Exercise on Dynamic Programming
2010 Spring
SHUFE

Question 1: An economy has identical households whose preferences are represented by the utility function:

$$\sum\nolimits_{t=0}^{\infty} \beta^t \left[\ln(c_t) - \frac{1}{2} l_t^2\right], \ 0 < \beta < 1.$$

Here c is consumption and l the amount of time the household spends shopping. Each household is endowed with one unit of time in each period. The total resource needed to consume c units of goods is $(c+m)$ units of goods, where m is the transactions cost. Assume that $m = m_0(1-l)c$, where $m_0 \in (0,1)$ is a constant. The production function in this economy is $F(k) = Ak^{\alpha}$, where k is the capital stock. The constants A and α satisfies: $A > 0$ and $0 < \alpha < 1$. Capital depreciates completely each period after production. Consider the optimization problem of the social planner who chooses (c, k_{+1}, l) every period, where k_{+1} is the capital stock for the next period.

1. Formulate the social planner's problem as a dynamic programming problem. Derive the optimality conditions for the social planner's choices.
2. Solve the policy functions and the value function for the social planner.

Question 2: A rational agent tries to solve the following dynamic problem:

$$\max_{\{c_t,\ k_{t+1}\}} \left[\sum_{t=0}^{\infty} \beta^t (c_t)^{1/2} : k_{t+1} = (k_t - c_t)R\right].$$

Here $\beta \in (0,1)$ is the discount factor, $R > 0$ is the constant gross real interest rate, c_t is consumption in period t and k_t is the level of wealth at the beginning of period t. The initial wealth k_0 is given. The parameters satisfy

$$0 < R < 1/\beta^2.$$

Complete the following parts.

1. Formulate the above decision as a dynamic programming problem, denoting the value funciton as $v(k)$.
2. Start with an initial value function $v_0 = 0$, use the Bellman equation to derive the value function for the next round of iteration.
3. Start with the value function obtained in 2, use the Bellmen equation to derive the value function for the next period.
4. Make a conjecture about the functional form of the fixed point of the Bellman equation. Verify the conjecture by solving the coefficients in the conjecture.