Advanced Macroeconomics I

Lecture 1 (2)

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Solow model

Most modern dynamic models of macroeconomics build on the framework described in Solow's (1956) paper

Assumption: An exogenous proportion of income is saving

$$C_t + I_t = Y_t = F(K_t, L)$$

$$I_t = sF(K_t, L), \qquad s \in (0,1)$$

Growth comes from increased saving

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_t, \qquad K_0 \text{ is given} \\ \delta &\in [0,1] \qquad\qquad \text{depreciation rate} \end{aligned}$$

Assumptions

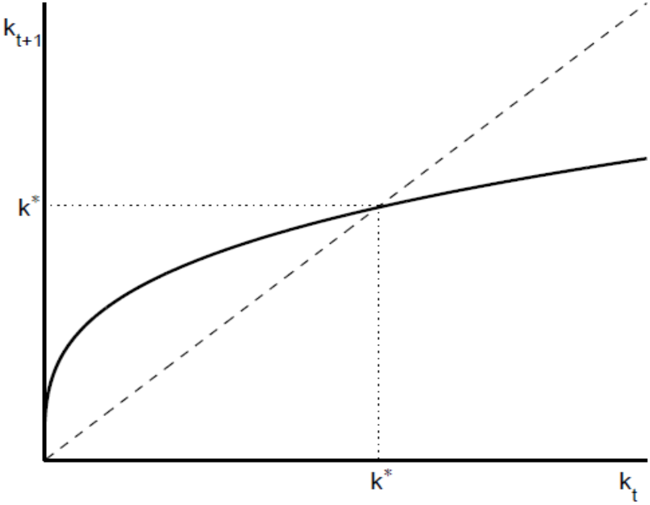
- Closed economy, fixed population
  - Production function: $F(.)$ is strictly concave in K and strictly increasing in K
  - Capital is necessary in production: $F(0, L) = 0$
  - When K is close to zero, the economy grows: $F_K(0, L) > \frac{\delta}{s}$
  - When K is very large, no more growth: $\lim_{K\to\infty} sF_K(K, L) + 1 - \delta < 1$

Example: Cobb-Douglas function

$$F(K, L) = AK^{\alpha}L^{1-\alpha}, \text{ capital share } \alpha \in (0, 1)$$

Convergence in the Solow model

The law of motion of K_t : $K_{t+1} = (1-\delta)K_t + sF(K_t, L)$



Global and monotonic convergence

Theorem

$\exists\ K^* > 0$ *such that* $K^* = (1-\delta)K^* + sF(K^*, L):\ \forall K_0 > 0,\ K_t \to K^*.$

Proof.

(1) Find a K^* candidate; show it is unique. (use strictly concave and strictly increasing of $F()$ on K)
(2) If $K_t > K^*$ show that $K^* < K_{t+1} < K_t$, $\forall\ t \geq 0$. (using $K_{t+1} - K_t = sF(K_t, L) - \delta K_t$, $\delta K^* = sF(K^*, L)$, $F(K_t, L) < K_t \frac{F(K^*,L)}{K^*}$); If $K_t < K^*$ show that $K^* > K_{t+1} > K_t$, $\forall\ t \geq 0$. □

Application 1 - Growth

- Why output grows in the long run and what forms that growth takes
  - what features of the production technology are important for long-run growth
  - the endogenous determination of productivity in a technological sense
  - example: $F(K, L) = AK^{\alpha}L^{1-\alpha}$
    - $\alpha \to 1 ==> K_{t+1} = (1-\delta)K_t + sF(K_t, L)$ is linear $==> K^*$ far to the right
    - $\alpha \to 0 ==> K_{t+1} = (1-\delta)K_t + sF(K_t, L)$ is non-linear $==>$ converge fast to K^*

AK Model

- $\alpha = 1 : F(K, L) = AK$
  - Saving rate $sA + 1 - \delta > 1$: over time output would keep growing, and it would grow at precisely rate $sA + 1 - \delta$
  - Saving rate $sA + 1 - \delta < 1$: the economy shrinks
- AK model is the simpliest "endogeneous growth" model
  - Individuals' preference of consumption over time determines an optimal growth rate, which determines their optimal choice of the saving rate
  - Keeping in mind that savings rates are probably influenced by government policy, such as taxation, this means that there would be a choice, both by individuals and government, of whether or not to grow

Endogeneous technology models

- The Ak model of growth emphasizes physical capital accumulation as the driving force of prosperity
- It is not the only way to think about growth, however. For example: one could model A more carefully and be specific about how productivity is enhanced over time via explicit decisions to accumulate R&D capital or human capital

Growth accounting

- In the context of understanding the growth of output, Solow also developed the methodology of 'growth accounting', which is a way of breaking down the total growth of an economy into components: input growth and technology growth (Solow residual)
- Growth accounting remains a central tool for analyzing output and productivity growth over time and also for understanding diffrences between different economies in the cross-section

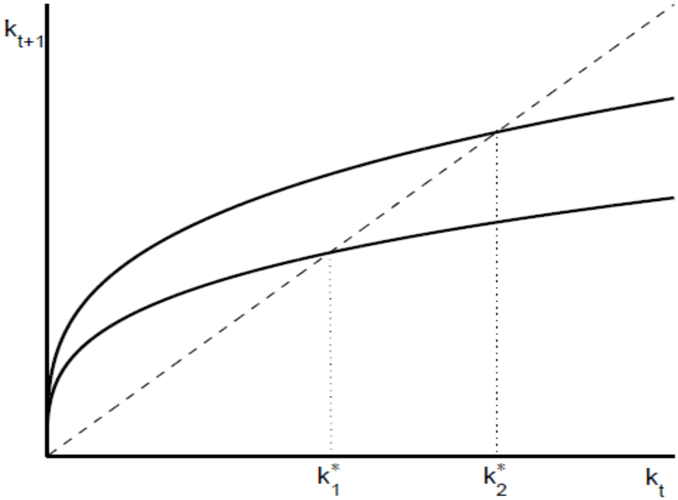
Application 2 - Business Cycles

- Many modern studies of business cycles also rely fundamentally on the Solow model

$$F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

where A_t is stochastic, for instance taking on two values: A_H, A_L

Will there be convergence to a steady state?



Will there be convergence to a steady state?

- In the sense of constancy of capital and other variables, steady states will clearly not be feasible here
- However, another aspect of the convergence in deterministic model is inherited here: over time, initial conditions (the initial capital stock) lose influence and eventually - "after an infinite number of time periods" - the stochastic process for the endogenous variables will settle down and become stationary

Stationary equilibrium and ergodic set

- One element of stationarity is that there will be a smallest compact set of capital stocks such that, once the capital stock is in this set, it never leaves the set: the "ergodic set".
- In the figure, this set is determined by the two intersections with the 45^o line