Advanced Macroeconomics I

Lecture 3 (4)

Zhe Li

SUFE

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Linearization for a general dynamic system

- The task is now to find $g'(k^*)$
- by linearization: linearize the Euler equation
- This will lead to a difference equation in k_t. One of the solutions to this difference equation will be the one we are looking for

Local convergence conditions

Theorem

Let $x_t \in R^n$*. Given* $x_{t+1} = h(x_t)$ *with a stationary point* $x^* : x^* = h(x^*)$ *if*
1. h is continuously differentiable with Jacobian $H(x^*)$ *around* x^*
2. $I - H(x^*)$ *is non-singular*
then there is a set of initial conditions x_0*, of dimension equal to the number of eigenvalues of* $H(x^*)$ *that are less than 1 in absolute value, for which* $x_t \longrightarrow x^*$

Converging Dynamics - n=1

Examples

There is only one eigenvalue: $\lambda = h'(x^*)$

1. $|\lambda| \geq 1$, no initial condition leads to x_t converging to x^*
   In this case, only for $x_0 = x^*$ will the system stay in x^* if $\lambda = 1$
2. $|\lambda| < 1$, $x_t \rightarrow x^*$ for any value of x_0

Converging Dynamics - n=2

Examples

There are two eigenvalues: $\lambda_1, \ \lambda_2$

1. $|\lambda_1|, |\lambda_2| \geq 1$, no initial condition x_0 leads to x_t converging to x^*
2. $|\lambda_1| < 1, |\lambda_2| \geq 1$, dimention of $x_0's$ leading to *convergence* is 1
3. $|\lambda_1| < 1, |\lambda_2| < 11$, dimention of $x_0's$ leading to *convergence* is 2, x_t $\rightarrow x^*$ for any value of x_0

Interpret the eigenvalues

- Let the number of eigenvalues less than 1 in absolute value be denoted by m. This is the dimension of the set of initial $x_0's$ leading to x^*
- We may interpret m as the degrees of freedom
- Let the number of economic restrictions on initial conditions be denoted by $\hat{m}$. These are the restrictions coming from physical conditions in our economic model
- Notice that an interpretation of this is that we have $\hat{m}$ equations and m unknowns. Then the issue of convergence boils down to the following cases

Solutions

1. $m = \hat{m}$: there is a unique convergent solution to the difference equation system
2. $m < \hat{m}$: No convergent solution obtains
3. $m > \hat{m}$: There is "indeterminacy", i.e. many solutions