Advanced Macroeconomics I

Lecture 2 (2)

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Example 1 - logarithmic utility function

- $u(c) = \log(c)$, $f(k) = Ak$
- Euler equation:

$$u'(c_t) = \beta u'(c_{t+1}) f'(k_{t+1})$$

$$\begin{aligned} \frac{1}{c_t} &= \frac{\beta}{c_{t+1}} A \\ c_{t+1} &= \beta A c_t \end{aligned}$$

Intertemporal budget constraint

- Resource constraints

$$
\begin{aligned}
c_0 + k_1 &= Ak_0 \\
c_1 + k_2 &= Ak_1 \\
&\ldots \\
c_T + k_{T+1} &= Ak_T \\
k_{T+1} &= 0
\end{aligned}
$$

- Intertemporal budget constraint

$$c_0 + \frac{1}{A}c_1 + \frac{1}{A^2}c_2 + \ldots + \frac{1}{A^T}c_T = Ak_0$$

present value of consumption stream = present value of income

Optimal consumption

Using the optimal consumption growth rule $c_{t+1} = \beta A c_t$,

$$c_0 + \frac{1}{A}\beta A c_0 + \frac{1}{A^2}(\beta A)^2 c_0 + ... + \frac{1}{A^T}(\beta A)^T c_0 = A k_0$$

$$c_0 \left[\beta + \beta^2 + ... + \beta^T\right] = A k_0$$

$$c_0 = \frac{A k_0}{\beta + \beta^2 + ... + \beta^T}$$

The Effects of productivity A

- An increase in A will cause a rise in consumption in all periods. Crucial to this result is the chosen form for the utility function: Logarithmic utility has the property that income and substitution effcts, when they go in opposite directions, exactly offset each other
- Changes in A have two components: a change in relative prices (of consumption in different periods) and a change in present-value income: Ak_0.

The Effects of productivity A - logarithmic utility

- With logarithmic utility, a relative price change between two goods will make the consumption of the favored good go up whereas the consumption of other good will remain at the same level
- The unfavored good will not be consumed in a lower amount since there is a positive income effct of the other good being cheaper, and that effect will be spread over both goods
- Thus, the period 0 good will be unfavored in our example (since all other goods have lower price relative to good 0 if A goes up), and its consumption level will not decrease
- The consumption of good 0 will in fact increase because total present-value income is multiplicative in A

Varying productivity A

Productivity stream $\{A_t\}$

$$c_0 + \frac{1}{A_1}c_1 + \frac{1}{A_1A_2}c_2 + \ldots + \frac{1}{A_1A_2\ldots A_T}c_T = A_0k_0$$

Plugging in the optimal path $c_{t+1} = \beta A_{t+1}c_t$,

$$c_0\left[\beta + \beta^2 + \ldots + \beta^T\right] = A_0k_0$$

$$\begin{aligned} c_0 &= \frac{A_0k_0}{\beta + \beta^2 + \ldots + \beta^T} \\ c_1 &= \frac{(A_1\beta)A_0k_0}{\beta + \beta^2 + \ldots + \beta^T} \\ &\ldots \\ c_T &= \frac{A_0A_1\ldots A_T\beta^Tk_0}{\beta + \beta^2 + \ldots + \beta^T} \end{aligned}$$

Comparative Statics: $A_t \uparrow \implies c_0, c_1, \ldots, c_{t-1}$ are unaffected

Example 2 - CES Utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

$$\begin{aligned}
\sigma &= 0: \text{ linear utility} \\
\sigma &> 0: \textit{ concave} \\
\sigma &= 1: \text{ limit: logarithmic utility}
\end{aligned}$$

Intertemporal Elasticity of Substitution

Let $R_{t,t+k}$ denote the interest rate: $R_{t,t+k} = A_{t+1}...A_{t+k}$

$$IES \equiv \frac{\frac{d\frac{c_{t+k}}{c_t}}{\frac{c_{t+k}}{c_t}}}{\frac{dR_{t,t+k}}{R_{t,t+k}}}$$

Intertemporal Elasticity of Substitution

$R_{t+1} = A_{t+1}$

$$u'(c_t) = \beta u'(c_{t+1}) R_{t+1}$$

Replacing repeatedly, we have

$$\left. \begin{array}{c} u'(c_t) = \beta^k u'(c_{t+k}) R_{t+1} \ldots R_{t+k} \\ u'(c) = c^{-\sigma} \end{array} \right\} \Longrightarrow c_t^{-\sigma} = \beta^k c_{t+k}^{-\sigma} R_{t,t+k}$$

$$IES \equiv \frac{\frac{d \frac{c_{t+k}}{c_t}}{\frac{c_{t+k}}{c_t}}}{\frac{dR_{t,t+k}}{R_{t,t+k}}} = \frac{d \log \frac{c_{t+k}}{c_t}}{d \log R_{t,t+k}} = \frac{1}{\sigma}$$

Intertemporal Elasticity of Substitution

- When $\sigma = 1$, expenditure shares do not change: this is the logarithmic case
- When $\sigma > 1$, an increase in $R_{t,t+k}$ would lead c_t to go up and savings to go down: the income effect, leading to smoothing across all goods, is larger than substitution effect
- Finally, $\sigma < 1$, the substitution effect is stronger: savings go up whenever $R_{t,t+k}$ goes up
- When $\sigma = 0$, the elasticity is infinite and savings respond discontinuously to $R_{t,t+k}$