# Advanced Macroeconomics I

## Lecture 4 (1)

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# Competitive equilibrium - consumers

- A "representative consumer"
  - there are a large number of consumers in the economy
  - all identical: same preference, same endowment, rational

# Prices

- Prices of commodities will adjust so that markets clear
    - prices will make all the identical consumers make the same decisions
    - prices will have to adjust so that consumers do not interact

# Prices

**Example**

The dynamic model without production gives a trivial allocation outcome: the consumer consumes the endowment of every product

The competitive mechanism ensures that this outcome is achieved by prices being set so that the consumer, when viewing prices as beyond his control, chooses to consume no more and no less than his endowments

# Competitive equilibrium - firms

- The production factors (capital and labor) are owned by many individual *households*
    - Households' decisions consist of the amount of factors to provide to firms, and the amount of consumption goods to purchase from them
- The technology to transform those factors into consumption goods is operated by firms
    - Firms have to choose their production volume and factor demand

# Competitive equilibrium - markets

- Markets: the device by which sellers and buyers are driven together
- Equilibrium: for some given prices, individual households' and firms' decisions show an aggregate consistency
  - the amount of factors that suppliers are willing to supply equals the amount that producers are willing to take
  - consumption goods: sypply = demand
  - we say that markets clear

# Competitive equilibrium

- The word "competitive" indicates that we are looking at the perfect competition paradigm, as opposed to economies in which firms might have some sort of "market power"
- A competitive equilibrium is a vector of prices and quantities that satisfy certain properties related to *the aggregate consistency* of individual decisions

# Competitive equilibrium properties

1. Households choose quantities so as to maximize the level of utility attained given their "wealth" (factor ownership evaluated at the given prices)
   - When making decisions, households take prices as given parameters
   - The maximum monetary value of goods that households are able to purchase given their wealth is called *the budget constraint*

# Competitive equilibrium properties

2. Firms chose the production volume that maximizes their profits at the given prices
3. Markets clear: the quantity choice is "feasible".
    - demand can be produced with the available technology using the amount of factors that suppliers are willing to supply

# Dynamic competitive equilibrium

## How trade takes place over time

- Are the economic agents using assets?
- Different arrangements for the same physical environment - same final allocation:
  - Firms rent their inputs from consumers every period
    - do not need assets to fulfill their profit maximization objective
  - Firms buy and own the long-lived capital they use in production
    - need to consider the relative values of profits in different periods

- Mathematically there are two alternative procedures:
    - equilibria can be defined and analyzed in terms of (infinite) sequences
    - or they can be expressed recursively, using functions

# Date 0 arrangement

## Arrow-Debreu-McKenzie

- Let goods be dated
  - for example, in a one-good per date context, there is an infinite sequence of commodities: consumption at $t = 0$, consumption at $t = 1$, etc.
  - like in a static model, let the trade in all these commodities take place once and for all
- In this arrangement, there is no need for assets

# Sequencial arrangement

- Assets are traded every period
- There are nontrivial decisions made in every future period

# The neoclassical growth model with date-0 trade

- The consumer is endowed with 1 unit of "time" each period, which he can allocate between labor and leisure
- The utility derived from the consumption and leisure stream $\{c_t,\ 1-n_t\}_{t=0}^{\infty}$ is given by

$$U\left(\{c_t,\ 1-n_t\}_{t=0}^{\infty}\right)=\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)$$

# The production

- The consumer rents his labor services at $t$ to the firm for a unit rental (or wage) rate of $w_t$
- The production function of the consumption / investment good is $F(K; n)$
    - $F$ is strictly increasing in each argument, concave, and homogeneous of degree 1

# The prices

- Price of consumption good $c_t$ at every $t$
  - $p_t$ intertemporal relative prices
  - if $p_0 = 1$, then $p_t$ is the price of $c_t$ at $t$ relative to (in terms of) $c_0$
- Price of capital services at $t$: $p_t r_t$ rental rate
  - price of capital services at $t$ interms of $c_0$
- Price of labor: $p_t w_t$ wage rate
  - price of labor at $t$ interms of $c_0$

# Definition of a competitive equilibrium

A competitive equilibrium is a set of sequences:
Prices: $\{p_t^*\}_{t=0}^{\infty}$, $\{r_t^*\}_{t=0}^{\infty}$, $\{w_t^*\}_{t=0}^{\infty}$
Quantities: $\{c_t^*\}_{t=0}^{\infty}$, $\{K_{t+1}^*\}_{t=0}^{\infty}$, $\{n_t^*\}_{t=0}^{\infty}$ such that

- $\{c_t^*\}_{t=0}^{\infty}$, $\{K_{t+1}^*\}_{t=0}^{\infty}$, $\{n_t^*\}_{t=0}^{\infty}$ solve the consumer's problem

# Equilibrium definition - Consumer

- Consumer's problem

$$\left\{c_t^*, K_{t+1}^*, n_t^*\right\}_{t=0}^{\infty} = \arg \max_{\{c_t, K_{t+1}, n_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \right\}$$

# Equilibrium definition - Consumer

- Budget constraint:

$$s.t. \sum_{t=0}^{\infty} p_t^* (c_t + K_{t+1})$$

$$= \sum_{t=0}^{\infty} p_t^* \left[ r_t^* K_t + (1-\delta) K_t + n_t w_t^* \right]$$

$$c_t \geq 0 \ \forall t, \ \ k_0 \text{ given}$$

  - $K_{t+1}$ is quoted in the same price as $c_t$
  - Labor has no utility cost: $w_t > 0$ implies that the consumer supplies all his time, $n_t^* = 1$

# Equilibrium definition - Firm

- $\{K^*_{t+1}\}_{t=0}^{\infty}$, $\{n^*_t\}_{t=0}^{\infty}$ solve the firm's problem:

$$\{K^*_t, 1\}_{t=0}^{\infty} = \arg\max_{K_t,\ n_t} \left\{ \begin{array}{c} p^*_t F(K_t, n_t) \\ -p^*_t r^*_t K_t - p^*_t w^*_t n_t \end{array} \right\}$$

  The firm's decision problem involves just a one-period choice
  - all of the model's dynamics come from the consumer's capital accumulation problem

# Equilibrium definition - market

- Firm's condition may equivalently be expressed as follows: $\forall t : (r_t^*, w_t^*)$ satisfy:

$$\begin{aligned} r_t^* &= F_K(K_t^*, 1) \\ w_t^* &= F_n(K_t^*, 1) \end{aligned}$$

- If the production function $F(K; n)$ is increasing in $n$, then $n_t^* = 1$ follows
- Feasibility (market clearing):

$$c_t^* + K_{t+1}^* = F(K_t^*, 1) + (1 - \delta) K_t^*$$

# Characterize the equilibrium

- intertemporal first-order conditions

$$c_t : \ \beta^t u'(c_t^*) = p_t^* \lambda^*$$

where $\lambda^*$ is the Lagrange multiplier corresponding to the budget constraint

$$c_{t+1} : \beta^{t+1} u'(c_{t+1}^*) = p_{t+1}^* \lambda^*$$

# Marginal rate of substitution

$$\frac{p_t^*}{p_{t+1}^*} = \frac{1}{\beta}\frac{u'(c_t^*)}{u'(c_{t+1}^*)}$$

- $\frac{p_t^*}{p_{t+1}^*}$ as the real interest rate
- $\frac{1}{\beta}\frac{u'(c_t^*)}{u'(c_{t+1}^*)}$ as the marginal rate of substitution of consumption goods between $t$ and $t+1$

# Technical substitution

- FOC with $K_{t+1}$

$$K_{t+1} : \lambda^* p_t^* = \lambda^* p_{t+1}^* \left[ r_{t+1}^* + 1 - \delta \right]$$

$$\begin{aligned} \frac{p_t^*}{p_{t+1}^*} &= r_{t+1}^* + 1 - \delta \\ &= F_K(K_t^*, 1) + 1 - \delta \end{aligned}$$

  - $F_K(K_t^*, 1) + 1 - \delta$ is the marginal return on capital
  - the marginal rate of technical substitution (transformation) between $c_t$ and $c_{t+1}$

# First welfare theorem

- Same as the Euler Equation from the planner's problem

$$u'(c_t^*) = \beta u'(c_{t+1}^*) \left[ F_K(K_t^*, 1) + 1 - \delta \right]$$

- Therefore a competitive equilibrium allocation satisfies the optimality conditions for the centralized economy: the competitive equilibrium is optimal
  - the *First Welfare Theorem*

# Second welfare theorem

- We have assumed that there is a single consumer, so in this case Pareto-optimality just means utility maximization
- In addition, with the appropriate assumptions on $F(K;n)$ (constant returns to scale), an optimum can be supported as a competitive equilibrium, which is the result of the *Second Welfare Theorem*