

# Advanced Macroeconomics I

## Lecture 7 (3) AK Model

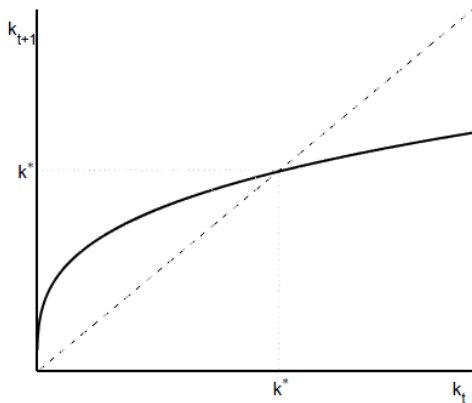
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# Global Dynamics

- New classical model:  $f(0) = 0$ ,  $f'(\cdot) > 0$ ,  $f''(\cdot) < 0$ ,  $\lim_{x \rightarrow 0} f'(\cdot) = \infty$ ,  $\lim_{x \rightarrow \infty} f'(\cdot) = 0$
- Long-run growth is not feasible



# Long-run growth - production function

- Dispose assumption:  $\lim_{x \rightarrow 0} f'(\cdot) = 0$ 
  - $f$  does not cross the 45° line
- $\lim_{x \rightarrow \infty} f'(\cdot) = 1$ ?
  - $f$  parallel to the 45° line
- Need  $\lim_{x \rightarrow \infty} f'(\cdot) > 1$ 
  - Example  $f(k) = Ak$ ,  $A > 1$
  - If depreciation rate  $\delta$ ,  $f(k) + (1 - \delta)k = Ak + (1 - \delta)k$ ,  $A > \delta$

- Utility function

$$U = \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right\}, \quad \sigma > 0$$

Euler Equation:

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} A$$

$$\frac{c_{t+1}}{c_t} = (\beta A)^{1/\sigma}$$

- The growth rate is constant as from  $t = 0$ : there are no transitional dynamics in this model
- Long-run growth if  $(\beta A)^{1/\sigma} > 1$

# Is utility bounded?

$$U = \sum_{t=0}^{\infty} \left[ \beta \left[ (\beta A)^{1/\sigma} \right]^{1-\sigma} \right]^t \frac{c_0^{1-\sigma}}{1-\sigma}$$

Sufficient condition for boundedness:

$$\beta \left[ (\beta A)^{1/\sigma} \right]^{1-\sigma} < 1$$

# What explains the international difference?

- Tax distortion on capital income

$$c_t + k_{t+1} = (1 - \tau_k)R_t k_t + \tau_t$$

Government budget

$$\tau_k R_t k_t = \tau_t$$

- Barriers to Growth

# Evaluate the AK model

- Explanatory power:
  - how realistic the assumptions
  - the results of the model
- The AK production function could be interpreted as a special case of the Cobb-Douglas function with  $\alpha = 1$  - then labor is not productive
  - This contradicts actual data, that shows that labor is a hugely significant component of factor input

# Results of the AK model

- Transitional dynamics: The AK model implies no transitional dynamics. However, we tend to see transitional dynamics in the data
- The growth has become a function of underlying parameters in the economy, affecting preferences and production. Could the dispersion in cross-country growth rates be explained by differences in these parameters? Country  $i$ 's Euler Equation (with a distortionary tax on capital income) would be:

$$\left( \frac{c_{t+1}}{c_t} \right)_i = (\beta_i A_i (1 - \tau_k^i))^{1/\sigma_i}$$

- Problem: if parameters are calibrated to mimic the data's dispersion in growth rates, the simulation results in too much divergence in output level (1960-1990 growth rates would result in a difference in output levels wider than the actual)