Advanced Macroeconomics I

Lecture 3 (3)

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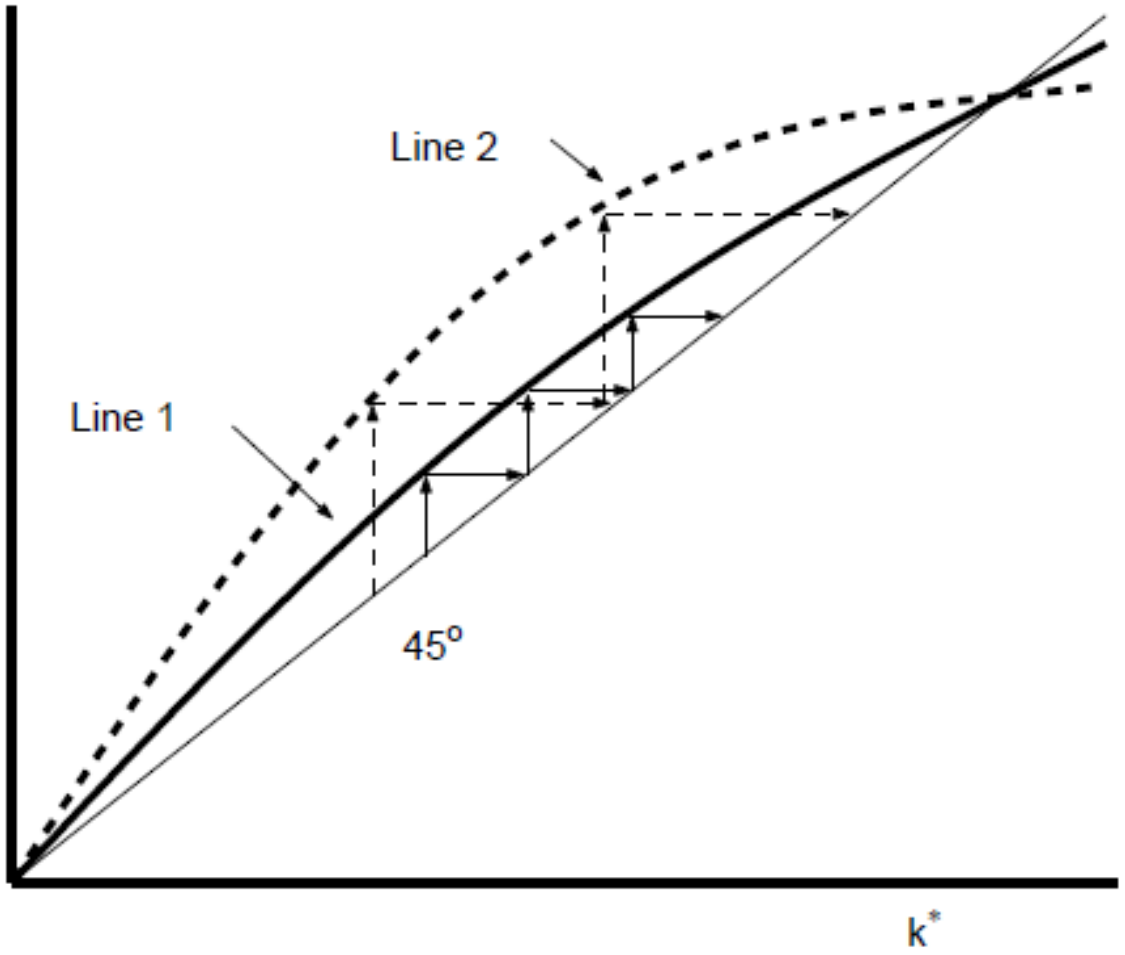
Dynamics: the speed of convergence



Speed of convergence

- There is no simple way to summarize, in a quantitative way, the speed of convergence for a general decision rule
- However, for a limited class of decision rules, it can be measured simply by looking at the slope. This is an important case, for it can be used locally to approximate the speed of convergence around the steady state k^*

Linear rule

- The accumulation path will spend infinite time arbitrarily close to the steady state
- In a very small region a continuous function can be arbitrarily well approximated by a linear function, using the first-order Taylor expansion of the function
- That is, for any capital accumulation path, we will be able to approximate the speed of convergence arbitrarily well as time passes
- If the starting point is far from the steady state, we will make mistakes that might be large initially, but these mistakes will become smaller and smaller and eventually become unimportant

Local dynamics for a dynamic system

- The global convergence theorem, in contrast, applies only for the one-sector growth model
- A general dynamic system
- The first-order Taylor series expansion of the decision rule gives

$$k' = g(k) \approx g(k^*) + g'(k^*)(k - k^*)$$

$$k' - k^* = g'(k^*)(k - k^*)$$

interpret $g'(k^*)$ as a measure of the rate of convergence. If $g'(k^*)$ is very close to zero, convergence is fast and the gap decreases significantly each period