Advanced Macroeconomics I

Lecture 5 (2)

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Spring 2011

Uncertain income

Example

- A simple 2-period model, an agent faces:
  - Consume and save in period 0
  - Consume and work in period 1
- The uncertainty arises in the income of period 1 through the stochasticity of the wage
  - $w \in \{w_1, w_2, ..., w_m\}$, where $\pi_i = \Pr\{w = w_i\}$, for $i = 1, 2, ..., m$

Utility function

- Von Neumann-Morgenstern utility function: an expected utility maximizer
- Leisure in the second period is valued:

$$U = u(c_0) + \beta \sum_{i=1}^{m} \pi_i \left(u(c_{1i}) + v(n_i)\right)$$

$$v'(n_i) < 0$$

Incomplete market

- Asset: assume that there is a "risk free" asset denoted by a, and priced q, such that every unit of a purchased in period 0 pays 1 unit in period 1, whatever the state of the world
  - Period 0 budget

$$c_0 + aq = I$$

  - Period 1 budget

$$c_{1i} = a + w_i n_i \qquad i = 1, 2, ..., m$$

Euler equation

$$\begin{aligned} u'(c_0) &= \beta \sum_{i=1}^{m} \pi_i u'(c_{1i}) R \\ &= \beta E\left[u'(c_{1i}) R\right] \end{aligned}$$

Interpretation: on the margin, the consumer's marginal utility from consumption at period 0 is equated to the discounted **expected** marginal utility from consuming R units in period 1

Complete market

- Arrow securities (state-contingent claims): n assets are traded in period 0, and each unit of asset i purchased pays off 1 unit if the realized state is i, and 0 otherwise
    - a risk free asset yielding the same payout in each state

Complete market

- The new budget constraint in period 0 is

$$c_0 + \sum_{i=1}^{m} q_i a_i = I$$

- Second period: if the realized state is i

$$c_{1i} = a_i + n_i w_i$$

Arrow securities vs risk free asset

- Assume that the total price paid for such a portfolio is the same as before, i.e.

$$q = \sum_{i=1}^{m} q_i$$

- A risk free asset can be constructed by purchasing one unit of each a_i, the return

$$\sum_{i=1}^{m} \pi_i a_i = 1$$

Complete vs incomplete market

- Whether the consumer will be better or worse off with complete market structure than incomplete one?
  - The Agent could not be worse off
    - the structure of wealth transfer across periods that was available before (namely, the risk free asset) is also available now at the same cost

Complete better?

- The complete market structure now allows the wealth transfer across periods
  - not only can the consumer reallocate his income between periods 0 and 1, but also move his wealth across states of the world
  - to be state-specific

Welfare improvement

- The added ability to move income across states will lead to a welfare improvement if
  - the w_i's are nontrivially random
  - and if preferences show risk aversion (i.e. if the utility index $u(c)$ is strictly concave)

One budget constraint

- Solving for a_i in the period-1 budget constraints and replacing in the period-0 constraint, we get

$$c_0 + \sum_{i=1}^{m} q_i c_i = I + \sum_{i=1}^{m} q_i w_i n_i$$

  - q_i is the price, in terms of c_0, of consumption goods in period 1 if the realized state is i
  - $q_i w_i$ is the money payment to labor if the realized state is i, measured in term of c_0
  - the price of c_0 has been normalized to 1

One budget constraint

- One budget constraint (complete market)

$$c_0 + \sum_{i=1}^{m} q_i c_i = I + \sum_{i=1}^{m} q_i w_i n_i$$

- n budget constraints (incomplete market)

$$c_{1i} = a + w_i n_i \qquad i = 1, 2, ..., m$$

This budget consolidation is a consequence of the free reallocation of wealth across states

Maximization problem

- Expected utility

$$U = \max_{\{c_0, c_i, n_i\}} u(c_0) + \beta \sum_{i=1}^{m} \pi_i \left(u(c_{1i}) + v(n_i) \right)$$

- Budget constraint

$$\lambda : c_0 + \sum_{i=1}^{m} q_i c_i = I + \sum_{i=1}^{m} q_i w_i n_i$$

Optimal conditions

First order conditions:

$$c_0 : u'(c_0) = \lambda$$

$$c_i : \beta \pi_i u'(c_{1i}) = q_i \lambda$$

$$n_i : \beta \pi_i v'(n_i) = -q_i w_i \lambda$$

Consumption-leisure choice

- Intra-state consumption-leisure choice (same as with incomplete markets)

$$u'(c_{1i}) = -\frac{1}{w_i} v'(n_i)$$

Consumption across state

- The added flexibility in allocation of consumption: the agent now not only makes consumption-saving decision in period 0, but also chooses consumption pattern across states of the world

$$u'(c_0) = \beta \frac{\pi_i}{q_i} u'(c_{1i})$$

Log preference

- $$c_{1i} = \beta \frac{\pi_i}{q_i} c_0$$
- Consumption in each period is proportional to consumption in c_0
- This proportionality is a function of the cost of insurance: the higher q_i in relation to π_i, the lower the wealth transfer into state i