Advanced Macroeconomics I

Lecture 7 (4) Endogenous Growth Models

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Romer's externality Model

- $\bar{K}$ is the aggregate level of capital, K is the capital operated by the firm

$$F(K, L, \bar{K}) = AK^{\alpha}L^{1-\alpha}\bar{K}^{\rho}$$

- There are externalities to capital accumulation, so that individual savers do not realize the full return on their investment
- If $\rho = 1 - \alpha$, AK-Model
- If $\rho > 1 - \alpha$, then the balanced growth path would not be possible

Romer's externality Model

- Suppose $\rho = 1 - \alpha$
  - CES utility function
$$\frac{c_{t+1}}{c_t} = (\beta R_{t+1})^{1/\sigma}$$
  - planner faces an AK model $g^o = (\beta A)^{1/\sigma}$
  - competitive market $g = (\beta \alpha A)^{1/\sigma}$
- Good: labor is relevant
- model leads to large divergence in output levels
- little evidence supporting capital externality

Lucas' human capital accumulation model

- Plain labor is replaced by Human capital

$$F(K,H) = AK^{\alpha}H^{1-\alpha}$$

- There are two distinct capital accumulation equations:

$$\begin{aligned} K_{t+1} &= (1-\delta_K) K_t + I_t^K \\ H_{t+1} &= (1-\delta_H) H_t + I_t^H \end{aligned}$$

- The resource constraint

$$c_t + I_t^K + I_t^H = AK_t^{\alpha}H_t^{1-\alpha}$$

Concluding Remarks

Exogenous Growth	AK Model
$AK^{\alpha}L^{1-\alpha}$	AK
Marginal productivity of K	
$\lim_{K\to\infty} A\alpha K^{\alpha-1}L^{1-\alpha} = 0$	A
Convergence	Divergence in relative income levels

Generating divergence

- Introduce a "mystery capital", so that the production function looks like:

$$F(K, L, \bar{K}) = AK^{\alpha}L^{1-\alpha}\bar{K}^{\rho}$$

- Or, alternatively introduce "human capital" as the third production factor, besides physical capital and labor:

$$F(K, L, H) = AK^{\alpha}L^{\beta}H^{1-\alpha-\beta}$$