma = \hat{q}$ are not worth investing from the banks' perspective. To maximize profit, the banks have already made the best use of equity and debt. The banks do not want to use more equity, because it is expensive, i.e., the marginal cost of equity ρ is high. Given the bank equity and the capital requirement, the bank has already used up the maximum amount of deposit it can get. In the equilibrium, the banks cannot finance the rationed projects through the traditional equity and debt. The coexistence of potential profitable projects and shortage of funding may motivate the banks to innovate "out of balance sheet" activities.

Of course, loan-sales is not just about raising more funds, and it is also about how to raise more funds (how to design and price loan-sales, see among others DeMarzo and Duffie [30], and Rahi and Duffie [31]), which is not the focus of the current model. It is important that through loan-sales banks can sell illiquid assets to raise funds, repackaging the assets of different levels of risk to fit the final investors's taste for risks. This whole process of loan-sales increases the supply for funding. But if there aren't any potentially profitable projects available, banks have no need to raise more "out of balance sheet" funds. Our focus is the creation of demand for funding via the competition among banks. By making some potentially profitable projects rationed, bank competition could be a trigger for loan-sales.

Let us add some remarks on the bank capital. In an environment of bank competition, capital requirement reduces the potential benefit from deviating the symmetric equilibrium. This is because the deposit at a bank is restricted by the capital requirement rate. With restricted size of deposit, the benefit from attracting additional projects is limited. So capital requirement has the effect of preventing excess bank competition, thus causing banks to earn positive profit in the equilibrium. Without this restriction, the equilibrium will be a Bertrand equilibrium with all the banks earning zero profit.

5 Symmetric equilibrium with loan-sales

In this paper, loan-sales is defined as pooling contractual debts that have different risk levels and selling them to households. To be consistent with the above model environment, we let loan-sales be the fourth stage of banks' decisions, after the banks post their loan contract in the third stage. Given the regulation on λ, the bank i is allowed to sell up to $1 - \lambda$ proportion of its loan. The fund from selling loans is used to finance the projects rationed in the third stage.

Recall that the information about the types of entrepreneurs is revealed after they visit an island. This information is a common knowledge to both the bank and the entrepreneurs in the island, but the information is sealed to other banks. Since we have assumed that banks must evaluate the projects before they can invest in them and the evaluation could be done only at the beginning of a period, the entrepreneurs do not move after their types have been revealed by one bank. This is because they cannot get funded by other banks in the same period. If the banks can raise funds and invest in the rationed projects, the banks can take the whole profit from these projects since the entrepreneurs do not have outside options. This assumption simplifies the analysis, but the general result should not be affected.

To solve the problem in the loan-sales stage, we first find the threshold value of θ, above which the projects are going to be financed, $\vec{\theta}_i^a$. The value of $\vec{\theta}_i^a$ depends on the profitability of projects and the availability of the marketable loans. We denote $\hat{\theta}_i^a$ as the threshold value of θ, above which projects are profitable, then $\hat{\theta}_i^a$ should satisfy $xr^a\mathring{\theta}_i = \mathring{q}\hat{\theta}_i^a$. Let $\tilde{\theta}_i^a$ be the threshold value of θ that satisfies $n_i\left[G(\vec{\theta}_i) - G\left(\tilde{\theta}_i^a\right)\right] = (1-\lambda)\,\Phi_i$, where $\vec{\theta}_i$ is the project quality above which projects were financed in the third stage. Recall that $\Phi_i = \min\left\{n_i\left[1 - G(\hat{\theta}_i)\right],\ D_i\right\}$ is the total lending that had gone to the investments in the third stage. So $(1-\lambda)\,\Phi_i$ is the maximum quantity of loan that could be securitized and sold by bank i. Then, $\vec{\theta}_i^a = \max\left\{\hat{\theta}_i^a, \tilde{\theta}_i^a\right\}$.

The projects with quality between $\vec{\theta}_i$ and $\vec{\theta}_i^a$ are not financed before the banks sell their loans. Bank competition causes $\vec{\theta}_i$ to be higher than $\vec{\theta}_i^*$, the later being the threshold quality in the one bank equilibrium. In the equilibrium with loan-sales, $\vec{\theta}_i^a$ might be higher or lower than $\vec{\theta}_i^*$. If the bank chooses to sell less than $1-\lambda$ proportion of its loan, it is for sure that $\vec{\theta}_i^a < \vec{\theta}_i^*$. This is because the bank can get enough funds to support all the projects that are profitable, i.e., the revenue from the project covers the cost of the project. Moreover, raising money through loan-sales is less costly than doing it through deposit, since there is no capital requirement on loan-sales. Of course, this might not be true if there are other costs associated with loan-sales. On the other hand, if the bank chooses to sell $1-\lambda$ proportion of its loan, then $\vec{\theta}_i^a$ might be higher than $\vec{\theta}_i^*$, due to insufficient funding.

The demand for funding from new projects is $n_i\left[G(\vec{\theta}_i) - G\left(\vec{\theta}_i^a\right)\right]$, which weakly decreases in r^a through $\vec{\theta}_i^a$. The supply of funding from the sale of loans is denoted by $S_i^a\left(r^a, \acute{\theta}\right)$, where r^a is the interest rate for loan-sales and $\acute{\theta}$ is the market success probability of loan-sales. The supply of funding is eventually the households' spending on loan-sales which strictly increases in r^a, given a fixed $\acute{\theta}$. Given r^c, $\vec{\theta}_i$, $\acute{\theta}$, and Φ_i determined in the third stage, there exists an r^a such that the demand for funding equals the supply of funding in the fourth stage, i.e.,

$$n_i \left[G(\vec{\theta}_i) - G\left(\vec{\theta}_i^a\right)\right] = S_i^a\left(r^a, \acute{\theta}\right). \tag{8}$$

Through loan-sales the economy can extend the number of financed projects from $N\left[1 - G(\vec{\theta}_i)\right]$ to $N\left[1 - G(\vec{\theta}_i^a)\right]$. As a result, the economy is going to have a higher level of the intermediate good, $m' = N\mathring{\theta}_i^a p_i^a$, where $\mathring{\theta}_i^a = \int_{\vec{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)/p_i^a$ and $p_i^a = 1 - G(\vec{\theta}_i^a)$. Recall that $\hat{q} = \hat{z}\ f'(m')$ and $w = y - \hat{z}\ f'(m')m'$, so the expected price of the intermediate good $\hat{q}$ will decrease, while the total output y and wage rate w will increase.

In the fourth stage, the bank's problem is to maximize the total profit subject to a "skin in the game" constraint

$$n_i \left[G(\vec{\theta}_i) - G\left(\vec{\theta}_i^a\right)\right] \leq (1 - \lambda)\,\Phi_i. \tag{9}$$

Knowing the problem the bank is going to face in the fourth stage, it chooses a pair of $(\gamma_i,\, \vec{\theta}_i)$ in the third stage to maximize the combined profit from the third and the fourth stages.

In the third stage, the indifference curve over $\left(\gamma_i,\ \vec{\theta}_i\right)$ for the entrepreneurs is still the same as in the case of no loan-sales, since we have assumed that the banks get all the profits in the loan-sales stage so that the entrepreneurs only need to consider their profits in the third stage. However, the banks may post a γ_i higher or lower than in the case of no loan-sales, since γ_i affects not only the bank's profit in the third stage, but also the bank's profit in the fourth stage. Given the interest rate r^c, the expected interest rate r^a, the total funding D_i, the indifference curve of the entrepreneurs, and the strategy of other banks, the bank i chooses a pair of $(\gamma_i,\, \vec{\theta}_i)$ to maximize the following combined profit from the third and the fourth stages:

$$\pi_{b,i} = \max_{\gamma_i} n_i \frac{\pi_e^c \gamma_i}{\hat{q} - \gamma_i} - \eta D_i + n_i\left(\hat{q} - \acute{\theta} r^a\right) \int_{\vec{\theta}_i^a}^{\vec{\theta}_i} \theta dG(\theta). \tag{10}$$

Here, $n_i = D_i / \left[1 - G\left(\vec{\theta}_i\right)\right]$, and $\vec{\theta}_i = \tilde{\theta}_i$, since the banks still have no need to raise more funding than necessary.

Note that it is possible that $\eta > \gamma_i \vec{\theta}_i$, i.e., the bank may invest in some projects with negative expected return in the third stage. The purpose of investing in these "non-profitable" projects is that the banks can sell their loans. The funds raised through loan-sales can be used to invest in new projects to earn more profit, which may cover the loss incurred by the projects with $\eta > \gamma_i \theta_i$.

The possibility of investing in "non-profitable" projects highlights the banks' motive for re-selling their loans: they have good originating opportunity. This originating opportunity is caused by excess competition across banks: a low γ_i increases the lending standard and cuts off funding for some good projects. The banks put themselves in a difficult situation: if a bank increases its

γ_i, it loses a pool of potential projects to its competitors; if the bank keeps a low γ_i, then only some very good projects can give the bank enough return to cover the cost of funding and many good projects cannot be financed. The banks may choose to post a relatively low γ_i to attract the potential projects to them, and then get additional funding from selling their loans to finance the projects that could not get financed in the first run.

Whether or not we have some "non-profitable" projects being invested in the third stage may depend on whether or not the "skin in the game" constraint (9) is binding and on the intensity of bank competition. If the constraint is binding, then the bank may invest in some projects that have negative expected return, i.e., $\eta > \gamma_i \vec{\theta}_i$. Since a binding "skin in the game" constraint makes selling additional loans valuable, the banks have an incentive to increase the volume of their loans. On the other hand, if the bank expects that the constraint (9) will not be binding, then it has no need to invest in "non-profitable" projects, and in that case $\eta \leq \gamma_i \vec{\theta}_i$. If the bank competition is intense, then banks may try to attract entrepreneurs using a very attractive contract, which may give themselves negative profit in the third stage.

Depending on whether or not the constraint (9) is binding, we have two possible cases. In the first case, if the constraint (9) is binding, then not all projects with $\theta \geq \hat{\theta}_i^a = r^a \mathring{\theta}_i / \hat{q}$ are invested, so $\vec{\theta}_i^a = \tilde{\theta}_i^a \geq \hat{\theta}_i^a$, where $\tilde{\theta}_i^a$ is determined by $n_i \left[G(\tilde{\theta}_i) - G\left(\tilde{\theta}_i^a\right) \right] = (1 - \lambda)\, \Phi_i$. In the second case, if the constraint (9) is not binding, then $\vec{\theta}_i^a = \hat{\theta}_i^a$ and $r^a \mathring{\theta}_i = \hat{q} \hat{\theta}_i^a$.

We define D_i^b by $N/B(1 - G(\tilde{\theta}_i^b)) = D_i^b$ and $N/B \left[G(\tilde{\theta}_i^b) - G\left(\hat{\theta}_i^{a,b}\right) \right] = (1 - \lambda)\, D_i^b$, where $\hat{\theta}_i^{a,b} = r^a \mathring{\theta}_i^b / \hat{q}$ and $\mathring{\theta}_i^b = \int_{\tilde{\theta}_i^b}^{\bar{\theta}} \theta dG(\theta) / \left[1 - G\left(\tilde{\theta}_i^b\right) \right]$. If $\Phi_i \leq D_i^b$, then the "skin in the game" constraint is binding.

We first consider the case in which the constraint (9) is binding. In this case, the binding "skin in the game" constraint gives

$$G(\tilde{\theta}_i) - G\left(\tilde{\theta}_i^a\right) = (1 - \lambda) \left[1 - G(\tilde{\theta}_i) \right]. \tag{11}$$

We differentiate completely the equation (11) to get

$$\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} = \frac{(2 - \lambda)\, g(\tilde{\theta}_i)}{g(\tilde{\theta}_i^a)}. \tag{12}$$

The term $\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} > 0$ in (12), indicating that if $\tilde{\theta}_i$ increases, then $\tilde{\theta}_i^a$ increases. It means that a higher quality of assets (a smaller volume of assets) in the third stage would cause a lower amount of loan sales in the fourth stage, due to the binding "skin in the game" constraint.

In the first case, a bank's problem is to choose a γ_i to maximize the total profit, from the investment in both the third stage and the fourth stage, taking D_i as given,

$$\pi_{b,i} = \max_{\gamma_i} n_i \frac{\pi_e^c \gamma_i}{\hat{q} - \gamma_i} - \eta D_i + n_i \hat{q} \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) - n_i \acute{\theta} r^a \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} dG(\theta). \tag{13}$$

We can show that there is a unique symmetric Nash equilibrium. This result is summarized in proposition 4.

Proposition 4 *In the case where the constraint (9) is binding, there exists a unique value of* γ_i *that maximizes the profit of the bank* i.

Proof. Given the profit function of a bank i from (13),

$$\pi_{b,i} = \max_{\gamma_i} \frac{D_i}{1-G(\tilde{\theta}_i)} \left[\frac{\gamma_i \pi_e^c}{(\hat{q}-\gamma_i)} + \hat{q} \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) \right] - \eta D_i - n_i \acute{\theta} r^a \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} dG(\theta).$$

As in the case without loan-sales, the banks take the indifference curve of entrepreneurs as given. The first order derivative of $\pi_{b,i}$ with respect to γ_i is

$$\psi(\gamma_i) = \frac{D_i \hat{q} \pi_e^c}{\left[1-G(\tilde{\theta}_i)\right](\hat{q}-\gamma_i)^2 \tilde{\theta}_i} \left[\tilde{\theta}_i^a \left[1 - G\left(\tilde{\theta}_i^a\right)\right] - \frac{\gamma_i}{\hat{q}} \int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta) - \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) \right]. \tag{14}$$

To derive (14), we have used $\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i}$ from (12), the entrepreneur's trade-off between γ_i and $\tilde{\theta}_i$ given by (6), π_e^c from (5), and the binding "skin in the game" constraint $G(\tilde{\theta}_i) - G\left(\tilde{\theta}_i^a\right) = (1-\lambda)\left[1-G(\tilde{\theta}_i)\right]$.

Now we will show that the solution to $\psi(\gamma_i) = 0$ exists, i.e., the equilibrium exists. We define

$$\phi(\gamma_i) = \left[\tilde{\theta}_i^a \left[1 - G\left(\tilde{\theta}_i^a\right)\right] - \frac{\gamma_i}{\hat{q}} \int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta) - \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) \right]. \tag{15}$$

The sign of $\phi(\gamma_i)$ determines the sign of $\psi(\gamma_i)$. We first look at the sign of $\phi(\gamma_i)$ when $\gamma_i \to \hat{q}$,

$$\lim_{\gamma_i \to \hat{q}} \phi(\gamma_i) = -\int_{\tilde{\theta}_i^a}^{\bar{\theta}} \left(\theta - \tilde{\theta}_i^a\right) dG(\theta) < 0.$$

Second, we look at the value of $\phi(\gamma_i)$ as $\gamma_i \to -\infty$. We have

$$\lim_{\gamma_i \to -\infty} \phi(\gamma_i) > 0. \tag{16}$$

Since $\phi(\gamma_i)$ is continuous, there exists a γ_i^c such that $\phi(\gamma_i^c) = 0$, i.e., there exists a γ_i^c such that $\psi(\gamma_i^c) = 0$. We have proved the existence of a symmetric equilibrium.

Next we will prove the uniqueness of the equilibrium. A sufficient condition is that the function $\phi(\gamma_i)$ is monotonically decreasing in γ_i. From (15) we can get

$$\phi'(\gamma_i) = -\frac{(2-\lambda)^2 \left[1-G(\tilde{\theta}_i)\right] \int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{(\hat{q}-\gamma_i)\tilde{\theta}_i g(\tilde{\theta}_i^a)} < 0.$$

As a result, the solution to $\phi(\gamma_i) = 0$ is unique. We have a unique stationary symmetric equilibrium. ■

Next, we need to figure out the optimal decision of D_i in the second stage. The total amount of funding D_i cannot be determined by equating $\hat{\theta}_i = \tilde{\theta}_i$, while it is true in the case without loan-sales. The optimal amount of D_i is obtained from maximizing the bank's total profit, i.e.,

$$\max_{D_i} \pi_{b,i}(\gamma_i, D_i) = \max_{D_i} \frac{N}{B}\gamma_i \int_{\vec{\theta}_i}^{\bar{\theta}} \theta dG(\theta) - \eta D_i + \frac{N}{B}\hat{q}\int_{\tilde{\theta}_i^a}^{\vec{\theta}_i} \theta dG(\theta) - \acute{\theta} r^a (1-\lambda) D_i.$$

This decision of D_i is made before γ_i. Using backward induction, the optimal pair $\left(\gamma_i, \vec{\theta}_i\right)$ is a function of D_i. Using envelop therem, we have the first order condition

$$\gamma_i \tilde{\theta}_i - \eta + \left[(2-\lambda)\tilde{\theta}_i^a - \tilde{\theta}_i\right]\hat{q} - (1-\lambda)\acute{\theta} r^a = 0. \tag{17}$$

To get (17), we have used $D_i = N/B\left[1 - G(\tilde{\theta}_i)\right]$ and so $\frac{d\tilde{\theta}_i}{dD_i} = -B/N/g(\tilde{\theta}_i)$, and $\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} = \frac{(2-\lambda)g(\tilde{\theta}_i)}{g(\tilde{\theta}_i^a)}$. Note that in the case without loan sales the optimal solution satisfies $\gamma_i\tilde{\theta}_i = \eta$. But in this case with loan sales, the optimal condition becomes $\gamma_i\tilde{\theta}_i = \eta - \left\{\left[(2-\lambda)\tilde{\theta}_i^a - \tilde{\theta}_i\right]\hat{q} - (1-\lambda)\acute{\theta} r^a\right\}$. The additional term $\left[(2-\lambda)\tilde{\theta}_i^a - \tilde{\theta}_i\right]\hat{q} - (1-\lambda)\acute{\theta} r^a$ is the net marginal benefit of additional investment in projects due to increase in funding. It is also easy to show that $\pi_{b,i}(\gamma_i, D_i)$ is concave in D_i.

In the case where the "skin in the game" constraint (9) is not binding, there is no stationary symmetric equilibrium. We will show that in lemma 5.

Lemma 5 *If the constraint (9) is not binding, there is no stationary symmetric equilibrium.*

Proof. If the constraint (9) is not binding, in the fourth stage, all the profitable projects will be financed, then the marginal project satisfies $r^a\acute{\theta} = \hat{q}\hat{\theta}_i^a$. As a result, $\frac{d\hat{\theta}_i^a}{d\gamma_i} = 0$.

The bank's maximization problem becomes:

$$\pi_{b,i} = \max_{\gamma_i} n_i \frac{\gamma_i \pi_e^c}{(\hat{q} - \gamma_i)} - \eta D_i + n_i \hat{q}\int_{\hat{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) - n_i r^a \acute{\theta}\int_{\hat{\theta}_i^a}^{\tilde{\theta}_i} dG(\theta), \tag{18}$$

where $n_i = D_i / \left\{\left[1 - G\left(\tilde{\theta}_i\right)\right]\right\}$. The first order derivative of $\pi_{b,i}$ with respect to γ_i is

$$\psi(\gamma_i) = \frac{D_i \hat{q}\pi_e^c}{\left[1 - G\left(\tilde{\theta}_i\right)\right]^2 (\hat{q} - \gamma_i)\tilde{\theta}_i}\left[-\frac{\gamma_i}{\hat{q}}\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta) - \int_{\hat{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) + \hat{\theta}_i^a\left[1 - G\left(\hat{\theta}_i^a\right)\right]\right].$$

We define

$$\phi(\gamma_i) = -\frac{\gamma_i}{\hat{q}}\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta) - \int_{\hat{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) + \hat{\theta}_i^a\left[1 - G\left(\hat{\theta}_i^a\right)\right]. \tag{19}$$

Then, $\phi'(\gamma_i) = 0$. As a result, the second derivative of profit with respect to γ_i is also zero.

It makes sense that $\frac{d^2\pi_{b,i}}{d\gamma_i} = 0$. Given a D_i, the bank's expected profit does not vary with different choices of $\left(\gamma_i, \vec{\theta}_i\right)$, as long as it makes the entrepreneurs indifferent. The combined benefit in the third and the fourth stage is determined by the threshold value $\hat{\theta}_i^a$ and the total expected profit of the entrepreneurs. However, if this is true, then the bank would reduce its D_i in the second stage. This is because it can use a lower γ_i and a higher $\vec{\theta}_i$ to get the same profit, and a reduction in D_i will give the same benefit but cause less cost. Evantually, D_i will goes to D_i^b, and the "skin in the game" constraint will become binding. In the case where the "skin in the game" constraint is not binding, we have no stationary symmetric equilibrium. ■

Combining the results from proposition 4 and lemma 5, we can expect a unique equilibrium with a binding constraint.

6 Example

In this example we solve a symmetric equilibrium with a uniform distribution of θ, i.e., $G(\theta) = (\theta - \underline{\theta}) / (\bar{\theta} - \underline{\theta})$. With a uniform distribution of θ, a unique symmetric equilibrium exists in the case of no loan-sales according to proposition 3 and a unique symmetric equilibrium exists in the case of loan-sales according to proposition 4 as well.

6.1 Equilibrium without loan-sales

We first derive the entrepreneur's indifference curve. Given the total funding D_i and that $n_i \left[1 - G\left(\tilde{\theta}_i\right)\right] = D_i$, the threshold value of θ above which the projects can be financed is $\tilde{\theta}_i = \bar{\theta} - \left(\bar{\theta} - \underline{\theta}\right) D_i / n_i$. Without loan-sales, $\hat{\theta}_i = \tilde{\theta}_i$, according to lemma 1. Since the marginal project breaks even, we have

$$r^c = \left[\gamma_i \tilde{\theta}_i - k\rho\right] / (1 - k). \tag{20}$$

Substituting $\tilde{\theta}_i$ into the entrepreneur's expected profit, we get

$$\pi_e^c = (\hat{q} - \gamma_i) \left[2\bar{\theta} D_i / n_i - \left(\bar{\theta} - \underline{\theta}\right) (D_i / n_i)^2\right] / 2. \tag{21}$$

Equation (21) gives an indifference curve over the choices of (γ_i, n_i) for an entrepreneur. We express γ_i in terms of n_i,

$$\gamma_i = \hat{q} - 2\pi_e^c / \left[2\bar{\theta} D_i / n_i - \left(\bar{\theta} - \underline{\theta}\right) (D_i / n_i)^2\right]. \tag{22}$$

Taking as given the indifference curve of the entrepreneurs (21) and their expected profit π_e^c, the expected profit of a bank is $\pi_{b,i}(n_i) = \max_{n_i} n_i \pi_e^c \gamma_i / (\hat{q} - \gamma_i) - \eta D_i$. Here we have made n_i

the choice variable, instead of γ_i, just for convenience. We substitute γ_i from (22) into the bank's profit function to get

$$\pi_{b,i}(n_i) = \max_{n_i} n_i \left\{ \hat{q} \left[\bar{\theta} D_i/n_i - \left(\bar{\theta} - \underline{\theta}\right) (D_i/n_i)^2 /2 \right] - \pi_e^c \right\} - \eta D_i. \tag{23}$$

The first order condition is $\psi(n_i) = \hat{q}\left(\bar{\theta} - \underline{\theta}\right)(D_i/n_i)^2/2 - \pi_e^c = 0$. We can solve for n_i^c, $n_i^c = D_i\sqrt{\hat{q}\left(\bar{\theta} - \underline{\theta}\right)/\left(2\pi_e^c\right)}$. In the symmetric equilibrium, we should have $n_i^c = N/B$. We can solve for π_e^c,

$$\pi_e^c = \hat{q}\left(\bar{\theta} - \underline{\theta}\right)\left[BD_i/N\right]^2/2. \tag{24}$$

Using (24) and (22), we can solve for γ_i given the amount of funding D_i,

$$\gamma_i = \hat{q} - \hat{q}\frac{\left(\bar{\theta} - \underline{\theta}\right)\left(\frac{BD_i}{N}\right)}{2\bar{\theta} - \left(\bar{\theta} - \underline{\theta}\right)\left(\frac{BD_i}{N}\right)}. \tag{25}$$

We have solved the problem of banks as credit suppliers and the entrepreneurs' problem. Given the total projects being invested, the quantity of the intermediate goods is $m = p_i\mathring{\theta}_i N$. The corresponding price of the intermediate good and wage rate are

$$\hat{q} = \hat{z}f'\left(p_i\mathring{\theta}_i N\right) \text{ and } w = \hat{z}f\left(p_i\mathring{\theta}_i N\right) - \hat{q}p_i\mathring{\theta}_i N. \tag{26}$$

Next, we are going to solve the problem of banks as credit demanders and the problem of households.

The representative young household takes the value of w, $\hat{q}$, and r^c as given. The optimal choice of deposit satisfies the following first order conditions:

$$-u'\left(w - \sum_{i=1}^{B} S_i\right) + \beta r^c E u'\left(r^c\sum_{i=1}^{B} S_i + \tilde{\pi}_e^c\right) \geq 0, \text{ if } 0 < S_i \leq S_i^h, \tag{27}$$

where S_i^h is the limit of deposit contract that bank i could provide since the bank i is restricted by the bank capital K_i through $K_i/\left(K_i + S_i^h\right) = k$. In the symmetric equilibrium,

$$S_i = S_i^h \text{ and } u'\left(w - BS_i\right) = \beta r^c E u'\left(r^c BS_i + \tilde{\pi}_e^c\right). \tag{28}$$

We can solve for the supply function $S_i(r^c)$ from (28).

In the equilibrium, we should have the total funding supply equals the total funding demand, that is

$$p_i N = BS_i(r^c)/(1-k). \tag{29}$$

In a steady state, we can solve the seven equations of (20), (24), (25), (26), (28) and (29) to get the equilibrium $\hat{q}$, w, r^c, S_i, γ_i, $\tilde{\pi}_e^c$, and $\tilde{\theta}_i$.

6.2 Equilibrium with loan-sales

6.2.1 Binding constraint

We first consider the case where the "skin in the game" constraint is binding, $(G(\tilde{\theta}_i)-G(\tilde{\theta}_i^a))n_i = (1-\lambda)\,D_i$. In the equilibrium the funding markets clear in both the stage three and stage four, i.e., $BS_i(r^c)/(1-k) = \left(1-G(\tilde{\theta}_i)\right)n_i$, and $BS_i^a(r^a,r^c) = (G(\tilde{\theta}_i)-G(\tilde{\theta}_i^a))n_i$. These conditions together imply that

$$S_i^a = (1-\lambda)\,D_i. \tag{30}$$

Use (30) and the above market clearing conditions, we can derive the following relationship between $\tilde{\theta}_i^a$ and $\tilde{\theta}_i$: $\tilde{\theta}_i^a = (2-\lambda)\,\tilde{\theta}_i - (1-\lambda)\,\bar{\theta}$. Accordingly, $\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} = (2-\lambda)$. Applying equilibrium symmetry and lemma 1, we have $N/B = D_i/\left[1-G\left(\tilde{\theta}_i\right)\right]$. The value of $\tilde{\theta}_i$ and $\tilde{\theta}_i^a$ are

$$\tilde{\theta}_i = \bar{\theta} - \left(\bar{\theta}-\underline{\theta}\right)BD_i/N \text{ and } \tilde{\theta}_i^a = \bar{\theta} - (2-\lambda)\left(\bar{\theta}-\underline{\theta}\right)BD_i/N. \tag{31}$$

The first order derivative of the bank's profit has the same sign as $\phi(\gamma_i)$ defined in (15). Substituting (31) into $\phi(\gamma_i)$, we have

$$\phi(\gamma_i) = \left(-\frac{\gamma_i}{\hat{q}}+1\right)\left[\bar{\theta}BD_i/N - \left(\bar{\theta}-\underline{\theta}\right)\left(BD_i/N\right)^2/2\right] - (2-\lambda)^2\left(\bar{\theta}-\underline{\theta}\right)\left(BD_i/N\right)^2/2.$$

So the first order condition is equivalent to $\phi(\gamma_i^a)=0$, which gives

$$\gamma_i^a = \hat{q} - \hat{q}\frac{(2-\lambda)^2\left(\bar{\theta}-\underline{\theta}\right)BD_i/N}{2\bar{\theta}-\left(\bar{\theta}-\underline{\theta}\right)BD_i/N}. \tag{32}$$

Using a uniform distribution of θ, we can analytically show some properties of the equilibrium with loan-sales. One important result is summarized in Lemma 6.

Lemma 6 *If $\lambda < 1$, there is loan-sales in the equilibrium. If the "skin in the game" constraint is binding, the lending rate is lower compared to an economy without loan-sales, i.e. $\gamma_i^a < \gamma_i$, given the same size of deposit S_i, with a uniform distribution of θ.*

The proof of Lemma 6 is apparent by comparing (32) to (25). We have $\gamma_i^a < \gamma_i$ if $(2-\lambda)^2 > 1$. A lower lending rate in the equilibrium with loan-sales comes from the fact that the banks have more incentive to compete for potential projects if they have access to loan-sales. Moreover, the smaller the value of λ (the looser the "skin in the game" constraint), the smaller the value of γ_i^a, indicating a more intense competition among banks.

Since the "skin in the game" constraint is binding, it is possible that $\hat{\theta}_i^a < \tilde{\theta}_i^a$, i.e., not all the projects can get funding even if they could make profit. So we have $\vec{\theta}_i^a = \tilde{\theta}_i^a \geq \hat{\theta}_i^a$. It is difficult to determine whether $\hat{\theta}_i > \tilde{\theta}_i$ or $\hat{\theta}_i \leq \tilde{\theta}_i$. It is possible that $\hat{\theta}_i > \tilde{\theta}_i$, i.e., banks make negative profit from some projects invested in the third stage. The banks may have incentive to invest in these projects bringing negative profit, because they can relax the "skin in the game" constraint such that they can invest in more projects by selling more loans. The negative profit should be compensated by the profit from increased investment in the fourth stage. Moreover, if a project is invested in the third stage, the bank gets a return rate of γ_i^a; if the project is invested in the fourth stage, the bank gets a return rate of $\hat{q} > \gamma_i^a$. The loss from this change should also be compensated by the additional profits from additional investment due to a relax of the "skin in the game" constraint.

Substituting γ_i^a, $\tilde{\theta}_i$, and $\tilde{\theta}_i^a$ into (17),

$$d^2 - \left[\frac{7-3\lambda}{2(2-\lambda)} - \frac{\vartheta}{2(2-\lambda)^2}\right]\bar{\theta}d + \frac{\bar{\theta}^2}{2-\lambda} - \frac{\bar{\theta}\vartheta}{(2-\lambda)^2} = 0, \tag{33}$$

where $d = \left(\bar{\theta} - \underline{\theta}\right) BD_i/N$ and $\vartheta = \left[\eta + (1-\lambda)\acute{\theta}r^a\right]/\hat{q}$. We can solve for the optimal $D_i \in (0, D_i^b]$ given $\hat{q}$, $\acute{\theta}$, r^a, and η.

The total amount of intermediate goods is $m = p_i^a \mathring{\theta}_i^a N$, where $p_i^a = 1 - G(\tilde{\theta}_i^a)$ and $\mathring{\theta}_i^a = \int_{\tilde{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)/p_i^a$. The price of the intermediate good and the wage rate are

$$\hat{q} = \hat{z}f'\left(p_i^a \mathring{\theta}_i^a N\right) \text{ and } w = \hat{z}f\left(p_i^a \mathring{\theta}_i^a N\right) - \hat{q}p_i^a \mathring{\theta}_i^a N. \tag{34}$$

The saving S_i satisfies

$$\begin{aligned} & u'\left(w - BS_i - BS_i^a\right) \\ = \quad & \beta r^c E\left\{\acute{\theta}u'\left(r^a BS_i^a + r^c BS_i + \tilde{\pi}_e^c\right) + \left(1-\acute{\theta}\right)u'(r^c BS_i + \tilde{\pi}_e^c)\right\}, \end{aligned} \tag{35}$$

and the supply of funds for loan-sales S_i^a satisfies

$$u'\left(w - BS_i - BS_i^a\right) = \beta r^a E\left[\acute{\theta}u'\left(r^a BS_i^a + r^c BS_i + \tilde{\pi}_e^c\right)\right]. \tag{36}$$

In a stationary symmetric equilibrium, we can solve equations (**??**) - (36) to get the equilibrium $\hat{q}$, w, r^c, r^a, S_i, S_i^a, γ_i, $\tilde{\theta}_i$ and $\tilde{\theta}_i^a$.

6.3 Numerical example

In order to see how the model with bank competition and loan-sales works, we do a numerical exercise. For simplicity, we demonstrate a static model here. We use an isoelastic utility function $\frac{c^{1-\sigma}}{1-\sigma}$, a Cobb–Douglas production function $y = \hat{z}m^\alpha$, and some plausible values of parameters: $\alpha = 0.36$, $\quad \hat{z} = 1$, $\bar{\theta} = 1$, $\underline{\theta} = 0$, $B = 2$, $N = 1200$, $k = 0.08$, $\lambda = 0.5$, $\beta = 0.9$ and $\rho = 1.3$. The value of risk aversion parameter has to be less than 1 in order for the deposit supply function to increase in interest rate, we let $\sigma = 0.3$.

First, we need to find an index to measure the competitiveness in the banking sector. The equilibrium lending rate normalized by $\hat{q}$, $\gamma/\hat{q}$ is supposed to decrease if the market becomes more competitive. We look at the thickness of the loan market, N/B. Here, the setup of the model makes the equilibrium result independent of the number of banks, as long as $B \geq 2$ so that the banks will compete with each other. So we vary N to see the response of equilibrium $\gamma/\hat{q}$. As the potential projects increase, the banks become less aggressive to steal from others and post a higher $\gamma/\hat{q}$, as shown in figure 1.

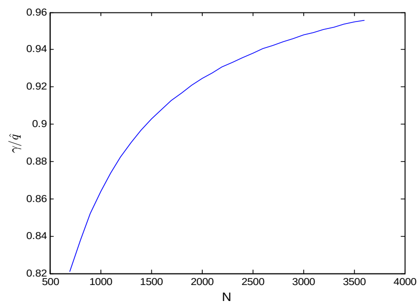

Figure 1 is about here

When N increases, there are two forces to increase the total investment in the third stage. First, the banks face less competition from each other and therefore they can increase the return of lending. Second, the invested projects will have a higher average quality given a larger pool of available projects. To separate the second effect of N from its role in decreasing bank's competition, we define the deviation of total investment in the third stage from a social planner's world (or a world with only one bank) as a measure of the imperfection of the banking industry. Specifically, we define a measure of bank's competitiveness, μ, by $\mu = \frac{\text{investment (in third stage) in bank competition equilibrium}}{\text{investment in one bank equilibrium}}$. A larger μ means that the banks are less competitive. We can see from figure 2 that μ is also increasing in N.

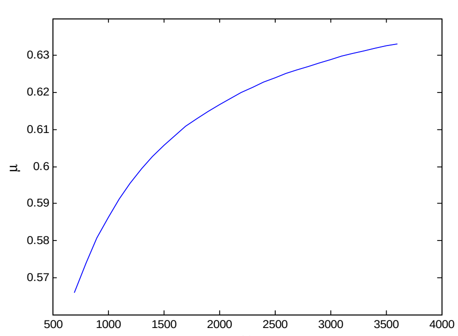

Figure 2 is about here

Second, we want to see the effect of capital requirement on the bank's lending rate and the investment in the third stage. As shown in figure 3 and figure 4, both $\gamma/\hat{q}$ and μ decrease in k. An increase in capital requirement does cause a stronger competition for loan sales. However, the magnitude of the effects is small. As k increases from 10^{-8} to 0.12, the change in lending rate is ignorable and the change in μ is small.

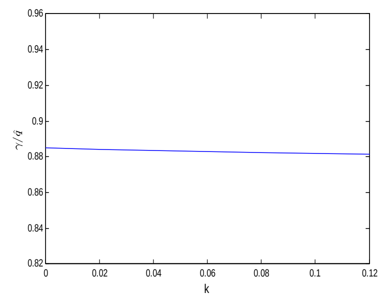

Figure 3 is about here

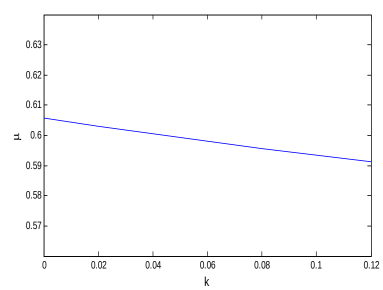

Figure 4 is about here

Finally, the possibility of loan sales can also have effects on bank competition. We show this by varying the value of λ. As λ decreases, the banks may have more flexibility to sell their loans. This increased possibility of loan sales strengthens the competition among banks and it drives down the lending rate.

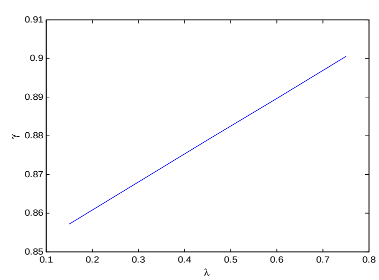

Figure 5 is about here

7 Conclusion

We have built a dynamic general equilibrium model with bank competition. The framework is a directed search model. Capital requirement imposes a short-run capacity constraint on banks' lending. Given the capacity constraint, the banks compete for projects using lending rate. The model is a Bertrand competition with capacity-constraint as in Peters [28] and BSW [32]. We focus on a stationary symmetric mixed strategy equilibrium. We find that bank competition can cause a low equilibrium lending rate and excess demand for funding. As a consequence, banks may seek funds through the sale of their loans. We show that loan-sales could be motivated by a purpose other than risk sharing.

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Variable list

t : time period
B : number of banks (islands)
i : index of a bank (island)
N : number of entrepreneurs
y_t : output of the consumption good in period t
z_t : aggregate productivity
$\hat{z}$: mean of the aggregate productivity
m_t : intermediate good in period t, per young household
m' : intermediate good in next period, per young household
M_t : aggregate intermediate good
q_t : price of intermediate good
$\hat{q}_{t+1}$: expected price of intermediate good
L_t : aggregate labor
w_t : wage rate
$f()$: production function of the consumption good
ξ_j : outcome of the project
j : index of the possible outcome of the projects
θ : quality of a project (success probability of a project)
$\bar{\theta}$: upper bound of θ
$\underline{\theta}$: lower bound of θ
$g(\theta)$: PDF of θ
$G(\theta)$: CDF of θ
$\vec{\theta}_i$: threshold value of θ, above which the project is financed in island i
$\hat{\theta}_i$: lower bound of project quality, above which projects are profitable
$\tilde{\theta}_i$: lower bound of θ above which the project can be financed given bank $i's$ capacity
$\acute{\theta}$: competitive market expected success probability of securities
n_i : total number of entrepreneurs visiting island i
n_i^c : total number of entrepreneurs visiting island i in a symmetric competitive equilibrium
n_i^d : total number of entrepreneurs visiting island i if bank i deviates from symmetric contracts
$\vec{\theta}_i(n_i)$: contract condition on selection threshold according to n_i
$\vec{\theta}_i^*$: contract selection criterion if all the banks post $\gamma = 1$
$\vec{\theta}_i^d$: contract selection criterion if bank i deviates from symmetric contracts
$\vec{\theta}_i^c$: contract selection criterion in a symmetric equilibrium
$\vec{\theta}_i^a$: threshold value of θ above which the projects are financed by selling securities
$\mathring{\theta}_i$: mean value of θ among all the invested projects by bank i without loan-sales
$\mathring{\theta}_i^a$: mean value of θ among all the invested projects by bank i with loan-sales
$\mathring{\theta}_i^d$: mean value of θ among all the invested projects by bank i if the bank deviates from symmetric contracts
$\hat{\theta}_i$: threshold value of θ above which projects are profitable for bank i
$\hat{\theta}_i^c$: threshold value of θ above which projects are profitable for bank i in a symmetric equilibrium
$\hat{\theta}_i^a$: threshold value of θ above which projects are profitable using funding from selling securities for bank i

$\tilde{\theta}_i$: threshold value of θ above which projects could have funding
$\tilde{\theta}_i^c$: threshold value of θ above which projects could have funding in a symmetric equilibrium
$\tilde{\theta}_i^a$: threshold value of θ above which projects could have funding from selling securities
p_i : probability of obtaining funding in island i without loan-sales
p_i^a : probability of obtaining funding in island i with loan-sales
p_i^d : probability of obtaining funding in island i if bank i deviates from symmetric contracts
k : capital requirement rate
K_i : bank capital at bank i
K : aggregate bank capital
a_0 : initial bank equity
$b_{i,t}$: debt of bank i from outside
b_t : debt of the banks from outside
λ : minimum proportion of security a bank must hold to itself
r_t : interest rate for deposit in period t
r_t^a : interest rate for loan-sales: interest rate in period t
r^c : interest rate for deposit in a competitive equilibrium
$S_{i,t}(r_t)$: deposit at bank i (saving function) in period t
$S_{i,t}^a(r_t^a, r_t)$: loans bought from bank i in period t
s_t : storage in period t
D_i : total funding available at bank i
D : aggregate funding
$S(r)$: aggregate supply for fund given interest rate r
$S^d(r)$: aggregate demand for fund given interest rate r
$\pi_{e,t}$: expected profit of one entrepreneur
$\tilde{\pi}_{e,t}$: expected total profit from all invested projects
π_e^c : expected profit of one entrepreneur in a symmetric competitive equilibrium
β : discounting factor of households
$U()$: utility function of households
c_t^y : consumption of young household in period t
c_t^o : consumption of old household in period t
$u_{b,i}$: utility of a bank i
$\pi_{b,i,t}$: expected profit of bank i
$\pi_{b,i}^c$: expected profit of bank i if it deviates from symmetric contracts
$V_{i,t}$: Franchise value of bank i at period t
ρ : time preference of a bank (marginal cost of bank equity)
η : marginal cost of funding through deposit
γ_i : bank's lending rate
γ_i^c : bank's lending rate in a symmetric equilibrium without loan-sales
γ_i^d : bank's lending rate if it deviates from symmetric contracts
γ_i^{d*} : bank's optimal lending rate if it deviates from symmetric contracts
γ_i^a : bank's lending rate in a symmetric equilibrium with loan-sales
$\Phi_i(\gamma_i)$: total loan the bank i would have if it posts γ_i
Φ : aggregate investment
$\psi(\gamma_i)$: first order derivative of a bank's profit with respect to γ_i
$\phi(\gamma_i)$: a term in $\psi(\gamma_i)$, the first order derivative of a bank's profit with respect to γ_i
φ : a term contains parameter λ

σ : risk aversion parameter in the household utility function
α : capital's share in the consumption good production function
μ : market competitiveness measure
κ : optimal proportion of securities