Advanced Macroeconomics I

Lecture 7 (2) Exogenous Growth

Zhe Li

SUFE

Spring 2011

Issue of Growth

- Technological: Is growth feasible with the assumed production technology?
- Decision making:
  - Will a social planner choose a growing path?
  - Which types of utility funciton allow for what we will call a "balanced growth path"?

The Technology of Exogenous Growth

- Labor-augmenting technological change:

$$c_t + i_t = F_t(K_t, n_t) = F(K_t, \gamma^t n_t)$$

$\gamma > 1$
$F()$ constant returns to scale technology
Law of motion for capital:

$$K_{t+1} = (1 - \delta) K_t + i_t$$

- Is sustained growth possible?

Balanced Growth Path

- All economic variables grow at constant rates (that could vary from one variable to another)

$$
\begin{aligned}
y_t &= y_0 g_y^t \\
c_t &= c_0 g_c^t \\
K_t &= K_0 g_K^t \\
i_t &= i_0 g_i^t \\
n_t &= n_0 g_n^t
\end{aligned}
$$

An analogue of a steady state

Growth rate

- Capital and investment must grow at the same rate

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + i_t \\ \frac{K_{t+1}}{K_t} &= (1-\delta) + \frac{i_t}{K_t} \end{aligned}$$

- $g_y = g_c$

$$\begin{aligned} c_t + i_t &= F(K_t, \gamma^t n_t) \equiv y_t \\ \frac{c_t}{K_t} + \frac{i_t}{K_t} &= F(1, \frac{\gamma^t n_t}{K_t}) = \frac{y_t}{K_t} \\ \frac{g_c^t c_0}{g_K^t K_0} + \frac{i_0}{K_0} &= \frac{g_y^t y_0}{g_K^t K_0}, \quad for \quad all \ \ t \end{aligned}$$

- $g_y = g_c = g_K$

$$\begin{aligned} \frac{g_c}{g_K} &= \frac{g_y}{g_K} = \alpha \\ \alpha^t &= \text{constant for all } t \end{aligned}$$

Growth rate

- $$F(1, \frac{\gamma^t n_t}{K_t}) = \frac{y_t}{K_t}$$

  $\frac{\gamma^t n_t}{K_t}$ is constant
- Since the time endowment is bounded, actual hours can not growth beyond a certain upper limit (usually normalized to 1); hence $g_n = 1$ must hold
- $g_K = \gamma$
- a balanced growth path is technologically feasible

Labor augmenting?

- $$c_t + \frac{i_t}{\gamma_i^t} = \gamma_z^t F(\gamma_K^t K_t, \gamma_n^t n_t)$$

  - γ_i : Investment-specific technological change
    - You could think of this as the relative price of capital goods showing a long term decreasing trend, relative to consumption goods
    - In fact this has been measured in the data, and in the case of the US this factor accounts for 60% of growth (for details see Greenwood et al. (1997))
  - γ_z : Neutral (or Hicks-neutral) technological change
  - γ_K : Capital augmenting technological change
  - γ_n : Labor augmenting technological change

Equivalent to labor augmenting

$$c_t + \frac{i_t}{\gamma_i^t} = (\gamma_K^t K_t)^\alpha (\gamma_n^t n_t)^{1-\alpha}$$

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + i_t \\ \frac{K_{t+1}}{\gamma_i^{t+1}} \gamma_i &= (1-\delta) \frac{K_t}{\gamma_i^t} + \frac{i_t}{\gamma_i^t} \end{aligned}$$

$$\tilde{K}_{t+1} \equiv \frac{K_{t+1}}{\gamma_i^{t+1}}, \tilde{\imath}_t \equiv \frac{i_t}{\gamma_i^t}$$

$$\tilde{K}_{t+1}\gamma_i = (1-\delta) \tilde{K}_t + \tilde{\imath}_t$$

$$c_t + \tilde{\imath}_t = (\gamma_K^t \gamma_i^t \tilde{K}_t)^\alpha (\gamma_n^t n_t)^{1-\alpha} = \tilde{K}_t{}^\alpha (\hat{\gamma}_n^t n_t)^{1-\alpha}$$

$$\hat{\gamma}_n = \gamma_n \gamma_K^{\frac{\alpha}{1-\alpha}} \gamma_i^{\frac{\alpha}{1-\alpha}}$$

Choosing Growth

- Exogenous (Solow):

$$i_t = sy_t, \quad s \in [0, 1]$$

- Endogenous: conditions of preference for constant growth (focus on time separable preference)

$$\max_{\{c_t, n_t, K_{t+1}, i_t\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right) \right\}$$

$$c_t + i_t = F\left(K_t, \gamma^t n_t\right)$$

$$\begin{aligned} K_{t+1} &= i_t + \left(1 - \delta\right) K_t \\ & K_0 \text{ given} \end{aligned}$$

Preference

Theorem

Balanced growth is possible as a solution to the central planner's problem if and only if $u(c,n) = \frac{c^{1-\sigma}v(1-n)-1}{1-\sigma}$, *where time endowment is normalized to one and* $v()$ *is a function with leisure as an argument.*

The isoelastic utility function $\frac{c^{1-\sigma}-1}{1-\sigma}$, the elasticity of intertemporal substitution $= -\frac{u'(c)}{cu''(c)} = 1/\sigma$

$$\frac{\frac{d(c_2/c_1)}{c_2/c_1}}{\frac{d(u'(c_2)/u'(c_1))}{u'(c_2)/u'(c_1)}} = \frac{dc_2/c_2}{du'(c_2)/u'(c_2)} = \frac{u'(c_2)}{c_2 u''(c_2)}$$

$u'(c_2)/u'(c_1) = 1 + r$
$y = k^{\alpha}n^{1-\alpha}$, $r = \alpha k^{\alpha-1}n^{1-\alpha}$
($CRRA$ exhibits constant relative risk aversion $-\frac{cu''(c)}{u'(c)} = \sigma$)