Advanced Macroeconomics I

Lecture 6 (2)

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Government debt

- Missing market across generations
- Enforcement of market
  - Assume same interest rate (market rate = government rate)
  - Return of debt

$$(1 + r_t) L_t b$$

  - New debt

$$L_{t+1} b$$

Competitive quilibrium

$$f'(k^*) = r^* = \frac{\alpha}{1-\alpha}\frac{1+n}{1-\beta}$$

Golden rule

$$f'(k^*) = n$$

$\alpha = 0.36$
$\beta = 0.6$
$n = 0.02$
$r^* = \frac{\alpha}{1-\alpha}\frac{1+n}{1-\beta} = \frac{0.36}{(0.36-1)(0.6-1)}(0.02+1) = 1.4344$

Government Tax

- If $r > n$, accumulate too little capital
- Tax on the young

$$(1 + r_t)L_t b = L_{t+1} b + T_t$$

$$T_t = L_t b(r_t - n)$$

- Debt holding of per young agent

$$\frac{L_{t+1} b}{L_t} = (1 + n)b$$

Budget constraint

- Young agent born at t

$$c_t^1 + k_{t+1}^s + (1+n)b \leq w_t - \frac{T_t}{L_t}$$

- Consumption when they become old

$$c_t^2 = (1 + r_{t+1})\left[k_{t+1}^s + (1+n)b\right]$$

- Intertemporal budget constraint

$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} = w_t - (r_t - n)b$$

Solution

$$\max_{c_t^1,\ c_t^2} u\left(c_t^1,\ c_t^2\right)$$

$$c_t^1 + \frac{c_t^2}{1+r_{t+1}} = w_t - (r_t - n)b$$

- Foc.

$$1 + r_{t+1} = \frac{u_1(w_t - (r_t - n)b - s_t, (1 + r_{t+1})s_t)}{u_2(w_t - (r_t - n)b - s_t, (1 + r_{t+1})s_t)}$$

- $$s_t = s(w_t - (r_t - n)b, r_{t+1})$$

- $$0 < s_w < 1,\ s_r < 0$$

The effect of debt

- Assumbe b can be used as capital
- Focus on normal case $\psi'\phi' > 1$
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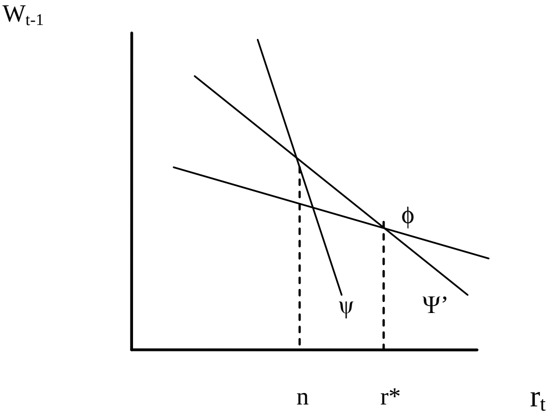
$$\begin{pmatrix} k_t = \frac{s(w_{t-1} - (r_t - n)b, r_t)}{1+n} \\ r_t = f'(k_t) \end{pmatrix} \implies r_t = f'(\frac{s(w_{t-1} - (r_t - n)b, r_t)}{1+n})$$
$$\implies r_t = \psi(w_{t-1} - (r_t - n)b)$$
-
$$w_t = \left[f(k_t) - k_t f'(k_t)\right]_{k_t = f'^{-1}(r_t)} \equiv \phi(r_t)$$

The effect of debt

$r_t = \psi(w_{t-1} - (r_{t-1} - n)b) \quad w_t = \phi(r_t)$
Arround the steady state, taken r_{t-1} as given



The welfare effect of debt

- Steady state utility $u^* = u(c^1, c^2)$, $c^1 = w - (r-n)b - s$, $c^2 = (1+r)s$
- Foc. $s: u_1 = u_2(1+r)$
-

$$\begin{aligned}\frac{du^*}{db} &= u_1\left[\frac{dw}{db} - (r-n) - b\frac{dr}{db} - \frac{ds}{db}\right] \\ &\quad + u_2\left[(1+r)\frac{ds}{db} + s\frac{dr}{db}\right]\end{aligned}$$

$$\frac{du^*}{db} = u_1\left[\frac{dw}{db} - (r-n)\right] + \left[u_2 s - u_1 b\right]\frac{dr}{db}$$

disposible income *interest rate*

The welfare effect of debt

$$\frac{dw}{db} = \frac{d\left[f\left(k_t\right) - k_t f'\left(k_t\right)\right]}{db} = -kf''\frac{dk}{db} = -k\frac{dr}{db}$$

$$\begin{aligned} f'(k) &= r \\ f''\frac{dk}{db} &= \frac{dr}{db} \end{aligned}$$

$$s = k^s + (1+n)b = (1+n)(k+b)$$

$$\begin{aligned} \frac{1}{u_1}\frac{du^*}{db} &= \frac{dw}{db} - (r-n) + \left[\frac{u_2}{u_1}s - b\right]\frac{dr}{db} \\ &= (n-r)\left(1 + \frac{b+k}{1+r}\frac{dr}{db}\right) < 0 \end{aligned}$$